

数值分析A课程第七次作业参考答案

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$$T_1$$

$p_0(x) = 1$, 由G-S正交化即得:

$$p_1(x) = x - \frac{(x,1)}{(1,1)} = x - 0 = x$$

$$p_2(x) = x^2 - \frac{(x^2,1)}{(1,1)} - \frac{(x^2,x)}{(x,x)}x = x^2 - \frac{1}{3}$$

$$\text{同理}, p_3(x) = x^3 - \frac{3}{5}x$$

$$T_2$$

由G-S正交, 有: $L_1(x) = x - \frac{(x,1)}{(1,1)} = x - \frac{\int_0^\infty e^{-x} x dx}{\int_0^\infty e^{-x} dx} = x - 1$

由递推定理, 此时 $\alpha_n = 1, \beta_n = -1 - 2n, \gamma_n = -n^2$

$$\therefore L_{n+1}(x) = (x - 1 - 2n)L_n(x) - n^2 L_{n-1}(x)$$

$$\text{于是立得 } L_2(x) = x^2 - 4x + 2, L_3(x) = x^3 - 9x^2 + 18x - 6$$

(注: 注意到 L_1 不是 $1 - x$, 所以不能直接用书本p237的递推公式, 这里很多同学都错了。不过有些同学计算出 L_1 后改令为 $1 - x$, 那就可以使用)

$$T_6$$

换元, 令 $y = 2x - 1$, 则 $dx = \frac{1}{2}dy$

$$(T_n^*(x), T_m^*(x)) = \int_{-1}^1 T_n(y) T_m(y) \frac{1}{\sqrt{1-y^2}} dy$$

由Chebyshev多项式正交性, 立得结论。

$$T_7$$

直接写出法方程, 可解得:

$$(1) p_1^*(x) = 1.175 + 1.104x$$

$$(2) p_2^*(x) = 0.996 + 1.104x + 0.537x^2$$

(注: 结果差的差不多就算正确)

$$T_{10}$$

$$p_0(x) = \frac{\sqrt{2}}{2}, p_1(x) = \sqrt{\frac{3}{2}}x$$

$$p_2(x) = \frac{1}{2}\sqrt{\frac{5}{2}}(3x^2 - 1), p_3(x) = \frac{1}{2}\sqrt{\frac{7}{2}}(5x^3 - 3x)$$

设 $S_n^*(x) = \sum a_j^* p_j(x)$ 为其最佳平方逼近

则 $a_j^* = \int_{-1}^1 f(x) p_j(x) dx$, 计算得:

$$a_0^* = 0, a_1^* = 0.9927408, a_2^* = 0, a_3^* = -0.1202095$$

$$\text{故代入计算可得 } S_n^*(x) = -0.5622285x^3 + 1.5531912x$$

(注: 结果差的差不多就算正确)

$$T_{13}$$

直接写出法方程, 可解得:

$$(1) n = 1, m = 5, p_1^*(x) = 1.2196x + 0.6209$$

$$(2) n = 2, m = 5, p_2^*(x) = -0.0109x^2 + 1.2533x + 0.5966$$

(注: 结果差的差不多就算正确)