Measurement models

Sensor fusion & nonlinear filtering

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MEASUREMENT MODELS

- A measurement model relates the measurement, $\mathbf{y}_k$, to the state vector, $\mathbf{x}_k$.
- Our models can often be expressed as

$$\mathbf{y}_k = h_k(\mathbf{x}_k) + \mathbf{r}_k, \qquad \mathbf{r}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_k).$$

  or, more generally,

$$p(\mathbf{y}_k | \mathbf{x}_k).$$

- The list of useful and important sensors is long:
  radar, laser scanners, GNSS (e.g., GPS), accelerometers, gyroscopes, cameras, etc.

EXAMPLES OF MEASUREMENT MODELS

Global navigation and satellite system (GNSS)

State: $\mathbf{x}_k = \begin{bmatrix} p_k^1 & p_k^2 & v_k^1 & v_k^2 \end{bmatrix}^T$

Observation: noisy position in 2D

$$\mathbf{y}_k = \begin{bmatrix} p_k^1 \\ p_k^2 \end{bmatrix} + \begin{bmatrix} r_k^1 \\ r_k^2 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}}_{H_k} \mathbf{x}_k + \mathbf{r}_k$$

Gyroscope (yaw-rate sensor)

State: $\mathbf{x}_k = \begin{bmatrix} p_k^1 & p_k^2 & v_k & \phi_k & \omega_k \end{bmatrix}^T$

Observation: noisy observation of yaw rate

$$\mathbf{y}_k = \omega_k + r_k = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{x}_k + r_k$$

EXAMPLES OF MEASUREMENT MODELS

Radar sensor

State: $\mathbf{x}_k = \begin{bmatrix} p_k^1 & p_k^2 & v_k^1 & v_k^2 \end{bmatrix}^T$

Observation: noisy observation of distance and angle

$$\mathbf{y}_k = \begin{bmatrix} \sqrt{(p_k^1)^2 + (p_k^2)^2} \\ \arctan\left(\frac{p_k^2}{p_k^1}\right) \end{bmatrix} + \begin{bmatrix} r_k^1 \\ r_k^2 \end{bmatrix}$$

Wheel speed encoders

State: $\mathbf{x}_k = \begin{bmatrix} p_k^1 & p_k^2 & v_k^1 & v_k^2 \end{bmatrix}^T$

Observation: noisy observation of speed

$$y_k = f_c\left(\sqrt{(v_k^1)^2 + (v_k^2)^2} + r_k\right)$$

SENSOR CALIBRATION AND BIAS FILTERING

- Suppose our sensor has an offset, or a bias, $\mathbf{s}$ such that we observe

$$\mathbf{y}_k = h_k(\mathbf{x}_k) + \mathbf{s} + \mathbf{r}_k, \qquad \mathbf{r}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_k).$$

  instead of just $\mathbf{y}_k = h_k(\mathbf{x}_k) + \mathbf{r}_k$.

- If $\mathbf{s}$ is constant, it can usually be estimated from a set of training data.

- For low-quality sensors it is common that the bias drifts significantly over time.

- Common solution: include $\mathbf{s}_k$ in the state vector and describe its motion as a random walk

$$\mathbf{s}_k = \mathbf{s}_{k-1} + \mathbf{q}^s_{k-1}.$$

  $\rightsquigarrow$ Our filter now jointly estimates the kinematic states and the bias.

SELF ASSESSMENT

In many cases, the sensor model has to be adjusted to the geometry of the particular problem at hand. Here is a simple example of that type.

Suppose that we have a radar sensor positioned at (p_s^1, p_s^2) that observes range (distance), y_k^r. Suppose again that we have a state vector $\mathbf{x}_k = \begin{bmatrix} p_k^1 & p_k^2 & v_k^1 & v_k^2 \end{bmatrix}^T$.

Select one of the following possible measurement models:

- $y_k = \sqrt{(p_k^1 - p_s^1)^2 + (p_k^2 - p_s^2)^2} + r_k$
- $y_k = \sqrt{(p_k^1)^2 + (p_k^2)^2} + r_k$
- $y_k = \sqrt{(p_s^1)^2 + (p_s^2)^2} + r_k$
- $y_k = \arctan(p_k^2 / p_k^1) + r_k$