

作业 (3):

1. 用两阶段法求解下列线性规划: (119 页第 2 题)

$$\begin{aligned} (3) \quad & \max 3x_1 - 5x_2 \\ & s.t. \quad -x_1 + 2x_2 + 4x_3 \leq 4 \\ & \quad \quad x_1 + x_2 + 2x_3 \leq 5 \\ & \quad \quad -x_1 + 2x_2 + x_3 \geq 1 \\ & \quad \quad x_1, x_2, x_3 \geq 0 \end{aligned}$$

$$(4) \quad \min x_1 - 3x_2 + x_3$$

$$\begin{aligned} & s.t. \quad 2x_1 - x_2 + x_3 = 8 \\ & \quad \quad 2x_1 + x_2 \geq 2 \\ & \quad \quad x_1 + 2x_2 \leq 10 \\ & \quad \quad x_1, x_2, x_3 \geq 0 \end{aligned}$$

2. 用大 M 法求解下列线性规划: (119 页第 2 题)

$$(5) \quad \max -3x_1 + 2x_2 - x_3$$

$$\begin{aligned} & s.t. \quad 2x_1 + x_2 - x_3 \leq 5 \\ & \quad \quad 4x_1 + 3x_2 + x_3 \geq 3 \\ & \quad \quad -x_1 + x_2 + x_3 = 2 \\ & \quad \quad x_1, x_2, x_3 \geq 0 \end{aligned}$$

$$(7) \quad \min 3x_1 - 2x_2 + x_3$$

$$\begin{aligned} & s.t. \quad 2x_1 - 3x_2 + x_3 = 1 \\ & \quad \quad 2x_1 + 3x_2 \geq 8 \\ & \quad \quad x_1, x_2, x_3 \geq 0 \end{aligned}$$

3. 给定原问题

$$\begin{aligned} & \min 4x_1 + 3x_2 + x_3 \\ & s.t. \quad x_1 - x_2 + x_3 \geq 1 \\ & \quad \quad x_1 + 2x_2 - 3x_3 \geq 2 \\ & \quad \quad x_1, x_2, x_3 \geq 0 \end{aligned}$$

已知对偶问题的最优解 $(w_1, w_2) = \left(\frac{5}{3}, \frac{7}{3}\right)$, 利用对偶性质求原问题的最优解。

4. 给定线性规划问题:

$$\begin{aligned} & \min 5x_1 + 21x_3 \\ & s.t. \quad x_1 - x_2 + 6x_3 \geq b_1 \\ & \quad \quad x_1 + x_2 + 2x_3 \geq 1 \\ & \quad \quad x_1, x_2, x_3 \geq 0 \end{aligned}$$

其中 b_1 是某一个正数, 已知这个问题的一个最优解为 $(x_1, x_2, x_3) = \left(\frac{1}{2}, 0, \frac{1}{4}\right)$ 。

- (1) 写出对偶问题。
- (2) 求对偶问题的最优解。

5. 考虑线性规划问题

$$\begin{array}{ll}\min & cx \\ \text{s.t.} & Ax = b \\ & x \geq 0\end{array}$$

其中 A 是 m 阶对称矩阵, $c^T = b$ 。证明若 $x^{(0)}$ 是上述问题的可行解, 则它也是最优解。

Homework (3):

1. Solve the following problems by two-phase method.

$$\begin{array}{ll}(3) & \max 3x_1 - 5x_2 \\ & \text{s.t.} \quad -x_1 + 2x_2 + 4x_3 \leq 4 \\ & \quad \quad x_1 + x_2 + 2x_3 \leq 5 \\ & \quad \quad -x_1 + 2x_2 + x_3 \geq 1 \\ & \quad \quad x_1, x_2, x_3 \geq 0\end{array}$$

$$\begin{array}{ll}(4) & \min x_1 - 3x_2 + x_3 \\ & \text{s.t.} \quad 2x_1 - x_2 + x_3 = 8 \\ & \quad \quad 2x_1 + x_2 \geq 2 \\ & \quad \quad x_1 + 2x_2 \leq 10 \\ & \quad \quad x_1, x_2, x_3 \geq 0\end{array}$$

2. Solve the following problems by big-M method.

$$\begin{array}{ll}(5) & \max -3x_1 + 2x_2 - x_3 \\ & \text{s.t.} \quad 2x_1 + x_2 - x_3 \leq 5 \\ & \quad \quad 4x_1 + 3x_2 + x_3 \geq 3 \\ & \quad \quad -x_1 + x_2 + x_3 = 2 \\ & \quad \quad x_1, x_2, x_3 \geq 0\end{array}$$

$$\begin{aligned}
(7) \quad & \min 3x_1 - 2x_2 + x_3 \\
& s.t. \quad 2x_1 - 3x_2 + x_3 = 1 \\
& \quad \quad 2x_1 + 3x_2 \geq 8 \\
& \quad \quad x_1, x_2, x_3 \geq 0
\end{aligned}$$

3. Consider the following problem:

$$\begin{aligned}
& \min 4x_1 + 3x_2 + x_3 \\
& s.t. \quad x_1 - x_2 + x_3 \geq 1 \\
& \quad \quad x_1 + 2x_2 - 3x_3 \geq 2 \\
& \quad \quad x_1, x_2, x_3 \geq 0
\end{aligned}$$

Suppose the optimal feasible solution of its dual is $(w_1, w_2) = \left(\frac{5}{3}, \frac{7}{3}\right)$. Give the optimal feasible solution of the problem.

4. Given the problem:

$$\begin{aligned}
& \min 5x_1 + 21x_3 \\
& s.t. \quad x_1 - x_2 + 6x_3 \geq b_1 \\
& \quad \quad x_1 + x_2 + 2x_3 \geq 1 \\
& \quad \quad x_1, x_2, x_3 \geq 0
\end{aligned}$$

Where b_1 is a positive integer. Assume $(x_1, x_2, x_3) = \left(\frac{1}{2}, 0, \frac{1}{4}\right)$ is an optimal solution of the problem..

- (3) Write the dual for the problem.
- (4) Give the optimal solution of the dual.

5. Given the problem:

$$\begin{aligned}
& \min \quad cx \\
& s.t. \quad Ax = b \\
& \quad \quad x \geq 0
\end{aligned}$$

Where A is m symmetry matrix, $c^T = b$. Show that if $x^{(0)}$ is a feasible solution of the problem, then it is optimal.