Model Based Engineering

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Part I

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Chapter 1

Model Based Engineering

In this chapter we will introduce the Reinforcement Learning, or RL for short, paradigm. Concretely, we will try to cover the fundamental points of the RL paradigm. This chapter will touch upon the following topics

1.1 Feedback Systems

What is a feedback system?

1.1.1 Feedback Principles

Let's quickly summarize some important feedback principles

- Control objectives that is specifications. We may have qualitative or quantitative objectives.
- Derivation of the mathematical model describing the system or the component we work on
- Design the controller
- Analyze the performance

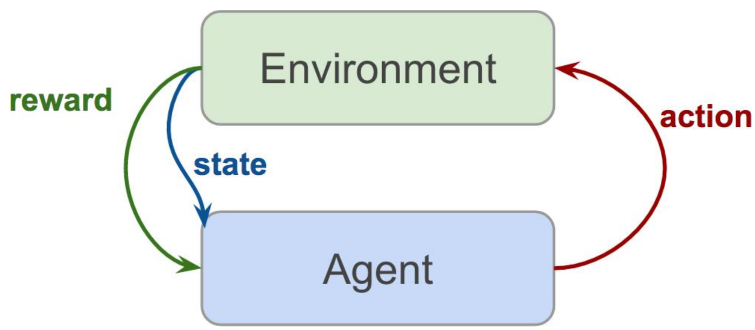

Figure 1.1: Schematics of Reinforcemtn Learning framwork.

Chapter 2

Linear Systems

Modeling of automotive systems (or any other system whatsoever) typically results in a mathematical description of the system. This mathematical description is most usually given in the form of differential equations that involve the input(s) and the output(s) of the system as well as other parameters that are of interests and affect the behavior of the system. In this chapter, we introduce the notion of a linear system (LS) and other useful terms that will be usueful in subsequent chapters. We will see that a LS can be represented in different forms susch as:

- Differential equations
- Transfer functions
- State-Space models

2.1 Linear ODEs

Let's consider the ODE:

$$\frac{d^n y}{dt^n} + \alpha_1 \frac{d^{n-1} y}{dt^{n-1}} + \cdots + \alpha_n y = \beta_1 \frac{d^{n-1} u}{dt^{n-1}} + \cdots + \beta_n u \tag{2.1}$$

where u represents the input to the system and y represents the output. α_i and β_i are coefficients which may or may not be time dependent. In the case that the coefficients do not depend on the time variable t we have a linear time invariant (LTI) system. Otherwise the system will be time variant.

Every solution to such a system can be written as the sum (because the system is linear superposition of solutions...) of a solution $y_h$to the homogeneous equation and a particular solution y_p:

$$y = y_h + y_p \tag{2.2}$$

The solution to the homogeneous solution will have the form:

$$y_h = \sum_{k=1}^{n} C_k e^{s_k t} \tag{2.3}$$

where s_k are the roots of the so called characteristic equation or polynomial. C_k can be determined from the initial conditions. Now that we have established the general form of the homogeneous solution, let's turn our attention to the particular solution. In order to find the particular solution y_p, we need the input signal u. Let's assume a damped sinusoidal input signal of the following form

$$u(t) = e^{st} \tag{2.4}$$

where s is a complex variable. The particular solution is assumed to have the follwoing general form:

$$y_p = G(s)e^{st} \tag{2.5}$$

The function $G(s)$ is called the transfer function TF. Thus, the general solution of the equation 2.1 is

$$y(t) = \sum_{k=1}^{n} C_k e^{s_k t} + G(s)e^{st} \tag{2.6}$$

The first part gives the dependency due to the initial conditions. The second part is due to the input signal.

2.1.1 Characteristic Equation

In equation 2.6 the s_ks are the roots of the characteristic equation or polynomial $\alpha(s)$. This polynomial is formed from the coefficients of the system 2.1. Hence, for the system

$$\frac{d^n y}{dt^n} + \alpha_1 \frac{d^{n-1} y}{dt^{n-1}} + \cdots + \alpha_n y = \beta_1 \frac{d^{n-1} u}{dt^{n-1}} + \cdots + \beta_n u$$

we will have

$$\alpha(s) = s^n + \alpha_1 s^{n-1} + \cdots + \alpha_n \tag{2.7}$$

Thus, the s_k will be the roots of the equation

$$\alpha(s) = 0 \tag{2.8}$$

Concretely, the solutions s_k are called the poles of the system and play a crucial role in the stability of the solution:

- If all the roots of $\alpha(s) = 0$ have $Re(s_k) < 0$ then the system is asymptotically stable
- If any of the roots of $\alpha(s) = 0$ have $Re(s_k) > 0$ then the system is unstable

Example

Consider the system

$$\dot{x} = x + u$$

Form the characteristic equation. Is the solution stable or not?

Answer

The characteristic equation of the system above is

$$\alpha(s) = s - 1 = 0$$

The only root to this equation is $s = 1$ and since $Re(s) > 0$ the system is not asymptotically stable.

2.1.2 Laplace Transformation

2.2 State-Space models

A state-space formulation of a linear system has the following form

$$\dot{x} = Ax + Bu \tag{2.9}$$
$$y = Cx + Du \tag{2.10}$$

where

- $x \in R^n$ is the state vector
- $y \in R^n$ is the output vector

- $u \in R^p$ is the input vector
- $A \in R^{n\times n}$ is the matrix describing the dynamics
- $B \in R^{n\times p}$ is the matrix describing the input
- $C \in R^{q\times n}$ is the output or sensor matrix
- $D \in R^{q\times p}$ is the direct matrix

Just like in equation 2.1, if the coefficient matrices do not depend on time, we have an LTI system.

2.3 Questions

Question 1

Consider the following model:

$$\dot{x} = x + u$$

We saw in example 2.1.1 that the solution to this system is asymptotically stable. Cast the system in the

$$\frac{dx}{dt} = Ax + Bu$$

form and argue again about its stability.

Answer

We can write the given system in the form above if we assume that

$$A = [1] \ \text{ and } \ B = [1]$$

The stability depends on the eignevalues of A here we have only one eigenvalue which is $\lambda = 1$. Thus, since $Re(\lambda) > 0$ the system is not asymptotically stable as we concleded in example 2.1.1

Question 2

Consider the following model:

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + x = u$$

compute the poles of the equation and argue about the stability.

Answer

The poles of the equation are the solutions of the characteristic equation:

$$\alpha(s) = s^2 + 2s + 1 = 0$$

The discriminant of the quadratic equation is

$$\Delta = b^2 - 4ac = 0$$

Hence, the characteristic polynomial has only one real solution given by

$$s = \frac{-b}{2a} = -1$$

Thus, since $Re(s) < 0$ the system is asymptotically stable.

Chapter 3

Examples I

In this chapter we will present some examples in order to reinforce and clarify various topics introduced in the previous chapters

3.1 PI Cruise Controller

In this example we will analyse the model of the closed loop cruise controller. This is shown in the image below

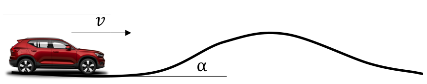

Figure 3.1: Schematics of PI close loop cruise controller.

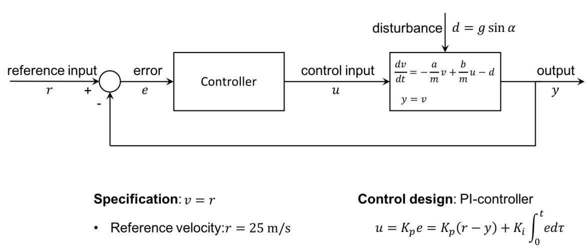

Figure 3.2: Schematics of PI close loop cruise controller.

3.1.1 Questions

Question 1:

Which features are representative for a closed loop controller?

- A) Acts on deviation
- B) No risk for instability
- C) Risk for instability
- D) Insensitive to measurement noise

Answer: A closed loop controller acts on the deviation from the reference signal in a measure-decide-act cycle. So option A is correct. Closed loop controllers are subject to instability problems; the closed loop dynamics can be shaped in such a way that the system might become unstable. So option C is also correct.

Question 2:

Which features are representative for an open loop controller?

- A) Acts on deviation
- B) No risk for instability
- C) Risk for instability
- D) Insensitive to measurement noise

Answer: In an open loop controller all control actions are preprogrammed (otherwise we have no control). Thus, option A is correct. Option B is also correct provided that the system is also stable. In this case the open loop controller will also be stable. For an open loop controller no measurements are required, so no sensitivity towards measurements (VERIFY THIS). Thus, option D is also correct.

Question 3:

Would you say that a driver is closed loop or open loop controller?

Answer: Since a driver typically aquires feedback via his/her senses and acts accordingly in order to adapt to the measured feedback, we can say that a driver is a closed loop controller.

Appendices