SAT Math Algebra

1.  For the linear function f, the graph of in the xy-plane has a slope of 5 and passes through the point . Which equation defines f?

A.

B.

C.

D.

2.  A charity organization is tracking the number of meals it can provide for the homeless each week based on the amount of funds raised. The equation that models this relationship is given by the function , where represents the number of meals provided and w is the amount of money raised in hundreds of dollars. Based on this model, approximately how many meals can be provided for each additional hundred dollars raised?

A.   1 meal

B.   4.04 meals

C.   98.7 meals

D.   100 meals

3.  In the xy-plane, which of the following regions does NOT contain any points that are part of the solution set to , ?

A.    The region where and

B.    The region where and

C.    The region where and

D.    The y-axis

4.  A certain school has a total of 46 students enrolled in a club. The number of students in the club is at least 32 more than the number of students who have signed up for a volunteer activity. If we let x represent the number of students signed up for the volunteer activity, which inequality represents this situation?

A.

B.

C.

D.

5.  In the xy-plane, line m has a slope of 4 and a y-intercept of (0, -54). What is the x-coordinate of the x-intercept of line m?

6.  A company produces eco-friendly products. The total revenue of the company in the year 2023 is represented by the equation , where x represents the number of years since 2015. If the revenue is expected to reach 831 dollars in 2023, how many years since 2015 has the company been operating?

A.   10 years

B.   15 years

C.   18 years

D.   20 years

7.  If , what is the value of ?

A.   -9

B.   -6

C.    3

D.    6

8.  What is the y-coordinate of the y-intercept of the graph of in the xy-plane, given that and ?

A.   -83

B.   -78

C.   -73

D.   -68

9.  For each real number r, which of the following points lies on the graph of each equation in the xy-plane for the given system? ,

A.

B.

C.

D.

10.  A research study shows that the average number of hours students spend on community service per year, x, can be estimated by the function . Which statement best interprets the value 89 in this context?

A.   Students will spend a total of 89 hours on community service after 5 years.

B.   The average number of community service hours is expected to grow by 5 hours each year.

C.   The average hours of community service increased from 89 hours after each year.

D.   The estimated number of community service hours was 89 hours when no additional hours are counted.

SAT Math Algebra Solutions

1.  For the linear function f, the graph of in the xy-plane has a slope of 5 and passes through the point . Which equation defines f?

A.

B.

C.

D.

## Answer

A

## Solution

Concept Check : The question intends for the student to understand the concept of linear functions, specifically how to use the slope-intercept form of a linear equation () to find the equation of a line. The student is expected to know how to identify the slope and y-intercept of a linear function.  
  
Solution Strategy : To solve this problem, the student should recognize that the slope-intercept form of a line is given by , where m represents the slope and b represents the y-intercept. Given the slope of 5 and the point (0, -45), the student should substitute these values into the equation to find the linear function f.  
  
Quick Wins : Remember that the slope-intercept form is very helpful in these types of problems. The slope (m) is the coefficient of x, and the y-intercept (b) is the constant term. If a point is given where x equals 0, it directly gives you the y-intercept. Be sure to correctly substitute the values and simplify the equation properly.  
  
Mistake Alert : Be careful not to confuse the point (0, -45) with the slope. It is easy to mistakenly use the y-coordinate as the slope or vice versa. Additionally, double-check your arithmetic when substituting values into the equation to avoid simple calculation errors.  
  
SAT Know-How : This problem falls under the category of Algebra, specifically focused on linear equations and functions. It assesses the student’s ability to apply the slope-intercept form of a linear equation to find a function. Mastering this skill is crucial for solving various types of algebraic problems on the SAT, showcasing the importance of understanding the relationship between slope, intercepts, and linear functions.  
  
1. Understand the General Form of a Linear Function: The equation of a line in slope-intercept form is , where m is the slope and b is the y-intercept.  
2. Identify the Given Slope: The problem states that the slope m is 5.  
3. Determine the Y-Intercept: Since the line passes through the point (0, -45), the y-intercept b is -45.  
4. Express the function : Substitute the values of m and b into the slope-intercept form: .

2.  A charity organization is tracking the number of meals it can provide for the homeless each week based on the amount of funds raised. The equation that models this relationship is given by the function , where represents the number of meals provided and w is the amount of money raised in hundreds of dollars. Based on this model, approximately how many meals can be provided for each additional hundred dollars raised?

A.   1 meal

B.   4.04 meals

C.   98.7 meals

D.   100 meals

## Answer

B

## Solution

Concept Check : The intent of the question is to assess the student’s understanding of linear equations and how they relate to real-world scenarios. Specifically, it requires knowledge of interpreting the slope of a linear function to determine the change in the output (number of meals) based on a change in the input (amount of money raised). The student is expected to recognize that the slope of the function indicates how many additional meals are provided for each unit increase in the independent variable.  
  
Solution Strategy : To solve this problem, the student should focus on the given linear equation f(w) = -4.04w + 98.7. They need to identify the slope of the function, which represents the rate of change of meals provided with respect to the amount of money raised. In this case, the slope is the coefficient of w, which is -4.04. The student should interpret this slope in the context of the problem to find out how many meals can be provided for each additional hundred dollars raised.  
  
Quick Wins : Remember that in a linear equation of the form y = mx + b, ’m’ represents the slope and indicates how much y changes for a one-unit increase in x. In this context, since w is in hundreds of dollars, the value of the slope directly tells you how many meals change per one hundred dollars raised. Pay close attention to the sign of the slope; a negative slope indicates a decrease in the number of meals with an increase in funds raised, which might seem counterintuitive at first.  
  
Mistake Alert : Be careful not to confuse the slope with the y-intercept. The y-intercept (98.7) tells you the starting point (the number of meals provided when no funds are raised), while the slope (-4.04) tells you how meals change with each additional hundred dollars raised. Also, ensure you correctly interpret the context of the problem; in this case, a negative slope means that raising more funds leads to fewer meals provided, which is an important aspect to recognize.  
  
SAT Know-How : This problem falls under the category of algebra, specifically linear equation word problems. It assesses the student’s ability to interpret the slope of a linear function in a real-world context. By understanding how to analyze the slope, students can effectively determine the impact of changes in one variable on another. Approaching SAT math problems with a clear understanding of linear relationships and careful interpretation of their components is key to success.  
  
Identify the meaning of the slope in the function .  
The slope (m) is -4.04, which indicates the number of additional meals provided decreases by 4.04 for each additional hundred dollars raised.  
Since the question asks for the number of meals provided, we consider the magnitude of the slope, which is 4.04.

3.  In the xy-plane, which of the following regions does NOT contain any points that are part of the solution set to , ?

A.    The region where and

B.    The region where and

C.    The region where and

D.    The y-axis

## Answer

A

## Solution

Concept Check : The intent of the question is to assess the student’s understanding of linear inequalities, specifically how to graph them and identify solution sets in the xy-plane. The student is expected to know how to manipulate and interpret inequalities and recognize the graphical representation of these inequalities.  
  
Solution Strategy : To approach this problem, the student should first convert the inequality into an equation () to find the boundary line. Then, they should determine the region that satisfies the inequality () by testing a point not on the line (often the origin is a convenient choice). After identifying the solution region, the student can analyze the given options to see which region does not include any points from this solution set.  
  
Quick Wins : To efficiently solve the problem, remember to always graph the boundary line of the inequality. Use a dashed line for ’<’ or ’>’ inequalities to indicate that points on the line are not included in the solution set. After identifying the solution region, test points in the regions to confirm if they satisfy the inequality. Keep in mind that the inequality can be rewritten in slope-intercept form () to easily identify the y-intercept and slope for graphing.  
  
Mistake Alert : Be cautious when determining the type of line to draw (dashed vs. solid) based on the inequality symbol. Also, when testing points, ensure that you select points that are definitely in the regions you are evaluating. It’s easy to mistakenly assume a region is part of the solution set without checking a point from that region against the inequality.  
  
SAT Know-How : This question falls under the category of Algebra, focusing on solving linear inequalities and understanding the graphical representation of their solutions. It assesses the student’s ability to manipulate inequalities and interpret their solution sets. Mastery of these concepts is crucial for performing well on the SAT, as it involves critical thinking and graphical analysis skills.  
  
Step 1: Write the inequality in slope-intercept form.  
 can be rearranged by subtracting 2x from both sides: .  
Step 2: Divide every term by 5 to solve for y: .  
This inequality tells us that the solution set is the region below the line in the xy-plane.  
Step 3: Analyze each option against this inequality:  
Option A: The region where and . The line has a y-intercept of 2, so is above the line, meaning this region does NOT contain any points of the solution set.  
Option B: The region where and . A negative y-value can satisfy the inequality because it is below the line, so this region may contain points of the solution set.  
Option C: The region where and . A y-value less than 2, especially for positive x, could satisfy the inequality as it is below the line.  
Option D: The y-axis (). Substituting x = 0 into the inequality gives , or , which can be satisfied for y-values less than 2 on the y-axis.

4.  A certain school has a total of 46 students enrolled in a club. The number of students in the club is at least 32 more than the number of students who have signed up for a volunteer activity. If we let x represent the number of students signed up for the volunteer activity, which inequality represents this situation?

A.

B.

C.

D.

## Answer

A

## Solution

Concept Check : The intent of the question is to assess the student’s understanding of linear inequalities, particularly in the context of a real-world scenario. The student is expected to be familiar with how to translate verbal descriptions into mathematical inequalities and understand the relationship between the total number of students in the club and those signed up for a volunteer activity.  
  
Solution Strategy : To approach this problem, the student should focus on identifying the key components of the situation presented. The total number of students in the club is 46, and there is a relationship described between the number of students in the club and those signed up for the volunteer activity. The student should express this relationship as an inequality, taking into account that the number of students in the club (which is 46) is at least 32 more than the number of students signed up (represented by x). This leads to the formulation of an inequality that captures this relationship.  
  
Quick Wins : When translating word problems to inequalities, it is helpful to break down the problem into smaller parts. Identify what you know (in this case, the total number of students and the relationship described) and define your variable clearly. Use phrases like ’at least’ to recognize that you will use greater than or equal to (≥) in your inequality. Remember to carefully consider the direction of the inequality based on the wording of the problem.  
  
Mistake Alert : Be careful with the mathematical symbols and terms used in the problem. Phrases like ’at least’ indicate that you should use a greater than or equal to symbol (≥), while ’more than’ would use a greater than symbol (>). Additionally, ensure you correctly interpret the relationship between the total number of students and those signed up for the volunteer activity to avoid misformulating the inequality.  
  
SAT Know-How : This problem belongs to the Algebra category, specifically focusing on linear inequality word problems. It assesses the student’s ability to interpret a real-world scenario and convert it into a mathematical inequality. Understanding how to extract relevant information and translate it accurately into inequalities is a critical skill for the SAT, and practicing this will aid in developing problem-solving proficiency.  
  
Identify the given relationship: The number of students in the club (46) is at least 32 more than x.  
Express this relationship mathematically: 46 is at least .  
This can be transformed into an inequality: .  
Rearrange the terms to find the correct inequality: .  
Therefore, the inequality representing the situation is .

5.  In the xy-plane, line m has a slope of 4 and a y-intercept of (0, -54). What is the x-coordinate of the x-intercept of line m?

## Answer

13.5

## Solution

Concept Check : The intent of the question is to assess the student’s understanding of linear equations, specifically how to identify the slope and y-intercept of a line. The student is expected to know how to use the slope-intercept form of a linear equation, which is , where m is the slope and b is the y-intercept. The goal is to find the x-coordinate of the x-intercept, where the line crosses the x-axis ().  
  
Solution Strategy : To approach this problem, the student should start by writing the equation of the line using the given slope and y-intercept. The slope (m) is 4, and the y-intercept (b) is -54, so the equation will take the form . Next, the student needs to find the x-coordinate of the x-intercept by setting y to 0 and solving for x. This involves rearranging the equation to isolate x and determine its value.  
  
Quick Wins : A useful tip is to remember that the x-intercept occurs where . To find this, simply substitute 0 for y in the linear equation. After substituting, be careful to simplify correctly. Also, it can help to visualize the graph of the line to understand where it crosses the x-axis.  
  
Mistake Alert : Students should be cautious not to misinterpret the slope and y-intercept, ensuring they are correctly inputting these values into the slope-intercept form. Additionally, they should not forget to isolate x correctly when solving for the x-intercept, as small arithmetic mistakes can lead to incorrect answers.  
  
SAT Know-How : This problem falls under the category of Algebra, specifically focusing on the graphs of linear equations and functions. It assesses the student’s ability to apply the slope-intercept form to find the x-intercept of a line. Mastery of this concept is essential for SAT math, as it tests both understanding and application of linear relationships.  
  
1. Write the equation of line m using the slope-intercept form of a line equation, which is .  
2. Substitute the given slope and y-intercept into the equation: .  
3. To find the x-coordinate of the x-intercept, set and solve for x:  
  
4. Rearrange the equation to solve for x:  
  
5. Divide both sides by 4 to isolate x:  
  
6. Simplify the fraction:  
  
7. Therefore, the x-coordinate of the x-intercept of line m is 13.5.

6.  A company produces eco-friendly products. The total revenue of the company in the year 2023 is represented by the equation , where x represents the number of years since 2015. If the revenue is expected to reach 831 dollars in 2023, how many years since 2015 has the company been operating?

A.   10 years

B.   15 years

C.   18 years

D.   20 years

## Answer

C

## Solution

Concept Check : The intent of the question is to assess the student’s ability to interpret a linear equation in the context of a real-life scenario. Students are expected to understand how to manipulate and solve linear equations, and they should be familiar with the concepts of revenue, time variables, and how to relate them in the context of the problem.  
  
Solution Strategy : To approach this problem, students should first identify what the variable ’x’ represents in the context of the problem, which is the number of years since 2015. Then, they should recognize that the equation given can be rearranged to isolate ’x’. This will require applying algebraic principles, such as distributing terms, combining like terms, and solving for the variable. It’s essential to keep track of the relationships between the years and the revenue stated in the equation.  
  
Quick Wins : A helpful tip is to break down the equation step-by-step. Start by simplifying the right side and isolating ’x’ on one side of the equation. It can also be beneficial to convert the equation into a more familiar linear form. Additionally, double-check your calculations at each step to ensure accuracy. Finally, remember to interpret the final value of ’x’ in the context of the problem to ensure it makes sense.  
  
Mistake Alert : Students should be careful not to make common mistakes such as misreading the equation or mistakenly adding or subtracting terms incorrectly. Additionally, pay attention to the meaning of the variable ’x’ to avoid misinterpreting what the solution represents. It’s also important to check that the context of the problem aligns with the solution you arrive at, ensuring that it is a reasonable answer in terms of the years since the company began operating.  
  
SAT Know-How : This problem falls under the category of Algebra, specifically focusing on linear equation word problems. It assesses the student’s skills in interpreting, manipulating, and solving linear equations in real-world contexts. Mastery of these skills is essential for success on the SAT, as it demonstrates the ability to connect mathematical concepts to practical situations.  
  
1. Begin by simplifying the equation: .  
2. Distribute 76 in the term : .  
3. The equation becomes: .  
4. Combine like terms on the right side: .  
5. The equation now is: .  
6. Add 537 to both sides to isolate the term with x: .  
7. This simplifies to: .  
8. Divide both sides by 76 to solve for x: .  
9. Calculate the division: .  
10. Therefore, the company has been operating for 18 years since 2015.

7.  If , what is the value of ?

A.   -9

B.   -6

C.    3

D.    6

## Answer

C

## Solution

Concept Check : The intent of this question is to assess the student’s ability to solve a linear equation using the substitution method. Students are expected to understand how to isolate variables and perform algebraic operations to find the value of ’x + 9.’  
  
Solution Strategy : To approach this problem, the student should start by recognizing that both sides of the equation contain the term ’(x + 9).’ The first step is to distribute the coefficients (86 and 36) to the term ’(x + 9)’ on both sides. After simplifying both sides, the student should then rearrange the equation to isolate ’x’ or directly solve for ’x + 9’ by manipulating the equation appropriately.  
  
Quick Wins : A helpful tip is to combine like terms after distributing the coefficients. This will make it easier to isolate the variable. Also, consider rewriting the equation in terms of ’x + 9’ directly, which could simplify your calculations. Always double-check your calculations after each step to ensure accuracy.  
  
Mistake Alert : Students should be cautious about making errors in distribution and combining like terms. It’s easy to miscalculate coefficients or to accidentally drop a term when rearranging the equation. Double-check your work to avoid these common mistakes, especially when dealing with negative numbers or when moving terms from one side of the equation to the other.  
  
SAT Know-How : This problem falls under the category of Algebra, specifically focusing on solving linear equations and inequalities through substitution. It assesses the student’s skills in distributing terms, combining like terms, and isolating variables. Mastering these techniques is essential for success in SAT math, as it reinforces the foundational skills needed for more complex algebraic concepts.  
  
Step 1: Simplify both sides by subtracting from both sides of the equation.  
  
Step 2: Combine like terms on the left side.  
  
Step 3: Isolate by dividing both sides by 50.  
  
Step 4: Simplify the fraction.

8.  What is the y-coordinate of the y-intercept of the graph of in the xy-plane, given that and ?

A.   -83

B.   -78

C.   -73

D.   -68

## Answer

A

## Solution

Concept Check : The question intends for the student to understand the concept of y-intercepts, particularly in the context of composite functions. Students should know how to find the y-intercept of a function and how transformations like vertical shifts (subtracting a constant) affect it. Additionally, familiarity with linear functions and their properties is essential.  
  
Solution Strategy : To solve the problem, the student should first determine the y-intercept of the function . This is achieved by substituting into the function to find . Once is found, the student can then calculate using the relationship , which will give the y-coordinate of the y-intercept for .  
  
Quick Wins : Remember that the y-intercept occurs where . When dealing with composite functions, it can be helpful to break the problem into smaller parts—first find the y-intercept of the inner function () and then apply any transformations to find the final y-intercept of the outer function (). Also, keep track of the signs when performing operations on constants.  
  
Mistake Alert : Students may forget to substitute in both functions or may miscalculate when applying the vertical shift (subtracting 5). It is also easy to lose track of negative signs or to confuse the order of operations, especially when handling the composite functions.  
  
SAT Know-How : This problem falls under the category of Algebra, specifically focusing on graphs of linear equations and functions. It assesses the student’s ability to find y-intercepts and understand the effects of transformations on functions. This type of question tests critical thinking and problem-solving skills, which are essential for success on the SAT.  
  
First, determine at :   
Calculate:   
Simplify:   
Calculate:   
Substitute :   
Simplify:   
Thus, the y-coordinate of the y-intercept of the graph is -83.

9.  For each real number r, which of the following points lies on the graph of each equation in the xy-plane for the given system? ,

A.

B.

C.

D.

## Answer

A

## Solution

Concept Check : The question aims to assess the student’s understanding of systems of linear equations, specifically focusing on recognizing equivalent equations and identifying points that satisfy both equations simultaneously.  
  
Solution Strategy : To approach this problem, students should first recognize that the given system of equations may represent the same line. This means that any point on one line will also be on the other line. Students should simplify or manipulate one of the equations to see if it can be transformed into the other form, or check if they are scalar multiples of each other. After confirming that the equations are equivalent, students can substitute the provided options for (x, y) into either equation to see if they satisfy it.  
  
Quick Wins : When working with systems of equations, it’s helpful to look for ways to manipulate the equations. You can multiply or divide entire equations by the same non-zero constant, or rearrange them to slope-intercept form (y = mx + b) to visually assess their relationship. Additionally, checking potential solutions can be done by substituting points back into the original equations, which allows you to confirm if they are valid solutions quickly.  
  
Mistake Alert : Be careful not to confuse equivalent equations with independent ones. Ensure that you check if both equations actually represent the same line before concluding that they have infinite solutions. Also, pay attention to arithmetic errors when manipulating the equations or substituting points. Miscalculations here can lead to incorrect conclusions about the solutions.  
  
SAT Know-How : This problem falls under the category of algebra, specifically in the unit of solving systems of linear equations. It assesses the student’s ability to recognize equivalent equations and find common solutions. Mastering this type of problem is crucial for the SAT, as it tests problem-solving skills and the understanding of linear relationships, which are key concepts in algebra.  
  
Step 1: Check for equivalence between given equations.  
Multiply the entire first equation by 8 to compare with the second equation: .  
This simplifies to: , which is exactly the same as the second equation.  
Step 2: Since the equations are equivalent, they represent the same line.  
Step 3: Express y in terms of x for the simplified equation .  
Rearrange to solve for y: .  
Divide the entire equation by -29 to solve for y: .  
Therefore, for any real number , the corresponding y is .  
Step 4: Match this result with the given options to find which point (r, y) corresponds to this expression.

10.  A research study shows that the average number of hours students spend on community service per year, x, can be estimated by the function . Which statement best interprets the value 89 in this context?

A.   Students will spend a total of 89 hours on community service after 5 years.

B.   The average number of community service hours is expected to grow by 5 hours each year.

C.   The average hours of community service increased from 89 hours after each year.

D.   The estimated number of community service hours was 89 hours when no additional hours are counted.

## Answer

D

## Solution

Concept Check : The intent of this question is to assess the student’s understanding of function interpretation, specifically the concept of the y-intercept in the context of a linear equation. The student is expected to recognize what the constant term in the linear equation represents regarding the average number of hours spent on community service.  
  
Solution Strategy : To approach this problem, the student should first identify the components of the linear equation provided, . The student needs to recognize that the term ’x’ represents the number of years, and the function f(x) represents the estimated average hours spent on community service. The constant term ’89’ should then be interpreted in relation to what it signifies in this context.  
  
Quick Wins : Consider breaking down the function into its parts: the slope (5) and the y-intercept (89). Remember that the y-intercept indicates the value of the function when x is 0. Think about what it means for students who have not yet spent any time on community service, and how that relates to the average hours in this scenario. This perspective can clarify the meaning of the constant term.  
  
Mistake Alert : Be cautious not to confuse the slope with the y-intercept. The slope (5) indicates how much the average number of hours increases with each additional year, while the y-intercept (89) has a specific meaning that should not be overlooked. Additionally, ensure you are interpreting ’89’ correctly in the context of the problem—this is not just a number but has relevance to the average hours spent on community service.  
  
SAT Know-How : This problem falls under the category of algebra, specifically focusing on interpreting linear equations and their components. It assesses the student’s ability to read and understand the context of mathematical functions. The ability to interpret the y-intercept in a real-world scenario is a valuable skill in SAT problem-solving, highlighting the importance of grasping how mathematical concepts apply beyond mere calculations.  
  
Step 1: Understand the Function .  
The function is in the form , where m is the slope and b is the y-intercept.  
Step 2: Identify the meaning of the constant term, 89.  
In a linear equation, the y-intercept (89 in this case) represents the starting value - the value of the function when .  
Step 3: Analyze the context provided by the problem.  
In this context, the value 89 represents the estimated initial average hours of community service before any additional time () is counted.  
Step 4: Check each option for alignment with the interpretation of 89.  
Option A is incorrect because it confuses the function’s application over 5 years.  
Option B is incorrect because it describes the slope (5) rather than the y-intercept (89).  
Option C is incorrect because it suggests 89 is a change rather than an initial value.  
Option D correctly identifies 89 as the initial estimate when no additional hours are factored in.