Prescott's RBC Model

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Introduction

- Finn Kydland and Edward Prescott were awarded the 2004 Nobel Memorial Prize in Economics, in part, for their contribution to our understanding of the driving forces behind business cycles.

- They pioneered a new approach to studying business cycles that became known as *real business cycle* or *RBC* theory.

Introduction

- Edward Prescott's 1986 article "Theory Ahead of Business Cycle Measurement" provides an overview of models he developed with Kydland.

- The model described on pages 11-17 – which I'll call *Prescott's model* – explains the endogenous co-movement of output, employment, consumption, and investment over the business cycle.

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- Below, I describe a *centralized* version of Prescott's model: the representative household owns all resources and makes all allocation decisions
- The only consequence of the difference is that Prescott is able to model the equilibrium prices of labor and capital.

Centralized RBC Model with Labor

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- Each period: consumes C_t and allocates share of available time L_t to labor activities.
- Flow of utility in period t:

$$\log(C_t) + \varphi \log(1 - L_t), \tag{1}$$

where $1 - L_t$ is the share of the household's time spent enjoying leisure.

Centralized RBC Model with Labor

- The *expected present value of lifetime utility* to the household from consuming $C_0, C_1, C_2, \ldots$ is denoted by U_0:

$$\begin{aligned} U_0 \quad = \quad & \log(C_0) + \varphi \log(1 - L_0) \\ & \quad + \beta \left[E_0 \log(C_1) + \varphi \log(1 - L_1) \right] \\ & \quad + \beta^2 \left[E_0 \log(C_2) + \varphi \log(1 - L_2) \right] \\ & \quad + \cdots \end{aligned} \tag{2}$$

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U_0 &= \log(C_0) + \varphi \log(1 - L_0) \\
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&\qquad + \beta^2\left[E_0 \log(C_2) + \varphi \log(1 - L_2)\right] \\
&\qquad + \cdots \qquad (2) \\
&= E_0 \sum_{t=0}^{\infty} \beta^t \left[\log(C_t) + \varphi \log(1 - L_t)\right], \qquad (3)
\end{aligned}
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where TFP A_t is stochastic:

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- The household's resource constraint is:

$$C_t + K_{t+1} = A_t K_t^{\alpha} L_t^{1-\alpha} + (1-\delta) K_t, \tag{6}$$

for $t = 0, 1, 2, 3, \ldots$

Centralized RBC Model with Labor

- **Optimization problem**: Each period the household chooses:
  1. Consumption for the current period
  2. Labor the current period
  3. Capital for the subsequent period

  to maximize its expected present value of lifetime utility.

- We'll solve the problem for period 0 and then generalize the solution to apply to all periods $0, 1, 2, \ldots$

Centralized RBC Model with Labor

- In period 0, the household solves:

$$\max_{C_0,L_0,K_1} E_0 \sum_{t=0}^{\infty} \beta^t \left[\log(C_t) + \varphi \log(1 - L_t) \right] \tag{7}$$

$$\text{s.t.} \quad C_t + K_{t+1} = A_t K_t^{\alpha} L_t^{1-\alpha} + (1 - \delta) K_t$$

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- The problem can be written as a choice of L_0 and K_1 only:

$$\max_{L_0, K_1} \; E_0 \sum_{t=0}^{\infty} \beta^t \Big[\log\left(A_t K_t^{\alpha} L_t^{1-\alpha} + (1-\delta) K_t - K_{t+1}\right) + \varphi \log(1 - L_t) \Big] \tag{8}$$

Centralized RBC Model with Labor

- The first-order condition with respect to L_0 is:

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- The first-order condition with respect to K_1 is:

$$\frac{1}{C_0} = \beta E_0\left[\frac{\alpha A_1K_1^{\alpha-1}L_1^{1-\alpha}+1-\delta}{C_1}\right] \tag{10}$$

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\frac{1}{C_t} &= \beta E_t\left[\frac{\alpha A_{t+1}K_{t+1}^{\alpha-1}L_{t+1}^{1-\alpha}+1-\delta}{C_{t+1}}\right] && (12)\\
C_t + K_{t+1} &= A_t K_t^{\alpha} L_t^{1-\alpha} + (1-\delta)K_t && (13)\\
\log A_{t+1} &= \rho \log A_t + \epsilon_{t+1} && (14)
\end{aligned}$$

Centralized RBC Model without Labor

- Computing numeric values for consumption and capital is not trivial. Recall the Euler equation:

$$\frac{1}{C_t} = \beta E_t \left[\frac{\alpha A_{t+1} K_{t+1}^{\alpha-1} L_{t+1}^{1-\alpha} + 1 - \delta}{C_{t+1}} \right] \tag{15}$$

- Consumption at date t depends on the *expectation* of consumption at date $t+1$ which in turn depends on the expectation of consumption at date $t+2$ and so on.

- Solving the problem requires numerical methods like those employed in the `linearsolve` Python package.

Figure 1: **Kydland and Prescott RBC model with labor.** Impulse responses to a one percent shock to TFP in period 5.

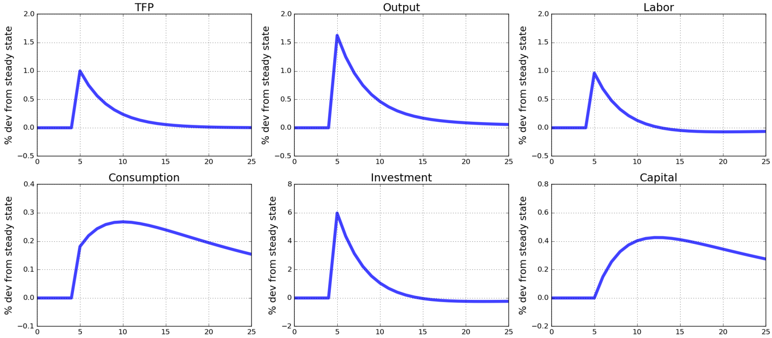

Figure 2: **GDP.** Actual and simulated data from Prescott's RBC model for the US from April 1948 to July 2018. Source: FRED.

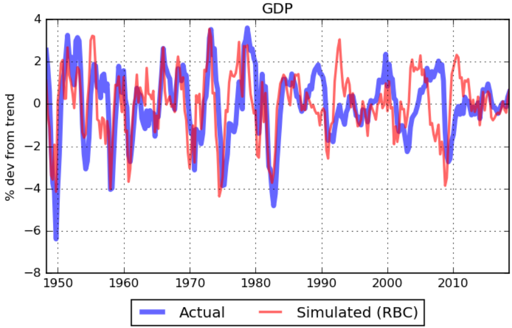

Figure 3: **Consumption.** Actual and simulated data from Prescott's RBC model for the US from April 1948 to July 2018. Source: FRED.

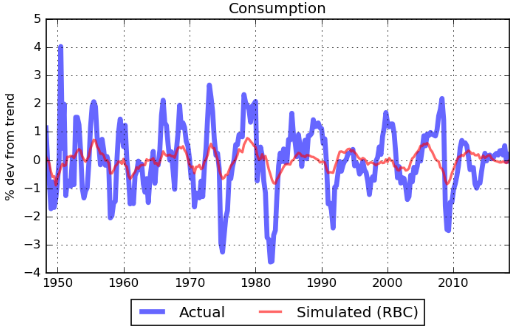

Figure 4: **Investment.** Actual and simulated data from Prescott's RBC model for the US from April 1948 to July 2018. Source: FRED.

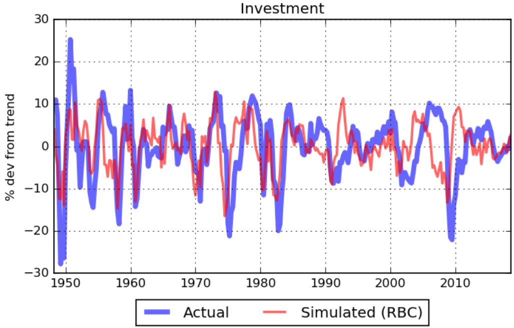

Figure 5: **Labor.** Actual and simulated data from Prescott's RBC model for the US from April 1948 to July 2018. Source: FRED.

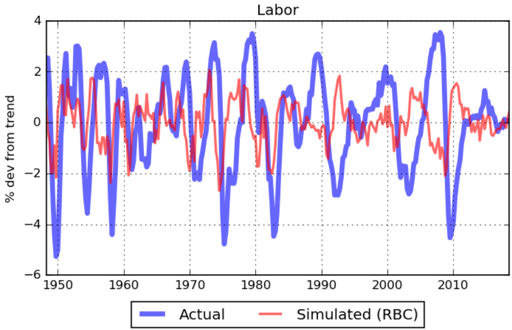

Table 1: **Standard deviations.** Actual data and data simulated from RBC models with and without labor. Units are percent deviations from trend. Source: FRED.

	Actual	RBC (w/o labor)	RBC (w/ labor)
Output	1.62	0.94	1.52
Consumption	1.16	0.22	0.32
Investment	7.50	3.39	5.56
Labor	1.89	–	0.90

Table 2: **Correlations with GDP.** Actual data and simulated data. Source: FRED.

	Actual	RBC (w/o labor)	RBC (w/ labor)
Consumption	0.79	0.62	0.59
Investment	0.85	0.99	0.99
Labor	0.87	–	0.98

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- Criticisms

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  1. Attributes *all* business cycle fluctuations to TFP shocks
  2. Attributes *all* labor fluctuations to *voluntary* changes in labor supply.

References

Prescott, Edward C., "Theory Ahead of Business Cycle Measurement," *Federal Reserve Bank of Minneapolis Quarterly Review*, Fall 1986, *10* (4).