Assumed density filters

Sensor fusion & nonlinear filtering

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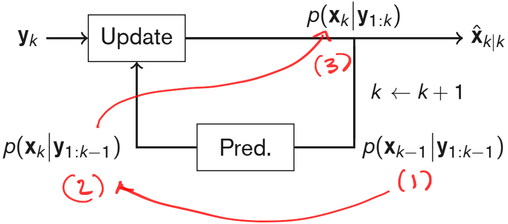
GENERAL FILTERING STRATEGIES

- Performing nonlinear filtering well is difficult!

Methodology

- Recursively compute $p(\mathbf{x}_k | \mathbf{y}_{1:k})$.
- Find an estimate $\hat{\mathbf{x}}_{k|k}$ based on $p(\mathbf{x}_k | \mathbf{y}_{1:k})$.

- Most methods follow the below strategy, but what else do they have in common?



GAUSSIAN FILTERING

Gaussian Filtering

1. Start each recursion with
   $p(\mathbf{x}_{k-1} | \mathbf{y}_{1:k-1}) \simeq \mathcal{N}(\mathbf{x}_{k-1}; \hat{\mathbf{x}}_{k-1|k-1}, \mathbf{P}_{k-1|k-1})$.
2. Prediction step: Find
   $p(\mathbf{x}_k | \mathbf{y}_{1:k-1}) \simeq \mathcal{N}(\mathbf{x}_k; \hat{\mathbf{x}}_{k|k-1}, \mathbf{P}_{k|k-1})$
3. Update step: Find
   $p(\mathbf{x}_k | \mathbf{y}_{1:k}) \simeq \mathcal{N}(\mathbf{x}_k; \hat{\mathbf{x}}_{k|k}, \mathbf{P}_{k|k})$.

Key property:

- Both $p(\mathbf{x}_{k-1} | \mathbf{y}_{1:k-1})$ and $p(\mathbf{x}_k | \mathbf{y}_{1:k})$ are Gaussian $\Rightarrow$ we have a recursive algorithm!

Note:

- KF, EKF, and IEKF are all examples of Gaussian filters.
- Although $p(\mathbf{x}_k | \mathbf{y}_{1:k-1})$ is also approximated as Gaussian, this is not a requirement.

ASSUMED DENSITY FILTERS

General solution

- A wide range of Bayesian filters can be written on the following form:
  1. Select a density parameterization $p(\mathbf{x}; \boldsymbol{\theta})$.
  2. Start from

$$p(\mathbf{x}_{k-1} \middle| \mathbf{y}_{1:k-1}) \approx p(\mathbf{x}_{k-1}; \boldsymbol{\theta}_{k-1|k-1})$$

and find $\boldsymbol{\theta}_{k|k}$ such that

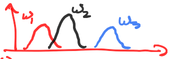
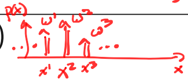
$$p(\mathbf{x}_k \middle| \mathbf{y}_{1:k}) \approx p(\mathbf{x}_k; \boldsymbol{\theta}_{k|k})$$

Note:

- The strategy is sometimes called assumed density filtering or canonical form filtering.

ASSUMED DENSITY FILTERS

Some important examples:

Assumed density	**Filters**
$p(\mathbf{x};\boldsymbol{\theta}) = \mathcal{N}(\mathbf{x};\boldsymbol{\mu},\mathbf{P})$ 	KF, EKF, UKF, CKF, NN, GNN, PDA, JPDA.
$p(\mathbf{x};\boldsymbol{\theta}) = \sum_{i=1}^{N} w_i \mathcal{N}(\mathbf{x};\boldsymbol{\mu}_i,\mathbf{P}_i)$ 	IMM, MHT.
$p(\mathbf{x};\boldsymbol{\theta}) = \sum_{i=1}^{N} w^{(i)} \delta(\mathbf{x}-\mathbf{x}^{(i)})$ 	Particle filters.
$p(\mathbf{x}_l,\mathbf{x}_n;\boldsymbol{\theta}) = \sum_{i=1}^{N} w^{(i)} \delta(\mathbf{x}_n-\mathbf{x}_n^{(i)}) \mathcal{N}(\mathbf{x}_l;\boldsymbol{\mu}^{(i)},\mathbf{P}^{(i)})$	Rao-Blackwellized particle filters.

- Algorithms in blue are covered in this course, whereas algorithms in red are related to data association problems.

SELF ASSESSMENT

Which of the following statements are true? Check all that apply!

- Our choice of density parametrization will both effect the performance of our filter and its computational complexity.
- In real-time applications, having a recursive filter means that our compute time for producing a new estimate is more or less constant with each new measurement.
- In nonlinear filtering, considering one measurement at a time (typical recursive filter) will produce better or equally good approximations of the true posterior as if we were allowed to consider the complete measurement history (non-recursive filter).
- A prerequisite for a recursive filter is that the prior and posterior density in each recursion can be described using the same density parametrization.