SAT Math Advanced

1.  For the given polynomial function , what is the value of if the graph of passes through the point (0, b)?

2.  An exponential function f is defined by the equation , where a is the initial value and b is the ratio. If , what is the value of b?

3.  The function f is a quadratic function. In the xy-plane, the graph of has a vertex at (6, -4) and passes through the points (-3, 239) and (-5, 359). What is the value of ?

A. -15

B. -12

C. -9

D. -5

4.  Which expression is equivalent to ?

A.

B.

C.

D.

5.  The function f is defined by . What is the value of ?

6.  What is an x-coordinate of an x-intercept of the graph in the xy-plane?

A.   -5

B.   -4

C.    0

D.    10

7.  A financial analyst is modeling the projected economic activity in a city using the equation for active jobs: , where represents the number of active jobs and is the number of months since January 2020. What is the best interpretation of the y-intercept of the graph of this equation in the xy-plane?

A.    At the end of January 2020, the projected number of active jobs was 0.

B.    At the end of January 2020, the projected number of active jobs was 86, suggesting a lack of job activity or data errors.

C.    At the end of July 2020, the projected number of active jobs was 86.

D.    At the end of July 2020, the projected number of active jobs was 0.

8.  The equation has exactly one solution for the variable . Which of the following could be the value of ?

A.   -55 only

B. 0 only

C.   -55 and 0

D.   -55 and 55

9.  A physicist is studying a certain radioactive substance whose quantity decreases over time. The equation representing the quantity of this substance in grams after x years is given by: . Which of the following is the best interpretation of 1100 in this context?

A.   The estimated amount of substance remaining after 1 year.

B.   The estimated amount of substance remaining after 2 years.

C.   The estimated initial quantity of the substance at time 0.

D.   The estimated quantity of substance after 6 years.

10.  A solution to the given system of equations is (x, y). What is a possible value of x?

A.   75

B.   76

C.   78

D.   80

SAT Math Advanced Solutions

1.  For the given polynomial function , what is the value of if the graph of passes through the point (0, b)?

## Answer

18

## Solution

Concept Check : The question is asking students to find the y-intercept of a polynomial function. Students should know that the y-intercept occurs where x equals zero, and they should be able to evaluate the function at that point. This requires understanding of polynomial evaluation.  
  
Solution Strategy : To find the y-intercept, substitute x = 0 into the polynomial function. This will involve evaluating each term of the polynomial at x = 0, which will simplify the calculation significantly since any term with x in it will equal zero.  
  
Quick Wins : Remember that the y-intercept is always found by setting x to zero in the equation. When evaluating polynomials, any term that includes x will drop out (become zero), making your calculations easier. Focus primarily on the constant term for the y-intercept.  
  
Mistake Alert : Be careful to correctly substitute x = 0 into each term of the polynomial. Double-check that you are not mistakenly performing operations on terms that should evaluate to zero. Also, ensure you properly account for the constant term, as it directly gives you the y-intercept value.  
  
SAT Know-How : This problem falls under the category of Advanced Math, specifically focusing on polynomial functions and their properties. It assesses a student’s ability to evaluate polynomials to find specific points, such as the y-intercept. Mastering this skill is crucial for solving similar SAT problems efficiently.  
  
Substitute x = 0 into the polynomial function f(x):  
  
Simplify each term after substitution:  
f(0) = 0 - 0 + 0 + 18  
Thus, f(0) = 18.  
Therefore, the value of b is 18.

2.  An exponential function f is defined by the equation , where a is the initial value and b is the ratio. If , what is the value of b?

## Answer

3

## Solution

Concept Check : The intent of this question is to assess the student’s understanding of exponential functions, specifically the properties of exponential growth and how to manipulate the function’s properties using given values. The student is expected to know how to apply the definition of exponential functions and possibly logarithmic properties to solve for the variable b.  
  
Solution Strategy : To approach this problem, the student should first substitute the values of x into the function to express and in terms of a and b. This will lead to an equation that can be simplified. The next step is to set up the equation based on the relationship given in the problem, which states that is 6561 times . This should lead to an equation that can be solved for b. The student should carefully track the bases and the exponents involved in the exponential function.  
  
Quick Wins : When dealing with exponential equations, remember that if two exponential expressions are equal, their bases and exponents can be related. It can be helpful to express both and in the same form, possibly factoring out common elements. If you encounter a number like 6561, it might be useful to express it as a power of a smaller number (for example, recognize that ) to simplify comparisons. This will help you leverage the properties of exponents effectively.  
  
Mistake Alert : Be cautious with the arithmetic when applying the exponential properties. Ensure that you accurately apply the exponent rules, especially when manipulating the equation to isolate b. Double-check your calculations when substituting values into the exponential function to avoid common mistakes, such as miscalculating powers or neglecting the coefficients.  
  
SAT Know-How : This problem is categorized under Advanced Math and focuses on nonlinear functions, specifically exponential functions. It assesses the student’s ability to analyze exponential relationships and manipulate equations effectively. Understanding the properties of exponents and being able to express relationships clearly is crucial for solving such problems in the SAT context.  
  
Use the definition of the exponential function to express and :  
  
Substitute these expressions into the given condition :  
  
Divide both sides by to isolate :  
  
To find b, take the eighth root of both sides:  
  
Since , we find:  
  
Therefore, the value of b is 3.

3.  The function f is a quadratic function. In the xy-plane, the graph of has a vertex at (6, -4) and passes through the points (-3, 239) and (-5, 359). What is the value of ?

A.   -15

B.   -12

C.   -9

D.   -5

## Answer

C

## Solution

Concept Check : The intent of this question is to assess the student’s understanding of quadratic functions, particularly in determining the function’s equation when given its vertex and specific points on the graph. Students are expected to know how to use the vertex form of a quadratic function and how to manipulate it based on known points.  
  
Solution Strategy : To solve the problem, the student should start by recalling the vertex form of a quadratic function, which is given by the equation: f(x) = a(x - h)² + k, where (h, k) is the vertex of the parabola. Here, the vertex is given as (6, -4), so h = 6 and k = -4. The next step is to substitute the coordinates of the points (-3, 239) and (-5, 359) into the equation to create two equations that can be solved to find the value of ’a’. Once ’a’ is determined, the full function can be established, and then f(7) and f(4) can be calculated to find their difference.  
  
Quick Wins : 1. Remember the vertex form of a quadratic function is essential in problems like this. 2. Substitute points carefully and ensure you maintain correct signs. 3. When substituting the points into the equation, simplify carefully to avoid arithmetic errors. 4. After finding the function, evaluate it at the required points to find the difference.  
  
Mistake Alert : 1. Be cautious when substituting the points; double-check that you plug the correct x and y values into the equation. 2. Pay special attention to the negative signs when working with the vertex’s coordinates and the points; it’s easy to make minor mistakes here. 3. Ensure that you correctly calculate the values of f(7) and f(4) before subtracting them.  
  
SAT Know-How : This problem belongs to the Advanced Math category and focuses on quadratic functions and their graphs. It assesses skills such as understanding the vertex form of a quadratic equation, substituting values correctly, and performing arithmetic operations accurately. Mastering these skills is crucial for solving quadratic-related problems on the SAT, as it enhances your problem-solving efficiency and reduces the chances of mistakes.  
  
Start by writing the quadratic function in vertex form: .  
Substitute the point (-3, 239) into the equation to solve for a: .  
Simplify: 239 = a(81) - 4.  
Add 4 to both sides: 243 = 81a.  
Solve for a: .  
Verify a with the point (-5, 359):  
.  
Simplify: 359 = 3(121) - 4.  
Calculate: 359 = 363 - 4.  
This confirms a = 3 is correct.  
Now, calculate f(7) and f(4):  
.  
.  
.  
Calculate f(7) - f(4): -1 - 8 = -9.

4.  Which expression is equivalent to ?

A.

B.

C.

D.

## Answer

D

## Solution

Concept Check : The question tests the student’s understanding of polynomial operations, specifically multiplication. Students should know how to distribute the term across each term in the polynomial () and apply the rules of exponents when multiplying terms with the same base.  
  
Solution Strategy : To solve this problem, the student should first recognize that they need to distribute to each term inside the parentheses. This means multiplying , then by , and finally by 84. The student should keep in mind how to handle the coefficients and the variables while applying the laws of exponents, specifically that when multiplying like bases, you add the exponents.  
  
Quick Wins : Start by rewriting the expression clearly; it helps to see each part. Remember to multiply the coefficients (the numbers in front) together and then handle the variable parts separately. When multiplying the variable parts, add the exponents of like bases. It can be helpful to write down intermediate steps to avoid confusion, ensuring that each term is calculated correctly before moving on to the next.  
  
Mistake Alert : Be careful with the signs—multiplying by -6 means that the sign of each resulting term will flip. Also, double-check the exponents; it’s easy to make a mistake when adding exponents, especially with higher-degree polynomials. Pay attention to the arrangement of your final expression to ensure all terms are included and properly simplified.  
  
SAT Know-How : This problem is an example of operations with higher-degree polynomials, specifically focusing on polynomial multiplication. It assesses skills such as distribution, combining like terms, and the laws of exponents. Mastering these concepts is crucial for success in the SAT’s math section, particularly in advanced math questions.  
  
Step 1: Distribute across each term in the parenthesis: , , and .  
Step 2: Calculate , , remains unchanged. So the first term becomes .  
Step 3: Calculate , , remains unchanged. So the second term becomes .  
Step 4: Calculate , remains unchanged, remains unchanged. So the third term becomes .  
Step 5: Combine all terms to form the expression: .

5.  The function f is defined by . What is the value of ?

## Answer

2

## Solution

Concept Check : The intent of this question is to assess the student’s understanding of evaluating a quadratic function. Students are expected to know how to substitute a given value into a function and compute the result, demonstrating their proficiency with basic algebraic operations.  
  
Solution Strategy : To solve this problem, the student should start by substituting the given value, which is 2, into the function . This involves replacing x with 2 in the expression . After substitution, they will need to follow the order of operations (PEMDAS/BODMAS) to simplify the expression step by step.  
  
Quick Wins : When substituting, it’s helpful to write out the function clearly and confirm each step as you go. Break down the calculation into manageable parts, calculating , , and then combining these results along with the constant term -38. This systematic approach can minimize errors.  
  
Mistake Alert : Be careful with arithmetic, especially when squaring numbers and performing addition or subtraction. It’s easy to make mistakes with signs, so double-check your calculations to ensure accuracy. Also, remember to follow the order of operations closely.  
  
SAT Know-How : This problem falls under the category of Advanced Math, focusing on nonlinear functions, particularly quadratics. It assesses the student’s skill in evaluating functions and performing algebraic operations. Mastering such problems enhances problem-solving capabilities and prepares students for the types of questions they may encounter on the SAT.  
  
Start with the function: .  
Substitute into the function: .  
Calculate : , so .  
Calculate 6(2): .  
Combine the terms: .  
Perform the arithmetic: .  
Subtract 38 from 40: .  
Thus, the value of is 2.

6.  What is an x-coordinate of an x-intercept of the graph in the xy-plane?

A.   -5

B.    -4

C.    0

D.    10

## Answer

A

## Solution

Concept Check : The question asks students to find the x-coordinate of an x-intercept of a polynomial function. Students should know that the x-intercepts occur where the value of y is zero, which means solving the equation set to zero. This requires understanding polynomial functions and their roots.  
  
Solution Strategy : To find the x-intercept, the student should set the polynomial expression equal to zero: 3(x - 4)(x - 7)(x + 5) = 0. This will lead to a product of factors equal to zero, which means setting each factor equal to zero to find the corresponding x-values. The student should be prepared to solve for x by isolating each factor.  
  
Quick Wins : First, remember that if a product of factors equals zero, at least one of the factors must be zero. This means you will need to consider each factor individually. In this case, the factors are (x - 4), (x - 7), and (x + 5). Set each factor to zero and solve for x to find the x-intercepts. It may help to write down each equation separately for clarity.  
  
Mistake Alert : Be careful not to overlook any factors when setting them to zero. It’s easy to accidentally skip one if you’re not thorough. Also, remember that the polynomial has real roots, so ensure that you are not making assumptions about the nature of the roots without checking the factors. Lastly, double-check your arithmetic when solving for x to avoid simple calculation errors.  
  
SAT Know-How : This problem falls under the category of Advanced Math, specifically dealing with polynomials and their roots. It tests the student’s ability to manipulate polynomial equations and solve for specific values (x-intercepts). Mastery of this skill is crucial for handling more complex polynomial functions in the SAT, as it demonstrates understanding of foundational algebraic concepts.  
  
Set y = 0 in the equation: 0 = 3(x - 4)(x - 7)(x + 5).  
Since the product is zero, at least one of the factors must be zero.  
Solve each factor for x:  
1. x - 4 = 0 x = 4  
2. x - 7 = 0 x = 7  
3. x + 5 = 0 x = -5  
Therefore, the x-coordinates of the x-intercepts are 4, 7, and -5.

7.  A financial analyst is modeling the projected economic activity in a city using the equation for active jobs: , where represents the number of active jobs and is the number of months since January 2020. What is the best interpretation of the y-intercept of the graph of this equation in the xy-plane?

A.    At the end of January 2020, the projected number of active jobs was 0.

B.    At the end of January 2020, the projected number of active jobs was 86, suggesting a lack of job activity or data errors.

C.    At the end of July 2020, the projected number of active jobs was 86.

D.    At the end of July 2020, the projected number of active jobs was 0.

## Answer

B

## Solution

Concept Check : The intent of this question is to assess the student’s understanding of quadratic functions, specifically the interpretation of the y-intercept in the context of a real-world application. Students should recognize that the y-intercept represents the value of y when x is zero, which corresponds to a specific point in time (January 2020 in this case).  
  
Solution Strategy : To approach this problem, students should first identify what the y-intercept represents in the given equation. This involves substituting into the equation to find the value of y. Then, they should interpret this value in the context of the problem, considering what it means in terms of active jobs at the starting point of the timeframe given (January 2020).  
  
Quick Wins : When interpreting the y-intercept, remember that it gives you the starting value of the dependent variable when the independent variable is zero. In this case, think about what ’months since January 2020’ means— will indicate January 2020, so the y-intercept will tell you the number of active jobs at that time. Always read the problem carefully to ensure you understand the real-world connection.  
  
Mistake Alert : Be careful not to confuse the y-intercept with other points on the graph. The y-intercept is specifically the point where the graph crosses the y-axis (x = 0). Additionally, ensure that you correctly substitute x = 0 into the equation; miscalculating the equation may lead to incorrect interpretations. Remember that negative values can have significant implications in real-world contexts.  
  
SAT Know-How : This problem falls under the category of Advanced Math, focusing on quadratic and exponential word problems. It tests the student’s ability to interpret mathematical models in real-world contexts, specifically through the lens of the y-intercept. Mastering this concept is crucial for SAT problem-solving, as it reinforces the connection between algebraic equations and their applications in various scenarios.  
  
1. Identify the y-intercept in the given equation .  
2. The y-intercept occurs when .  
3. Substitute into the equation: .  
4. Simplify: .  
5. The y-intercept of 86 means that at the start of January 2020, the projected number of active jobs is 86.

8.  The equation has exactly one solution for the variable . Which of the following could be the value of ?

A.   -55 only

B.    0 only

C.   -55 and 0

D.   -55 and 55

## Answer

A

## Solution

Concept Check : The intent of this question is to assess the student’s understanding of absolute value equations and how the number of solutions can be determined based on the properties of the equations involved. Students should be familiar with the concept of absolute values and the conditions under which an equation has one solution, particularly in relation to quadratic functions.  
  
Solution Strategy : To solve this problem, students should start by isolating the absolute value expression on one side of the equation. They will need to recognize that for the equation to have exactly one solution, the expression inside the absolute value must equal zero at that solution, and the quadratic expression on the right side must also take a specific form. This typically involves finding the vertex of the quadratic function and ensuring it touches the x-axis at exactly one point. Students should think about how to manipulate the equation to find values of k that meet these conditions.  
  
Quick Wins : Consider rewriting the equation in a form that allows you to analyze the absolute value term. Remember that an absolute value equation has one solution when the expression inside the absolute value is zero, or when the graph of the quadratic touches the line represented by the absolute value. You may want to identify the vertex of the quadratic equation on the right side and find values of k that make it equal to the expression represented by the absolute value. Calculating the discriminant of the quadratic can also provide insight into the number of solutions.  
  
Mistake Alert : Be careful not to overlook the conditions for the absolute value to yield exactly one solution. It’s crucial to ensure that the quadratic’s vertex aligns correctly for it to touch the axis at just one point. Additionally, pay attention to any algebraic manipulations and signs when isolating the absolute value term, as mistakes in these steps can lead to incorrect conclusions about the number of solutions.  
  
SAT Know-How : This problem falls under the category of Advanced Math, specifically focusing on radical, rational, and absolute value equations. It assesses skills related to manipulating and analyzing equations to determine the number of solutions. The key takeaway is to understand how absolute values interact with quadratic functions and to use properties of both to solve for possible values of k. Mastering these concepts is essential for effective problem-solving in SAT math.  
  
Step 1: Start with the absolute value equation: .  
Step 2: Isolate the absolute value term: .  
Step 3: Divide both sides by 4: .  
Step 4: For |x - 97| to have exactly one solution, the expression must be 0.  
Step 5: Solve the equation: = 0.  
Step 6: Factor the quadratic equation: .  
Step 7: Set the factored expression to 0: .  
Step 8: Solve for k: k + 55 = 0, so k = -55.

9.  A physicist is studying a certain radioactive substance whose quantity decreases over time. The equation representing the quantity of this substance in grams after x years is given by: . Which of the following is the best interpretation of 1100 in this context?

A.   The estimated amount of substance remaining after 1 year.

B.   The estimated amount of substance remaining after 2 years.

C.   The estimated initial quantity of the substance at time 0.

D.   The estimated quantity of substance after 6 years.

## Answer

C

## Solution

Concept Check : The question is designed to test the student’s understanding of exponential decay, particularly in the context of real-world applications such as radioactive decay. The student is expected to know how to interpret the parameters of an exponential function, specifically the initial quantity and the decay factor.  
  
Solution Strategy : To approach this problem, the student should identify the components of the given exponential function. They should recognize that in the equation of the form , ’a’ represents the initial value of the quantity when . The student should think through the meaning of the parameters in relation to the problem context to determine what 1100 signifies.  
  
Quick Wins : A good strategy is to substitute into the function to see what value you get. This will help clarify what the initial quantity is. Also, it can be helpful to visualize the process of radioactive decay and how it relates to the parameters in the equation. Remember that the base of the exponent (0.41 in this case) indicates the rate of decay, whereas the coefficient provides the starting amount.  
  
Mistake Alert : Be careful not to confuse the initial quantity with the decay factor. The number 1100 is significant at the start of the observation (when ), while the base of the exponent, 0.41, tells you how much of the substance remains after each year. Misinterpreting these can lead to incorrect conclusions.  
  
SAT Know-How : This problem falls under the category of advanced math, specifically focusing on exponential word problems related to real-life scenarios like radioactive decay. It assesses the student’s ability to interpret mathematical models and understand the significance of parameters. Mastering this type of problem enhances your skills in applying mathematical concepts to analyze and interpret real-world data effectively.  
  
Step 1: Understand the general form of an exponential decay function. It is typically given by , where ’a’ is the initial quantity, ’b’ is the decay factor, and ’x’ is the time elapsed.  
Step 2: Compare the given function with the general form. Here, and .  
Step 3: Interpret the meaning of . This value represents the initial quantity of the substance before any decay has occurred, i.e., at time .  
Step 4: Verify by setting in . The equation becomes .  
Step 5: Therefore, 1100 represents the initial amount of the substance, confirming the interpretation that it is the quantity present at time .

10.  A solution to the given system of equations is (x, y). What is a possible value of x?

A.   75

B.   76

C.   78

D.   80

## Answer

B

## Solution

Concept Check : The intent of the question is to assess the student’s understanding of systems of equations, specifically a combination of linear and quadratic equations. Students should know how to manipulate both types of equations and recognize how to find possible solutions for x based on the given equations.  
  
Solution Strategy : To solve the problem, the student should first substitute the expression for y from the linear equation into the quadratic equation. This will create a single equation in terms of x, which can then be solved for possible values of x. The student should be prepared to explore the nature of the quadratic equation resulting from this substitution, which may yield one or two solutions for x.  
  
Quick Wins : 1. Always start by identifying one equation that is easier to manipulate—here, the linear equation is simpler. 2. Substitute expressions carefully to avoid mistakes. 3. When working with quadratic equations, remember to check for both possible solutions, as some quadratics can yield two values for x. 4. Use the quadratic formula or factoring techniques if applicable.  
  
Mistake Alert : Be cautious about sign errors when rearranging the equations. Pay special attention to the quadratic form; sometimes students overlook negative solutions or fail to correctly simplify expressions. Ensure you check both possible solutions after solving the quadratic equation, as it can have two valid x-values.  
  
SAT Know-How : This problem falls under the category of Advanced Math and specifically tests skills in solving systems of linear and quadratic equations. It requires the ability to manipulate equations and recognize the relationship between linear and quadratic functions. Developing these problem-solving skills is essential for success on the SAT, especially in recognizing how different types of equations can interact.  
  
1. Substitute equation 1 into equation 2, since both are equal to y.  
2. Set up the equation: .  
3. Rearrange the equation: .  
4. Factor the equation: (x - 76)(x - 76 - 1) = (x - 76)(x - 77) = 0.  
5. Solve the factors individually:  
- If (x - 76) = 0, then x = 76.  
- If (x - 77) = 0, then x = 77.  
6. Thus, possible values for x are 76 and 77.