Kydland and Prescott's RBC Model

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Introduction

- Finn Kydland and Edward Prescott were awarded the 2004 Nobel Memorial Prize in Economics, in part, for their contribution to our understanding of the driving forces behind business cycles.

- They pioneered a new approach to studying business cycles that became known as *real business cycle* or *RBC* theory.

Introduction

- Edward Prescott's 1986 article "Theory Ahead of Business Cycle Measurement" provides an overview of models he developed with Kydland.
- The model described on pages 11-17 – which I'll call *Prescott's model* – explains the endogenous co-movement of output, employment, consumption, and investment over the business cycle.

Introduction

- Prescott's model is *decentralized*: the household makes some decisions and profit-maximizing firms make others and allocations are settled in markets with prices.
- Below, I describe a *centralized* version of Prescott's model: the representative household owns all resources and makes all allocation decisions
- The only consequence of the difference is that Prescott is able to model the equilibrium prices of labor and capital.

Centralized RBC Model with Labor

- A *representative household* lives for an infinite number of periods.
- Each period: consumes C_t and allocates share of available time L_t to labor activities.
- Flow of utility in period t:

$$\log(C_t) + \varphi \log(1 - L_t), \tag{1}$$

where $1 - L_t$ is the share of the household's time spent enjoying leisure.

Centralized RBC Model with Labor

- The *expected present value of lifetime utility* to the household from consuming $C_0, C_1, C_2, \ldots$ is denoted by U_0:

$$
\begin{aligned}
U_0 &= \log(C_0) + \varphi \log(1 - L_0) \\
&\qquad + \beta\left[E_0 \log(C_1) + \varphi \log(1 - L_1)\right] \\
&\qquad + \beta^2\left[E_0 \log(C_2) + \varphi \log(1 - L_2)\right] \\
&\qquad + \cdots \qquad (2) \\
&= E_0 \sum_{t=0}^{\infty} \beta^t \left[\log(C_t) + \varphi \log(1 - L_t)\right], \qquad (3)
\end{aligned}
$$

Centralized RBC Model with Labor

- The household enters period 0 with capital $K_0 > 0$.
- Production in period t:

$$F(A_t, K_t, L_t) = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{4}$$

where TFP A_t is stochastic:

$$\log A_{t+1} = \rho \log A_t + \epsilon_{t+1} \tag{5}$$

- The household's resource constraint is:

$$C_t + K_{t+1} = A_t K_t^{\alpha} L_t^{1-\alpha} + (1-\delta) K_t, \tag{6}$$

for $t = 0, 1, 2, 3, \ldots$

Centralized RBC Model with Labor

- **Optimization problem**: Each period the household chooses:
  1. Consumption for the current period
  2. Labor the current period
  3. Capital for the subsequent period

  to maximize its expected present value of lifetime utility.

- We'll solve the problem for period 0 and then generalize the solution to apply to all periods $0, 1, 2, \ldots$

Centralized RBC Model with Labor

- In period 0, the household solves:

$$\max_{C_0,L_0,K_1} E_0 \sum_{t=0}^{\infty} \beta^t \left[\log(C_t) + \varphi \log(1 - L_t) \right] \tag{7}$$

$$\text{s.t.} \quad C_t + K_{t+1} = A_t K_t^{\alpha} L_t^{1-\alpha} + (1-\delta) K_t$$

- The problem can be written as a choice of L_0 and K_1 only:

$$\max_{L_0,K_1} E_0 \sum_{t=0}^{\infty} \beta^t \Big[\log(A_t K_t^{\alpha} L_t^{1-\alpha} + (1-\delta) K_t - K_{t+1}) + \varphi \log(1 - L_t) \Big] \tag{8}$$

Centralized RBC Model with Labor

- The first-order condition with respect to L_0 is:

$$\frac{(1-\alpha)A_0 K_0^{\alpha} L_0^{-\alpha}}{C_0} = \frac{\varphi}{1-L_0} \tag{9}$$

- The first-order condition with respect to K_1 is:

$$\frac{1}{C_0} = \beta E_0 \left[\frac{\alpha A_1 K_1^{\alpha-1} L_1^{1-\alpha} + 1 - \delta}{C_1} \right] \tag{10}$$

Centralized RBC Model with Labor

- The household solves the same problem in periods $1, 2, 3, \ldots$
- So given $K_0 > 0$ and A_0, the equilibrium paths for labor, consumption, capital, and TFP are described described by:

$$\begin{aligned}
\frac{\varphi}{1 - L_t} &= \frac{(1-\alpha)A_t K_t^{\alpha} L_t^{-\alpha}}{C_t} && (11) \\
\frac{1}{C_t} &= \beta E_t \left[\frac{\alpha A_{t+1} K_{t+1}^{\alpha-1} L_{t+1}^{1-\alpha} + 1 - \delta}{C_{t+1}} \right] && (12) \\
C_t + K_{t+1} &= A_t K_t^{\alpha} L_t^{1-\alpha} + (1-\delta) K_t && (13) \\
\log A_{t+1} &= \rho \log A_t + \epsilon_{t+1} && (14)
\end{aligned}$$

Centralized RBC Model without Labor

- Computing numeric values for consumption and capital is not trivial. Recall the Euler equation:

$$\frac{1}{C_t} = \beta E_t \left[\frac{\alpha A_{t+1} K_{t+1}^{\alpha-1} L_{t+1}^{1-\alpha} + 1 - \delta}{C_{t+1}} \right] \tag{15}$$

- Consumption at date t depends on the *expectation* of consumption at date $t+1$ which in turn depends on the expectation of consumption at date $t+2$ and so on.

- Solving the problem requires numerical methods like those employed in the `linearsolve` Python package.

Figure 1: **Kydland and Prescott RBC model with labor.** Impulse responses to a one percent shock to TFP in period 5.

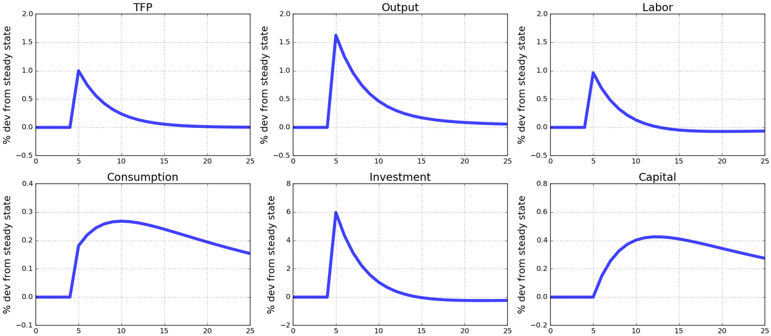

Figure 2: **GDP.** Actual and simulated data from Prescott's RBC model for the US from April 1948 to July 2018. Source: FRED.

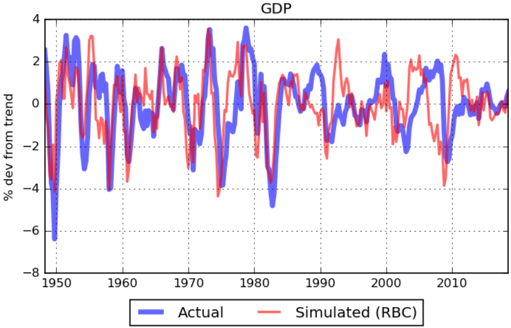

Figure 3: **Consumption.** Actual and simulated data from Prescott's RBC model for the US from April 1948 to July 2018. Source: FRED.

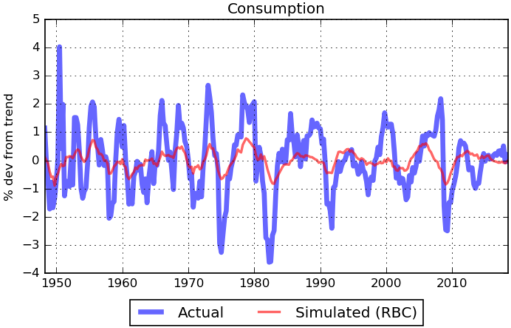

Figure 4: **Investment.** Actual and simulated data from Prescott's RBC model for the US from April 1948 to July 2018. Source: FRED.

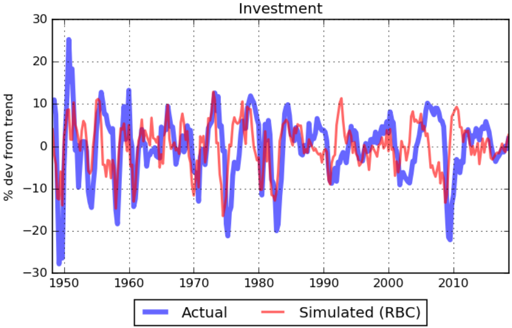

Figure 5: **Labor.** Actual and simulated data from Prescott's RBC model for the US from April 1948 to July 2018. Source: FRED.

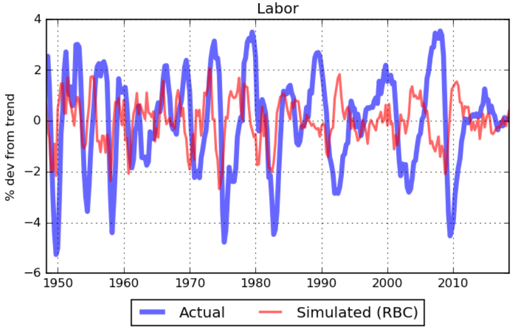

Table 1: **Standard deviations.** Actual data and data simulated from RBC models with and without labor. Units are percent deviations from trend. Source: FRED.

	Actual	RBC (w/o labor)	RBC (w/ labor)
Output	1.62	0.94	1.52
Consumption	1.16	0.22	0.32
Investment	7.50	3.39	5.56
Labor	1.89	–	0.90

Table 2: **Correlations with GDP.** Actual data and simulated data. Source: FRED.

	Actual	RBC (w/o labor)	RBC (w/ labor)
Consumption	0.79	0.62	0.59
Investment	0.85	0.99	0.99
Labor	0.87	–	0.98

Centralized RBC Model without Labor

- Summary analysis of the Prescott RBC Model with labor:
  - Substantial improvement in explaining output, consumption, investment, and labor volatility
  - Still under-predicts consumption-GDP correlation and over-predicts investment-GDP correlation

- Criticisms
  1. Attributes *all* business cycle fluctuations to TFP shocks
  2. Attributes *all* labor fluctuations to *voluntary* changes in labor supply.

References

Prescott, Edward C., "Theory Ahead of Business Cycle Measurement," *Federal Reserve Bank of Minneapolis Quarterly Review*, Fall 1986, *10* (4).