Introduction to New-Keynesian Business Cycle Modeling

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- The model is *new* because it is built upon a solid microeconomic foundation.

- New-Keynesian models are useful for explaining the equilibrium relationships between inflation, output, and interest rates.

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- The household receives utility from consuming goods in each period.
- The *lifetime utility* to the household from consuming C_0 in the first period and C_1 in the second is denoted by $U(C_0, C_1)$ and is written as:

$$U(C_0, C_1) = u(C_0) + \beta u(C_1), \tag{1}$$

where $u(\cdot)$ is the period utility function with $u'(\cdot) > 0$, $u''(\cdot) < 0$ and $\beta \leqslant 1$.

Figure 1: **The household's flow of utility** in a single period as a function of consumption.

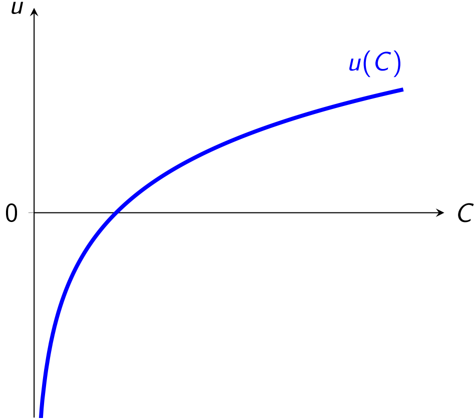
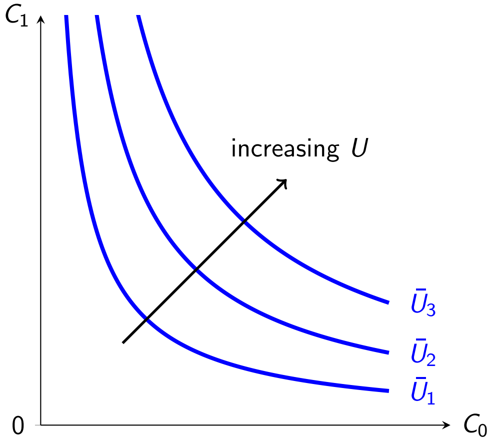


Figure 2: **The household's indifference curves** over combinations of consumption in periods 0 and 1.



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- Receives an income *endowment* of Y_0 in period 0 and Y_1 in period 1.
- In period 0, household chooses an amount of Y_0 to save S_1 given a real interest rate r_0.

The Euler Equation

- The household's budget constraints for periods 0 and 1 are:

$$C_0 = Y_0 - S_1, \tag{2}$$
$$C_1 = Y_1 + (1 + r_0)S_1. \tag{3}$$

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- Combine the two period budget constraints to obtain the intertemporal budget constraint:

$$C_1 = Y_1 + (1 + r_0)Y_0 - (1 + r_0)C_0. \tag{4}$$

The Euler Equation

- The household's optimization problem is:

$$\max_{C_0, C_1, S_1} \quad u(C_0) + \beta u(C_1) \tag{5}$$

$$\text{s.t.} \quad C_0 = Y_0 - S_1 \tag{6}$$

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- Equivalent to:

$$\max_{S_1} \; u(Y_0 - S_1) + \beta u\left[Y_1 + (1 + r_0)S_1\right] \tag{8}$$

The Euler Equation

- Setting derivative of the objective function with with respect to S_1 equal to zero:

$$-u'(Y_0 - S_1) + (1 + r_0)\beta u'\left[Y_1 + (1 + r_0)S_1\right] = 0, \quad (9)$$

where $u'(\cdot)$ denotes the period marginal utility of consumption.

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- Given Y_0, Y_1, and r_0, the Euler equation (14) determines whether the household will borrow or save.
- If Y_0 is relatively large or if r_0 is relatively large, then the household will want to save.

Figure 3: **A Household that saves**. The period 0 endowment is large relative to the period 1 endowment

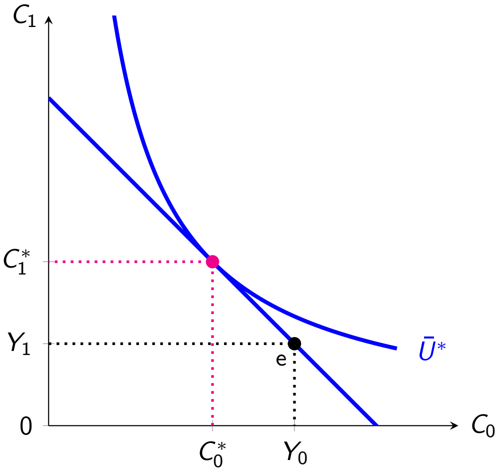
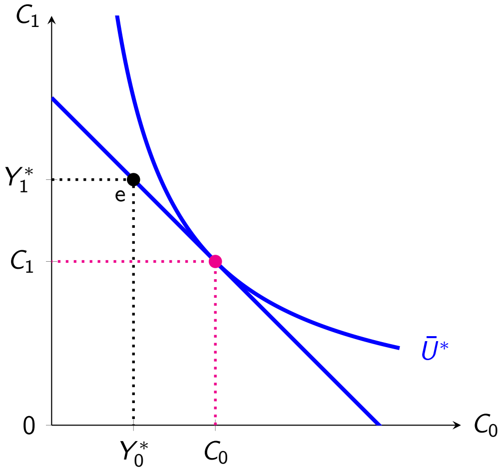


Figure 4: **A Household that borrows**. The period 0 endowment is small relative to the period 1 endowment



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- With this utility function, the household's Euler equation is expressed as:

$$\boxed{\frac{1}{C_0} = \frac{\beta(1+r_0)}{C_1}} \tag{14}$$

- Assume no investment, government purchases, or net exports, so $Y = C$ in every period:

$$\frac{1}{Y_0} = \beta \frac{1}{Y_1}(1 + r_0) \tag{15}$$

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- Next, take the log of each side to obtain a linear representation of the Euler equation:

$$\log Y_0 = \log \beta - \log Y_1 + \log (1 + r_0) \tag{16}$$

The Demand for Goods

- Rearranging and defining $\bar{r} \equiv -\log \beta$ as the natural rate of interest and $y \equiv \log Y$, obtain:

$$y_0 = y_1 - (r_0 - \bar{r}). \tag{17}$$

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  - Append an exogenous demand shock
- Finally:

$$\boxed{y_t = E_t y_{t+1} - (r_t - \bar{r}) + g_t} \tag{18}$$

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- The equations are: dynamic IS equation, Fisher equation, monetary policy rule, new-Keynesian Phillips curve or dynamic AS equation.

References