

Introduction to Real Business Cycle Modeling

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- Recall the Solow growth model with stochastic TFP:

$$Y_t = A_t K_t^\alpha \quad (1)$$

$$C_t = (1 - s) Y_t \quad (2)$$

$$Y_t = C_t + I_t \quad (3)$$

$$K_{t+1} = I_t + (1 - \delta) K_t \quad (4)$$

$$\log A_{t+1} = \rho \log A_t + \epsilon_{t+1}, \quad (5)$$

where $\epsilon_{t+1} \sim \mathcal{N}(0, \sigma^2)$.

- The model generates business cycle-like fluctuations in output, consumption, and investment.

Introduction

- Let's evaluate the performance of the model by doing the following:
 - ① Use actual TFP values for the US for A_t in equation (1) and simulate the other variables in the model.
 - ② Compare simulated output, consumption, and investment data with the actual data
- But keep in mind that the Solow model was not designed to explain business cycles.

Figure 1: GDP. The stochastic Solow growth model does a *reasonably* good job matching GDP fluctuations for the US from April 1948 to July 2018. Source: FRED.

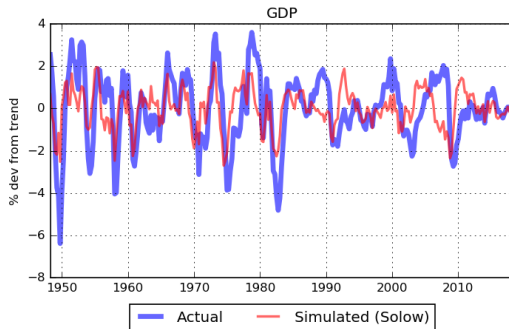


Figure 2: Consumption. The stochastic Solow growth model also does a *reasonably* good job matching consumption fluctuations for the US from April 1948 to July 2018. Source: FRED.

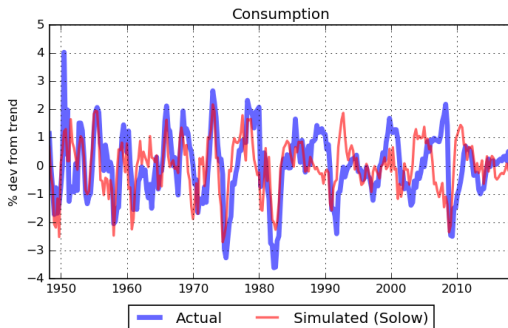


Figure 3: Investment. The stochastic Solow growth model *under-predicts* the magnitude of investment fluctuations for the US from April 1948 to July 2018. Source: FRED.

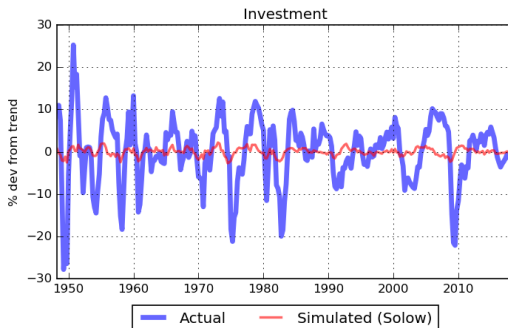


Table 1: Standard deviations. Actual data and data simulated from stochastic Solow model. Units are percent deviations from trend. The model under-predicts the volatility of investment in particular. Source: FRED.

	Actual Data	Simulated Data (Solow)
Output	1.62	0.93
Consumption	1.16	0.93
Investment	7.50	0.93

Table 2: Correlations with GDP. Actual data and data simulated from stochastic Solow model. The model over-predicts the correlation of investment and consumption with GDP. Source: FRED.

	Actual Data	Simulated Data (Solow)
Consumption	0.79	1.0
Investment	0.85	1.0

- Summary analysis of the stochastic Solow growth model:
 - Simulated fluctuations are comparable in scale to observed fluctuations
 - Under-predicts investment volatility
 - Over-predicts consumption-GDP and investment-GDP correlation
 - No explanation for why labor fluctuates over the business cycle.
- **Objective:** Extend the model to improve performance.

Introduction

- The model that Prescott (1986) describes is an extension of the stochastic Solow growth model.
- The most important differences are:
 - 1 **Endogenous saving rate.** Consumption and investment decisions are made by a utility-maximizing *representative household*.
 - 2 **Endogenous labor supply.** The utility-maximizing household chooses how much to work and therefore faces a labor-leisure tradeoff.
- Prescott's model is a **real business cycle (RBC)** model because it has no role for nominal quantities like inflation or nominal interest rates.

- We will ease into Prescott's RBC model in stages.
 - ① The **baseline** RBC model described in these slides:
 - **No labor supply choice:** household only chooses how much to consume and save.
 - **Centralized:** Household makes all production and allocation decisions. I.e., no firms, markets, or prices
 - ② In the next lecture, we'll add labor choice to the baseline model

Centralized RBC Model without Labor

- A *representative household* lives for an infinite number of periods.
- The *expected present value of lifetime utility* to the household from consuming C_0, C_1, C_2, \dots is denoted by U_0 :

$$U_0 = \log(C_0) + \beta E_0 \log(C_1) + \beta^2 E_0 \log(C_2) + \dots \quad (6)$$

$$= E_0 \sum_{t=0}^{\infty} \beta^t \log(C_t), \quad (7)$$

- $0 < \beta < 1$ is the household's *subjective discount factor*. (For a quarterly model: $\beta \approx 0.99$ makes usually sense)
- E_0 denotes the *expectation with respect to all information available as of date 0*.

Centralized RBC Model without Labor

- The household enters period 0 with capital $K_0 > 0$.
- Production in period t :

$$F(A_t, K_t) = A_t K_t^\alpha \quad (8)$$

where TFP A_t is stochastic:

$$\log A_{t+1} = \rho \log A_t + \epsilon_{t+1} \quad (9)$$

- Capital depreciates at the constant rate δ per period.

Centralized RBC Model without Labor

- The household faces the following sequence of budget constraints:

$$C_0 + K_1 = A_0 K_0^\alpha + (1 - \delta) K_0 \quad (10)$$

$$C_1 + K_2 = A_1 K_1^\alpha + (1 - \delta) K_1 \quad (11)$$

$$C_2 + K_3 = A_2 K_2^\alpha + (1 - \delta) K_2 \quad (12)$$

$$\vdots \quad (13)$$

- Express the constraints more concisely as

$$C_t + K_{t+1} = A_t K_t^\alpha + (1 - \delta) K_t, \quad (14)$$

for $t = 0, 1, 2, 3, \dots$

- **Optimization problem:** Each period the household chooses:
 - ① Consumption for the current period
 - ② Capital for the subsequent periodto maximize its expected present value of lifetime utility.
- We'll solve the problem for period 0 and then generalize the solution to apply to all periods $0, 1, 2, \dots$

Centralized RBC Model without Labor

- In period 0, the household solves:

$$\max_{C_0, K_1} E_0 \sum_{t=0}^{\infty} \beta^t \log(C_t) \quad (15)$$

$$\text{s.t.} \quad C_t + K_{t+1} = A_t K_t^{\alpha} + (1 - \delta)K_t$$

- The problem can be written as a choice of K_1 only:

$$\max_{K_1} E_0 \sum_{t=0}^{\infty} \beta^t \log(A_t K_t^{\alpha} + (1 - \delta)K_t - K_{t+1}) \quad (16)$$

Centralized RBC Model without Labor

- Note:

$$\begin{aligned} E_0 \sum_{t=0}^{\infty} \beta^t \log(AK_t^\alpha + (1-\delta)K_t - K_{t+1}) \\ = \log(A_0K_0^\alpha + (1-\delta)K_0 - K_1) \\ + \beta E_0 \log(A_1K_1^\alpha + (1-\delta)K_1 - K_2) \\ + [\text{terms independent of } K_1] \end{aligned} \quad (17)$$

- So:

$$\begin{aligned} \frac{\partial}{\partial K_1} U_0 = & -\frac{1}{A_0K_0^\alpha + (1-\delta)K_0 - K_1} \\ & + \beta E_0 \left[\frac{\alpha A_1 K_1^{\alpha-1} + 1 - \delta}{A_1 K_1^\alpha + (1-\delta)K_1 - K_2} \right] \end{aligned} \quad (18)$$

Centralized RBC Model without Labor

- Therefore the first-order condition for the optimal choice of K_1 is:

$$\frac{1}{\underbrace{A_0 K_0^\alpha + (1 - \delta) K_0 - K_1}_{C_0}} = \beta E_0 \left[\frac{\alpha A_1 K_1^{\alpha-1} + 1 - \delta}{\underbrace{A_1 K_1^\alpha + (1 - \delta) K_1 - K_2}_{C_1}} \right] \quad (19)$$

- Or more concisely:

$$\frac{1}{C_0} = \beta E_0 \left[\frac{\alpha A_1 K_1^{\alpha-1} + 1 - \delta}{C_1} \right] \quad (20)$$

Centralized RBC Model without Labor

- The household solves the same problem in periods $1, 2, 3, \dots$
- So given $K_0 > 0$ and A_0 , the equilibrium paths for consumption, capital, and TFP are described by:

$$\frac{1}{C_t} = \beta E_t \left[\frac{\alpha A_{t+1} K_{t+1}^{\alpha-1} + 1 - \delta}{C_{t+1}} \right] \quad (21)$$

$$C_t + K_{t+1} = A_t K_t^\alpha + (1 - \delta) K_t \quad (22)$$

$$\log A_{t+1} = \rho \log A_t + \epsilon_{t+1} \quad (23)$$

Centralized RBC Model without Labor

- Computing numeric values for consumption and capital is not trivial. Recall the Euler equation:

$$\frac{1}{C_t} = \beta E_t \left[\frac{\alpha A_{t+1} K_{t+1}^{\alpha-1} + 1 - \delta}{C_{t+1}} \right] \quad (24)$$

- Consumption at date t depends on the *expectation* of consumption at date $t + 1$ which in turn depends on the expectation of consumption at date $t + 2$ and so on.
- Solving the problem requires numerical methods like those employed in the `linearsolve` Python package.

Figure 4: Baseline RBC model without labor. Impulse responses to a one percent shock to TFP in period 5.

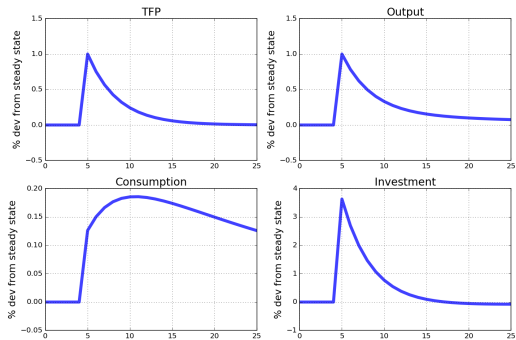


Figure 5: GDP. Like the stochastic Solow model, the baseline RBC model without labor does a *reasonably* good job matching GDP fluctuations for the US. Source: FRED.

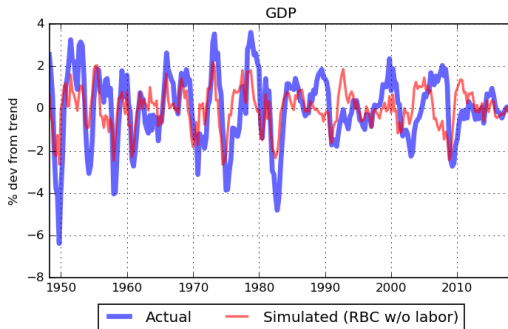


Figure 6: Consumption. In contrast to the stochastic Solow model, the baseline RBC model without labor *under-predicts* the magnitude of consumption fluctuations for the US. Source: FRED.

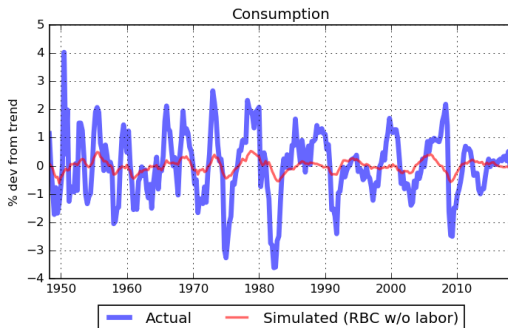


Figure 7: Investment. The baseline RBC model does a *reasonably* good job matching the magnitude of investment fluctuations better than the stochastic Solow model. Source: FRED.

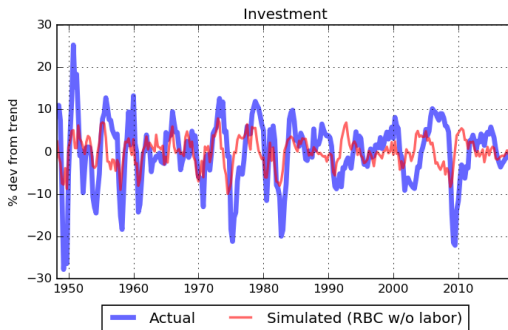


Table 3: Standard deviations. Actual data and simulated data. Units are percent deviations from trend. Source: FRED.

	Actual Data	Solow	RBC w/o Labor
Output	1.62	0.93	0.94
Consumption	1.16	0.93	0.22
Investment	7.50	0.93	3.39

Table 4: Correlations with GDP. Actual data and simulated data. Source: FRED.

	Actual Data	Solow	RBC w/o Labor
Consumption	0.79	1.0	0.62
Investment	0.85	1.0	0.99

- Summary analysis of the RBC Model without labor:
 - Substantial improvement over the stochastic Solow model for explaining investment volatility
 - Substantially under-predicts consumption volatility (i.e., consumption is too smooth)
- **Next:** Add a labor-leisure tradeoff to the household's problem.

Prescott, Edward C., "Theory Ahead of Business Cycle Measurement," *Federal Reserve Bank of Minneapolis Quarterly Review*, Fall 1986, 10 (4).