Kalman filter tuning and consistency

Sensor fusion & nonlinear filtering

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A MATHEMATICAL RESULT BEFORE WE START

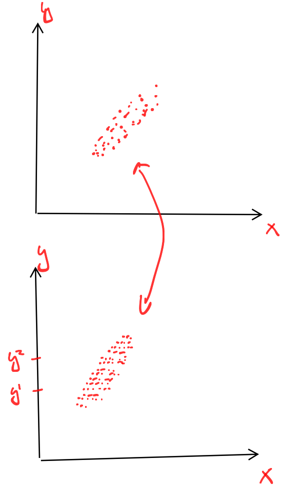
Decomposing joint expectations (Product rule)

For any two random variables x and y, it holds that

$$\mathbb{E}\{g(x,y)\} = \mathbb{E}\{\underbrace{\mathbb{E}\{g(x,y)|y\}}_{h(y)}\} = \mathbb{E}\{h(y)\}$$

$$\iint g(x,y)\underbrace{p(x,y)}_{p(x|y)p(y)}dx\,dy = \int\underbrace{\int g(x,y)p(x|y)dx}_{h(y)}\,p(y)dy$$

$$= \int h(y)p(y)dy = E\{h(y)\}$$



THE KALMAN FILTER

$x_k = \begin{bmatrix} x_1 \\ x_2 \\ \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}$

Prediction

$$\begin{cases} \hat{\mathbf{x}}_{k|k-1} = \mathbf{A}_{k-1}\hat{\mathbf{x}}_{k-1|k-1} \\ \mathbf{P}_{k|k-1} = \mathbf{A}_{k-1}\mathbf{P}_{k-1|k-1}\mathbf{A}_{k-1}^T + \mathbf{Q}_{k-1} \end{cases}$$

Update

$$\begin{cases} \hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k\mathbf{v}_k \\ \mathbf{P}_{k|k} = \mathbf{P}_{k|k-1} - \mathbf{K}_k\mathbf{S}_k\mathbf{K}_k^T \end{cases}$$

$$\begin{cases} \mathbf{K}_k = \mathbf{P}_{k|k-1}\mathbf{H}_k^T\mathbf{S}_k^{-1} \\ \mathbf{v}_k = \mathbf{y}_k - \mathbf{H}_k\hat{\mathbf{x}}_{k|k-1} \\ \mathbf{S}_k = \mathbf{H}_k\mathbf{P}_{k|k-1}\mathbf{H}_k^T + \mathbf{R}_k \end{cases}$$

Does the filter perform well?

- Have we implemented the filter correctly?
- Have we selected good model types?
- Are the covariance matrices properly tuned?

IDEAL PROPERTIES OF FILTER OUTPUTS

- The filter output is the posterior mean and covariance:

$$p(\mathbf{x}_k | \mathbf{y}_{1:k}) = \mathcal{N}(\mathbf{x}_k; \hat{\mathbf{x}}_{k|k}, \mathbf{P}_{k|k})$$

Both over x_k and $y_{1:n}$

Deterministic function of $y_{1:n}$

A well performing filter should satisfy

$$E\{E\{x_k - \hat{x}_{k|k} | y_{1:k}\}\} = \mathbb{E}\{\mathbf{x}_k - \hat{\mathbf{x}}_{k|k}\} = 0$$

$$\underbrace{\mathbb{E}\left\{\left(\mathbf{x}_k - \hat{\mathbf{x}}_{k|k}\right)\left(\mathbf{x}_k - \hat{\mathbf{x}}_{k|k}\right)^T \middle| \mathbf{y}_{1:k}\right\}}_{\mathbf{P}_{k|k}} = \underbrace{\mathbb{E}\left\{\left(\mathbf{x}_k - \hat{\mathbf{x}}_{k|k}\right)\left(\mathbf{x}_k - \hat{\mathbf{x}}_{k|k}\right)^T\right\}}_{\text{MSE}}$$

Ind. of $y_{1:k}$

- Weakness: need to know $\mathbf{x}_k$ to check these conditions!
  - $\rightsquigarrow$ simulations?
  - $\rightsquigarrow$ reference sensors in test environment?

SELF-ASSESSMENT

Why is it often difficult to check if $\mathbb{E}\{\mathbf{x}_k - \hat{\mathbf{x}}_{k|k}\}$ using real data (measurements that we have not simulated in a computer):

- It is not a good idea to approximate expected values using ensemble averaging.
- It is difficult to compute $\hat{\mathbf{x}}_{k|k}$.
- We do not know the values of $\mathbf{x}_k$.

Check all that apply.