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ORIGINAL ARTICLE

The solar differential rotation: a historical view

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Abstract This brief historical review covers the period preceding the advent of helioseismology, when the author actively worked in the field of solar differential rotation and knowledge of the Sun's dynamics was limited to the observations of surface phenomena, while the interpretation of the internal rotation was demanded solely to theoretical conjectures based on fluid dynamic of convective, rotating fluids. After a survey of the observations, based on both sunspot motions and spectroscopy, which led to the laws of differential rotation, the various attempts to interpret observations on theoretical grounds and to infer the internal dynamics are discussed, also within the dynamo requirements of the velocity field. We point out that the many efforts for a correct description of the Sun's internal dynamics failed in the light of the helioseismic results. Finally, the recent theoretical efforts, essentially based on powerful computer simulations, for modeling the solar internal dynamic with the aim of reproducing the helioseismic results are outlined.

Keywords Sun: rotation · Sun: convection · Sun: differential rotation

1 Introduction

The discovery of solar differential rotation happened in 1630 when Christoph Scheiner, after Galileo's sunspot observations, first noticed that the equatorial sunspots showed a shorter rotation period than those present at higher latitudes. Since then, other scientists, like Carrington (1863), Spörer (1874), and Maunder and Maunder (1905), studied the phenomenon and determined some rotation laws. But it was not until the second half of the XXth century that the systematic studies of Newton and Nunn (1951) and Ward (1966) on sunspot motions led to a precise formulation of the law of solar differential rotation. On starting from the early years of the past century the observations of sunspot motions were integrated with the spectroscopic ones by Adams (1909), Plaskett and De Lury (1913), Plaskett (1915), and in more recent times by Cimino and Rainone (1951), Livingston (1969), and Howard and Harvey (1970).

The attempts to understand the basic mechanisms of solar differential rotation in the pre-helioseismology era were based solely on theoretical conjectures for describing the behavior of rotation in the solar convection zone with the only constraint of fitting (sometimes) the observed differential rotation at the Sun's surface. The pioneer of these studies was Biermann (1951) who introduced the basic ideas of the problem. Then Kippenhahn (1963), followed later by Sakurai (1966), Cocke (1967), Köhler (1970), and Rüdiger (1974), established a theory based on the anisotropy of viscosity that depends on the preferred action of gravity on the convective motion, causing a large scale meridian circulation that transports angular momentum toward the equator.

In the early seventies, theories based on the effect of the interaction of convection with rotation were developed, in the more treatable approximations by Boussinesq and Hering, by Busse (1970), Durney (1970, 1971), and Yoshimura and Kato (1971), who computed the onset and growing of convection in a rotating spherical shell. Gilman (1972) and later Gilman and Glatzmaier (1981) performed a direct integration of the relevant fluid dynamics equations with approximations regarding both the physics and geometry of

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the solar model. This approach demonstrated that a net angular momentum transport toward the equator by Reynolds stresses is possible, but these models could barely be applied to the real Sun.

A different approach, based on the parameterization of the effect of the interaction of convection with rotation, applied to a calculated stratification of the solar convection zone was introduced, following an original idea of Weiss (1965), by Durney and Roxburgh (1971), Belvedere and Paternò (1976, 1977), and Belvedere et al. (1980a). In these models the unknown coupling constant between convection and rotation was determined by fitting the results of calculations to the observed surface differential rotation. Later, Pidatella et al. (1986) compared the characteristics of the models based on the anisotropy of viscosity with those based on the interaction of convection with rotation.

In almost the same epoch also dynamo models of the solar activity cycle were developed by virtue of the efforts of Parker (1955), Steenbeck and Krause (1969), Roberts and Stix (1972), and Belvedere et al. (1980b). Since nonuniform rotation is one of the main ingredients of the dynamo action and this needs a particular shape of the isorotation surfaces inside the convection zone for displaying the correct butterfly diagram, the behavior of the internal rotation as calculated by theoretical models could be verified by means of dynamo model results. Most of the theories of differential rotation predicted isorotation surfaces aligned on cylinders, as a consequence of the Taylor–Proudman theorem, consistent with an angular velocity increasing outward, while the requirements of the correct dynamo action claimed an angular velocity increasing inward.

The advent of precise helioseismology in the early 80s, with the possibility of reconstructing the detailed behavior of rotation inside the convection zone, and even below it, demonstrated that none of the differential rotation theories developed in the previous twenty years predicted the correct shape of the isorotation surfaces in the solar convection zone.

2 Observations

Two main methods were used for determining the surface angular velocity of the Sun: the tracer method and the spectroscopic method. The first method is based on the assumption that the long-lived structures observed at the Sun's surface, such as sunspots and plages, play the same role as tracers in fluid dynamics experiments, with the caution that these magnetic structures do not behave as ideal tracers because of the interaction with the surrounding plasma. The second method is based on the Doppler effect measured on spectral lines observed near the east and west solar limbs at different latitudes. While the tracer method is limited to

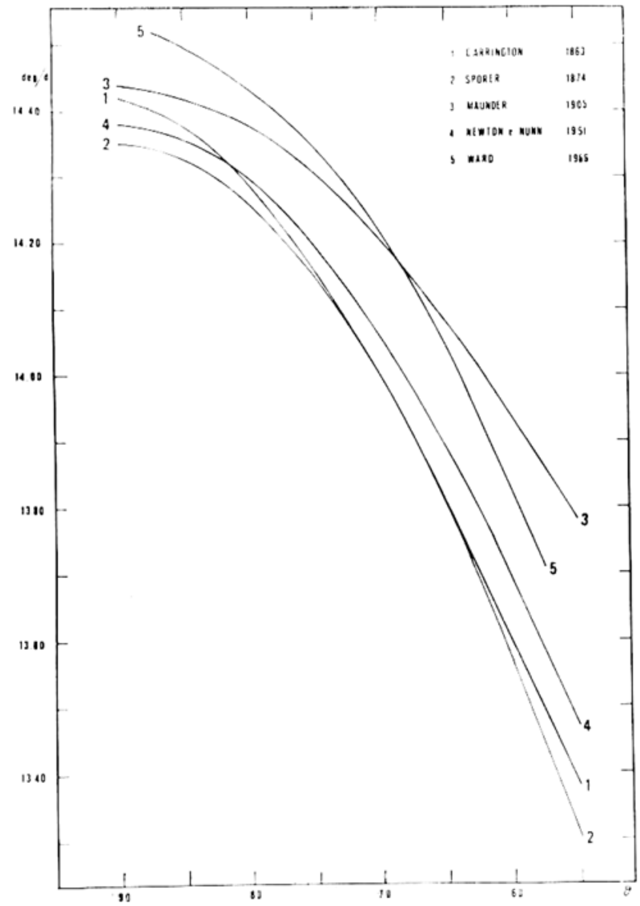

Fig. 1 Angular velocity of rotation $\Omega(\theta)$ is plotted vs. co-latitude θ as deduced from sunspot observations by various authors: (1) Carrington (1863); (2) Spörer (1874); (3) Maunder and Maunder (1905); (4) Newton and Nunn (1951); (5) Ward (1966) average value. The plot is taken from Belvedere et al. (1972). The vertical scale is expressed in deg day^{-1} from 14.40 (*top*) to 13.40 (*bottom*) with marks every 0.1 deg day^{-1}; the horizontal scale is expressed in deg from 90 (*left*) to 60 (*right*) with marks every 10 deg. Unfortunately the original of this old figure is also of bad quality

the latitudinal activity band, the spectroscopic one may be extended to higher latitudes.

Results obtained by various authors with the tracer method based on the observations of sunspot displacements are summarized in Fig. 1, where the angular velocity in deg d^{-1} is plotted vs. the co-latitude.

The differences of a few percent between the various curves may be ascribed either to measurements carried out in different epochs or to the use of different kinds of sunspots (small or large areas, single, bipolar, groups), as pointed out by Ward (1966), who analyzed the rotation laws of various type of sunspots finding a significant spread of values. The most accepted rotation law was deduced by Newton and Nunn (1951) and based on almost 70 year of sunspot observations:

$$\Omega(\varphi) = 14.38 - 2.44 \sin^2 \varphi \text{ deg d}^{-1} \quad (1)$$

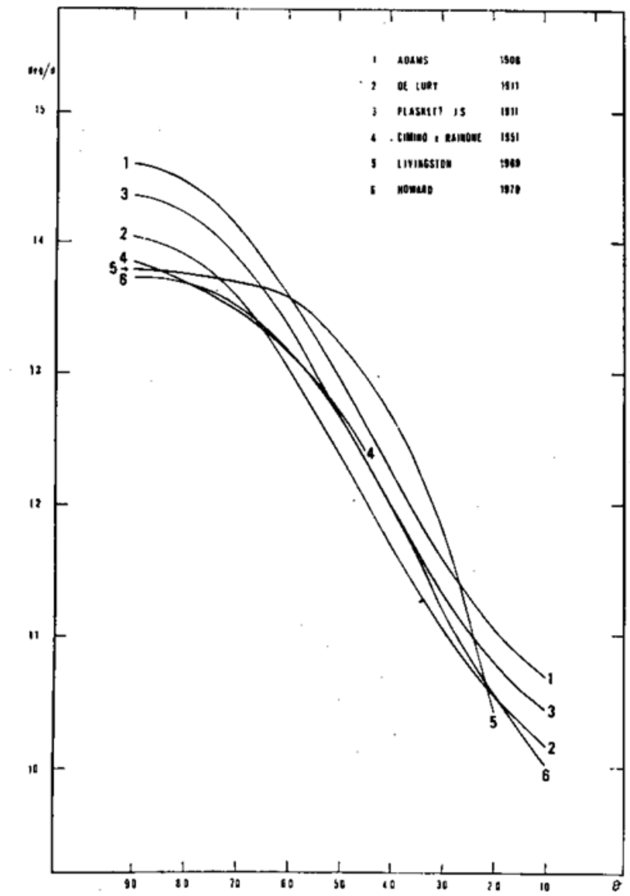

Fig. 2 Angular velocity of rotation $\Omega(\theta)$ plotted vs. co-latitude θ as deduced from spectroscopic observations by various authors: (1) Adams (1909); (2) Plaskett and De Lury (1913); (3) Plaskett (1915); (4) Cimino and Rainone (1951); (5) Livingston (1969); (6) Howard and Harvey (1970). The plot is taken from Belvedere et al. (1972). The vertical scale is expressed in deg day^{-1} from 15 (*top*) to 10 (*bottom*) with marks every 1 deg day^{-1}; the horizontal scale is expressed in deg from 90 (*left*) to 10 (*right*) with marks every 10 deg. Unfortunately the original of this old figure is also of bad quality

where φ is the latitude.

Results obtained by various authors with the spectroscopic method, either by using two points at solar limbs (old technique) or by scanning each latitude strip (modern technique), are summarized in Fig. 2, where the angular velocity in deg d^{-1} is plotted vs. the co-latitude. As in the case of sunspot measurements, a spread of a few percent is evident between the various observations, probably due either to the different techniques used or to different epochs of observation. The most accepted rotation law as deduced from spectroscopic observations was derived by Howard and Harvey (Howard and Harvey 1970):

$$\Omega(\varphi) = 13.76 - 1.74\sin^2\varphi - 2.19\sin^4\varphi \text{ deg d}^{-1} \qquad (2)$$

By comparing both curves shown in Figs. 1 and 2 and (1) and (2) it appears that rotation deduced from sunspot motions is systematically faster than that deduced from spectroscopic observations as illustrated in Fig. 3. In the complete absence of any information about the internal rotation, this was attributed to the fact that the magnetic structures, like the sunspots, were anchored to the sub-surface layers which so demonstrated to have a higher rotation rate than the surface ones.

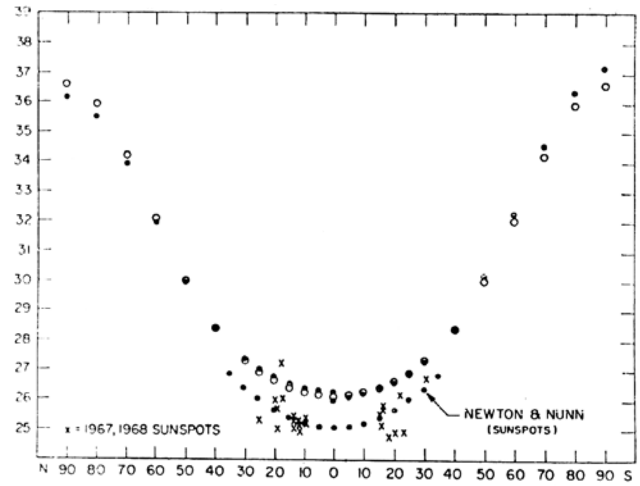

Fig. 3 Sideral rotation period as deduced from spectroscopic observations is plotted vs. solar latitude. *Open circles* represent an average between northern and southern hemispheres, while black dots near them refer to the two hemispheres separately. The *lower black dots* are the results of observations by Newton and Nunn (1951) and *crosses*—sunspot results from Mount Wilson data (from Howard and Harvey 1970)

Time variations of the solar equatorial velocity as determined by spectroscopic measurements were noted by Howard and Harvey (1970) in the period 1966–1969. These variations were attributed by the authors to local velocity fields and surface currents. In this regard, Belvedere and Paternò (1975) analyzed the spectroscopic observations carried out since the first years of 1900 and found a periodic variation of the equatorial velocity with a period of about 34 years, which they attributed to the interaction of rotation with the non-axisymmetric convection (Fig. 4).

Another argument of importance for differential rotation was the determination of the temperature difference between poles and equator. This is connected with the ideas of those times, that a temperature gradient could drive a thermal wind, thus forcing a large scale meridian circulation. In Table 1 are summarized the results of these measurements.

Though the evidence of large scale meridian circulation is scarce (Lustig and Wöhl 1990), and with an upper limit of 10 m s^{-1} difficult to be measured, correlated motions were detected at the Sun's surface by studying the displacements of structures like the faculae and sunspots. Belvedere et al. (1976) analyzed the facular motions as recorded by the Catania Astrophysical Observatory solar patrol in the period 1967–1970 and found a good correlation between the merid-

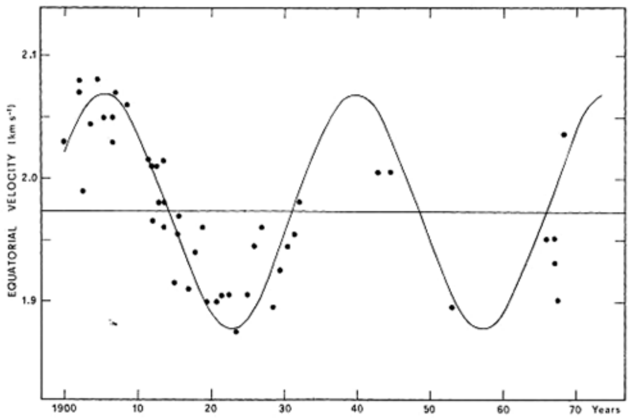

Fig. 4 Spectroscopic determinations of the solar equatorial velocity as functions of time are plotted together with the best-fit curve indicating a 34 year oscillation period (from Belvedere and Paternò 1975)

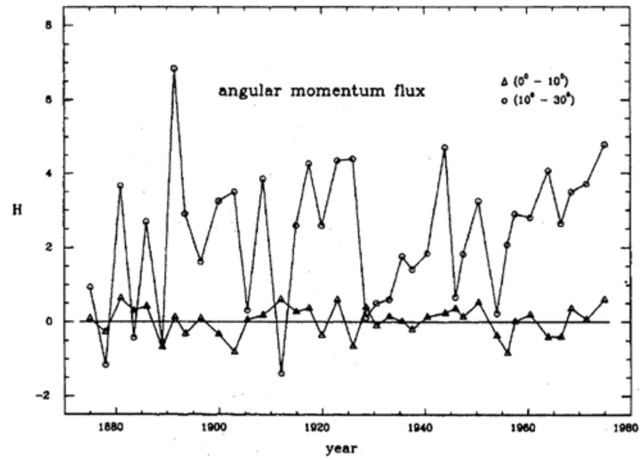

Fig. 5 Angular momentum flux H (10^6 cm^2 s^{-1}) as a function of time as deduced from the analysis of 100 year sunspot motions. *Open circles* refer to the latitudinal strip $\pm(10°–30°)$, while *triangles* to the latitudinal strip $\pm(0°–10°)$. It is evident that there is a net flux of angular momentum toward the equator ($H > 0$), especially at higher latitudes, where the action of the Coriolis force is dominant (from Paternò et al. 1991)

Table 1 Various spectroscopic determinations of the pole-equator temperature difference in the solar photosphere

Authors	ΔT
Appenzeller and Schröter (1967)	<6 K
Caccin et al. (1970)	<1%
Plaskett (1970)	≈5%
Altrock and Canfield (1972)	<1.5 K
Caccin et al. (1973)	≈0 K
Noyes et al. (1973)	<7 K
Rutten (1973)	<3 K

ian and latitudinal displacements of these structures. Later, Paternò et al. (1991) analyzed the Greenwich sunspot data in the 100 year period from 1874 to 1976 and found a firm correlation between perpendicular velocities. This correlation was attributed to the action of Reynolds stresses that demonstrated the presence of a net transport of angular momentum towards the equator able to maintain the differential rotation, as illustrated in Fig. 5.

3 Theory

The observations of the surface rotation, either deduced with the tracer method or spectroscopically, the correlated motions of surface structures, and the determinations of pole–equator temperature differences were the only information the scientists of the sixties and seventies had in their hands for constructing theories that could explain the mechanism of solar differential rotation. In addition, there was a theoretical knowledge of the internal stratification of the Sun as given by the evolutionary models that indicated the base of the convection zone placed at about $0.8R_{\odot}$, shallower than that deduced from helioseismology and modern theoretical models. No great computing resources were available at that time to approach complex fluid dynamic problems.

The proved existence of a quasi-stationary equatorial acceleration implies the action of a mechanism that transfers angular momentum toward the equator, balanced by an equal amount of viscous momentum transferred from the equator to the poles. Two plausible mechanisms can provide this action: (i) large scale meridian circulation directed toward the equator; (ii) Reynolds stresses caused by the proper correlation of perpendicular velocities. The efforts of theoreticians were devoted to investigate how this mechanism is produced and maintained in the solar convection zone. In fact, the highly speculative attempt to explain differential rotation as a fossil relic (Cowling 1953) appeared to be untenable owing to the highly turbulent character of the convection zone that would cancel any fossil effect in a short time as demonstrated by Chiu and Paternò (1992). With regard to proposed exotic mechanisms, Schatten (1973) claimed a mechanism related to the differential action of the solar wind torque with latitude. However, Gilman (1974) pointed out that these torques are negligible with respect to the internal ones caused by turbulence. Attempts to explain the differential rotation invoking Rossby waves or similar motions were made by Plaskett (1959), Ward (1964), and Gilman (1969) but without success. The Rossby waves, analogously to those propagating in the Earth's atmosphere, require a latitudinal temperature gradient not observed in the Sun's atmosphere as outlined in Table 1.

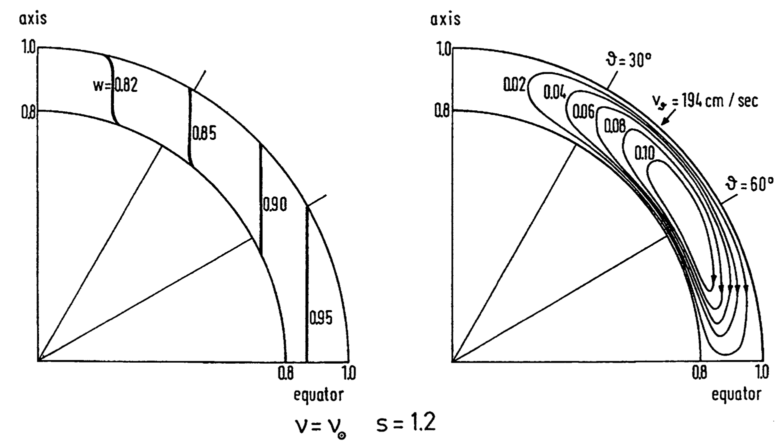

Fig. 6 The surfaces of constant angular velocity (*left*) and the stream lines of meridian circulation (*right*) as derived by Köhler (1970)

3.1 Anisotropy of viscosity

On starting from an idea of Wasiutynski (1946) on turbulent momentum exchange, Biermann (1951) first introduced the concept of anisotropy of viscosity in a turbulent fluid such as the solar convection zone claiming the preferred action of gravity on convective eddies.

Later, Kippenhahn (1963) demonstrated that, in the case of anisotropic viscosity, a state of pure rotation cannot be maintained and the conservation of angular momentum leads to the generation of a large scale meridian circulation from the poles to the equator that in turn transports angular momentum toward the equator compensated by viscous momentum transferred toward the poles in order to attain a stationary state. The correct direction of circulation was achieved only if viscosity was dominated by horizontal motions and the amount of viscous anisotropy (the parameter s for Kippenhahn) was adjusted in order to reproduce the observed differential rotation.

Following Kippenhahn (1963), Sakurai (1966), Cocke (1967), and Köhler (1970) elaborated models of differential rotation obtaining essentially the same results. To this regard, Rüdiger (1974) introduced the concept of the Λ-effect, namely the dominance of latitudinal Reynolds stresses with respect to the radial ones. In Fig. 6 are shown the typical results of one of these models based on the anisotropy of viscosity.

There are two fundamental objections to this kind of theories: the first is that the energy equation is not considered but only the momentum balance; the second is that the vertical momentum exchange seems to be more effective than the horizontal one since the convective motions extend over several pressure scale heights as first suggested by Simon and Weiss (1968). Probably the effect of momentum anisotropy is not the main mechanism responsible for differential rotation even if it should be present in a convective fluid.

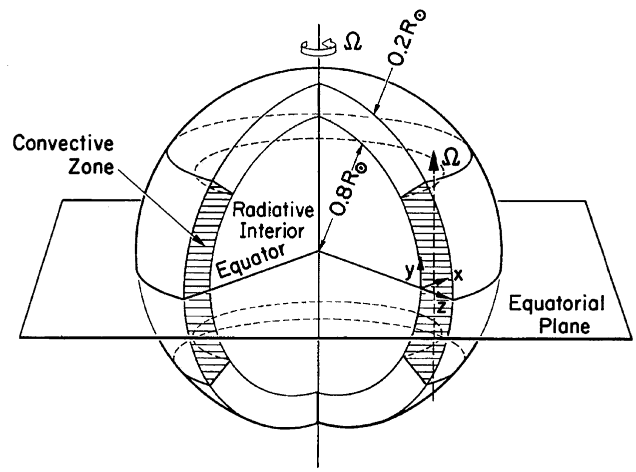

Fig. 7 The geometry of the first numerical Boussinesq model by Gilman (1972)

3.2 Global convection

A completely different approach was based on a detailed numerical study of the onset and growing of convection in a rotating spherical shell. These models, at least initially, made use of many approximations the most important of which were the Boussinesq and Herring ones. The first considers the variation of density with depth only when it is coupled with gravity, so neglecting the highly compressible character of the convection zone. The second, called also a quasi-linear approximation, considers only the coupling of contiguous convective modes, so ignoring all the other non-linear couplings.

The pioneers of this kind of approach were Busse (1970), Durney (1970, 1971) and Yoshimura and Kato (1971) who demonstrated the action of Reynolds stresses for transporting angular momentum toward the equator.

Gilman (1972) solved the relevant fluid dynamic equation in the Boussinesq approximation with a simplified geometry based on an equatorial annulus with a latitude extent of about ±40°, as shown in Fig. 7. The results are shown

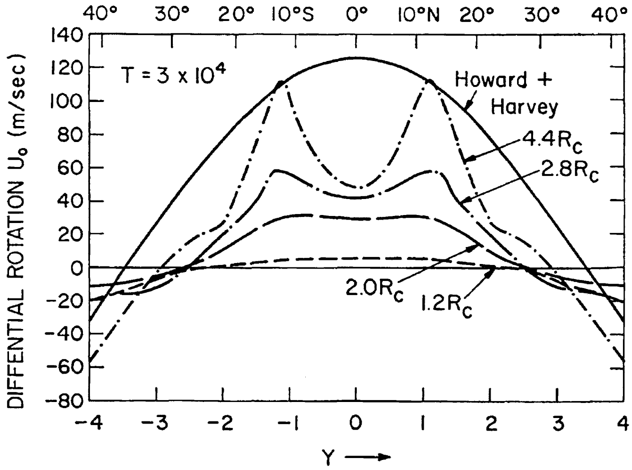

Fig. 8 The behavior of rotation is plotted vs. latitude for different Rayleigh numbers at fixed Taylor number, $T = 3 \times 10^4$; for comparison the rotation curve as deduced by spectroscopic observations of Howard and Harvey is also plotted (from Gilman 1972)

in Fig. 8 where the theoretical rotation curves obtained for different Rayleigh numbers are compared with observations denoting weak agreement. Gilman also found a meridian circulation directed toward the poles contrarily to the models based on anisotropic viscosity. Gilman (1977) and Gilman and Glatzmaier (1981) improved the model, making use of a more reliable geometry and considering compressibility and all the non-linear effects, but with essentially similar results.

These models based on numerical simulations of convection in a rotating spherical shell, though approaching the problem globally, could barely be applied to the real Sun and moreover they tended to produce significant temperature differences between the poles and the equator. However, they were particularly instructive for understanding the behavior of rotating, convective fluids, and in particular the suppression of convection for high rotation rates, an argument related to the magnetic activity in stars, and the non-axisymmetric and time-dependent character of convection; features that were used by Belvedere and Paternò (1975) to interpret the periodic time variations of equatorial angular velocity as discussed in Sect. 2.

3.3 Interaction of rotation with turbulent convection

The class of theories of interaction of rotation with turbulent convection was based on an original idea of Weiss (1965) who claimed that the effect was caused by the latitudinal variation of the angle between the rotation axis and the direction of gravity, which affects the convective heat flux, thus generating a meridian circulation that in turn transfers angular momentum toward the equator against the viscous dissipation.

Durney and Roxburgh (1971) first constructed a model of differential rotation in a very thin ($\approx 0.02 R_\odot$), compressible, spherical shell where both radiative and convective energy transports were considered. They performed a perturbation analysis of a spherically symmetric solar model in terms of Legendre polynomials, determining the perturbation parameter by fitting the theoretical solution to the observed differential rotation. They found a single cell meridian circulation of about 0.35 $\mathrm{m\,s^{-1}}$ in the outer convection zone directed toward the equator with a pole–equator temperature difference of about 70 K, the equator being hotter.

Following the same kind of approach, Belvedere and Paternò (1976) constructed a Boussinesq model of differential rotation in a deep ($0.2 R_\odot$), spherical shell. With respect to the previous work of Durney and Roxburgh they did not consider compressibility and radiative transport but used a more realistic convection zone. The results, similarly to what was obtained by Durney and Roxburgh, showed a single meridian circulation of about 0.15 $\mathrm{m\,s^{-1}}$ directed from the poles to the equator, but contrarily to them a pole–equator temperature difference of about 300 K, the poles being hotter. The surfaces of constant angular velocity were inclined smoothly with respect to the equatorial plane and shaped in such a way as to produce an angular velocity increasing toward the interior. Later, Belvedere and Paternò (1977) refined their model introducing the compressibility. They obtained results essentially similar to those of the previous model, except for the large scale circulation and pole–equator temperature difference. In the new, compressible model circulation in the convection zone was organized in three cells, the shallowest and most superficial of them with a flow of about 0.2 $\mathrm{m\,s^{-1}}$ directed toward the poles, and any pole–equator heat flux and temperature differences were smoothed out by the particular structure of circulation.

In a successive model, also in view of the first results of helioseismology, Belvedere et al. (1980a) improved the model by extending the convection zone depth to $0.3 R_\odot$, and using a more robust numerical code. The surfaces of constant angular velocity were similar to those found for the previous models and are illustrated in Fig. 9. Later on Pidatella et al. (1986) compared models based on the two approaches outlined in Sects. 3.1 and 3.3. They found that the angular velocity increased outward for all the models based on the anisotropy of viscosity, while it increased inward for most of the models based on the interaction of convection with rotation, except for a set of parameters for which it increased outward.

In the class of models based on the interaction of convection with rotation the interaction effect was taken into account through a perturbation of the convective flux coefficient:

$$k(r,\theta) = k_0(r)\big[1 + \varepsilon T_a(r) P_2(\theta)\big]$$

where $T_a = 4\Omega^2 \ell^4/\nu^2$ is the Taylor number, the ratio of centrifugal to viscous terms, with Ω a typical solar angular velocity, ℓ the mixing length, and ν the viscosity, $P_2(\theta)$

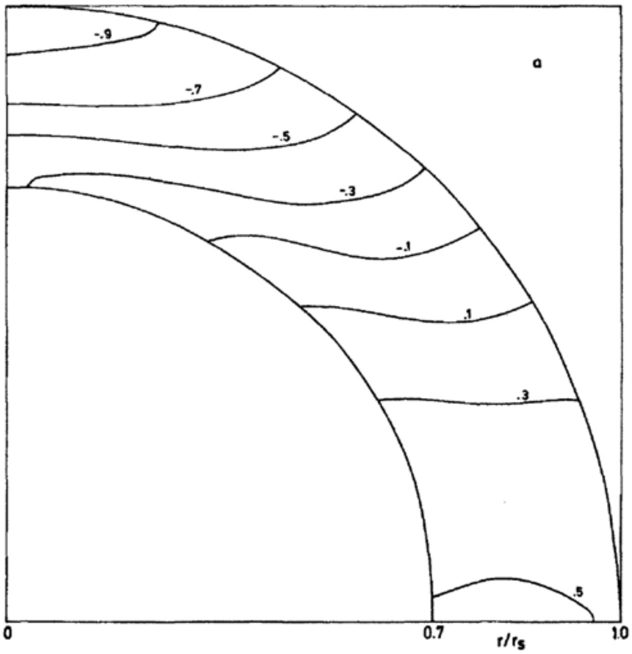

Fig. 9 Contours of constant angular velocity as derived from the model of Belvedere et al. (1980a)

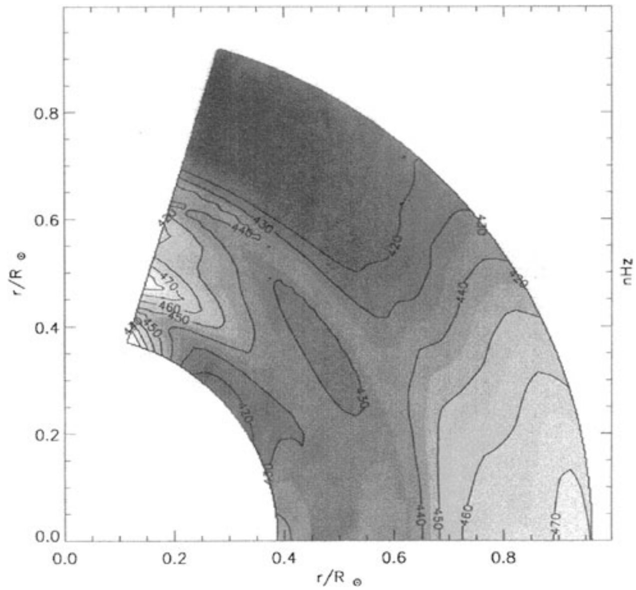

Fig. 10 Contours of constant angular velocity as obtained from an inversion of helioseismic data (from Di Mauro et al. 1998)

is the 2nd Legendre polynomial, and ε represents the coupling constant between convection and rotation that mimics all the physical processes underlying this phenomenon. It is clear that this kind of approach is highly parameterized with respect to the global convection approach and cannot capture the detailed mechanisms of the interaction, but it was better applicable to the real Sun in that it used a computed stratification of the convection zone and gave results consistent with the observations of pole–equator temperature differences (at least the latest models) and contours of angular velocity in agreement with the dynamo requirements.

4 The problem of the isorotation surfaces

After an intuition of Parker (1955), Yoshimura (1975) established the more realistic case that, during the solar activity cycle, the magnetic field migrates along the surfaces of isorotation so that, in order to have a correct dynamo action, the product $\alpha \times \partial\Omega/\partial r$, where α is the dynamo regenerative effect, should be negative in the northern hemisphere independently of the sign of the single factors. Considerations based on the helicity of convective motions and the observed phase relation between poloidal and toroidal fields indicated that $\alpha > 0$ in the northern hemisphere, and therefore $\partial\Omega/\partial r$ should have been negative, indicating an angular velocity increasing inwards as pointed out by Stix (1981). Though some models of differential rotation, essentially those based on the parameterization of the convection–rotation interaction, produced isorotation surfaces consistent with the correct dynamo action; however, full three-dimensional calculations of the dynamic of convection in a rotating, deep, compressible spherical shell tended to produce isorotation surfaces aligned on cylinders parallel to the rotation axis, which implies an angular velocity decreasing inward (see the review article by Gilman 1986).

Helioseismology has shown that the surfaces of isorotation in the solar convection zone are mainly radial with no possible dynamo action in a region where $\partial\Omega/\partial r \approx 0$. The rotational stress is concentrated in a thin overshoot layer (tachocline) at the base of the convection zone as shown in Fig. 10. Here the isorotation surfaces are aligned along cylinders at low latitudes, with Ω decreasing inward, while they are inclined toward the equator at high latitudes with Ω increasing inward. However, in the overshoot layer motions are essentially directed downward, with the consequence that α changes sign and $\alpha \times \partial\Omega/\partial r$ is negative at low latitudes, with the magnetic field migrating toward the equator, and positive at high latitudes, with the field migrating toward the poles, as the surface observations indicate.

5 The modern times

On starting from the beginnings of the XXIst century several efforts have been done for modeling the internal dynamics of the Sun by the use of full three-dimensional computer simulations in order to reproduce the differential rotation profiles as deduced from helioseismology, a task that was not imaginable with the computer facilities of the pioneering era of differential rotation studies.

Miesch et al. (2000) carried out three-dimensional simulations of the solar convection in a spherical shell considering both the case of laminar and of turbulent flow. They found that the interaction of rotation with convection induces velocity correlations and drives the differential rotation by the transport of angular momentum. The nature of the transport seems to be ascribed to the effect of an inverse cascade from the small scale turbulence to coherent, larger scale eddy structures. In the laminar regime they found that convection was structured in banana shaped cells as in the old simulations, while in the turbulent regime they noticed a weak similarity of the differential rotation profiles with the helioseismic radial ones, though the common tendency of these simulations of producing cylindrical alignment was persistent.

Brun and Toomre (2002) carried out similar simulations and established the dominant role of Reynolds stresses in transporting angular momentum toward the equator. In the case of strong turbulence, they found good agreement with the helioseismic data as far as the pole–equator rotational stress was concerned and obtained rotational profiles nearly radial at mid-latitudes.

More recently Miesch and Brun (2006), by using the same simulation approach as the previous two mentioned above, varied the thermal boundary conditions in order to ascertain whether a latitudinal entropy variation imposed at the lower boundary, which mimics the coupling between the convective envelope and radiative interior through the tachocline, can destroy the tendency of the numerical simulations to produce isorotation surfaces aligned on cylinders. They found that, as the strength of the entropy variation imposed was increased, the cylindrical profiles tended to change in conical ones with nearly radial angular velocity contours at mid-latitudes.

6 Conclusion

This review covers essentially the period from the early fifties to the early eighties during which were made significant theoretical efforts to describe the internal solar dynamic on the basis of the sole information derived from the observation of the surface motions. The advent of helioseismology, with the possibility of reconstructing the detailed behavior of the rotation inside the convection zone and even below it, demonstrated that none of the theories predicted the correct shape of the isorotation surfaces.

Some thirty years of theoretical efforts were nullified by the severe law of observations; nevertheless it was an amusing period of the life for many scientists who, like me, with the scarce computing facilities of that time, approached a very difficult problem.

However, in recent times, thanks to the availability of powerful computers, the modeling of the solar internal dynamic made remarkable progresses obtaining differential rotation profiles similar to those inferred from helioseismology.

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