Kalman filter tuning and consistency – Innovation

Sensor fusion & nonlinear filtering

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INNOVATION CONSISTENCY

Innovation consistency

The innovation $\mathbf{v}_k = \mathbf{y}_k - \mathbf{H}_k \hat{\mathbf{x}}_{k|k-1}$ should satisfy

$$p(\mathbf{v}_k | \mathbf{y}_{1:k-1}) = \mathcal{N}(\mathbf{v}_k; \mathbf{0}, \mathbf{S}_k)$$

$$\mathrm{Cov}(\mathbf{v}_k, \mathbf{v}_{k-l}) = \begin{cases} \mathrm{Cov}\{\mathbf{v}_k\} & \text{if } l = 0 \\ \mathbf{0} & \text{otherwise.} \end{cases}$$

- Note: it is enough to have a filter and a measurement sequence to compute $\mathbf{v}_1, \mathbf{v}_2, \ldots$

Proof:

$$E\{\underbrace{v_k}_{y_k - H_k \hat{x}_{k|k-1}} | y_{1:k-1}\} = \underbrace{E\{y_k | y_{1:k-1}\}}_{H_k \hat{x}_{k|k-1}} - H_k \hat{x}_{k|k-1}$$

$$= 0$$

$$\Rightarrow E\{v_k\} = 0$$

$$E\{v_k \cdot v_{k-l}^T | y_{1:k-1}\} = 0$$

$$\hookrightarrow y_{k-l} + H_{k-l} \cdot \hat{x}_{k-l|k-l-1}$$

$$\Rightarrow E\{v_k v_{k-l}^T\} = 0 \quad l > 0$$

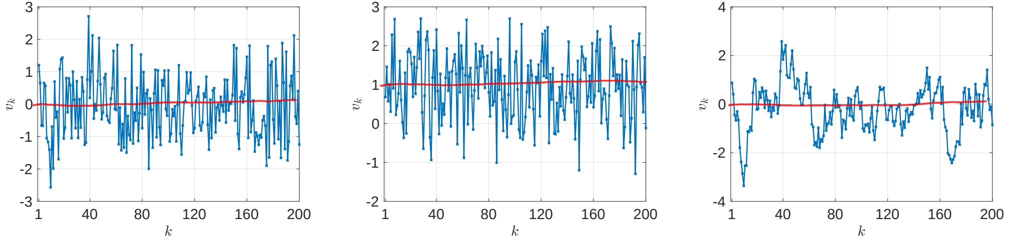
$$l = 0 \Rightarrow E\{v_k \cdot v_k^T\} = \mathrm{Cov}\{v_k\}$$

TEST OF INNOVATION PROPERTIES – VISUAL INSPECTION

- There are ways to test the properties of the innovation.

  ⇝ we will look at three methods.

- Visual inspection:
  - Zero mean?
  - uncorrelated?



TEST OF INNOVATION PROPERTIES – CONSISTENCY

Consistency

- Ideally $\mathbf{v}_k \big| \mathbf{y}_{1:k-1} \sim \mathcal{N}(\mathbf{0}, \mathbf{S}_k)$ and then

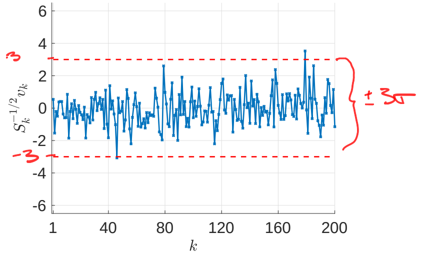
$$\mathbf{S}_k^{-1/2}\mathbf{v}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{I}) \Rightarrow \mathbf{v}_k^T\mathbf{S}_k^{-1}\mathbf{v}_k \sim \chi^2_{n_y}$$

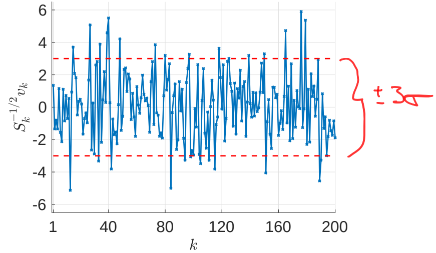
dim. of y_k

- Given a sequence $v_1, v_2, \ldots, v_K$ we can compute

$$\xi_K = \sum_{k=1}^{K} \mathbf{v}_k^T\mathbf{S}_k^{-1}\mathbf{v}_k \sim \mathcal{N}(Kn_y, 2Kn_y)$$

Within 3σ-region?





TEST OF INNOVATION PROPERTIES – CORRELATION

Whiteness

- Estimate the autocorrelation function (autocov. normalised to 1 at lag 0):

$$\rho(l) = \frac{\sum_{k=l+1}^{K} \mathbf{v}_k^T \mathbf{v}_{k-l}}{\sum_{\tau=l+1}^{K} \mathbf{v}_\tau^T \mathbf{v}_\tau}$$

and check if $\rho(l) \approx 0$ for $l > 0$.