ECE 51018: Project 3
Nazmuzzaman Khan

Objective:

The objective of the third project is to develop a Simulink-based semi-detailed simulation of a permanent-magnet ac motor drive that utilizes a sine-triangle modulator with third-harmonic injection. The equivalent circuit/block diagram is depicted in Fig. 1, and Fig. 2 shows the corresponding Simulink model. We need to observe how the model response to develop 100 Nm of torque (Te_star) at 3000 rpm (mechanical). Then repeat the process for -100 Nm torque development at 3000 rpm.

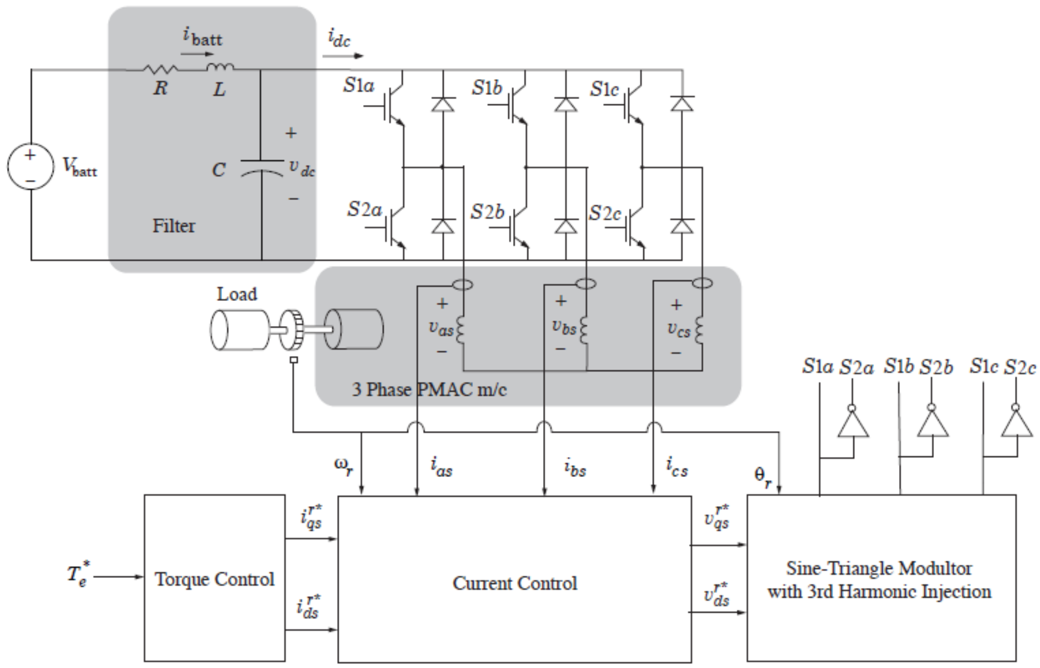

Fig 1: Block diagram of PMAC motor drive.

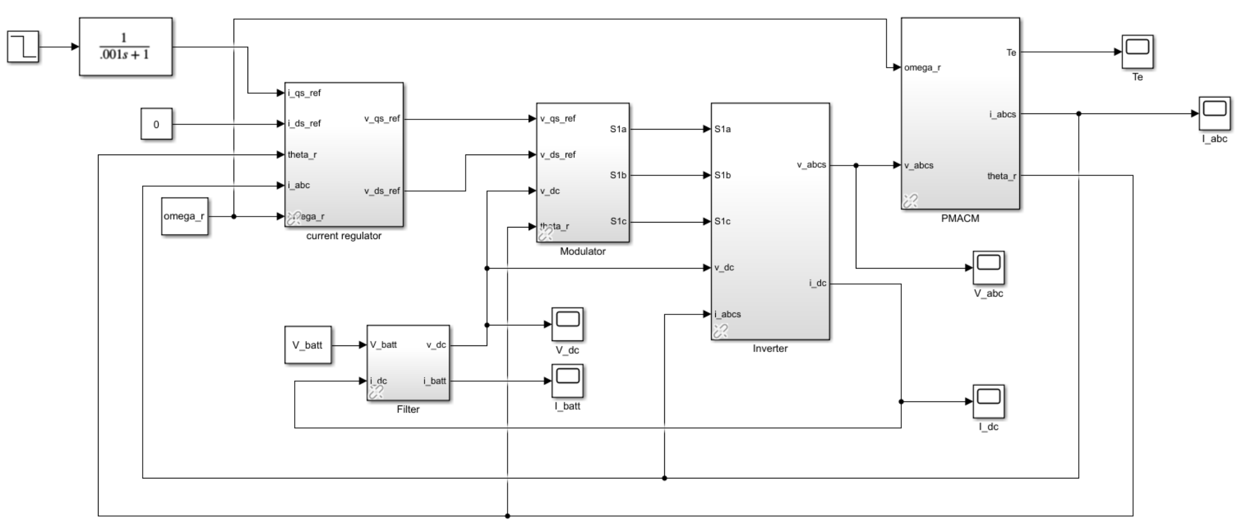

Fig 2: Simulink model of PMAC motor drive.

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Project Description:

Motor and Filter parameters are given as follows:

```
% motor parameters
P = 6; % number of poles
lambda_m = 0.1062;  %flux constant V-s/rad
rs = 0.01;  % stator resistance in ohms
Ld = 0.3e-3; %d-axis inductance in H
Lq = 0.3e-3; %q-axis inductance in H
V_batt = 350; %V

%Filter parameters
L = 5e-6; %inductance in H
R = 0.01; %resistance in ohms
C = 1e-3; %capacitance in F

%Current Regulator Parameters
Kq = 0.5;
Kd = 0.5;
```

Given response parameters:

Te_star = 100 Nm

ω_{rm} = 3000 rpm

From them electrical rotor speed is calculated:

```
% define electrical rotor speed
omega_r = 3000*2*pi/60 * P/2 = 942.47; %rad/s
```

Following SS equations are used to calculate motor voltage and current:

$$\left.\begin{aligned} V_{qs}^r &= r_s I_{qs}^r + \omega_r L_d I_{ds}^r + \omega_r \lambda_m' \qquad (SS-1) \\ V_{ds}^r &= r_s I_{ds}^r - \omega_r L_q I_{qs}^r \qquad (SS-2) \\ T_e &= \frac{3}{2}\frac{P}{2}\left[\lambda_m' I_{qs}^r + (L_d - L_q) I_{qs}^r I_{ds}^r\right] \qquad (SS-3) \end{aligned}\right\} \text{All variables constant in steady state}$$

```
%determine q-axis current from Te equation
I_qs_star = T_e_star * 2/P * 2/3 /lambda_m; = 209.24 A

%set d-axis current to zero for maximum efficiency
I_ds_star = 0;
%calculate q- and d-axis components of stator voltages
```

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```
V_qs = rs*I_qs_star + omega_r*Ld*I_ds_star + omega_r*lambda_m; =
102.18 V
V_ds = rs*I_ds_star - omega_r*Lq*I_qs_star ; = -59.16 V

%calculate peak ac voltage
V_p = sqrt(V_qs^2+V_ds^2);= 118.07 V
```

From the above calculations it is clear that, to get 100 Nm of torque at 3000 rpm 209.24 A of I_qs_star current is needed. As a result step input is set between +I_qs_star (100 Nm torque) to -I_qs_star (-100 Nm torque).

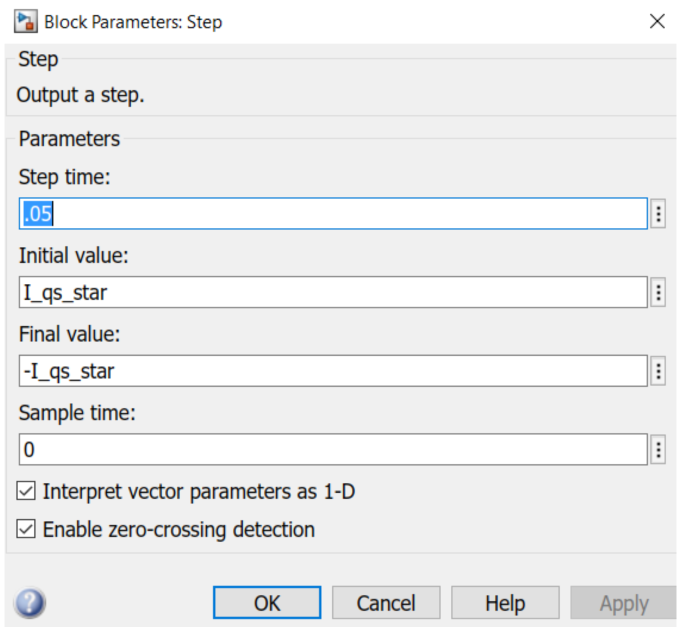

Fig 3: Step input parameters.

The step input goes through a low-pass filter to control disturbances in V_dc output. This low-pass filter is arbitrarily chosen. Step input goes through “current regulator” block (Fig. 4). This is a voltage based current modulator.

$$V_{qs}^{r*} = \omega_r L_d i_{ds}^r + \omega_r \lambda_m + K_q (i_{qs}^{r*} - i_{qs}^r) \cong V_{qs}^r \qquad (1)$$

$(\omega_r L_d i_{ds}^r + \omega_r \lambda_m)$ is the feed forward term which comes from Park’s transformation. $(K_q (i_{qs}^{r*} - i_{qs}^r))$ is the feedback term. We can achieve this if V_{qs}^{r*} doesn’t change much between switching (low modulation voltage). From eqn. (1) we get:

$$i_{qs}^r = K_q - i_{qs}^{r*}/(r_s+L_q)+pL_q \qquad (2)$$

if $K_q >> r_s$, then $i_{qs}^r \cong - i_{qs}^{r*}$ (current is controlled using voltage equations).
The same logic applies to i_{ds}^r.

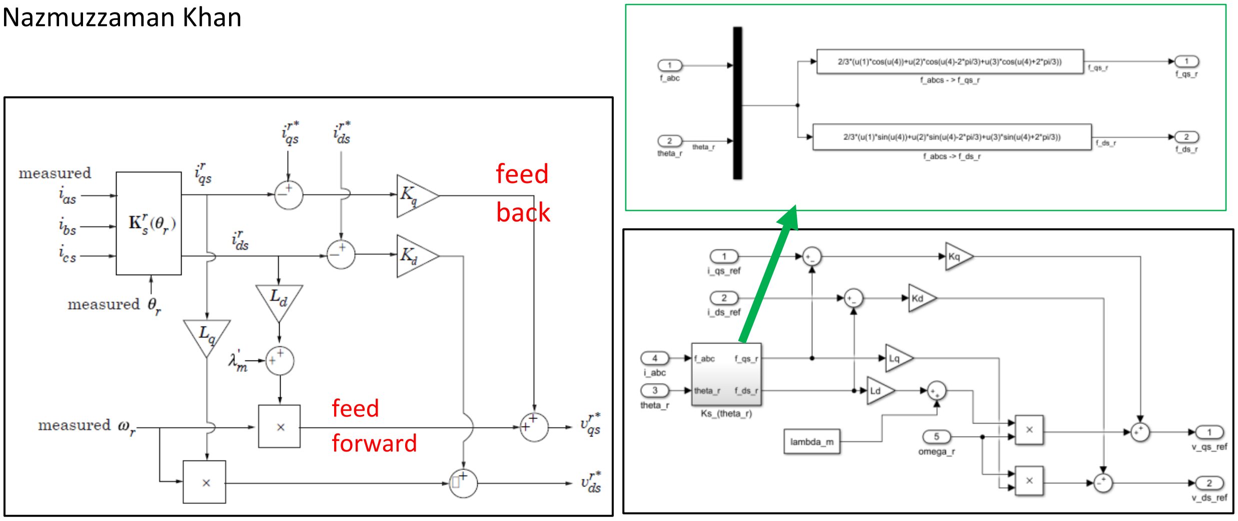

Fig 4: Current regulator block diagram (left) and Simulink block (right). Park's transformation block shown at top-right.

The "modulator" block (Sine triangle PWM with 3rd harmonic injection) showed in Fig 6. The input of the modulator block is the desired stator voltages (v_qs_ref and v_ds_ref), rotor position and DC voltage. Outputs are switching signals for transistor. 3rd harmonic is needed so that ea stays within [+1 to -1]. If ea goes beyond 1, then our assumption for PWM becomes void. Fig 5 shows the generated switching signal. Signal values are within (+-) 1.5 and they are 120 degree apart.

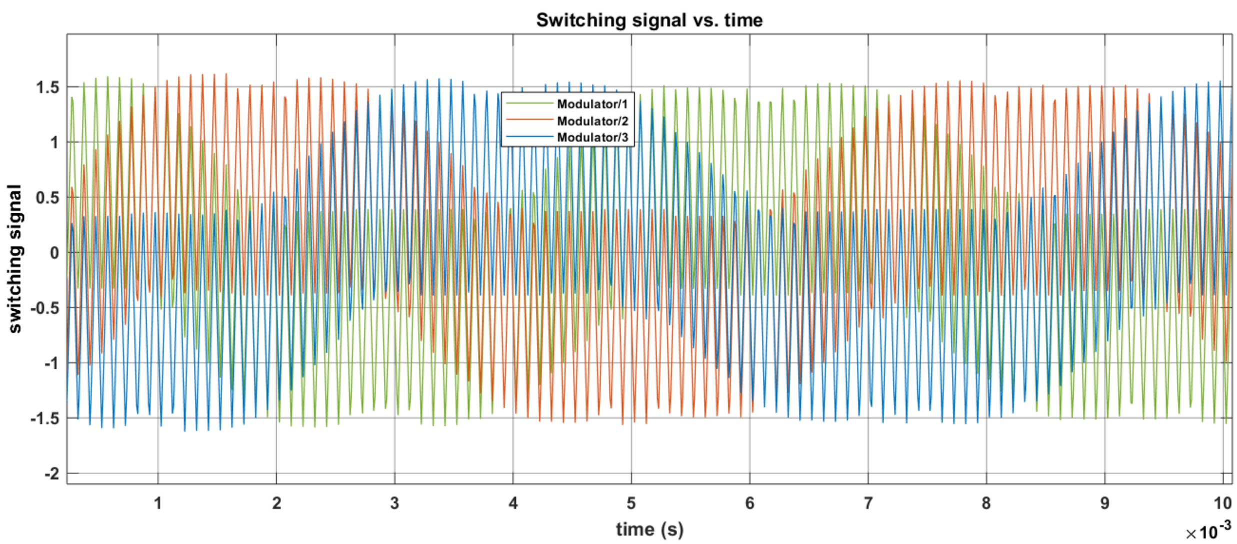

Fig 5: modulator block output (s1a, s1b, s1c).

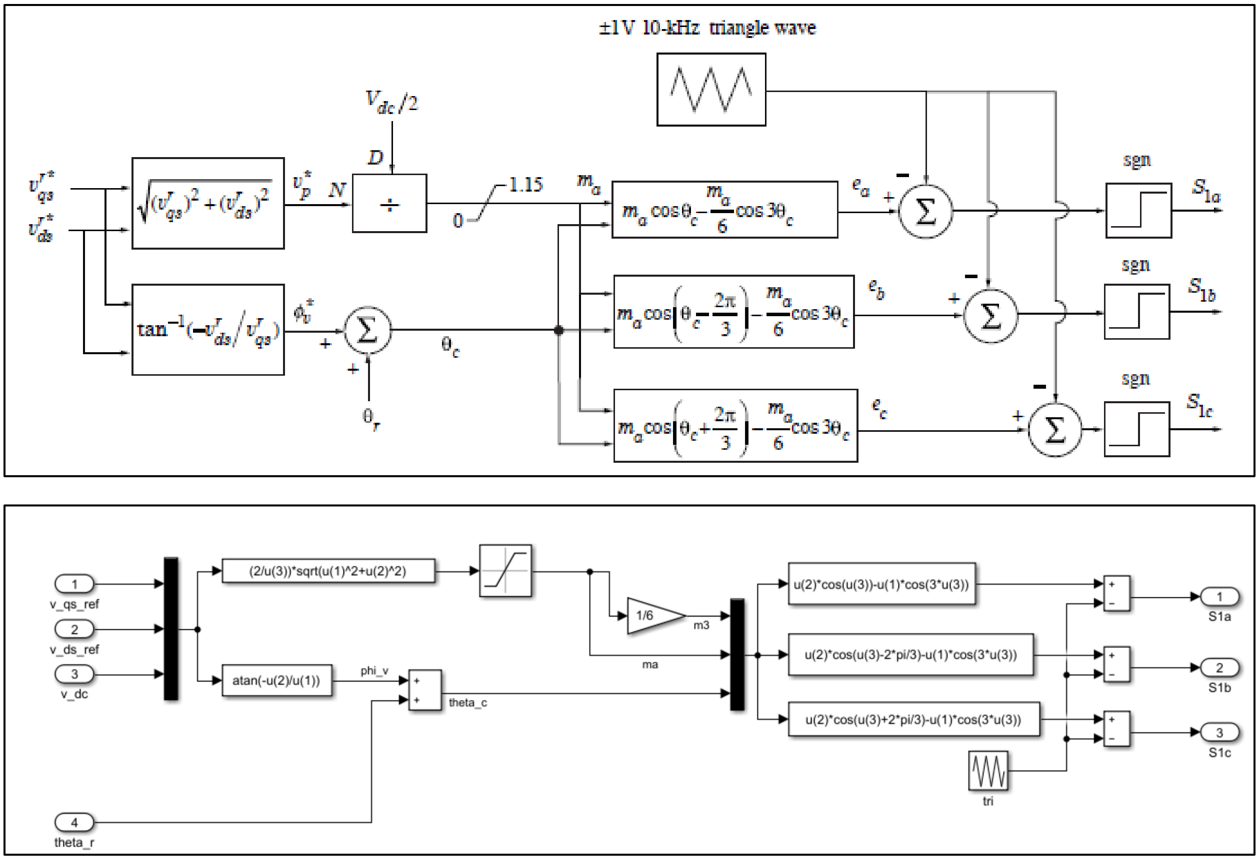

Fig 6: "modulator" block. Sine triangle PWM with 3rd harmonic injection. Block diagram (top) and Simulink model (bottom).

Switching signals are input to "inverter" block which is shown in Fig 8. This block generates the sinusoidal stator voltage v_{abcs}, which is the input of PMAC. For v_{as}, if the switching magnitude (s1a) is greater than 0, then upper switch closes in switch Va. The inverter block only cares about the sign of S1a, S1b, and S1c. This results in v_ag = V_dc. For symmetric PMAC v_ng = (v_ag+v_bg+v_cg)/3. This results in v_as = v_ag – v_ng. Similar logic applies to v_bs and v_cs. For no modulation, switching should create the following stator voltage:

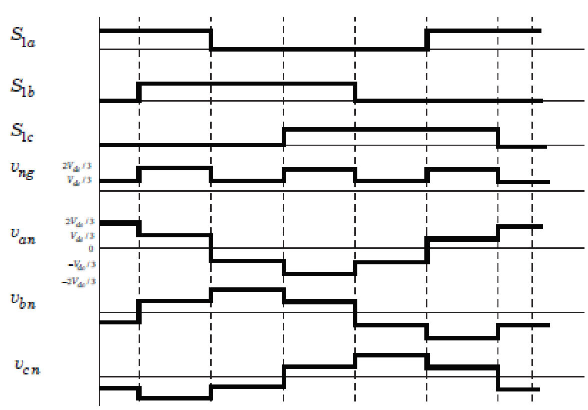

Fig 7: Basic Six-Step Waveform (No Modulation)

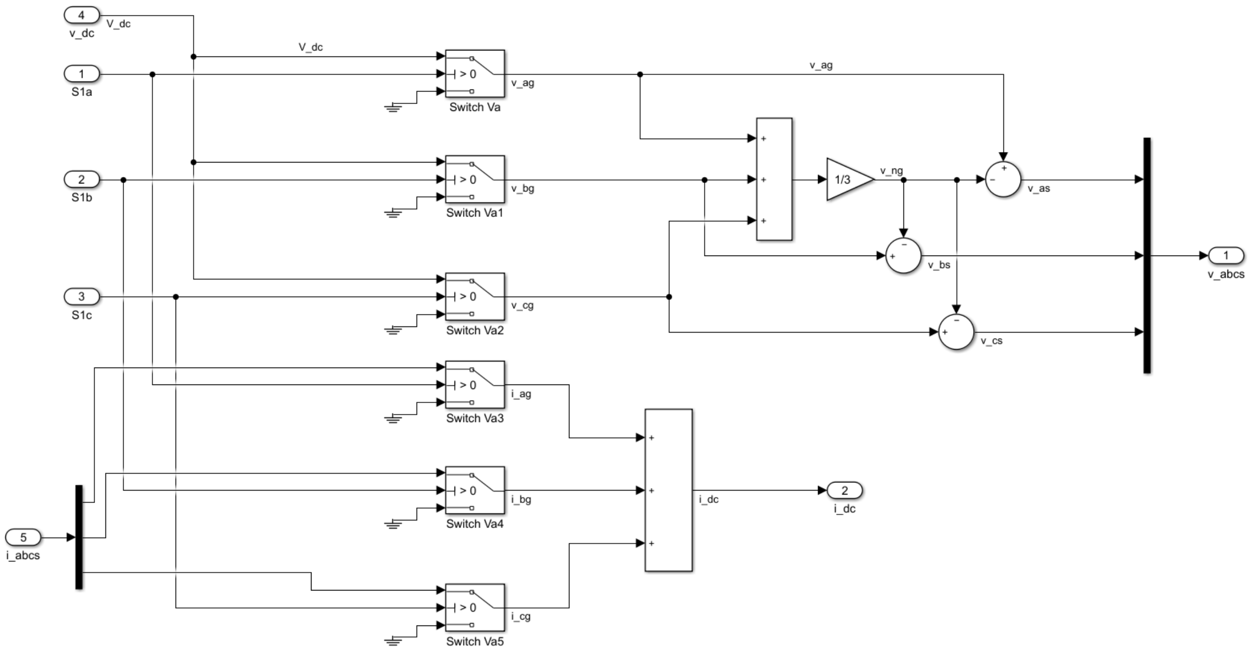

Fig 8: Simulink model of “inverter” block.

V_abcs is used as input in PMACM block. Using the equations in Fig 9, Iqs_r and Ids_r is calculated, which are used to calculate the Torque.

$$p\lambda_{qs}^{r} = v_{qs}^{r} - r_s i_{qs}^{r} - \omega_r \lambda_{ds}^{r} \qquad \lambda_{qs}^{r} = L_q i_{qs}^{r} \qquad T_e = \frac{3}{2}\frac{P}{2}\left[\lambda_m' i_{qs}^{r} + (L_d - L_q) i_{qs}^{r} i_{ds}^{r}\right]$$

$$p\lambda_{ds}^{r} = v_{ds}^{r} - r_s i_{ds}^{r} + \omega_r \lambda_{qs}^{r} \qquad \lambda_{ds}^{r} = L_d i_{ds}^{r} + \lambda_m'$$

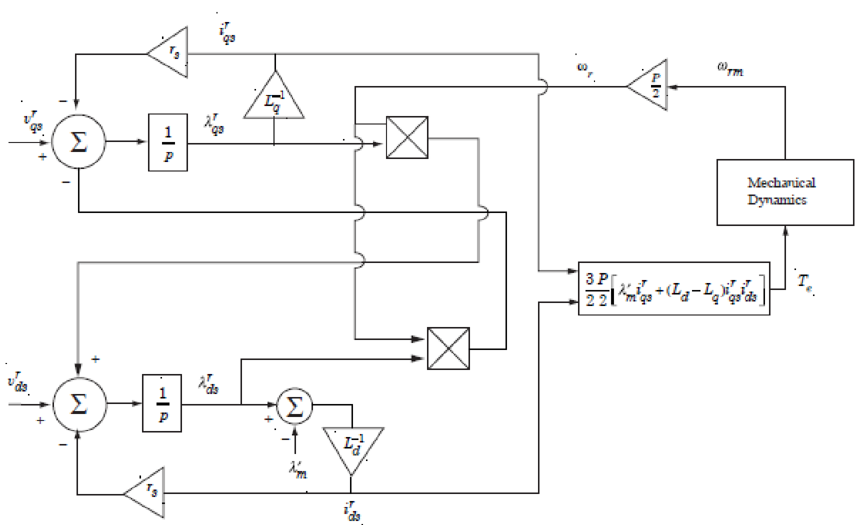

Fig 9: Block diagram of PMAC machine.

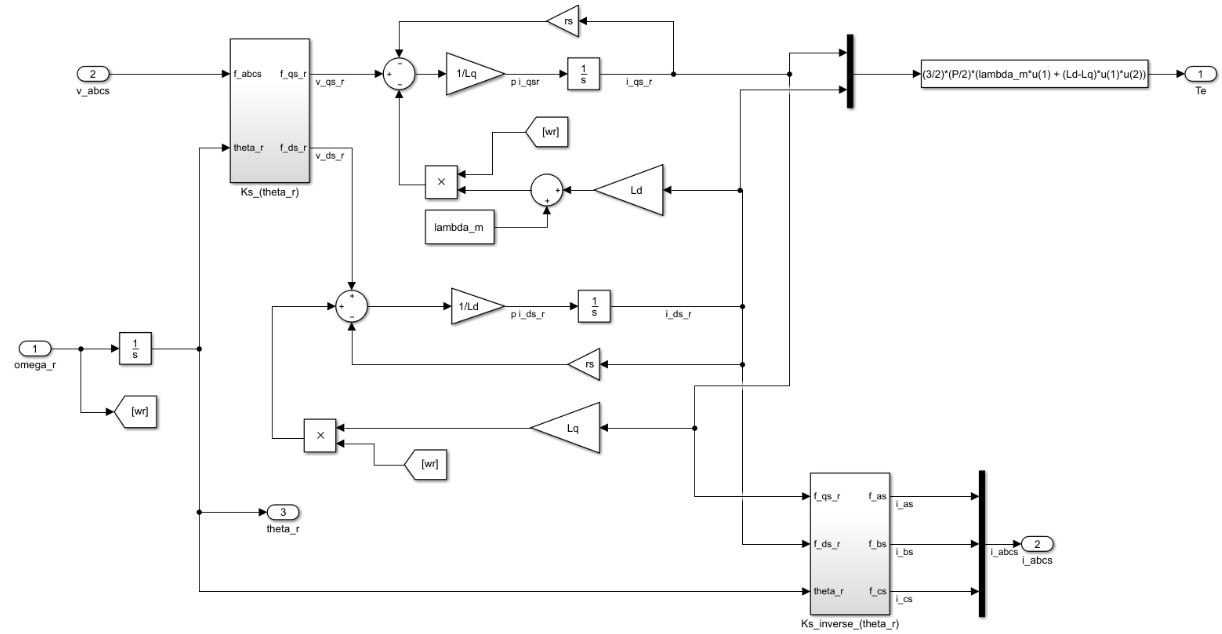

Fig 10: Simulink diagram of PMAC machine.

The filter block (Fig 11) sits between inverter and battery. I_dc (output of filter) changes rapidly. The cables associated with I_dc can work as antenna. The capacitance, inductance or resistance of cable is modeled as filter.

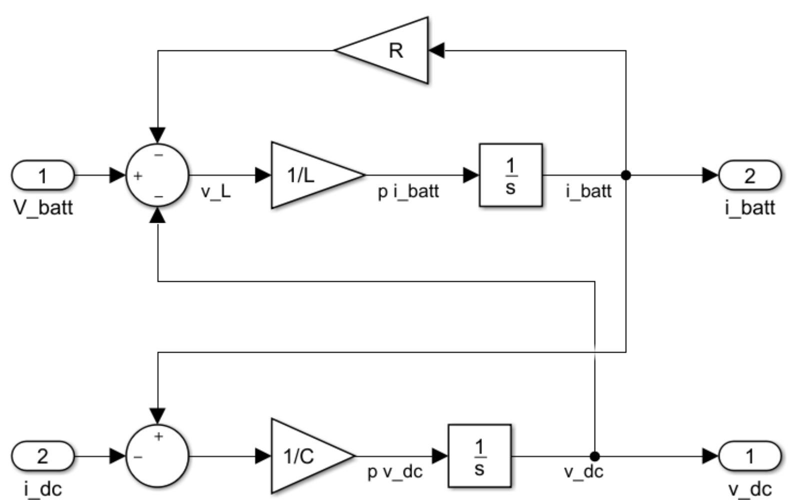

Fig 11: Simulink diagram for Filter block.

This concludes our description of the simulink model and blocks.

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Results:

Problem 1

Write a Matlab script that calculates and plots the first-quadrant maximum torque-versus-speed envelope of the given drive system. Consider the following code snippet. Plot maximum torque (in N-m) versus mechanical speed (in rad/s) and maximum mechanical power (in kW) versus mechanical speed (in rad/s).

Matlab code:

```
P = 6; % number of poles
lambda = 0.1062;  %flux constant V-s/rad
rs = 0.01;  % stator resistance in ohms
Ld = 0.3e-3; %d-axis inductance in H
Lq = 0.3e-3; %q-axis inductance in H
V_batt = 350;

%Filter parameters
L = 5e-6; %inductance in H
R = 0.01; %resistance in ohms
C = 1e-3; %capacitance in F

N_w = 10000;
N_i = 10000;
I_max = 210; %I_qs_star in amp
V_max = V_batt/sqrt(3);
%I_max = V_max/R;


w_r = linspace ( 0 , 5000 , N_w); % in radians per second
i_ds = - linspace ( 0 , I_max , N_i ); % in A


for i =1:N_w
    Te_max = 0 ;
    %perform one dimensional search for optimal ids , iqs
    for j = 1 : N_i
        i_qs = sqrt(I_max^2 - i_ds(j)^2) ;
        v_qs = rs * i_qs + w_r(i) * ( Ld * i_ds(j) + lambda ) ;
        v_ds = rs * i_qs - w_r(i) * Lq * i_ds(j);
        v_p = sqrt ( v_qs^2 + v_ds^2 ) ;
        if ( v_p < V_max) % viable but not necessarily optimal
solution
            T = 1.5 * (P/2) * lambda * i_qs ;
            if (T > Te_max )
                Te_max = T;
            end
        end
    end
    T_e (i) = Te_max ;
```

```
    w_rm (i) = 2*w_r(i)/P;
    P_mech (i) = Te_max * w_rm(i);

end

figure
plot (w_rm, T_e)
title('Maximum torque vs mechanical speed')
xlabel('w_{rm} (rad/s)')
ylabel('Te_{max} (Nm)')

figure
plot (w_rm, P_mech/1000)
title('Maximum P_{mechanical} vs mechanical speed')
xlabel('w_{rm} (rad/s)')
ylabel('Max P_{mech} (KW)')
```

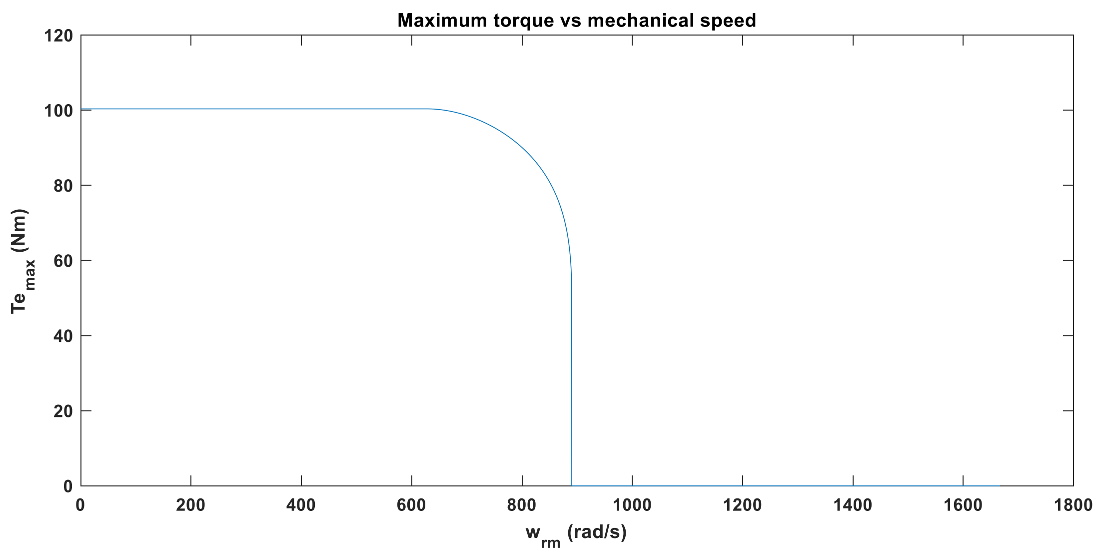

Fig 12: Maximum torque vs mechanical speed for the given system. Upto around 700 rad/s, Te is current limites. After that -i_ds_r is injected and Te is curent-voltage limited.

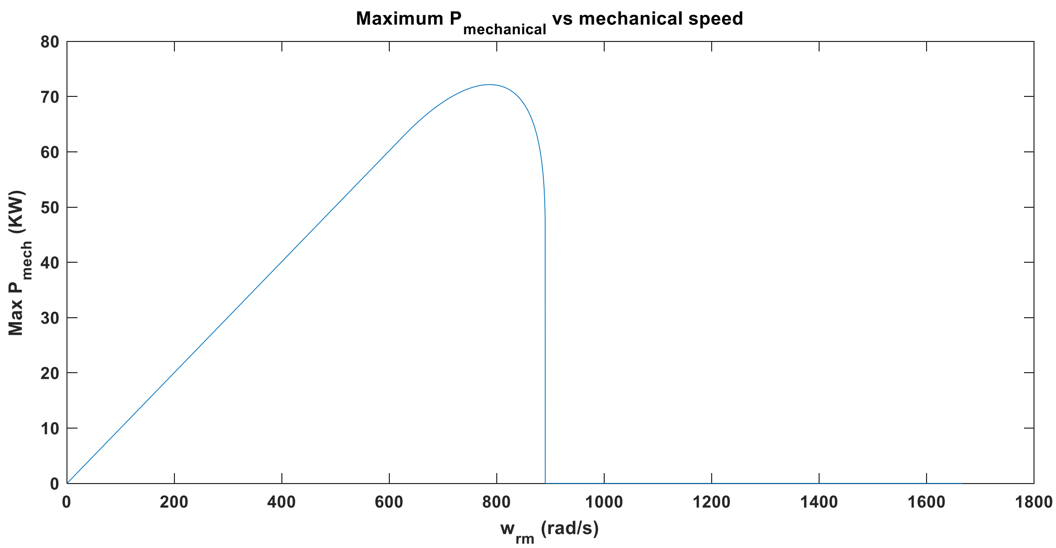

Fig 12: Maximum power vs mechanical speed for the given system. Upto 800 rad/s (7600rpm) P_mech goes upto around 75 KW. After that Te goes down rapidly and P_mech also goes to zero. So for the given configuration, at 800 rad/s motor provides the maximum power.

Problem 2

Determine Iqs^{r*} needed to develop 100 N-m at a mechanical speed of 3000 rpm. Assume I_{ds}^{r*} = 0. Using steady-state equations, calculate required Vqs^{r*} and Vds^{r*}. Verify that given Vbatt is sufficient. Calculate the average power supplied to the motor and the average steady-state battery current I_{batt} (avg). Repeat for Iqs^{r*}set to negative value (corresponding to -100 N-m).

Matlab code:

```
% motor parameters
P = 6; % number of poles
lambda_m = 0.1062;  %flux constant V-s/rad
rs = 0.01;  % stator resistance in ohms
Ld = 0.3e-3; %d-axis inductance in H
Lq = 0.3e-3; %q-axis inductance in H
V_batt = 350; %V

%Filter parameters
L = 5e-6; %inductance in H
R = 0.01; %resistance in ohms
C = 1e-3; %capacitance in F

% define electrical rotor speed
omega_r = 3000*2*pi/60 * P/2; %rad/s
omega_rm = 3000*2*pi/60; %rad/s

%Target torque +ve
%T_e_star = 100; %Nm
%Target torque -ve
```

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```
T_e_star = -100; %Nm

%set d-axis current to zero for maximum efficiency
I_ds_star = 0;

%%%  solution  %%%

%determine q-axis current
I_qs_star = T_e_star * 2/P * 2/3 /lambda_m;

%calculate q- and d-axis components of stator voltages
V_qs_r = rs*I_qs_star + omega_r*Ld*I_ds_star + omega_r*lambda_m
V_ds_r = rs*I_ds_star - omega_r*Lq*I_qs_star

%calculate peak ac voltage
V_max = sqrt ( V_qs_r^2 + V_ds_r^2 ) ;
sprintf('Is V_max %.2f V smaller than Vbatt/sqrt(3) = 202 V? It should!', V_max)

%Calculate electrical and mechanical power
P_mech = T_e_star * omega_rm;
P_elec = 3/2 * V_qs_r * I_qs_star;
sprintf('Is P_mech %.2f W smaller than P_elec %.2f W? It should!', P_mech,P_elec)

% Assuming no inverter loss and R not-negligible
% P_dc = P_elec+Ploss
%I_batt = I_dc
I_batt = P_elec/V_batt %Amp
P_loss = R*I_batt^2
P_dc = P_elec+P_loss %W
```

For Te = 100 Nm:
V_{qs}^{r*} = 102.1836 V

V_{ds}^{r*} = -59.1637 V

V_{batt} sufficient:
'Is V_max 118.08 V smaller than Vbatt/sqrt(3) = 202 V? It should!'

Pmech follows 1st law:
ans = 'Is P_mech 31415.93 W smaller than P_elec 32072.70 W? It should!'

Average power supplied to motor: P_dc = 3.21e+04 W (This is close to the value we get from simulation avg(V_dc*I_batt) ≅ 31.5 KW)

Average steady-state battery current: I_batt = 91.6363 A (This is close to the value we get from simulation avg(I_batt) ≅ 91 A)

For Te = - 100 Nm:
V_{qs}^{r*} = 97.9987 V

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Vds^{r*} = 59.1637 V

V_{batt} sufficient:
ans = 'Is V_max 114.47 V smaller than Vbatt/sqrt(3) = 202 V? It should!'

Pmech follows 1st law:
ans = 'Is P_mech -31415.93 W smaller than P_elec -30759.15 W? It should!'

Average power supplied to motor: P_dc = -3.06e+04 W W (This is close to the value we get from simulation avg(V_dc*I_batt) ≅ -30.5 KW)

Average steady-state battery current: I_batt = -87.8 A (This is close to the value we get from simulation avg(I_batt) ≅ - 87 A)

Problem 3

Simulate a step change in Iqs^r from 0 to the value calculated in (2) to minus the value calculate in (2) allowing the system to reach steady state before applying each step change. Assume that the mechanical speed is constant (3000 rpm). Plot vas, ias, vdc, idc, ibatt, and Te. Each plot should include a discussion of the associated results. Compare average ibatt and Te with calculated values from (2).

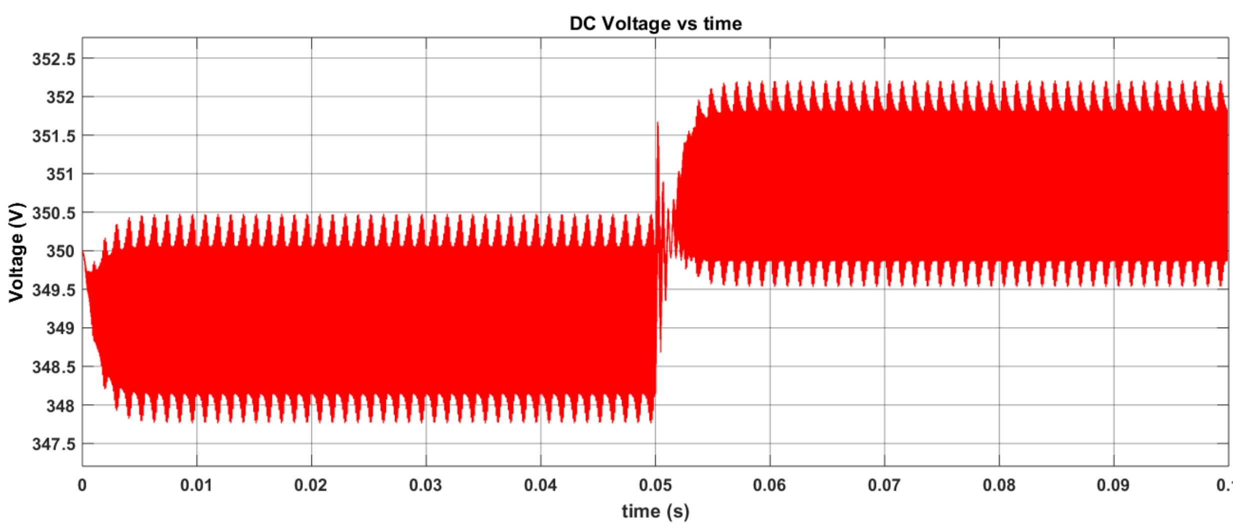

Fig 13: DC voltage output V_dc(from filter block) vs. time. As V_batt = 350V, this should be equal to 350V. But the instantaneous voltage fluctuates between 348V and 352V due to the high-frequency harmonic content in inverter current i_dc. But important point to notice, when i_qs_r is negative, voltage is increasing instantly (almost). Which indicates we can charge the battery instantly during regenerative braking.

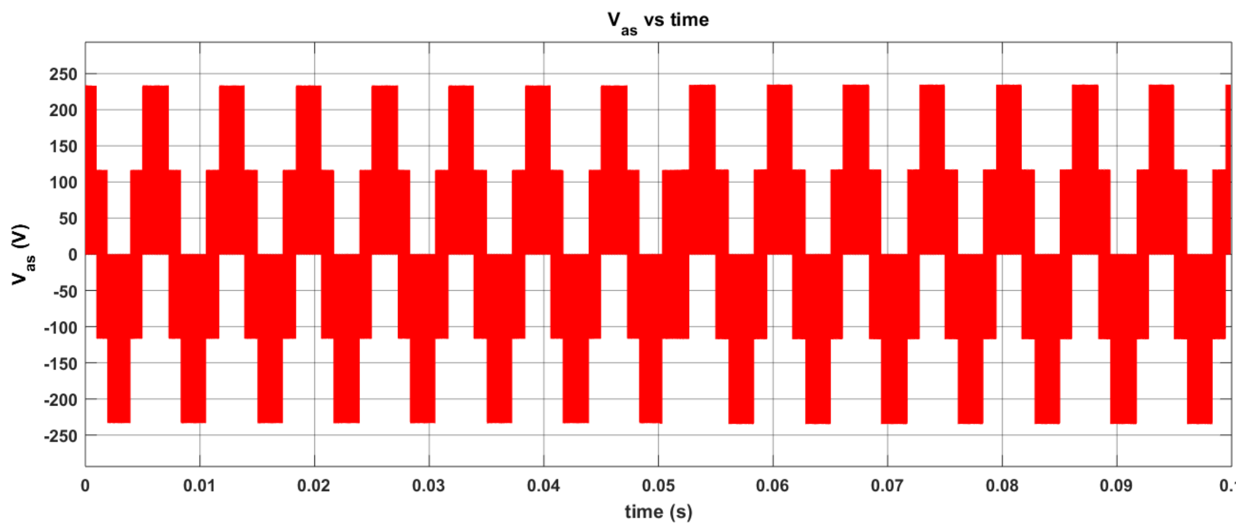

Fig 14: Stator a-phase voltage (Vas) vs. time. The peak value should be 2/3V_dc ≅233 V, which is what we get here. The values change in 5 different steps (2/3, 1/3, 0, -1/3, -2/3) of Vdc which is expected. At 0.05 s, we observe a shift in phase (due to change in direction of i_qs_r) but the magnitude remains the same. This contains only high frequency harmonics and all low frequency harmonics are filtered. As a result, outputs (Te, I_dc, I_batt) has high frequency harmonics into them. Same phenomenon happens for V_bs and V_cs with 120-degree and 240-degree phase shift respectively.

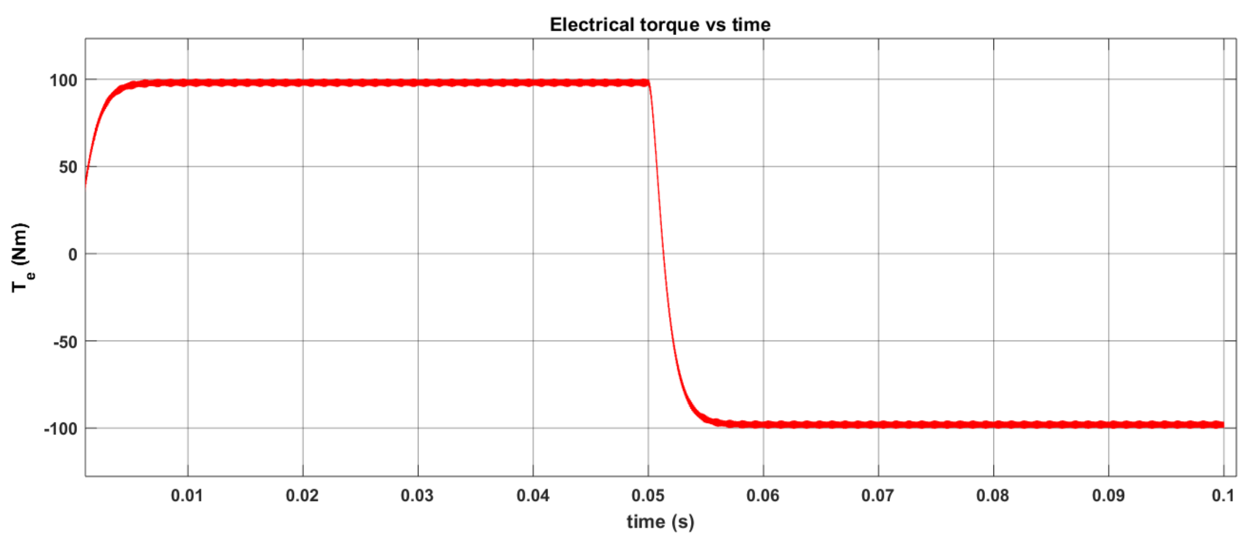

Fig 15: Electrical torque (Nm) vs. time. At the beginning of step change, Te takes a certain time (3τ, where τ=Kq/(rs+Lq)) to reach ss 100 Nm (almost) value. There is a small ss error due to proportional (Kd, Kq) feedback gain in current regulator block. That is why it is lower (around 98 Nm avg) than the value from prob 2 (100 Nm). At 0.5s with negative step change, we see the same incidents.

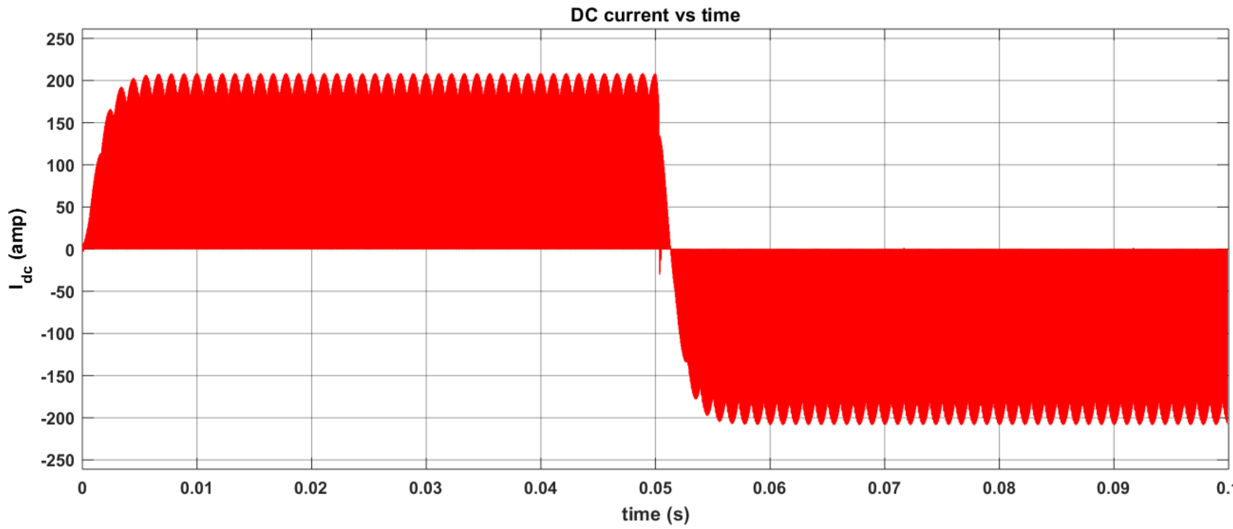

Fig 16: DC current Idc (from inverter block) vs. time. The maximum value is around 210 A. But it is very noisy. Values are changing between Maximum value to zero instantaneously. If we didn't have the capacitor in the filter, we would have faced a lot of electrical interferences.

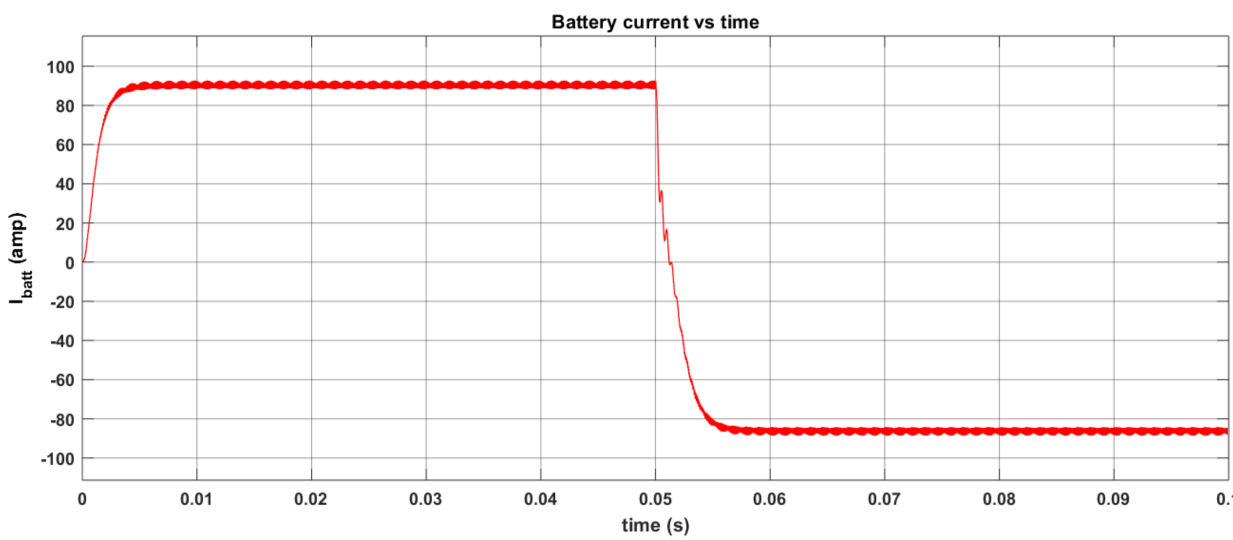

Fig 17: Battery current (I_{batt}) vs. time. During positive I_qs_r, we get maximum value around 91 amp. Which very close to the value we calculated from prob 2. When I_qs_r changes to negative we get near instant response and current flows into the battery. At this point, the maximum value is around - 87 A. Due to loss in motor (rs = 0.01) we are not getting the energy back that we supplied.

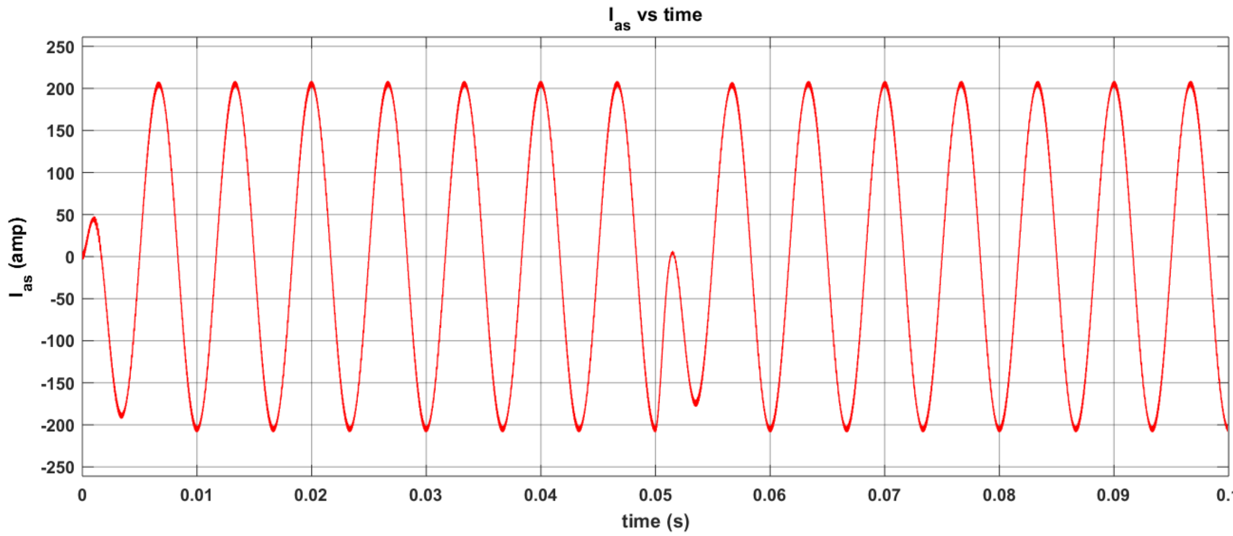

Fig 18: Stator a-phase current (Ias) vs. time. This is a sinusoid. The peak amplitude is 208 A, doesn't reach 210 A (as our input). At 0.05 s, the phase shifts to 180 degree (due to negative step change) but the magnitude remains the same. Same phenomenon happens for I_bs and I_cs with 120-degree and 240-degree phase shift respectively.

Conclusions:

1. A Simulink model was created to simulate a PMACM. Matlab codes are also written to validate (1^{st} law) results from Simulink model. All the questions asked in the project are answered.
2. Simulation results showed acceptable ranges in PMACM outputs. Some losses (eddy current, hysteresis) are not modeled. But model outputs are within range.
3. Dissecting the voltage and current plots from the simulation helped to grow intuitions about this machine. Some efficiency analysis may have been helpful to get real world outputs.
4. The simulation results showed that we can control the Torque near instantly if we don't ask for too much.
5. The changes in torque and output values in this project also validated our assumption in project 1 and 2 that, changes in electrical system is instantaneous and losses are negligible.