

# Deterministic Time Series Models

Brian C. Jenkins

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University of California, Irvine

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- Macroeconomic data evolve over time and there is good evidence that:
  - ① Historical events matter for the current economy
  - ② Current events will matter for the future economy
- We model the *dynamic* nature of the economy using *time series* models
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- Time series models can be:
  - ① **Deterministic**: non-random and purely mechanical
  - ② **Stochastic**: Well-defined mathematical structure, but with random elements
- Deterministic models are well-suited for modeling long-run aspects of the economy
- Stochastic models are ideal for modeling business cycle fluctuations because the consensus is that cycles are caused by unpredictable disturbances

# Discrete Versus Continuous Time

- Let  $y$  denote a variable that takes on the value  $y_t$  at date  $t$ .
- If  $t$  takes on values from a countable sequence, e.g.,  $t \in [0, 1, 2, \dots)$ , then  $y_t$  is a **discrete time** variable.
- Otherwise, if  $t$  takes on values from an uncountable sequence, e.g.,  $t \in [0, \infty)$ , then  $y_t$  is a **continuous time** variable.
- We will focus exclusively on discrete time models.

# First-Order Difference Equations

- Suppose that the variable  $y_t$  is determined by a linear function of  $y_{t-1}$  and some other exogenously given variable  $w_t$ :

$$y_t = \rho y_{t-1} + w_t, \tag{1}$$

where  $\rho$  is some constant.

- Equation (1) is an example of a **linear first-order difference equation**.

## Example: Compounding Interest

- Suppose that you have an initial balance of  $b_0$  dollars in an account that pays an interest rate  $i$  per compounding period.
- Your period  $t$  balance depends on your period  $t - 1$  balance:

$$b_t = (1 + i) b_{t-1}. \quad (2)$$

- Equation (2) is a linear first-order difference equation in the same form as Equation (1). You can see this by setting  $y_t = b_t$ ,  $\rho = 1 + i$ , and  $w_t = 0$  in Equation (1).

## Example: Physical Capital Accumulation

- In the Solow growth model, the law of motion for physical capital is:

$$K_{t+1} = I_t + (1 - \delta)K_t, \quad (3)$$

where  $K_t$  is the capital stock in period  $t$ ,  $\delta$  is the rate of capital depreciation, and  $I_t$  is investment in new capital

- Treating investment  $I_t$  as exogenous, Equation (3) is a linear first-order difference equation in the same form as Equation (1). You can see this by setting  $y_t = K_{t+1}$ ,  $\rho = 1 - \delta$ , and  $w_t = I_t$  in Equation (1).