Deterministic Time Series Models

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Introduction

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- Stochastic models are ideal for modeling business cycle fluctuations because the consensus is that cycles are caused by unpredictable disturbances

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- We will focus exclusively on discrete time models.

First-Order Difference Equations

- Suppose that the variable y_t is determined by a linear function of y_{t-1} and some other exogenously given variable w_t:

$$y_t = (1-\rho)\mu + \rho y_{t-1} + w_t, \tag{1}$$

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- Equation (1) is an example of a **linear first-order difference equation**.

Example: Compounding Interest

- Suppose that you have an initial balance of b_0 dollars in an account that pays an interest rate i per compounding period. In period t you balance is:

$$b_t = (1 + i)\, b_{t-1}. \tag{2}$$

- Equation (2) is linear first-order difference equation in the same form as Equation (1). You can see this by setting $y_t = b_t$, $\rho = 1 + i$, $\mu = 0$, and $w_t = 0$ in Equation (1).