# Sequential Importance Resampling (SIR)

Sensor fusion & nonlinear filtering

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- One can show that all SIS filters suffer from degeneracy:

  *after a few time steps all but one particle will have negligible weight.*

- Consequences of degeneracy:
  - the filter believes that it knows $\mathbf{x}_k$ exactly,
  - we obtain very poor state estimates,
  - most of our calculations are wasted on insignificant particles.

  These are very serious drawbacks!

- A key technique to improve performance is resampling.

- Challenge: we have $p(\mathbf{x}_k|\mathbf{y}_{1:k}) \approx \sum_{i=1}^{N} w_k^{(i)} \delta(\mathbf{x}_k - \mathbf{x}_k^{(i)})$ where most weights $w_k^{(i)}$ are very small.

**Idea: use Monte Carlo sampling**

- Generate independent samples $\tilde{\mathbf{x}}_k^{(1)}, \ldots, \tilde{\mathbf{x}}_k^{(N)}$ from $p(\mathbf{x}_k|\mathbf{y}_{1:k})$ and set

$$p(\mathbf{x}_k|\mathbf{y}_{1:k}) \approx \sum_{i=1}^{N} \frac{1}{N} \delta(\mathbf{x}_k - \tilde{\mathbf{x}}_k^{(i)}).$$

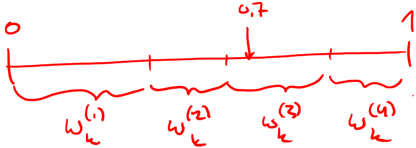
- After resampling we get
  - equal weights (they are all $1/N$),
  - multiple copies of high probability particles.

## Resampling algorithm

1) Draw $N$ samples with replacement from $\mathbf{x}_k^{(1)}, \mathbf{x}_k^{(2)}, \ldots, \mathbf{x}_k^{(N)}$, where the probability of selecting $\mathbf{x}_k^{(i)}$ is $w_k^{(i)}$.

2) Replace the old sample set with the new one and set all weights to $1/N$.

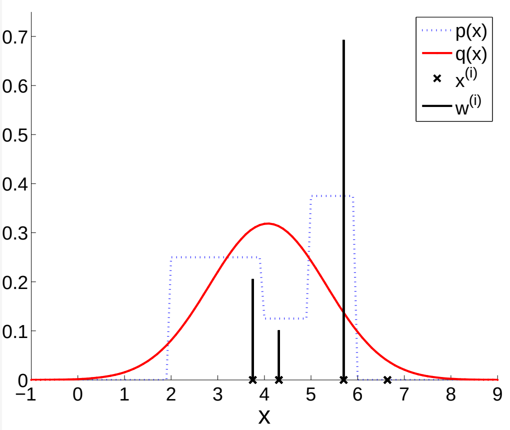
- A few remarks:
  - We use $\mathbf{x}_k^{(i)}$ and $w_k^{(i)}$ to denote the particles and their weights also after resampling.
  - We can use samples from the uniform distribution, unif[0, 1], to draw samples from the discrete distribution $p(\mathbf{x}_k | \mathbf{y}_{1:k})$.

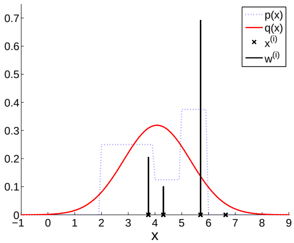


CHALMERS

## Self-assessment – Resampling

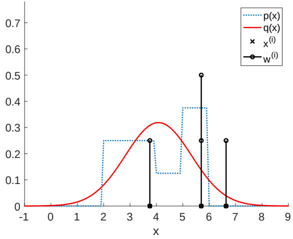
- Perform resampling on the density to the right and illustrate the result.
- Assume that the numbers 0.65, 0.03, 0.84 and 0.93 are drawn from unif[0, 1].

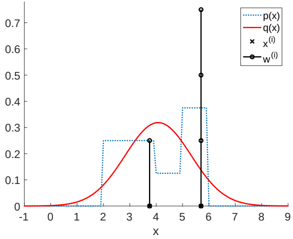


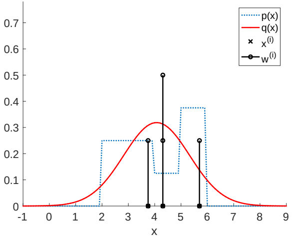



- Perform resampling on the density to the right and illustrate the result.
- Assume that the numbers 0.65, 0.03, 0.84 and 0.93 are drawn from unif[0, 1].

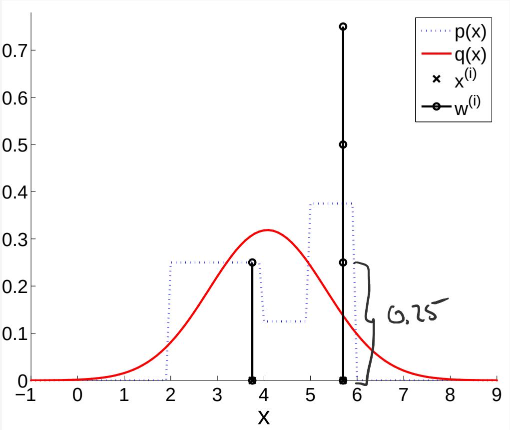
- Choose the figure below that illustrates the resampled particles:



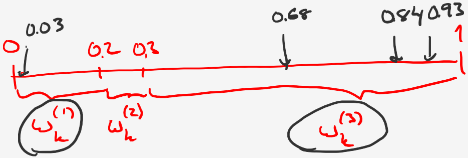



## Self-assessment – Solution




- If particles where ordered in ascending order, $x^{(1)} < \cdots < x^{(4)}$, resampling gives $x^{(1)} = 3.8$ and $x^{(2)} = x^{(3)} = x^{(4)} = 5.7$.



- Resampling costs some calculations and introduces some errors, but improves performance immensely over time.
- An estimate for the *effective number of particles* is

$$N_{eff} = \frac{1}{\sum_{i=1}^{N} \left( w_k^{(i)} \right)^2}.$$

- Many algorithms only resample when $N_{eff}$ is below some threshold, e.g., $N/4$.