STATISTICAL WEIGHTS AND SELECTIVE HEIGHT CORRECTIONS IN THE DETERMINATION OF THE SOLAR ROTATION VELOCITY

R. BRAJŠA[1], V. RUŽDJAK[1], B. VRŠNAK[1], H. WÖHL[2], S. POHJOLAINEN[3,4] and S. UPRO[4]

[1]*Hvar Observatory, Faculty of Geodesy, University of Zagreb, Kačićeva 26, HR-10000 Zagreb, Croatia*

[2]*Kiepenheuer-Institut für Sonnenphysik, Schöneckstr. 6, D-79104 Freiburg, Germany*

[3]*Observatoire de Paris, DASOP, 5 Place Jules Janssen, F-92195 Meudon, France*

[4]*Metsähovi Radio Observatory, Helsinki University of Technology, Otakaari 5A, FIN-02150 Espoo, Finland*

(Received 6 January 1999; accepted 10 June 2000)

Abstract. Observations of the Sun performed at 37 GHz with the 14-m radio telescope of the Metsähovi Radio Observatory were analyzed. Rotation velocities were determined, tracing Low Temperature Regions (LTRs) in the years 1979–1980, 1981–1982, 1987–1988, and 1989–1991. Statistical weights were ascribed to the determined rotation velocities of LTRs, according to the number of tracing days. Measured changes of the rotation velocity during the solar activity cycle, as well as a north–south rotation asymmetry, are discussed. The results obtained with and without the statistical weights procedure are compared, and it was found that the statistical significance of the solar differential rotation parameters' changes is higher when the statistical weights procedure is applied. A selective application of the height correction on LTR's positions has not removed the cycle-related changes nor the north–south asymmetry of the solar rotation measured tracing LTRs. So, projection effects cannot explain these changes. The differential rotation of LTRs is more rigid than the differential rotation obtained tracing magnetic features and measuring Doppler shifts, which can be explained by the association rate of the LTRs' positions with rigidly rotating 'pivot points'. The observed cycle-related changes and the north–south asymmetry of the rotation velocity of LTRs are consistent with the cycle-related changes and the north–south asymmetry of the association rate between LTRs and pivot points.

1. Introduction

There are two methods to determine the solar surface rotation velocity: the Doppler method and the tracer method (Howard, 1984; Schröter, 1985; Stix, 1989; Wöhl, 1990, 1997; Snodgrass, 1983, 1984, 1992). The solar angular rotation velocity can be determined tracing objects whose positions can be measured with sufficient accuracy. Visual identification of tracers can be supplemented by an application of the cross-correlation and auto-correlation methods. For stable tracers, the time span between two successive passages across the central meridian can be used (Bruzek, 1961) to avoid errors, caused by the center-to-limb effects (Balthasar and Wöhl, 1983; Lustig, 1984; Balthasar, Vázquez, and Wöhl, 1986; Pulkkinen and Tuominen, 1998; Roša *et al.*, 1998). The rotation velocity can be determined from

Solar Physics **196:** 279–297, 2000.

at least two positions of an identified tracer during its disc passage. If more than two positions of a tracer are measured, the statistical weights procedure can be applied (Bruzek, 1961; Lustig, 1984; Ternullo, 1990; Brajša *et al.*, 1991; Godoli, Mazzucconi, and Piergianni, 1998).

Solar microwave measurements performed at 37 GHz with the 14-m radio telescope of the Metsähovi Radio Observatory (Urpo, Pohjolainen, and Teräsranta, 1992) were used to determine rotation velocities tracing Low brightness Temperature Regions (LTRs) in the years 1979, 1980, 1981, 1982, 1987, 1988, 1989, 1990, and 1991 (Brajša *et al.*, 1997, hereafter denoted as Paper I). LTRs' heights in the solar atmosphere were estimated, measuring their rotation velocity by Brajša *et al.* (1999a), hereafter denoted as Paper II. In Papers I and II, as well as in the present Paper, the same LTRs data set was used. The measurements and the position determination of LTRs, were described in detail in Papers I, and II. In Paper I, changes of the solar differential rotation velocity during the activity cycle and a north–south rotational asymmetry were found. In Paper II, a difference in the measured rotation velocity for two classes of LTRs, associated and not associated with Hα filaments, was found. Based on this in Paper II, we proposed an interpretation that projection effects can explain the cycle-related changes and the north–south asymmetry of the solar rotation velocity reported in Paper I. In this paper we apply the statistical weights procedure and a selective height correction on LTR's positions to test this possibility, with an improvement of the data reduction procedure used in Papers I and II. The new results will be compared with the ones obtained in Papers I and II, as well as with the ones reported in the literature. The dependence of the obtained results on the applied procedure will be discussed also. It will be shown that projection effects cannot explain the cycle-related changes and the north–south asymmetry of the solar rotation velocity measurements tracing LTRs, but that these changes and the asymmetry are consistent with the cycle-related changes and the north–south asymmetry of the association rate between LTRs and rigidly rotating 'pivot points'.

2. Statistical Weights Procedure

2.1. Data Reduction

The identified LTRs were traced during their passage over the solar disc, and for each identified LTR the central meridian distance (l) was measured as a function of time (t). The value of the rotation velocity was obtained by the linear least-squares fit $l_i(t_i)$. Statistical weights (SW) were assigned to the measured rotation velocities according to the duration of the tracings, e.g., two days: SW $=$ 1, three days: SW $=$ 2, four days: SW $=$ 3, etc. So, SW $= d - 1$, where d represents the number of tracing days. A similar procedure was introduced by Bruzek (1961). Occasionally, more than one LTR's position measurement per day was used, but in

TABLE I
The number of identified LTRs (n_{LTR}) traced during d days.

d	2	3	4	5	6	7	8	9
n_{LTR}	138	145	74	62	47	19	7	1

calculating statistical weights, only the number of tracing days was counted. The values of the measured rotation velocities are then multiplied by statistical weights. So, the effective number of velocity values is given by

$$n_{\text{SW}} = \sum_{j=1}^{n_{\text{LTR}}} (\text{SW})_j \ , \tag{1}$$

where $)\text{SW})_j$ represents the statistical weight of the differential rotation velocity value for the jth identified LTR and n_{LTR} is the number of identified LTRs in a group of measurements. The assumption $\text{SW} = d - 1$ was used to have the effective number of obtained velocity values equal to the number of velocity values, which can be obtained from successive 2-day measurements.

In total, 582 LTRs were identified and traced during their disc passages. As in Paper I, the extreme values of rotation velocities (sidereal rotation velocities smaller than 11 deg per day and larger than 16 deg per day, for all latitudes under consideration) were excluded, so that 493 LTRs remained for further analysis. These 493 LTRs were traced during 2 to 9 days (Table I), in the range of central meridian distances $\leq \pm 60$ deg. The visibility of LTRs as a function of the central meridian distance shows a sharp drop of LTRs observed at central meridian distances larger than ± 50 deg. The visibility function was presented for a subset of the available LTR data by Vršnak *et al.* (1992) and the distribution for the complete data set looks very similar to it. The effective number of velocity values for all LTRs was 1304 (Table II). So, on the average $n_{\text{SW}}/n_{\text{LTR}} = 1304/493 = 2.645$ rotation velocity values per every identified individual LTR were calculated.

As usual, the solar differential rotation velocity is represented by

$$\omega(b) = A + B \sin^2 b + C \sin^4 b \ , \tag{2}$$

where ω is the sidereal angular rotation velocity in deg per day, b is the heliographic latitude, and A, B, C are the solar differential rotation velocity parameters. The additional assumption $B = C$ (Scherrer, Wilcox, and Svalgaard, 1980) was adopted to avoid the crosstalk between the parameters B and C in the three-parameter fits, which is especially important when the changes in time of the solar rotation velocity are studied and results from different intervals are compared (Paper I). The

TABLE II

Solar sidereal rotation velocity determined tracing LTRs in both solar hemispheres. The parameters from the expression (2), with $B = C$, are given for different years in deg per day. M – the standard error, n_{SW} – the effective number of velocity values defined by (1), b – the latitude range in deg.

Time interval	A	$\pm M_A$	$-B$	$\pm M_B$	$-C$	$\pm M_C$	n_{SW}	$\|b\|$
1979–1980	14.41	0.09	0.85	0.27	0.85	0.27	162	0–49
1981–1982	14.45	0.07	1.74	0.24	1.74	0.24	184	0–53
1987–1988	13.77	0.15	0.65	0.38	0.65	0.38	87	0–53
1989–1991	14.30	0.04	1.18	0.14	1.18	0.14	871	0–55
All years	14.31	0.03	1.18	0.11	1.18	0.11	1304	0–55

statistical significance of changes in the measured solar rotation velocities can be tested using the expression

$$\Delta\omega \equiv \omega_1 - \omega_2 > N(M(\omega_1) + M(\omega_2)) , \tag{3}$$

where ω_1 and ω_2 represent rotation velocities (which may be expressed by rotation parameters too) for two classes of tracers, or rotation velocities measured in two different phases of the solar cycle, M represents the standard error and $N = 1, 2, 3$. A difference of measured rotation velocities is statistically significant on the $N\sigma$ level if the criterion (3) is fulfilled for N (N defines the confidence interval with the corresponding confidence level).

2.2. Cycle-related changes of the solar rotation velocity for both solar hemispheres treated together

The mean values of the measured rotation velocity and latitude are presented for procedures with and without the statistical weights in Figures 1–4 for the years 1979–1980, 1981–1982, 1987–1988, and 1989–1991, respectively, and for all these years in Figure 5. The results for 10 deg intervals in latitude are displayed at slightly different latitudes when the statistical weights procedure is used, while in both cases the real mean values are used, and not just the centres of the intervals. These mean values binned for 10 deg latitude bands are presented in a tabular form for the procedures with (Brajša *et al.*, 1999b) and without (Paper I) the statistical weights procedure. The standard errors of the mean rotation velocity values are smaller in all cases when the statistical weights procedure is applied. Although some differences below the 1σ level can be noticed, the general trends are the same and systematically higher mean rotation velocity values are obtained when the statistical weights procedure is applied (Figures 1–5). This can be interpreted as a consequence of the LTRs' heights above the photosphere (Paper II). The influence

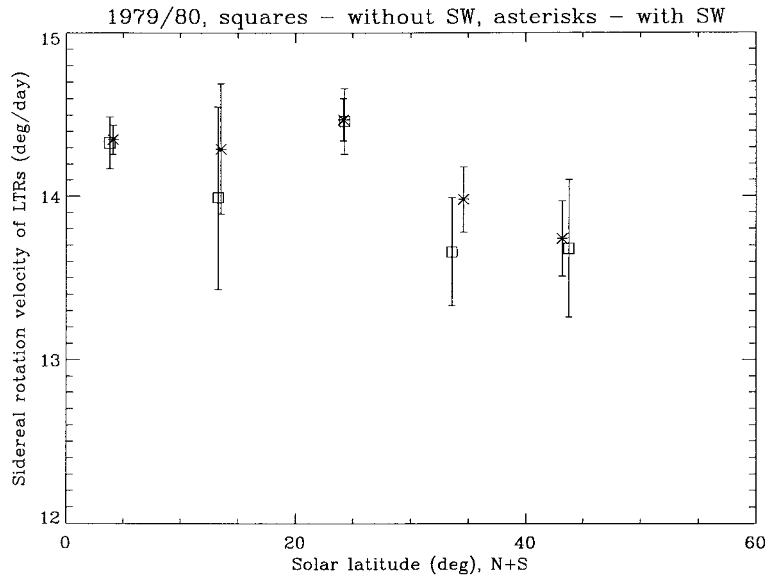

Figure 1. Asterisks and squares represent the mean rotation velocity values calculated with and without the statistical weights procedure, respectively. The mean values of the solar sidereal rotation velocities were obtained, tracing LTRs in 10 deg latitude bands with the error bars indicating the standard errors of the means. Data for both solar hemispheres were treated together for the years 1979–1980.

of projection effects (Liu and Kundu, 1976), yielding higher apparent rotation velocity values for tracers situated higher in the solar atmosphere, is more prominent when the tracer's central meridian distance increrases (Roša *et al.*, 1998). In the statistical weights procedure, the rotation velocity values of LTRs traced during several days contribute more to the mean values than the rotation velocity values of LTRs traced only on two days. The probability to observe LTRs with sufficient accuracy at large central meridian distances, is higher for stable LTRs traced more than 3 days. Such LTRs constitute 43% of our data sample (Table I). So, it follows that the mean rotation velocity values in the statistical weights procedure are systematically higher (Figures 1–5).

The solar differential rotation velocity parameters defined by expression (2) and calculated applying the statistical weights procedure are presented for different time intervals in Table II. All rotation velocity values were used in the fitting procedure, not only the mean values. The solar differential rotation velocity curves, characterized by these parameters for different time intervals, are presented together in Figure 6. A higher rotation velocity tracing LTRs was measured on the average in the intervals 1979–1980 and 1989–1991 (maxima of the solar activity cycles 21 and 22) than during the years 1981–1982 and 1987–1988. The changes of the rotation parameter A were found to be statistically significant up to the 3σ level (compare the interval 1987–1988 with other time intervals) and the changes

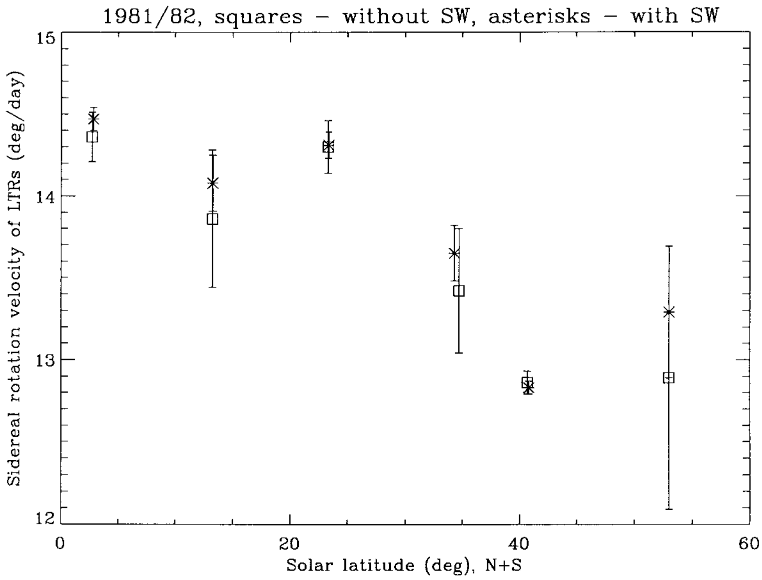

Figure 2. The same as in Figure 1, for the interval 1981–1982.

of the rotation parameters B and C on the 1σ level (Table II). The results obtained without applying the statistical weights procedure (Paper I, Table II) showed a statistical significance of changes on the 1σ level only.

2.3. North–south rotational asymmetry

The solar differential rotation velocity parameters, as defined by expression (2) and calculated by applying the statistical weights procedure, are presented in Table III for the northern and southern solar hemispheres, separately. When a LTR was situated at the solar equator ($b = 0$ deg), it was counted in both hemispheres, so that the sum of the effective numbers of velocity values for north and south ($n_{SW} = 690 + 622$, Table III) is larger than the effective number of velocity values for both solar hemispheres together ($n_{SW} = 1304$, Table II). The data for the northern and southern hemispheres are presented separately for all years and for the interval 1989–1991 only, as in the other three intervals, 1979–1980, 1981–1982, and 1987–1988, the number of separated velocity values is too small to allow an independent treatment.

The solar differential rotation velocity curves are presented in Figure 7 for the northern and southern solar hemispheres separately, revealing that the northern solar hemisphere had a higher rotation velocity than the southern one. A north–south rotational asymmetry was found to be significant on the 1σ level when the statistical weights procedure is applied (Table III, e.g., the 1989–1991 interval), which would not be the case when the statisical weights procedure had not been applied (Paper I, Table II; the statistical significance is below the 1σ level).

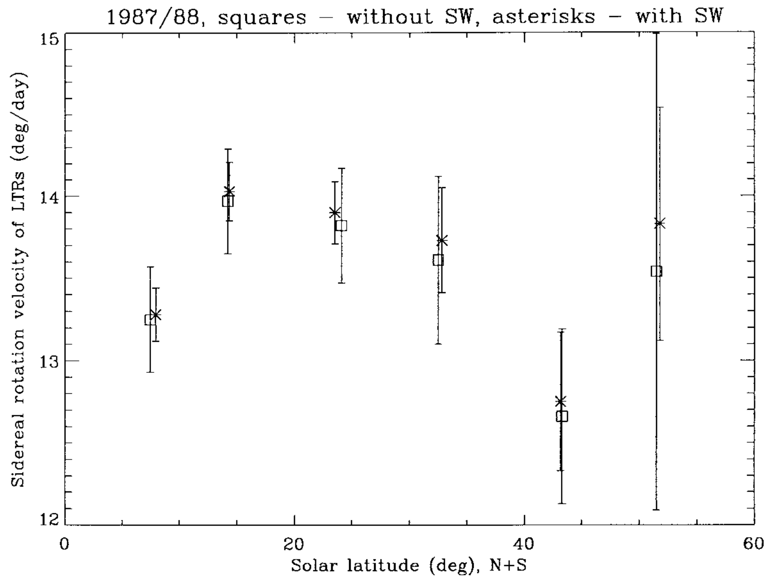

Figure 3. The same as in Figure 1, for the interval 1987–1988.

TABLE III

Similar as in Table II, separately for the northern (N) and the southern (S) hemisphere; $B = C$.

Time interval	A	$\pm M_A$	$-B$	$\pm M_B$	$-C$	$\pm M_C$	n_{SW}
N 1989–1991	14.38	0.05	1.18	0.17	1.18	0.17	461
N all years	14.34	0.04	1.14	0.13	1.14	0.13	690
S 1989–1991	14.22	0.06	1.22	0.22	1.22	0.22	415
S all years	14.27	0.05	1.27	0.17	1.27	0.17	622

2.4. Applicability of the Statistical Weights Procedure

The tracer method can be performed in different ways: either by calculating a rotation velocity value from only two pairs of central meridian distances (l) and times (t) (as $\Delta l/\Delta t$, Method I), or, when the tracer can be identified more than twice, by the linear least-squares fit $l_i(t_i)$ (Method II). In the latter case, from d measurements $l_i(t_i)$ only one rotation velocity value is obtained, although $d - 1$ rotation velocity values can be calculated from the data. So, the statistical weights procedure (Method III) can be applied.

The Method I is most commonly used in solar rotation studies (Dupree and Henze, 1972; Balthasar and Wöhl, 1980; Howard, Gilman, and Gilman, 1984; Balthasar, Vázquez, and Wöhl, 1986; Aini Kambry and Nishikawa, 1990; Howard,

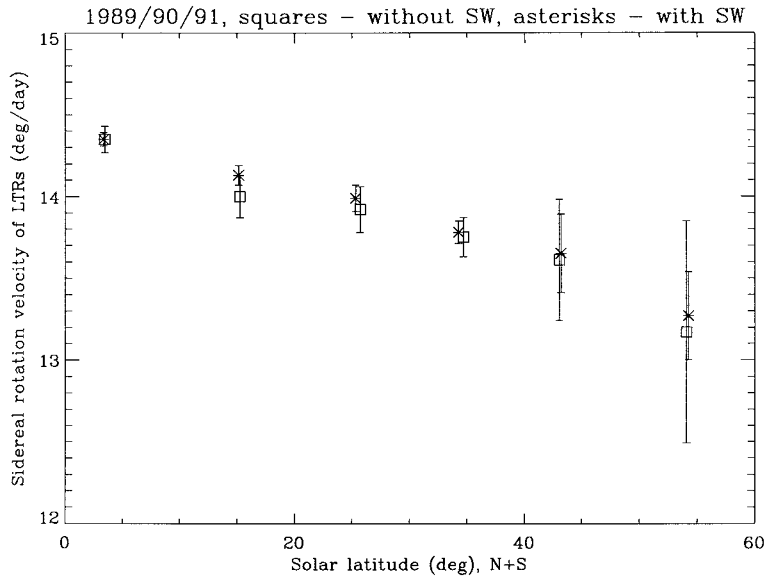

Figure 4. The same as in Figure 1, for the interval 1989–1991.

1990; Howard, Harvey, and Forgach, 1990; Zappala and Zuccarello, 1991; Sivaraman, Gupta and Howard, 1993; Godoli, Mazzucconi, and Piergianni, 1993; Javaraiah and Gokhale, 1997; Meunier, Nesme-Ribes, and Grosso, 1997; Suzuki, 1998). The Method II was applied by Liu and Kundu (1976), Balthasar, Schüssler, and Wöhl (1982), Wöhl (1983), and Dezsö and Kovács (1994), while the Method III was used by Bruzek (1961) and Brajša *et al.* (1991). However, there are some special cases which cannot be simply classified as Methods I–III. For instance, Schröter and Wöhl (1975) used a linear least-squares fit $l_i(t_i)$ for the identification of tracers (series of measurements in one day), and then calculated the rotation velocities from only two selected position measurements. A similar procedure was applied by Adams and Tang (1977) in which identified parts of tracers were followed during several days (generally more than two), but the rotation velocity was calculated from the two most widely separated measured positions. Further, Lustig (1984) applied Method II, but individual daily positions of the tracers were weighted according to the image quality and the distance from the central meridian. Godoli, Mazzucconi, and Piergianni (1998) calculated individual rotation velocity values from two daily position measurements (Method I). These velocities were then averaged for one transit of an identified tracer and the average velocity values were weighted, according to the numbers of individual rotation velocity values.

It might appear that there are no significant differences between these three approaches (Methods I–III), but we argue that it is better to apply the statistical weights procedure (Method III), because it gives more correct and reliable results than the other two, especially when tracers in microwaves are used. Namely, microwave features should be traced at least 2–4 days, since shorter tracing times,

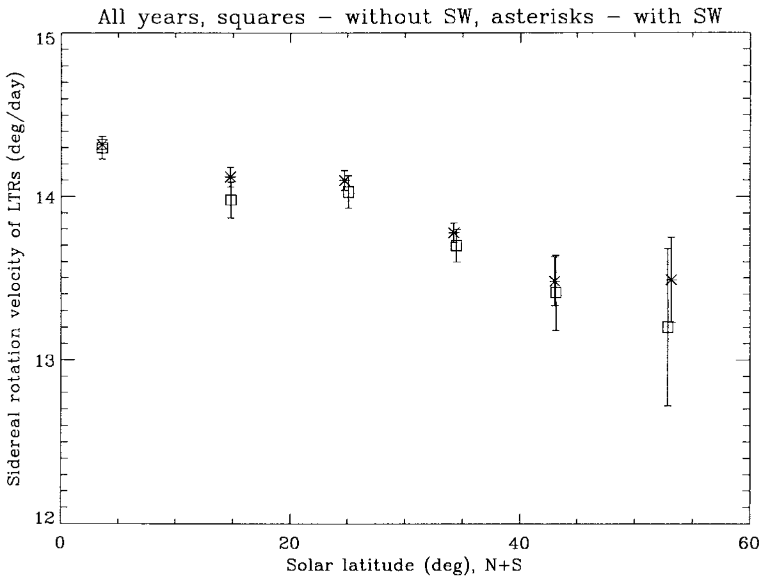

Figure 5. The same as in Figure 1, for all years considered.

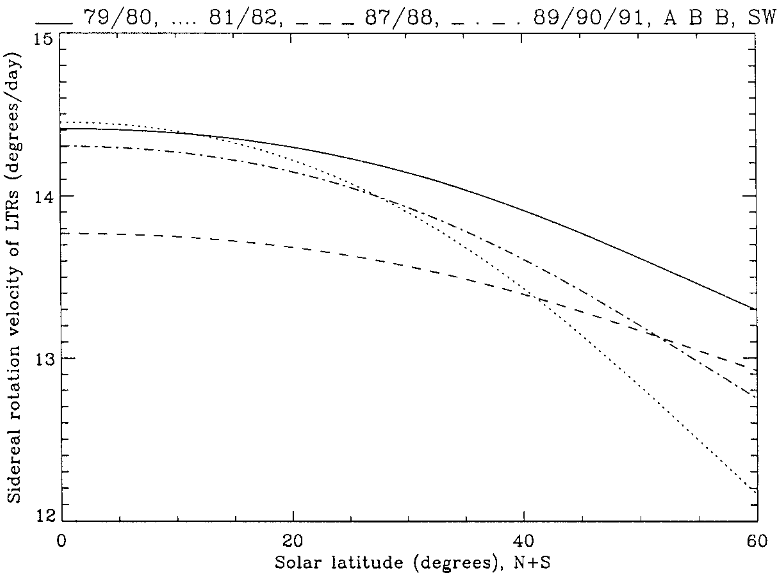

Figure 6. Solar rotation velocity determined tracing LTRs in four time intervals for both solar hemispheres. The solid line represents the data from the years 1979–1980, the dotted line the data from 1981–1982, the dashed line the data from 1987–1988, and the dash-dotted line the data from 1989–1991. Here all the parameters from Equation (2) were used ($B = C$, Table II).

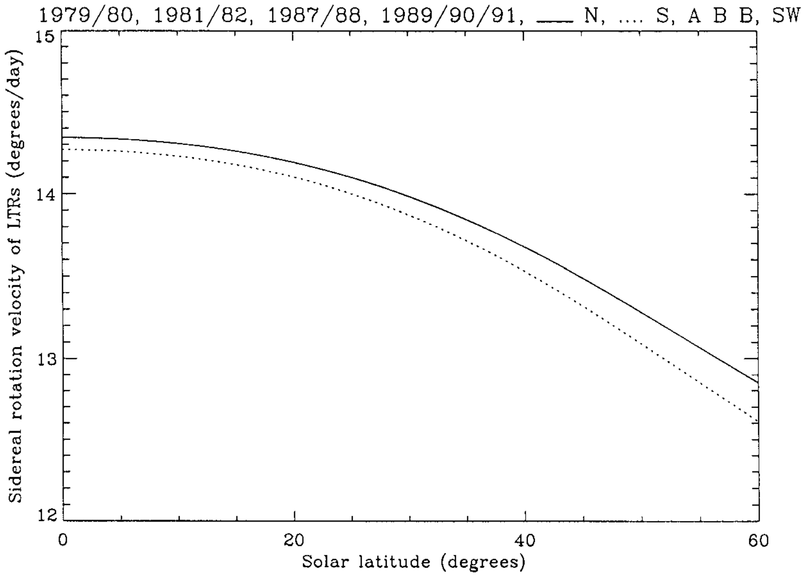

Figure 7. Measured solar sidereal rotation velocity given separately for the two solar hemispheres for all four time intervals together. The full line represents the solar rotation velocity in the northern hemisphere, and the dotted line gives the solar rotation velocity in the southern one. Both fits were obtained using Equation (2) with all three parameters different from zero, but requiring $B = C$ (Table III).

combined with the uncertainty in coordinate determination (3−5 deg at medium latitudes) due to the poor spatial resolution of the radio telescope, can give unreliable results. So, when only measurements $l_i(t_i)$ on two consecutive days are used (Method I), very uncertain rotation velocities (extremely low or high) can be obtained. This problem can be avoided, using more than two measurements $l_i(t_i)$, when possible. In our data set, LTRs were traced up to 9 days, but most frequently 2−4 days, see Table I, and an application of the statistical weights procedure (Method III) provides a higher statistical significance of the results using the complete amount of information at our disposal. In any case, all relevant influences should be taken into account (e.g., projection effects due to the height of tracers, subsection 3.1.), regardless of the method used.

3. Selective Height Correction

3.1. The Hypothesis and the Method

An attempt was made to explain the changes of the measured solar rotation velocity during the activity cycle, using LTRs by the observed cycle-dependence of the association rate between LTRs and Hα filaments (Paper II). The assumption was that a higher average rotation velocity in some time intervals can be the consequence of a larger contribution of LTRs associated with filaments in the same time intervals, which, due to projection effects, yield a higher rotation velocity than those LTRs which are not associated with filaments. This assumption looked plausible, since both relevant quantities, average rotation velocity and association rate, had qualitatively the same behaviour during the solar cycle. In addition, the situation with the north–south asymmetry was the same, namely, a higher average rotation velocity was obtained for the northern hemisphere where also the association rate with filaments was larger.

The most natural implication of this hypothesis is that the cycle-related changes and the north–south asymmetry of the measured rotation velocity should be removed if the projection effects can be excluded. Generally, this hypothesis can be tested in two different ways: to follow the time-dependence of only one LTR class (with or without associated filaments) or to apply the selective height correction on one LTR class only. The corrected results can then be merged with the ones from the other LTR class. To investigate the cycle-related changes of the rotation velocity, it is not important for which LTR class the rotation velocities are corrected. Both ways of treatment should yield a more homogeneous data sample, with respect to the influence of tracers' heights on the measured rotation velocity.

The LTR data sample was divided into two groups, LTRs with associated filaments and LTRs without associated filaments. Now, the cycle-dependence of the rotation velocity was inspected for these two subgroups separately. The samples are now smaller, especially for LTRs without associated filaments.

The selective height correction was applied in two ways. In Paper II we found that LTRs associated with filaments yield on the average a 0.2 deg per day higher rotation velocity than the LTRs, which are not associated with filaments. So, adding 0.2 deg per day to the velocities obtained tracing LTRs without associated filaments, would raise the velocities to the level of the obtained velocity values of LTRs with associated filaments (selective height correction procedure I).

However, the height correction is generally dependent on the central meridian distance and the latitude, and in the following treatment every rotation velocity value and the corresponding latitude were corrected individually. Now, we use a method to determine the solar synodic rotation velocity and the height of tracers (Roša *et al.*, 1998). From their Equation (21b), for small central meridian distances we obtain

$$\omega = \omega'/\beta \, , \tag{4}$$

where ω' represents the observed and ω the corrected rotation velocity. The parameter β is defined by the Equation (20b) from Roša *et al.* (1998):

$$\beta = \frac{\sqrt{(1+\epsilon)^2 - \sin^2\, b'}}{\cos\, b'} \, , \tag{5}$$

where $\epsilon = h/R$ is the height parameter (h – the height above the solar photosphere, R – the solar radius), and b' is the observed latitude. Finally, the corrected latitude b can be found by the Equation (17b) from Roša *et al.* (1998):

$$\cos\ b = \beta \frac{\cos\ b'}{1+\epsilon}\ . \tag{6}$$

According to the results of Paper II we set $h = 7000$ km ($\epsilon = 0.01$). The selective height correction procedure II is an application of Equations (4)–(6) to individual pairs of latitudes and rotation velocities. In this procedure, only the dependence of the height correction on the latitude was taken into account and the dependence of the height correction on the central meridian distance was neglected. This is justified as the height correction does not change very much for central meridian distances up to ± 50 deg (see Figure 4 in the paper by Roša *et al.*, 1998). This is valid for low heights (e.g., 7000 km) and non-polar regions, i.e., for our data sample. In the whole LTR data sample, only 2.4 % of all 1900 LTRs' positions were measured at central meridian distances $\geq \pm 50$ deg. The differences in the visibility function of LTRs at central meridian distances $\geq \pm 50$ deg are in the order of a few per cent for the four time intervals and below 1% treating the northern and the southern solar hemisphere separately.

3.2. CYCLE-RELATED CHANGES AND NORTH–SOUTH ASYMMETRY OF THE ROTATION VELOCITY

The procedures described in the previous subsection were tested, with and without applying the statistical weights procedure. The cycle-related changes and the north–south asymmetry of the rotation velocity could not be removed. As an example, we present here the results obtained by the selective height correction procedure I (Tables IV and V) and by the selective height correction procedure II (Tables VI and VII and Figures 8 and 9). The rotation velocities, obtained by the selective height correction procedure I, are systematically higher than the rotation velocities obtained by the selective height correction procedure II. This is so, because in the first case all velocities are corrected to the values which have the LTRs associated with filaments (having a higher rotation velocity on the average) and in the second case to the values of the LTRs not associated with filaments (having a lower rotation velocity on the average). Now, we can compare the cycle-related changes and the north–south asymmetry of the rotation velocity, obtained without any height correction with the ones obtained after applying the selective height corrections. In both cases, a similar qualitative behaviour is revealed, i.e., a higher rotation velocity during the solar activity maxima (1979–1980 and 1989–1991) and a lower one in the years 1981–1982 and 1987–1988. Also, the higher rotation velocity obtained for the northern solar hemisphere remained. Let us note that the statistical significance of the rotation velocity parameters, obtained by applying the selective height correction (Tables IV and V), is on the same level as in the case without the selective height correction (Tables II and III).

TABLE IV

The same as in Table II, applying the selective height correction procedure I. The statistical weights procedure was applied.

Time interval	A	$\pm M_A$	$-B$	$\pm M_B$	$-C$	$\pm M_C$	n_{SW}	$\|b\|$
1979–1980	14.49	0.09	0.95	0.27	0.95	0.27	162	0–49
1981–1982	14.53	0.07	1.83	0.24	1.83	0.24	184	0–53
1987–1988	13.90	0.15	0.66	0.39	0.66	0.39	87	0–53
1989–1991	14.35	0.04	1.23	0.14	1.23	0.14	871	0–55
All years	14.37	0.03	1.23	0.11	1.23	0.11	1304	0–55

TABLE V

The same as in Table III, applying the selective height correction procedure I. The statistical weights procedure was applied.

Time interval	A	$\pm M_A$	$-B$	$\pm M_B$	$-C$	$\pm M_C$	n_{SW}
N 1989–1991	14.42	0.05	1.22	0.18	1.22	0.18	461
N all years	14.40	0.04	1.19	0.13	1.19	0.13	690
S 1989–1991	14.28	0.06	1.27	0.22	1.27	0.22	415
S all years	14.34	0.05	1.31	0.17	1.31	0.17	622

TABLE VI

The same as in Table II, applying the selective height correction procedure II. The statistical weights procedure was not applied.

Time interval	A	$\pm M_A$	$-B$	$\pm M_B$	$-C$	$\pm M_C$	n	$\|b\|$
1979–1980	14.30	0.15	1.28	0.46	1.28	0.46	70	0–49
1981–1982	14.24	0.15	2.02	0.46	2.02	0.46	59	0–53
1987–1988	13.76	0.26	1.07	0.64	1.07	0.64	41	0–53
1989–1991	14.16	0.08	1.33	0.26	1.33	0.26	323	0–55
All years	14.16	0.06	1.41	0.19	1.41	0.19	493	0–55

TABLE VII

The same as in Table III, applying the selective height correction procedure II. The statistical weights procedure was not applied.

Time interval	A	$\pm M_A$	$-B$	$\pm M_B$	$-C$	$\pm M_C$	n
N 1989–1991	14.21	0.10	1.32	0.33	1.32	0.33	167
N all years	14.16	0.08	1.30	0.25	1.30	0.25	256
S 1989–1991	14.10	0.11	1.32	0.40	1.32	0.40	159
S all years	14.16	0.09	1.55	0.29	1.55	0.29	242

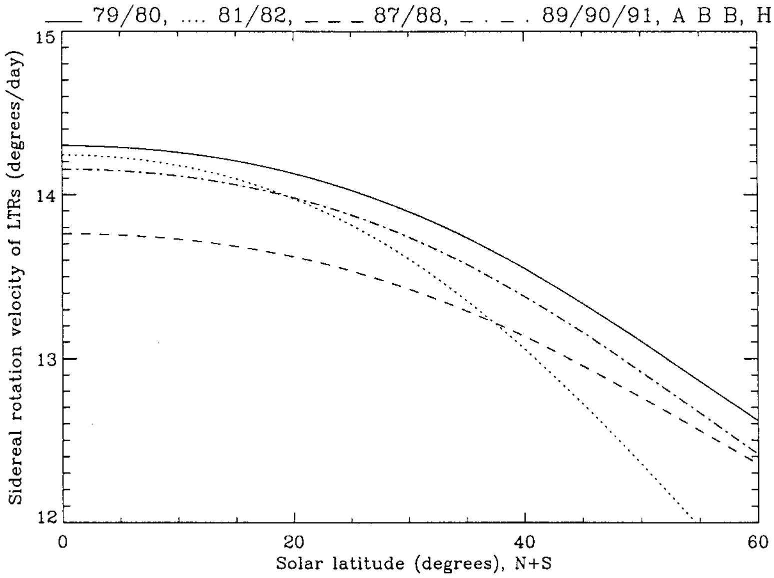

Figure 8. The same as in Figure 6, applying the selective height correction procedure II (Table VI).

4. Discussion

Let us compare the rotation velocity deduced tracing LTRs with the rotation velocity obtained by some other tracers and methods. Such a comparison with the rotation velocity of Hα filaments and sunspots was performed in Papers I and II, and now a comparison with the rotation velocity of magnetic tracers and the Doppler-determined velocities will be performed. The rotation velocities obtained by tracing LTRs in all time intervals and in both solar hemispheres are treated together and the solar rotation parameters are calculated with and without the statistical weights procedure, as well as with and without the selective height correction. The results are summarized in Table VIII. Let us first discuss the different

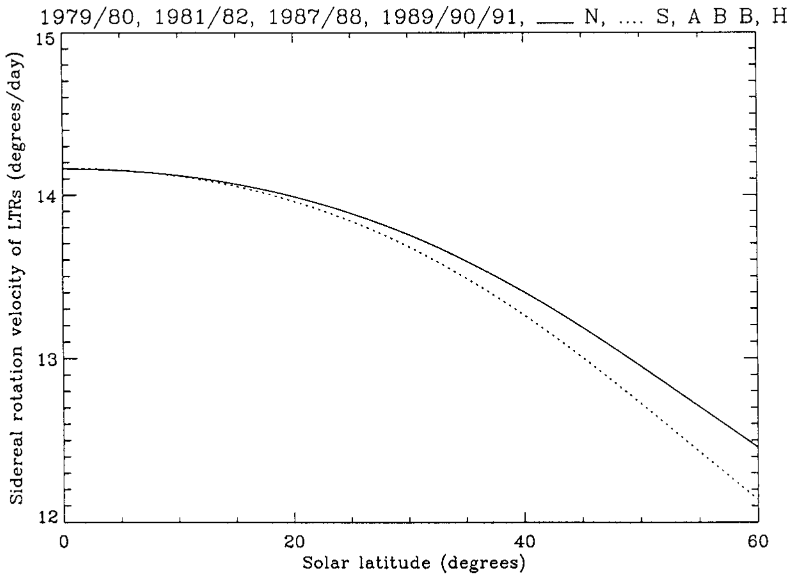

Figure 9. The same as in Figure 7, applying the selective height correction procedure II (Table VII).

obtained values of the parameter A (describing the solar equatorial rotation velocity) obtained by tracing LTRs. While the difference between the values of the parameter A obtained with and without the statistical weights procedure (first two rows in Table VIII) is not significant, the difference between the next two values of the parameter A is significant and is a consequence of the two different selective height correction procedures, as already discussed in subsection 3.2. The values of the rotation parameter A obtained tracing LTRs (2nd and 3rd rows in Table VIII, without the selective height correction and corrected to the height of filaments, respectively) are consistent with the values of the parameter A obtained tracing magnetic features (5th and 6th rows in Table VIII) but are higher than the values of Doppler measurements (rows 8–12 in Table VIII). On the other hand, the value of the parameter A obtained tracing LTRs and selectively corrected to a lower elevation of LTRs not associated with filaments (row 4 in Table VIII) is closer to, but still higher than the values of Doppler measurements. The solar rotation parameters B and C, having the same negative sign throughout the whole of Table VIII, describe the differentiality - rigidity of the solar rotation curve. The rotation velocity determined tracing LTRs is more rigid than any velocity obtained by magnetic tracers or Doppler measurements. In some cases, the difference between the values of these two parameters for LTRs (rows 2 and 3 in Table VIII) and for magnetic features (rows 5 and 6 in Table VIII) are statistically significant above the 3σ level.

To comprehend this rigid component in the rotation velocity of LTRs, we checked to see if some of the LTRs associated with filaments are located at ‘pivot points’. Pivot points (Escaut-Soru, Martres, and Mouradian, 1984) are limited areas which rotate rigidly with the Carrington rotation velocity, as defined by the Central

TABLE VIII

Solar sidereal rotation parameters determined tracing LTRs, magnetic features and measuring Doppler shifts. Rotation velocities in both solar hemispheres and from all time intervals of the measurement series are combined together. Abbreviations: SW – statistical weights, SHC – selective height correction, Rem. – remark.

Tracer/method, time	A	$\pm M_A$	$-B$	$\pm M_B$	$-C$	$\pm M_C$	Rem.
LTRs, 1979–1991	14.25	0.06	1.25	0.19	1.25	0.19	1
LTRs, 1979–1991	14.31	0.03	1.18	0.11	1.18	0.11	2
LTRs, 1979–1991	14.37	0.03	1.23	0.11	1.23	0.11	3
LTRs, 1979–1991	14.16	0.06	1.41	0.19	1.41	0.19	4
Magnetic, 1967–1980	14.307	0.005	1.98	0.06	2.15	0.11	5
Magnetic, 1975–1991	14.42	0.02	2.00	0.13	2.09	0.15	6
Magnetic, 1997	14.00	0.54	2.24	1.22	1.78	0.79	7
Doppler, 1966–1968	13.76		1.74		2.19		8
Doppler, 1967–1982	14.11		1.70		2.35		9
Doppler, 1967–1984	14.05		1.49		2.61		10
Doppler, 1981–1982	13.99	0.06					11
Doppler, 1983–1986	13.92	0.12					12

1. Without SW, without SHC, Paper I, Table IIc.
2. With SW, without SHC, present work, Table II.
3. With SW, with SHC procedure I, present work, Table IV.
4. Without SW, with SHC procedure II, present work, Table VI.
5. Photospheric magnetic fields, Snodgrass (1983).
6. Photospheric magnetic fields, Komm, Howard, and Harvey (1993).
7. Photospheric high-latitude magnetic fields, Deng, Wang, and Harvey (1999).
8. Photosphere, spectroscopic, Howard and Harvey (1970).
9. Photosphere, spectroscopic, Snodgrass, Howard, and Webster (1984).
10. Photosphere, spectroscopic, Snodgrass (1984).
11. Photosphere, spectroscopic, Küveler and Wöhl (1983).
12. Photosphere, spectroscopic, Lustig and Wöhl (1989).

Meridian rotation. Pivot points can be identified when the traces of two filaments cross in two successive rotations. We followed the convention (Mouradian *et al.*, 1987) to include only the filaments with at least three rotations to exclude random effects. It was found that 39 % of the whole LTR sample were located at pivot points, i.e., areas of rigid rotation. So, certainly the influence of this rigid rotation component should show up in the results of the performed calculations.

Now, we can check how the percentage of LTRs associated with rigidly rotating pivot points behaves during several time intervals and in the northern and southern solar hemisphere respectively. In the first time interval 1979–1980, the association rate amounts to 40%, in the second one (1981–1982) it is 31%, in the third one

(1987–1988) it is 22%, and in the fourth one (1989–1991) it amounts to 43%. We see that the higher association rate in the intervals 1979–1980 and 1989–1991 coincides with the higher obtained rotation velocity of LTRs at medium latitudes in these time intervals. Analogously, the lower association rate in the time intervals 1981–1982 and 1987–1988 coincides with the lower measured rotation velocity of LTRs at medium latitudes. Further, the north–south asymmetry in the association rate of LTRs and pivot points (42% in the northern hemisphere and 38% in the southern one for the whole data sample and 47% in the northern hemisphere and 42% in the southern one for the 1989–1991 interval) is responsible for the observed north–south asymmetry in the rotation velocity of LTRs. So, the observed cycle-related changes and the north–south asymmetry of the rotation velocity of LTRs are consistent with the cycle-related changes and the north–south asymmetry of the association rate between LTRs and rigidly rotating pivot points. This rigid component in the rotation velocity of LTRs is in agreement with the long persistence of some large-scale patterns, outlined by LTRs (Vršnak *et al.*, 1992).

5. Conclusions

The results, obtained with (present work) and without the statistical weights procedure (Paper I), were compared. Cycle-related changes and the north–south asymmetry of the deduced rotation velocity were confirmed and it was found that the statistical significance of the solar differential rotation parameters' changes is higher when the statistical weights procedure is applied. The statistical weights procedure is more reliable and accurate, especially when there is an uncertainty in the determination of the coordinates due to any reason (e.g., low spatial resolution in microwaves). Possible differences between the results obtained by these two procedures can be interpreted as a consequence of a higher contribution of LTRs traced at larger central meridian distances in the statistical weights procedure. So, the results of rotation velocity studies can be sensitive to the chosen way of the data reduction, and we should keep in mind all steps in the reduction procedure. This enables the deconvolution of various systematic influences on measured rotation velocities (e.g., the height correction due to projection effects) and allows for a comprehensive interpretation.

A higher angular rotation velocity was measured for LTRs associated with Hα filaments than for LTRs without associated filaments. This was interpreted as a consequence of a different height of these two types of LTR tracers (Paper II). A selective application of the height correction on LTR's positions has not removed the cycle-related changes, nor the north–south asymmetry of the solar rotation measured tracing LTRs. So, projection effects cannot explain these changes. The observed cycle-related changes and the north–south asymmetry of the rotation velocity of LTRs cannot be explained by the cycle-related changes and the north–south asymmetry of the association rate between LTRs and filaments by themselves, but rather by the cycle-related changes and the north–south asymmetry of the association rate between LTRs and rigidly rotating pivot points connected to some filaments.

The average equatorial rotation velocity of LTRs is consistent with the values obtained tracing magnetic features, but is higher than the values of Doppler measurements. The mean differential rotation of LTRs is more rigid than the differential rotation obtained tracing magnetic features and measuring Doppler shifts, which can be explained by the association rate of the LTRs' positions with pivot points.

Acknowledgements

R. B. acknowledges support from the Deutsche Forschungsgemeinschaft and Ministry for Science and Technology, Republic of Croatia. The authors are very thankful to D. Špoljarić and D. Švehla (Faculty of Geodesy, University of Zagreb) for valuable help in the reduction of the data and to M. P. Rast (HAO, Boulder) for an important comment. Finally, we would like to thank the anonymous referee, whose comments and suggestions have led to a substantial improvement of this paper.

References

Adams, W. M. and Tang, F.: 1977, *Solar Phys.* **55**, 499.
Aini Kambry, M. and Nishikawa, J.: 1990, *Solar Phys.* **126**, 89.
Balthasar, H. and Wöhl, H.: 1980, *Astron. Astrophys.* **92**, 111.
Balthasar, H. and Wöhl, H.: 1983, *Solar Phys.* **88**, 71.
Balthasar, H., Schüssler, M., and Wöhl, H.: 1982, *Solar Phys.* **76**, 21.
Balthasar, H., Vázquez, M., and Wöhl, H.: 1986, *Astron. Astrophys.* **155**, 87.
Brajša, R., Vršnak, B., Ruždjak, V., Schroll, A., Pohjolainen, S., Urpo, S., and Teräsranta, H.: 1991, *Solar Phys.* **133**, 195.
Brajša, R., Ruždjak, V., Vršnak, B., Pohjolainen, S., Urpo, S., Schroll, A., and Wöhl, H.: 1997, *Solar Phys.* **171**, 1 (Paper I).
Brajša, R., Ruždjak, V., Vršnak, B., Wöhl, H., Pohjolainen, S., and Urpo, S.: 1999a, *Solar Phys.* **184**, 281 (Paper II).
Brajša, R., Ruždjak, V., Vršnak, B., Wöhl, H., Pohjolainen, S., and Urpo, S.: 1999b, in A. Antalová, H. Balthasar, and A. Kučera (eds.), *JOSO Annual Report 1998*, 156.
Bruzek, A.: 1961, *Z. Astrophys.* **51**, 75.
Deng, Y., Wang, J., and Harvey, J.: 1999, *Solar Phys.* **186**, 13.
Dezsö, L. and Kovács, A.: 1994, *Solar Phys.* **151**, 385.
Dupree, A. K. and Henze, W.: 1972, *Solar Phys.* **27**, 271.
Escaut-Soru, I., Martres, M. J., and Mouradian, Z.: 1984, *C. R. Acad. Sci. Paris* **299**, 545.
Godoli, G., Mazzucconi, F., and Piergianni, I.: 1993, *Solar Phys.* **148**, 195.
Godoli, G., Mazzucconi, F., and Piergianni, I.: 1998, *Solar Phys.* **181**, 295.
Howard, R.: 1984, *Ann. Rev. Astron. Astrophys.* **22**, 131.
Howard, R. F.: 1990, *Solar Phys.* **126**, 299.
Howard, R. and Harvey, J.: 1970, *Solar Phys.* **12**, 23.
Howard, R., Gilman, P. A., and Gilman, P. I.: 1984, *Astrophys. J.* **283**, 373.

Howard, R. F., Harvey, J. W., and Forgach, S.: 1990, *Solar Phys.* **130**, 295.
Javaraiah, J. and Gokhale, M. H.: 1997, *Solar Phys.* **170**, 389.
Komm, R. W., Howard, R. F., and Harvey, J. W.: 1993, *Solar Phys.* **145**, 1.
Küveler, G. and Wöhl, H.: 1983, *Astron. Astrophys.* **123**, 29.
Liu, S.-Y. and Kundu, M. R.: 1976, *Solar Phys.* **46**, 15.
Lustig, G.: 1984, *Die Sterne* **60**, 295.
Lustig, G. and Wöhl, H.: 1989, *Astron. Astrophys.* **218**, 299.
Meunier, N., Nesme-Ribes, E., and Grosso, N.: 1997, *Astron. Astrophys.* **319**, 673.
Mouradian, Z., Martres, M. J., Soru-Escaut, I. and Gestelyi, L.: 1987, *Astron. Astrophys.* **183**, 129.
Pulkkinen, P. and Tuominen, I.: 1998, *Astron. Astrophys.* **332**, 748.
Roša, D., Vršnak, B., Božić, H., Brajša, R., Ruždjak, V., Schroll, A., and Wöhl, H.: 1998, *Solar Phys.* **179**, 237.
Scherrer, P. H., Wilcox, J. M., and Svalgaard, L.: 1980, *Astrophys. J.* **241**, 811.
Schröter, E.-H.: 1985, *Solar Phys.* **100**, 141.
Schröter, E.-H. and Wöhl, H.: 1975, *Solar Phys.* **42**, 3.
Sivaraman, K. R., Gupta, S. S., and Howard, R. F.: 1993, *Solar Phys.* **146**, 27.
Snodgrass, H. B.: 1983, *Astrophys. J.* **270**, 288.
Snodgrass, H. B.: 1984, *Solar Phys.* **94**, 13.
Snodgrass, H. B.: 1992, in K. L. Harvey (ed.), 'The Solar Cycle', *12th Sacramento Peak Summer Workshop, ASP Conf. Ser.* **27**, 205.
Snodgrass, H. B., Howard, R., and Webster, L.: 1984, *Solar Phys.* **90**, 199.
Stix, M.: 1989, in G. Klare (ed.), *Rev. Modern Astron.* **2**, 248.
Suzuki, M.: 1998, *Solar Phys.* **178**, 259.
Ternullo, M.: 1990, *Solar Phys.* **127**, 29.
Urpo, S., Pohjolainen, S., and Teräsranta, H.: 1992, *Solar Observations at Metsähovi in January – June 1992*, Helsinki University of Technology, Metsähovi Radio Research Station, Series A, Report 12.
Vršnak, B., Pohjolainen, S., Urpo, S., Teräsranta, H., Brajša, R., Ruždjak, V., Mouradian, Z., and Jurač, S.: 1992, *Solar Phys.* **137**, 67.
Wöhl, H.: 1983, *Solar Phys.* **88**, 65.
Wöhl, H.: 1990, in L. Dezső (ed.), 'The Dynamic Sun', *6th European Meeting on Solar Physics, Publ. Debrecen Obs.* **7**, 19.
Wöhl, H.: 1997, *Hvar Obs. Bull.* **21**, 1.
Zappala, R. A. and Zuccarello, F.: 1991, *Astron. Astrophys.* **242**, 480.