# Reinforcement Learning

## Function approximation and policy gradient methods

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# Disclaimer

All this material is a free re-arrangement of David Silver's UCL Course on RL.

You are also encouraged to take a look to his Youtube lectures.

# Introduction

# Large scale reinforcement learning



Reinforcement learning can be used to solve large problems, e.g.

- Backgammon: $10^{20}$ states
- Computer Go: $10^{170}$ states
- Helicopter: continuous state space

# Large scale reinforcement learning

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- Helicopter: continuous state space

How can we scale up the model-free methods for prediction and control from the last lecture?

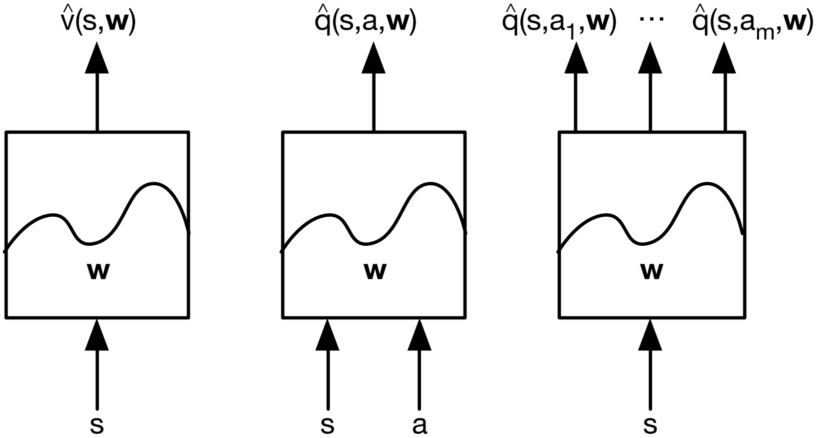
# Value function approximation

- So far we have represented value function by a lookup table
  - Every state $s$ has an entry $V(s)$
  - Or every state-action pair $s, a$ has an entry $Q(s, a)$
- Problem with large state spaces:
  - There are too many states and/or actions to store in memory
  - It is too slow to learn the value of each state individually
- Solution for large state spaces:
  - Estimate value function with function approximation

$$\hat{v}(s, \mathbf{w}) \approx v_\pi(s)$$
$$\hat{q}(s, a, \mathbf{w}) \approx q_\pi(s, a)$$

  - Generalise from seen states to unseen states
  - Update parameter $w$ using MC or TD learning

# Types of value function approximation



# Which function approximator?

There are many function approximators, e.g.

- Linear combinations of features
- Neural network
- Decision tree
- Nearest neighbour
- Fourier / wavelet bases
- ...

# Which function approximator?

We consider differentiable function approximators, e.g.

- Linear combinations of features
- Neural network
- Decision tree
- Nearest neighbour
- Fourier / wavelet bases
- ...

Furthermore, we require a training method that is suitable for non-stationary, non-iid data

# Incremental methods

# Gradient descent

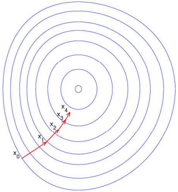
- Let $J(\mathbf{w})$ be a differentiable function of parameter vector $\mathbf{w}$
- Define the gradient of $J(\mathbf{w})$ to be

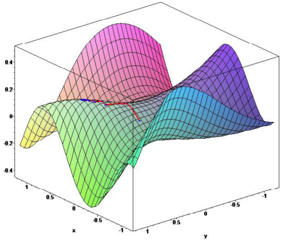
$$\nabla_{\mathbf{w}} J(\mathbf{w}) = \begin{pmatrix} \frac{\delta J(\mathbf{w})}{\delta \mathbf{w}_1} \\ \vdots \\ \frac{\delta J(\mathbf{w})}{\delta \mathbf{w}_n} \end{pmatrix}$$

- To find a local minimum of $J(\mathbf{w})$
- Adjust $\mathbf{w}$ in direction of -ve gradient

$$\Delta \mathbf{w} = -\frac{1}{2} \alpha \nabla_{\mathbf{w}} J(\mathbf{w})$$

where $\alpha$ is a step-size parameter



# Value function approximation by SGD

- Goal: find parameter vector $\mathbf{w}$ minimising mean-squared error between approximate value function $\hat{v}(s, \mathbf{w})$ and true value function $v_\pi(s)$

$$J(\mathbf{w}) = \mathbb{E}_\pi[(v_\pi(S) - \hat{v}(S, \mathbf{w}))^2]$$

- Gradient descent finds a local minimum

$$\begin{aligned} \Delta \mathbf{w} &= -\frac{1}{2}\alpha \nabla_{\mathbf{w}} J(\mathbf{w}) \\ &= \alpha E_\pi[(v_\pi(S) - \hat{v}(S, \mathbf{w}))\nabla_{\mathbf{w}} \hat{v}(S, \mathbf{w})] \end{aligned}$$

- Stochastic gradient descent samples the gradient

$$\Delta \mathbf{w} = \alpha(v_\pi(S) - \hat{v}(S, \mathbf{w}))\nabla_{\mathbf{w}} \hat{v}(S, \mathbf{w})$$

- Expected update is equal to full gradient update

# Feature vectors



- Represent state by a feature vector

$$\mathbf{x}(S) = \begin{pmatrix} \mathbf{x}_1(S) \\ \vdots \\ \mathbf{x}_n(S) \end{pmatrix}$$

- For example:
  - Distance of robot from landmarks
  - Trends in the stock market
  - Piece and pawn configurations in chess

# Linear value function approximation

- Represent value function by a linear combination of features

$$\hat{v}(S, \mathbf{w}) = \mathbf{x}(S)^T \mathbf{w} = \sum_{j=1}^{n} \mathbf{x}_j(S)\mathbf{w}_j$$

- Objective function is quadratic in parameters $\mathbf{w}$

$$J(\mathbf{w}) = \mathbb{E}_\pi[(v_\pi(S) - \mathbf{x}(S)^T \mathbf{w})^2]$$

- Stochastic gradient descent converges on global optimum
- Update rule is particularly simple

$$\nabla_{\mathbf{w}} \hat{v}(S, \mathbf{w}) = \mathbf{x}(S)$$

$$\Delta w = \alpha(v_\pi(S) - \hat{v}(S, \mathbf{w}))\mathbf{x}(S)$$

$$\text{Update} = \textit{stepsize} \times \textit{predictionerror} \times \textit{featurevalue}$$

# Table Lookup Features

- Table lookup is a special case of linear value function approximation
- Using table lookup features

$$\mathbf{x}^{table}(S) = \begin{pmatrix} \mathbf{1}(S = s_1) \\ \vdots \\ \mathbf{1}(S = s_n) \end{pmatrix}$$

- Parameter vector **w** gives value of each individual state

$$\hat{v}(S, \mathbf{w}) = \begin{pmatrix} \mathbf{1}(S = s_1) \\ \vdots \\ \mathbf{1}(S = s_n) \end{pmatrix} \cdot \begin{pmatrix} \mathbf{w}_1 \\ \vdots \\ \mathbf{w}_n \end{pmatrix}$$

# Incremental prediction algorithms

- Have assumed true value function $v_\pi(s)$ given by supervisor
- But in RL there is no supervisor, only rewards
- In practice, we substitute a *target* for $v_\pi(s)$
    - For MC, the target is the return $G_t$

$$\Delta \mathbf{w} = \alpha(G_t - \hat{v}(S_t, \mathbf{w}))\nabla_{\mathbf{w}} \hat{v}(S_t, \mathbf{w})$$

    - For TD(0), the target is the TD target $R_{t+1} + \gamma \hat{v}(S_{t+1}, \mathbf{w})$

$$\Delta \mathbf{w} = \alpha(R_{t+1} + \gamma \hat{v}(S_{t+1}, \mathbf{w}) - \hat{v}(S_t, \mathbf{w}))\nabla_{\mathbf{w}} \hat{v}(S_t, \mathbf{w})$$

# Monte-Carlo with value function approximation

- Return $G_t$ is an unbiased, noisy sample of true value $v_\pi(S_t)$
- Can therefore apply supervised learning to "training data":

$$\langle S_1, G_1 \rangle, \langle S_2, G_2 \rangle, \ldots, \langle S_T, G_T \rangle$$

- For example, using *linear Monte-Carlo policy evaluation*

$$\begin{aligned} \Delta \mathbf{w} &= \alpha({\color{red}G_t} - \hat{v}(S_t, \mathbf{w})) \nabla_{\mathbf{w}} \hat{v}(S_t, \mathbf{w}) \\ &= \alpha(G_t - \hat{v}(S_t, \mathbf{w})) \mathbf{x}(S_t) \end{aligned}$$

- Monte-Carlo evaluation converges to a local optimum
- Even when using non-linear value function approximation

# TD learning with value function approximation

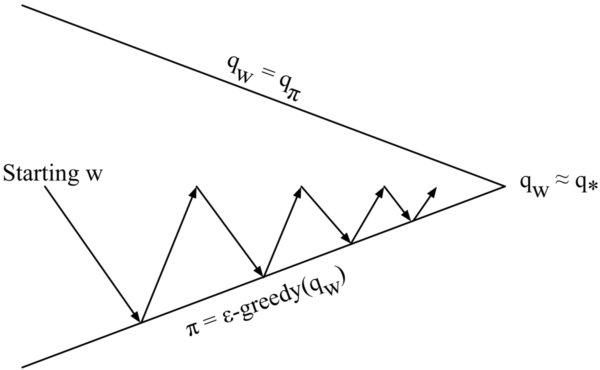
- The TD-target $R_{t+1} + \gamma \hat{v}(S_{t+1}, \mathbf{w})$ is a *biased* sample of true value $v_\pi(S_t)$
- Can still apply supervised learning to "training data":

$$\langle S_1, R_2 + \gamma \hat{v}(S_2, \mathbf{w}) \rangle, \langle S_2, R_3 + \gamma \hat{v}(S_3, \mathbf{w}) \rangle, \ldots, \langle S_{T-1}, R_T \rangle$$

- Linear TD(0) update is

$$\begin{aligned} \Delta \mathbf{w} &= \alpha (R + \gamma \hat{v}(S', \mathbf{w}) - \hat{v}(S, \mathbf{w})) \nabla_{\mathbf{w}} \hat{v}(S, \mathbf{w}) \\ &= \alpha \delta \mathbf{x}(S) \end{aligned}$$

- Linear TD(0) converges (close) to global optimum




Policy evaluation Approximate policy evaluation, $\hat{q}(\cdot, cdot, \mathbf{w}) \approx q_\pi$

Policy improvement $\epsilon$-greedy policy improvement

# Action-value function approximation

- Approximate the action-value function

$$\hat{q}(S, A, \mathbf{w}) \approx q_\pi(S, A)$$

- Minimise mean-squared error between approximate action-value function $\hat{q}(S, A, \mathbf{w})$ and true action-value function $q_\pi(S, A)$

$$J(\mathbf{w}) = \mathbb{E}_\pi[(q_\pi(S, A) - \hat{q}(S, A, \mathbf{w}))^2]$$

- Use stochastic gradient descent to
nd a local minimum

$$-\frac{1}{2}\nabla_\mathbf{w} J(\mathbf{w}) = (q_\pi(S, A) - \hat{q}(S, A, \mathbf{w}))\nabla_\mathbf{w}\hat{q}(S, A, \mathbf{w})$$

$$\Delta\mathbf{w} = \alpha(q_\pi(S, A) - \hat{q}(S, A, \mathbf{w}))\nabla_\mathbf{w}\hat{q}(S, A, \mathbf{w})$$

# Linear action-value function approximation

- Represent state *and* action by a *feature* vector

$$\mathbf{x}(S,A) = \begin{pmatrix} \mathbf{x}_1(S,A) \\ \vdots \\ \mathbf{x}_n(S,A) \end{pmatrix}$$

- Represent action-value function by linear combination of features

$$\hat{q}(S,A,\mathbf{w}) = \mathbf{x}(S,A)^T\mathbf{w} = \sum_{j=1}^{n} \mathbf{x}_j(S,A)\mathbf{w}_j$$

- Stochastic gradient descent update

$$\nabla_w \hat{q}(S,A,\mathbf{w}) = \mathbf{x}(S,A)$$

$$\Delta\mathbf{w} = \alpha(q_\pi(S,A) - \hat{q}(S,A,\mathbf{w}))\mathbf{x}(S,A)$$

# Incremental control algorithms

- Like prediction, we must substitute a target for $q_\pi(S, A)$
  - For MC, the target is the return $G_t$

$$\Delta \mathbf{w} = \alpha(G_t - \hat{q}(S_t, A_t, \mathbf{w}))\nabla_{\mathbf{w}}\hat{q}(S_t, A_t, \mathbf{w})$$

  - For TD(0), the target is the TD target $R_{t+1} + \gamma Q(S_{t+1}, A_{t+1})$

$$\Delta \mathbf{w} = \alpha(R_{t+1} + \gamma\hat{q}(S_{t+1}, A_{t+1}, \mathbf{w}) - \hat{q}(S_t, A_t, \mathbf{w}))\nabla_{\mathbf{w}}\hat{q}(S_t, A_t, \mathbf{w})$$