The extended Kalman filter – Examples and remarks

Sensor fusion & nonlinear filtering

Lars Hammarstrand

EKF: THE UPDATE STEP

$p(x|y) \propto p(x,y) = p(y|x)p(x)$

EKF update step – Example 1

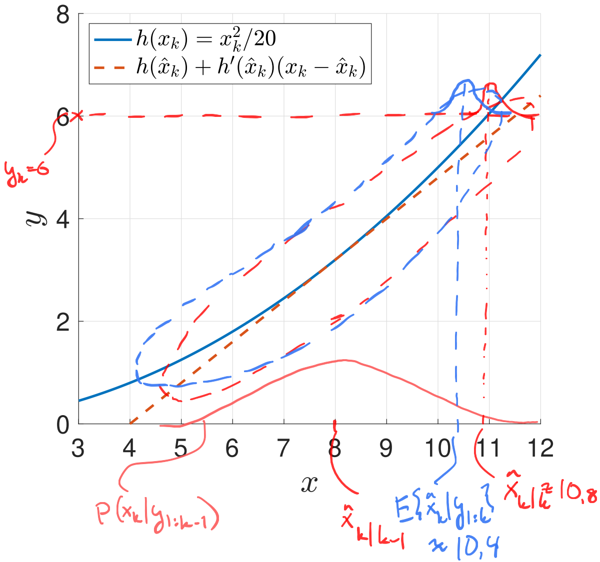
- Prediction:

$$x_k | y_{1:k-1} \sim \mathcal{N}(8, 3.5^2)$$

- Measurements: $y_k = 6$

$$y_k = h(x_k) + r_k$$
$$h(x_k) = x_k^2/20$$
$$r_k \sim \mathcal{N}(0, 2)$$

- Find the posterior mean $\mathbb{E}\{x_k | y_{1:k}\}$ and the approximate posterior mean $\hat{x}_{k|k}$ given by EKF.



EKF: THE UPDATE STEP

EKF update step – Example 2

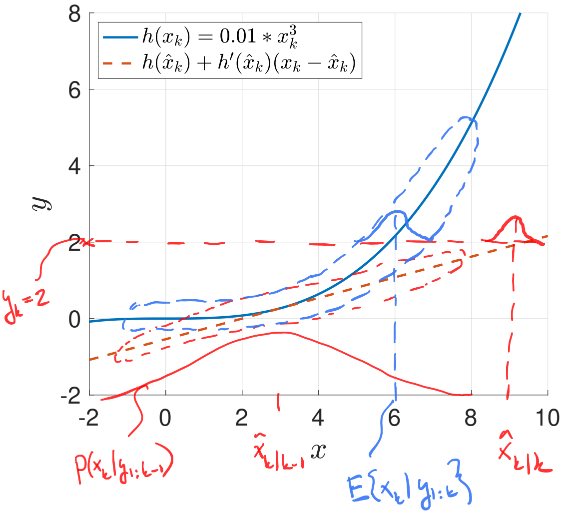
- Prediction:

$$x_k | y_{1:k-1} \sim \mathcal{N}(3, 3.5^2)$$

- Measurement: $y_k = 2$

$$y_k = h(x_k) + r_k$$
$$h(x_k) = 0.01 x_k^3$$
$$r_k \sim \mathcal{N}(0, 0.3)$$

- Find the posterior mean $\mathbb{E}\{x_k | y_{1:k}\}$ and the approximate posterior mean $\hat{x}_{k|k}$ given by EKF.



EKF: REMARKS

- The EKF recursively computes Gaussian approximations to $p(\mathbf{x}_k | \mathbf{y}_{1:k-1})$ and $p(\mathbf{x}_k | \mathbf{y}_{1:k})$.
- The EKF is also approximately LMMSE.
- As long as the systems are not too nonlinear, the EKF tends to perform well.
- To implement an EKF you need to compute derivatives of the nonlinear functions.
- Compared to other nonlinear filters that we study, it has the lowest computational complexity.

SELF ASSESSMENT

Which of the following statements are true? Check all that apply!

- The ordinary Kalman filter can be used also for nonlinear models but it will perform worse than, e.g., the extended Kalman filter.
- If we have a mix of linear and nonlinear models, let's say that, e.g., the motion model is linear while the measurement model is nonlinear. In this case, it is appropriate to combine the Kalman filter prediction step with the EKF update step to approximate the posterior.
- For linear and Gaussian state space models, the EKF is equivalent to the Kalman filter.
- We can use the EKF for all nonlinear state space models, but it will only be accurate for "mildly" nonlinear models.