

Introduction to Real Business Cycle Modeling

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- Recall the Solow growth model with stochastic TFP:

$$Y_t = A_t K_t^\alpha \quad (1)$$

$$C_t = (1 - s) Y_t \quad (2)$$

$$Y_t = C_t + I_t \quad (3)$$

$$K_{t+1} = I_t + (1 - \delta) K_t \quad (4)$$

$$\log A_{t+1} = \rho \log A_t + \epsilon_{t+1}, \quad (5)$$

where $\epsilon_{t+1} \sim \mathcal{N}(0, \sigma^2)$.

- The model generates business cycle-like fluctuations in output, consumption, and investment.

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 - 1 Use actual TFP values for the US for A_t in equation (1) and simulate the other variables in the model.
 - 2 Compare simulated output, consumption, and investment data with the actual data
- But keep in mind that the Solow model was not designed to explain business cycles.

Figure 1: GDP. The stochastic Solow growth model does a *reasonably* good job matching GDP fluctuations for the US from April 1948 to July 2018. Source: FRED.

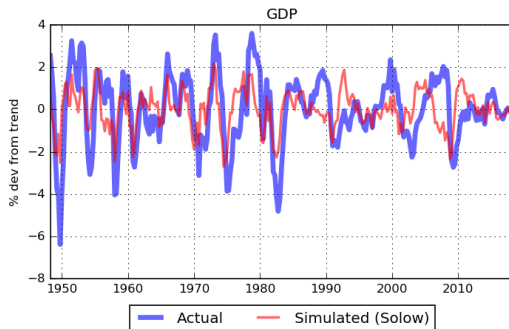


Figure 2: Consumption. The stochastic Solow growth model also does a *reasonably* good job matching consumption fluctuations for the US from April 1948 to July 2018. Source: FRED.

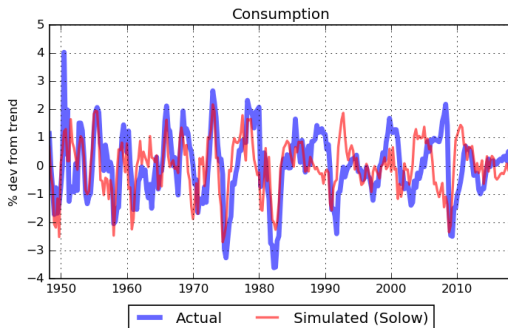


Figure 3: Investment. The stochastic Solow growth model *under-predicts* the magnitude of investment fluctuations for the US from April 1948 to July 2018. Source: FRED.

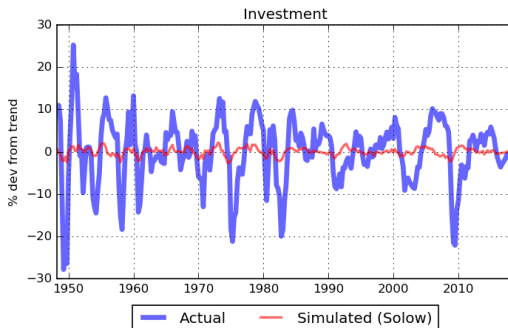


Table 1: Standard deviations. Actual data and data simulated from stochastic Solow model. Units are percent deviations from trend. The model under-predicts the volatility of investment in particular. Source: FRED.

	Actual Data	Simulated Data (Solow)
Output	1.62	0.93
Consumption	1.16	0.93
Investment	7.50	0.93

Table 2: Correlations with GDP. Actual data and data simulated from stochastic Solow model. The model over-predicts the correlation of investment and consumption with GDP. Source: FRED.

	Actual Data	Simulated Data (Solow)
Consumption	0.79	1.0
Investment	0.85	1.0

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 - Over-predicts consumption-GDP and investment-GDP correlation
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- **Objective:** Extend the model to improve performance.

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 - ① **Endogenous saving rate.** Consumption and investment decisions are made by a utility-maximizing *representative household*.
 - ② **Endogenous labor supply.** The utility-maximizing household chooses how much to work and therefore faces a labor-leisure tradeoff.
- Prescott's model is a **real business cycle (RBC)** model because it has no role for nominal quantities like inflation or nominal interest rates.

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 - **Centralized:** Household makes all production and allocation decisions. I.e., no firms, markets, or prices
 - 2 In the next lecture, we'll add labor choice to the baseline model

Centralized RBC Model without Labor

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$$= E_0 \sum_{t=0}^{\infty} \beta^t \log(C_t), \quad (7)$$

- $0 < \beta < 1$ is the household's *subjective discount factor*. (For a quarterly model: $\beta \approx 0.99$ makes usually sense)
- E_0 denotes the *expectation with respect to all information available as of date 0*.

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- Capital depreciates at the constant rate δ per period.

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- Express the constraints more concisely as

$$C_t + K_{t+1} = A_t K_t^\alpha + (1 - \delta) K_t, \quad (14)$$

for $t = 0, 1, 2, 3, \dots$

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 - 1 Consumption for the current period
 - 2 Capital for the subsequent periodto maximize its expected present value of lifetime utility.
- We'll solve the problem for period 0 and then generalize the solution to apply to all periods $0, 1, 2, \dots$

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- In period 0, the household solves:

$$\max_{C_0, K_1} E_0 \sum_{t=0}^{\infty} \beta^t \log(C_t) \quad (15)$$

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- The problem can be written as a choice of K_1 only:

$$\max_{K_1} E_0 \sum_{t=0}^{\infty} \beta^t \log(A_t K_t^{\alpha} + (1 - \delta)K_t - K_{t+1}) \quad (16)$$

Centralized RBC Model without Labor

- Note:

$$\begin{aligned} E_0 \sum_{t=0}^{\infty} \beta^t \log(AK_t^\alpha + (1 - \delta)K_t - K_{t+1}) \\ = \log(A_0K_0^\alpha + (1 - \delta)K_0 - K_1) \\ + \beta E_0 \log(A_1K_1^\alpha + (1 - \delta)K_1 - K_2) \\ + [\text{terms independent of } K_1] \end{aligned} \quad (17)$$

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- So:

$$\begin{aligned} \frac{\partial}{\partial K_1} U_0 = & -\frac{1}{A_0K_0^\alpha + (1-\delta)K_0 - K_1} \\ & + \beta E_0 \left[\frac{\alpha A_1 K_1^{\alpha-1} + 1 - \delta}{A_1 K_1^\alpha + (1-\delta)K_1 - K_2} \right] \end{aligned} \quad (18)$$

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- Therefore the first-order condition for the optimal choice of K_1 is:

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- Or more concisely:

$$\frac{1}{C_0} = \beta E_0 \left[\frac{\alpha A_1 K_1^{\alpha-1} + 1 - \delta}{C_1} \right] \quad (20)$$

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- So given $K_0 > 0$ and A_0 , the equilibrium paths for consumption, capital, and TFP are described by:

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- Computing numeric values for consumption and capital is not trivial. Recall the Euler equation:

$$\frac{1}{C_t} = \beta E_t \left[\frac{\alpha A_{t+1} K_{t+1}^{\alpha-1} + 1 - \delta}{C_{t+1}} \right] \quad (24)$$

- Consumption at date t depends on the *expectation* of consumption at date $t + 1$ which in turn depends on the expectation of consumption at date $t + 2$ and so on.
- Solving the problem requires numerical methods like those employed in the `linearsolve` Python package.

Figure 4: Baseline RBC model without labor. Impulse responses to a one percent shock to TFP in period 5.

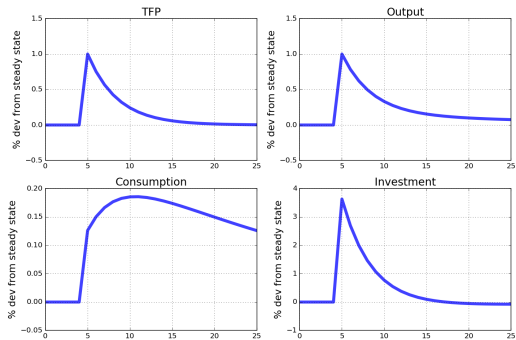


Figure 5: GDP. Like the stochastic Solow model, the baseline RBC model without labor does a *reasonably* good job matching GDP fluctuations for the US. Source: FRED.

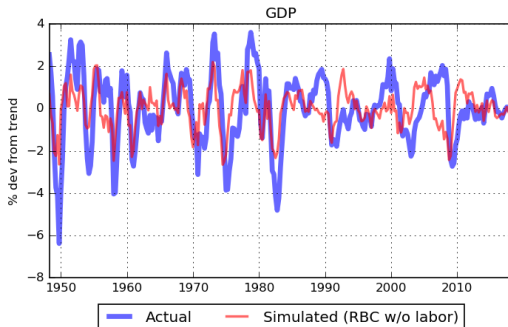


Figure 6: Consumption. In contrast to the stochastic Solow model, the baseline RBC model without labor *under-predicts* the magnitude of consumption fluctuations for the US. Source: FRED.

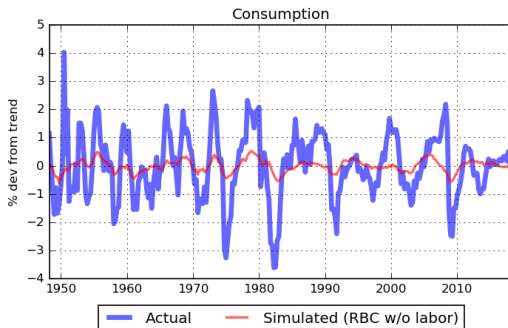


Figure 7: Investment. The baseline RBC model does a *reasonably* good job matching the magnitude of investment fluctuations better than the stochastic Solow model. Source: FRED.

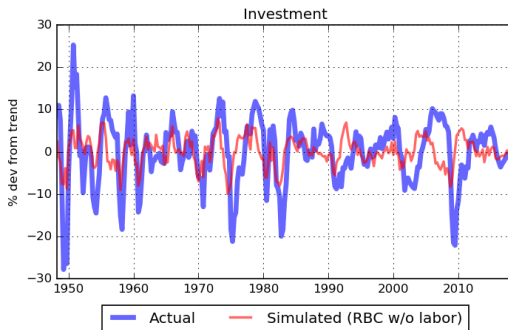


Table 3: Standard deviations. Actual data and simulated data. Units are percent deviations from trend. Source: FRED.

	Actual Data	Solow	RBC w/o Labor
Output	1.62	0.93	0.94
Consumption	1.16	0.93	0.22
Investment	7.50	0.93	3.39

Table 4: Correlations with GDP. Actual data and simulated data. Source: FRED.

	Actual Data	Solow	RBC w/o Labor
Consumption	0.79	1.0	0.62
Investment	0.85	1.0	0.99

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- **Next:** Add a labor-leisure tradeoff to the household's problem.

Prescott, Edward C., "Theory Ahead of Business Cycle Measurement," *Federal Reserve Bank of Minneapolis Quarterly Review*, Fall 1986, 10 (4).