SAT Math Geometry and Trigonometry

1.  A circle in the xy-plane has its center at (-4, -7) and has a radius of 3. An equation of this circle is represented by the standard form equation , where a, b, and c are constants. What is the value of c?

2.  In the xy-plane, circle N is the graph of the equation . Circle Q has the same center as circle N but has a radius that is three times the radius of circle N. Which equation represents circle Q?

A.

B.

C.

D.

3.  Circle A has a radius of 3x and circle B has a radius of 63x. The area of circle B is how many times the area of circle A?

4.  In triangle XYZ, the measure of angle X is 53°, the measure of angle Y is 90°, and the measure of angle Z is °. What is the value of m?

A.   60

B.   70

C.   74

D.   76

5.  Triangles PQR and STU are similar. Each side length of triangle PQR is 7 times the corresponding side length of triangle STU. The area of triangle PQR is 3479 square centimeters. What is the area, in square centimeters, of triangle STU?

A.   50

B.   71

C.   90

D.   100

6.  Triangles GHI and JKL are congruent, where G corresponds to J, H corresponds to K, and both angles H and K are right angles. The measure of angle G is 48°. What is the measure, in degrees, of angle L?

A.   Angle L = 42°

B.   Angle L = 48°

C.   Angle L = 60°

D.   Angle L = 90°

7.  What is the equation of the circle in the xy-plane that has a center at (8, -2) and a radius of 8?

A.

B.

C.

D.

8.  A hemisphere is half of a sphere. If a hemisphere has a radius of 21 inches, which of the following is closest to the volume, in cubic inches, of this hemisphere?

A.   4,800

B.   8,400

C.   15,600

D.   19,385

9.  For two angles, ∠A and ∠B, it is given that cos(A) = sin(B). The measures, in degrees, of ∠A and ∠B are 4x + 6 and 2x + 30, respectively. What is the value of x?

A.   5

B.   7

C.   9

D.  11

10.  Circle A has a radius of 96 millimeters (mm). Circle B has an area of . What is the total area, in , of circles A and B?

A.   9400π

B.   9800π

C.   10000π

D.   10200π

SAT Math Geometry and Trigonometry Solutions

1.  A circle in the xy-plane has its center at (-4, -7) and has a radius of 3. An equation of this circle is represented by the standard form equation , where a, b, and c are constants. What is the value of c?

## Answer

56

## Solution

Concept Check : The intent of the question is to assess the student’s understanding of the equation of a circle in the standard form and their ability to manipulate it to find specific constants. The student is expected to know how to derive the equation from the center and radius of the circle, and how to rearrange it into a specific form to isolate the constant c.  
  
Solution Strategy : To solve this problem, the student should start with the standard form of the equation of a circle, which is given by (x - h)² + (y - k)² = r², where (h, k) is the center and r is the radius. In this case, the center is (-4, -7) and the radius is 3. The student should substitute these values into the standard equation, expand it, and then rearrange it to match the provided form x² + y² + ax + by + c = 0 to identify the value of c.  
  
Quick Wins : A helpful approach is to first write down the standard equation of the circle using the given center and radius. After substituting the values, carefully expand the squared terms. Keep in mind that when expanding, you’ll have to distribute correctly and combine like terms. To find c, ensure that you clearly identify all the terms after rearranging the equation.  
  
Mistake Alert : Students should be cautious while expanding the squared terms and combining constants. It’s easy to make sign errors, especially when dealing with negative values. Also, remember to carefully isolate the constant c at the end of the rearrangement. Double-check your arithmetic to avoid small mistakes that could lead to an incorrect answer.  
  
SAT Know-How : This problem falls under the category of Geometry and Trigonometry, specifically focusing on circle equations and their representation. It assesses skills related to understanding the geometry of circles, manipulating algebraic expressions, and applying formulas correctly. Mastering these concepts is crucial for efficient SAT problem-solving, as it helps in recognizing patterns and applying relevant mathematical principles.  
  
Start with the standard form equation of a circle: (x - h)² + (y - k)² = r², where (h, k) is the center and r is the radius.  
Substitute the given center (-4, -7) and radius 3: (x - (-4))² + (y - (-7))² = 3².  
This simplifies to (x + 4)² + (y + 7)² = 9.  
Expand (x + 4)² and (y + 7)²:  
(x + 4)² = x² + 8x + 16  
(y + 7)² = y² + 14y + 49  
Combine these expansions: x² + 8x + 16 + y² + 14y + 49 = 9.  
Rearrange to the general form: x² + y² + 8x + 14y + 16 + 49 - 9 = 0.  
Combine constants: 16 + 49 - 9 = 56.  
Thus, the equation becomes: x² + y² + 8x + 14y + 56 = 0.  
Therefore, the value of c is 56.

2.  In the xy-plane, circle N is the graph of the equation . Circle Q has the same center as circle N but has a radius that is three times the radius of circle N. Which equation represents circle Q?

A.

B.

C.

D.

## Answer

C

## Solution

Concept Check : The question intends to assess the student’s understanding of circle equations in the Cartesian plane, specifically how to represent a circle using its standard equation format. Students should know that the standard form of a circle’s equation is given by , where (h, k) is the center and r is the radius. The problem also tests the ability to manipulate and scale the radius accordingly.  
  
Solution Strategy : To approach this problem, students should first identify the center and radius of circle N from the provided equation. The center can be determined from the values inside the parentheses, while the radius can be found by taking the square root of the constant on the right side of the equation. Once the radius of circle N is established, the student should then calculate the radius of circle Q, which is three times that of circle N. Finally, the student needs to write the equation for circle Q using the same center and the new radius.  
  
Quick Wins : Remember that the center of the circle is derived from the equation’s components, which are given in the form (x + 9) and (y + 4). The center will be (-9, -4). When scaling the radius, simply multiply the original radius by 3. After finding the new radius, substitute both the center coordinates and the new radius into the standard circle equation format.  
  
Mistake Alert : Be careful with the signs when identifying the center from the equation; they can sometimes cause confusion. Double-check the calculations when scaling the radius to ensure you multiply correctly. Additionally, when rewriting the equation, ensure that you maintain the correct format of and that you accurately substitute the center and the new radius squared.  
  
SAT Know-How : This problem falls under the category of Geometry and Trigonometry, specifically focusing on circle equations. It assesses the student’s ability to manipulate equations and understand the relationship between the center and radius of a circle. Mastery of these concepts is crucial for success in SAT problem-solving, as it demonstrates a student’s proficiency in working with geometric figures and their equations.  
  
Step 1: Determine the center of circle N from its equation , which is (-9, -4).  
Step 2: Calculate the radius of circle N: .  
Step 3: Since circle Q has a radius three times that of circle N, calculate the radius of circle Q: .  
Step 4: Use the standard form equation for a circle to find circle Q’s equation with center (-9, -4) and radius 12: .

3.  Circle A has a radius of 3x and circle B has a radius of 63x. The area of circle B is how many times the area of circle A?

## Answer

441

## Solution

Concept Check : The intent of this question is to assess the student’s understanding of the formula for the area of a circle and their ability to apply it to find the ratio of the areas of two circles based on their radii. The student is expected to know the formula for the area of a circle, which is A = πr², and how to compute ratios.  
  
Solution Strategy : To solve this problem, the student should first calculate the area of both circles using the given radii. This involves substituting the values of the radii (3x for circle A and 63x for circle B) into the area formula. After finding the areas, the next step is to set up a ratio of the area of circle B to the area of circle A. This will involve dividing the area of circle B by the area of circle A.  
  
Quick Wins : Remember that when calculating the area of a circle, the radius must be squared. Be careful with your calculations to avoid errors in squaring the radius. Once you have both areas, simplify the ratio as much as possible. It can also be helpful to keep track of the π terms, as they will cancel out when forming the ratio.  
  
Mistake Alert : A common mistake is to forget to square the radius when calculating the area. Additionally, ensure that you correctly set up the ratio and simplify it properly. Double-check to make sure you are dividing the correct quantities (area B over area A) and that you handle the variables correctly.  
  
SAT Know-How : This problem falls under the category of Geometry and Trigonometry, specifically focusing on the area of circles and the concept of ratios. It assesses the student’s ability to apply formulas and manipulate algebraic expressions. Mastering this type of problem requires a solid understanding of geometric concepts and the ability to perform algebraic operations accurately, which is an essential skill for success on the SAT.  
  
Step 1: Find the area of circle A using the formula: .  
.  
Step 2: Find the area of circle B using the same formula.  
.  
Step 3: Find the ratio of the area of circle B to circle A.  
.  
Step 4: Cancel out common factors (π and ) in the ratio.  
.  
Step 5: Simplify the ratio:  
.  
Thus, the area of circle B is 441 times the area of circle A.

4.  In triangle XYZ, the measure of angle X is 53°, the measure of angle Y is 90°, and the measure of angle Z is °. What is the value of m?

A.   60

B.   70

C.   74

D.   76

## Answer

C

## Solution

Concept Check : The intent of the question is to assess the student’s understanding of the properties of triangles, specifically that the sum of the interior angles in a triangle is always 180 degrees. The student is expected to know how to set up an equation based on this property to solve for the unknown angle, which is expressed in terms of m.  
  
Solution Strategy : To approach this problem, the student should first recall that the sum of the angles in any triangle is 180 degrees. Given the measures of angles X and Y, the student should set up an equation that incorporates the measure of angle Z, which is expressed as . The equation can be formed as: 53° + 90° + ° = 180°. From here, the student can simplify the equation to isolate m and solve for its value.  
  
Quick Wins : A good tip is to always remember the triangle angle sum property (the sum of angles in a triangle is 180°). When you have an angle expressed as a fraction or variable, substitute it into the equation carefully. It may also help to rewrite the equation step by step to avoid confusion. Keep track of your calculations and double-check your work to ensure accuracy.  
  
Mistake Alert : Students should be cautious not to miscalculate the sum of the known angles. Adding angles incorrectly can lead to an incorrect equation. Also, when solving for m, make sure to clearly isolate the variable and not confuse it with the angles themselves. Be careful with units and ensure all angles are in degrees.  
  
SAT Know-How : This problem falls under the category of Geometry and Trigonometry, specifically focusing on congruence, similarity, and angle relationships in triangles. It is designed to assess a student’s understanding of triangle properties and their ability to set up and solve equations based on given information. Mastering these skills is crucial for success in SAT mathematics.  
  
Step 1: Recall the triangle angle sum property: ∠X + ∠Y + ∠Z = 180°.  
Step 2: Substitute the known values into the equation: 53° + 90° + ° = 180°.  
Step 3: Simplify the equation: 143° + ° = 180°.  
Step 4: Isolate ° by subtracting 143° from both sides: ° = 180° - 143°.  
Step 5: Calculate: ° = 37°.  
Step 6: Solve for m by multiplying both sides by 2: m° = 37° .  
Step 7: Calculate: m = 74.

5.  Triangles PQR and STU are similar. Each side length of triangle PQR is 7 times the corresponding side length of triangle STU. The area of triangle PQR is 3479 square centimeters. What is the area, in square centimeters, of triangle STU?

A.   50

B.   71

C.   90

D.   100

## Answer

B

## Solution

Concept Check : The question tests the student’s understanding of the properties of similar triangles, particularly how the ratio of side lengths relates to the ratio of areas. Students should know that if two triangles are similar, the ratio of their areas is equal to the square of the ratio of their corresponding side lengths.  
  
Solution Strategy : To solve the problem, the student should first recognize that if triangle PQR is 7 times larger in side lengths than triangle STU, the ratio of the side lengths is 7:1. Consequently, the ratio of the areas will be the square of the side length ratio, which is or 49:1. Then, the student will need to set up an equation to find the area of triangle STU by dividing the area of triangle PQR by 49.  
  
Quick Wins : Remember the key property that the ratio of the areas of similar triangles is the square of the ratio of their corresponding side lengths. When calculating the area of triangle STU, ensure you perform the division carefully, and it might help to write down the relationship between the areas clearly before calculating.  
  
Mistake Alert : Be cautious with squaring the ratio of the side lengths; it’s easy to mistakenly use the side length ratio directly instead of squaring it. Also, double-check calculations when dividing the area of triangle PQR by 49 to ensure that no arithmetic errors occur.  
  
SAT Know-How : This problem falls under geometry, specifically focusing on the concepts of similarity and area relationships in triangles. It assesses the student’s ability to apply knowledge of ratios and areas, which is an essential skill in geometry. Mastering these concepts will not only help in SAT problem-solving but also in understanding broader mathematical principles.  
  
1. Determine the linear scale factor between the triangles: The scale factor k between corresponding side lengths is 7.  
2. Understand how area scales with the linear dimensions: Since the triangles are similar, the ratio of their areas is the square of the scale factor.  
3. Apply the area scaling formula: divided by .  
4. Plug in the values: .  
5. Calculate : .  
6. Divide 3479 by 49 to find the area of triangle STU.  
7. Perform the division: .  
8. Conclude that the area of triangle STU is 71 square centimeters.

6.  Triangles GHI and JKL are congruent, where G corresponds to J, H corresponds to K, and both angles H and K are right angles. The measure of angle G is 48°. What is the measure, in degrees, of angle L?

A.   Angle L = 42°

B.   Angle L = 48°

C.   Angle L = 60°

D.   Angle L = 90°

## Answer

A

## Solution

Concept Check : The intent of the question is to assess the student’s understanding of congruent triangles and angle relationships. The student is expected to know that corresponding angles in congruent triangles are equal and that the sum of angles in any triangle is 180 degrees.  
  
Solution Strategy : To solve this problem, the student should first recognize that since triangles GHI and JKL are congruent, the corresponding angles are equal. Given that angle H and angle K are right angles, both measure 90°. The student should then use the fact that the sum of the angles in a triangle is 180° to find the measure of angle L by calculating the remaining angle in triangle JKL after accounting for angles J and K.  
  
Quick Wins : Remember that in congruent triangles, corresponding angles are equal. Make sure to recall that the sum of the angles in any triangle is always 180°. This can help you find missing angles when you already know some of the angles. Also, clearly label which angles correspond to which triangles to avoid confusion.  
  
Mistake Alert : Be careful not to confuse the angles when identifying which angles correspond to one another. It’s easy to mistakenly assign the wrong angle values, especially if you’re writing them down. Additionally, remember that a right angle measures 90°; make sure to apply this correctly when calculating the remaining angles.  
  
SAT Know-How : This problem falls under the category of geometry, specifically focusing on congruence and angle relationships in triangles. It assesses the student’s ability to apply properties of congruent triangles and use the sum of angles in a triangle to find missing angle measures. Mastering these concepts is crucial for solving similar SAT problems effectively and efficiently.  
  
Triangular Angle Sum Property: The sum of angles in a triangle is 180°.  
Given that triangles GHI and JKL are congruent, and H corresponds to K being right angles, both measure 90°.  
Since angle G corresponds to angle J, both are 48°.  
Apply the triangle angle sum property to triangle JKL: J + K + L = 180°.  
Substitute known values: 48° + 90° + L = 180°.  
Simplify: L = 180° - 48° - 90°.  
Calculate: L = 42°.  
Thus, the measure of angle L is 42°.

7.  What is the equation of the circle in the xy-plane that has a center at (8, -2) and a radius of 8?

A.

B.

C.

D.

## Answer

A

## Solution

Concept Check : The intent of the question is to assess the student’s understanding of the standard form of the equation of a circle. The student is expected to know how to apply the formula for the equation of a circle given its center and radius, which is (x - h)² + (y - k)² = r², where (h, k) is the center and r is the radius.  
  
Solution Strategy : To solve the problem, the student should start by identifying the center coordinates (h, k) from the given point (8, -2) and recognize that the radius r is given as 8. Then, they should substitute these values into the standard form of the circle’s equation and simplify it to present the final equation.  
  
Quick Wins : Remember the standard form of a circle’s equation: (x - h)² + (y - k)² = r². Identify the center (h, k) and radius r clearly before substituting them into the formula. It might help to write the formula down first and then fill in the values step by step to avoid confusion.  
  
Mistake Alert : Be careful when substituting the center values; the signs are important (use a minus sign for h and k in the formula). Also, ensure that you square the radius correctly, as missing this step could lead to an incorrect equation.  
  
SAT Know-How : This problem falls under the category of Geometry and Trigonometry, specifically focusing on circle equations. It assesses the student’s ability to apply the formula for a circle based on its center and radius. Mastering this type of problem requires familiarity with the geometric concepts involved and careful attention to detail when substituting values into the equation.  
  
1. Recall the standard form of a circle’s equation: (x-h)² + (y-k)² = r².  
2. Identify the values of h and k from the center (8, -2), giving h = 8 and k = -2.  
3. Substitute h = 8, k = -2, and r = 8 into the standard equation.  
4. The equation becomes (x-8)² + (y+2)² = 8².  
5. Calculate 8² = 64.  
6. Therefore, the equation of the circle is (x-8)² + (y+2)² = 64.

8.  A hemisphere is half of a sphere. If a hemisphere has a radius of 21 inches, which of the following is closest to the volume, in cubic inches, of this hemisphere?

A.   4,800

B.   8,400

C.   15,600

D.   19,385

## Answer

D

## Solution

Concept Check : The question tests the student’s understanding of the formula for the volume of a sphere and how to apply it to find the volume of a hemisphere. Students are expected to know that the volume of a sphere is found using the formula , and since a hemisphere is half of a sphere, they will need to divide the result by 2.  
  
Solution Strategy : To approach this problem, students should first recall the formula for the volume of a sphere. Then, they should substitute the given radius (21 inches) into the formula to calculate the volume of the full sphere. After finding that volume, they will divide the result by 2 to find the volume of the hemisphere. It is important to ensure that they are using the correct value of π, which is often approximated as 3.14 or , depending on the level of precision required by the problem.  
  
Quick Wins : A useful tip is to remember that the volume of a hemisphere is half the volume of a sphere, so you can quickly find the volume of the sphere and then divide by 2. Also, keep in mind that when working with the radius, you will be cubing it, which can sometimes lead to larger numbers, so be careful with your calculations. Using a calculator can help avoid arithmetic mistakes, especially with the π value.  
  
Mistake Alert : Students should be careful not to confuse the formulas for different shapes. Ensure that you correctly apply the formula for the volume of a sphere and remember to divide by 2 for the hemisphere. Also, double-check the calculations when cubing the radius, as this is a common area for errors. Lastly, be mindful of rounding; if the problem asks for the closest estimate, ensure your final answer is rounded appropriately.  
  
SAT Know-How : This problem is a geometry and trigonometry question focused on finding the volume of a hemisphere. It assesses the student’s ability to apply formulas for volume accurately and understand the relationship between a sphere and a hemisphere. Mastering these concepts is crucial for solving similar SAT problems efficiently and accurately.  
  
Step 1: Calculate the volume of the sphere using .  
With , we have .  
Step 2: Calculate .  
  
  
Step 3: Substitute into the volume formula.  
  
Simplify:   
Step 4: Calculate the volume of the hemisphere.  
Hemisphere volume =   
Simplify:   
Step 5: Approximate to 3.14.  
Hemisphere volume ≈   
Calculate:   
  
Step 6: Round 19385.16 to the nearest integer.  
.

9.  For two angles, ∠A and ∠B, it is given that cos(A) = sin(B). The measures, in degrees, of ∠A and ∠B are 4x + 6 and 2x + 30, respectively. What is the value of x?

A.   5

B.   7

C.   9

D.   11

## Answer

C

## Solution

Concept Check : The question is designed to assess the student’s understanding of the relationship between cosine and sine functions, particularly how they relate angles. Students should know that cos(A) = sin(B) implies that A and B are complementary angles, meaning A + B = 90 degrees. Additionally, students should be able to set up and solve an equation based on the expressions given for angles A and B.  
  
Solution Strategy : To approach this problem, students should first recognize that if cos(A) = sin(B), then the angles A and B are complementary. This means that the sum of the measures of angles A and B should equal 90 degrees. Therefore, the student should set up the equation: (4x + 6) + (2x + 30) = 90. Next, the student would combine like terms and solve for x.  
  
Quick Wins : When solving problems involving angle relationships, always remember to look for complementary or supplementary angles. In this case, knowing that cos(A) = sin(B) leads to the conclusion about the angles summing to 90 degrees is crucial. Additionally, when setting up the equation, make sure to combine like terms carefully and double-check your arithmetic before solving for x.  
  
Mistake Alert : Be cautious when combining like terms and ensure that the equation is set up correctly. It’s easy to make a mistake in adding the constants or coefficients, which can lead to an incorrect solution. Also, ensure that the final value for x is reasonable within the context of angle measures, as angles must fall within the range of 0 to 180 degrees.  
  
SAT Know-How : This problem falls under the category of Geometry and Trigonometry, specifically focusing on the concepts of angle relationships and trigonometric functions. It assesses the student’s ability to apply knowledge of complementary angles and how to translate that into algebraic equations. Mastery of these skills is essential for SAT problem-solving, as it demonstrates the ability to connect geometric principles with algebraic manipulation.  
  
Start with the identity cos(A) = sin(B), which implies A = 90° - B.  
Substitute the given expressions: 4x + 6 = 90 - (2x + 30)  
Simplify the equation: 4x + 6 = 90 - 2x - 30  
Combine like terms: 4x + 6 = 60 - 2x  
Add 2x to both sides: 4x + 2x + 6 = 60  
Simplify: 6x + 6 = 60  
Subtract 6 from both sides: 6x = 54  
Divide by 6: x = 9  
Thus, the value of x is 9.

10.  Circle A has a radius of 96 millimeters (mm). Circle B has an area of . What is the total area, in , of circles A and B?

A.   9400π

B.   9800π

C.   10000π

D.   10200π

## Answer

C

## Solution

Concept Check : The intent of this question is to assess the student’s understanding of the concept of area, specifically the area of circles. Students are expected to know the formula for the area of a circle, which is A = πr², and how to apply it to find the area of Circle A using its radius and Circle B using its given area.  
  
Solution Strategy : To solve this problem, the student should first calculate the area of Circle A using its radius of 96 mm. They will apply the formula A = πr², substituting r with 96 mm. Next, they will consider Circle B, which already provides its area as 784π mm². The final step will involve adding the areas of Circle A and Circle B together to find the total area.  
  
Quick Wins : Remember the formula for the area of a circle. When calculating the area of Circle A, carefully square the radius before multiplying by π. For Circle B, since the area is already given in terms of π, you can directly add it to the area of Circle A once you compute it. It might be helpful to express both areas in a similar format before adding them.  
  
Mistake Alert : Be cautious while squaring the radius for Circle A; it’s easy to make a mistake in calculations. Double-check your multiplication and ensure that you are adding like terms (both areas should be in mm²). Also, be careful not to confuse the radius with the area when dealing with Circle B.  
  
SAT Know-How : This problem falls under the category of Geometry and Trigonometry, specifically focusing on the area of circles. It assesses the student’s ability to apply the area formula effectively and perform basic arithmetic operations. Mastery of these skills is crucial for solving SAT problems efficiently, as they often require quick calculations and a clear understanding of geometric concepts.  
  
1. Calculate the area of Circle A using the formula for the area of a circle, which is A = πr².  
2. Given that the radius of Circle A is 96 mm, substitute it into the formula: A = π(96)².  
3. Calculate 96 squared: .  
4. Thus, the area of Circle A is 9216π square millimeters.  
5. Add the area of Circle A and Circle B: 9216π + 784π.  
6. Perform the addition: 9216π + 784π = 10000π.  
7. Therefore, the total area of circles A and B is 10000π square millimeters.