Deterministic Time Series Models

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Introduction

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- Stochastic models are ideal for modeling business cycle fluctuations because the consensus is that cycles are caused by unpredictable disturbances

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- We will focus exclusively on discrete time models.

First-Order Difference Equations

- Suppose that the variable y_t is determined by a linear function of y_{t-1} and some other exogenously given variable w_t:

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- Equation (1) is an example of a **linear first-order difference equation**.

Examples

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- Your period t balance depends on your period $t-1$ balance:

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- Equation (2) is a linear first-order difference equation in the same form as Equation (1). You can see this by setting $y_t = b_t$, $\rho = 1 + i$, and $w_t = 0$ in Equation (1).

Examples

Example: Physical Capital Accumulation

- In the Solow growth model, the law of motion for physical capital is:

$$K_{t+1} = I_t + (1-\delta)K_t, \tag{3}$$

where K_t is the capital stock in period t, δ is the rate of capital depreciation, and I_t is investment in new capital

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- Treating investment I_t as exogenous, Equation (3) is a linear first-order difference equation in the same form as Equation (1). You can see this by setting $y_t = K_{t+1}$, $\rho = 1-\delta$, and $w_t = I_t$ in Equation (1).