

# Kydland and Prescott's RBC Model

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- Finn Kydland and Edward Prescott were awarded the 2004 Nobel Memorial Prize in Economics, in part, for their contribution to our understanding of the driving forces behind business cycles.
- They pioneered a new approach to studying business cycles that became known as *real business cycle* or *RBC* theory.

# Introduction

- Edward Prescott's 1986 article "Theory Ahead of Business Cycle Measurement" provides an overview of models he developed with Kydland.
- The model described on pages 11-17 – which I'll call *Prescott's model* – explains the endogenous co-movement of output, employment, consumption, and investment over the business cycle.

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- Below, I describe a *centralized* version of Prescott's model: the representative household owns all resources and makes all allocation decisions
- The only consequence of the difference is that Prescott is able to model the equilibrium prices of labor and capital.



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- Each period: consumes  $C_t$  and allocates share of available time  $L_t$  to labor activities.
- Flow of utility in period  $t$ :

$$\log(C_t) + \varphi \log(1 - L_t), \quad (1)$$

where  $1 - L_t$  is the share of the household's time spent enjoying leisure.

# Centralized RBC Model with Labor

- The *expected present value of lifetime utility* to the household from consuming  $C_0, C_1, C_2, \dots$  is denoted by  $U_0$ :

$$\begin{aligned} U_0 = & \log(C_0) + \varphi \log(1 - L_0) \\ & + \beta [E_0 \log(C_1) + \varphi \log(1 - L_1)] \\ & + \beta^2 [E_0 \log(C_2) + \varphi \log(1 - L_2)] \\ & + \dots \end{aligned} \tag{2}$$

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$$= E_0 \sum_{t=0}^{\infty} \beta^t [\log(C_t) + \varphi \log(1 - L_t)], \tag{3}$$

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- The household's resource constraint is:

$$C_t + K_{t+1} = A_t K_t^\alpha L_t^{1-\alpha} + (1 - \delta)K_t, \quad (6)$$

for  $t = 0, 1, 2, 3, \dots$



# Centralized RBC Model with Labor

- **Optimization problem:** Each period the household chooses:
  - 1 Consumption for the current period
  - 2 Labor the current period
  - 3 Capital for the subsequent periodto maximize its expected present value of lifetime utility.
- We'll solve the problem for period 0 and then generalize the solution to apply to all periods  $0, 1, 2, \dots$

# Centralized RBC Model with Labor

- In period 0, the household solves:

$$\max_{C_0, L_0, K_1} E_0 \sum_{t=0}^{\infty} \beta^t [\log(C_t) + \varphi \log(1 - L_t)] \quad (7)$$

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- The problem can be written as a choice of  $L_0$  and  $K_1$  only:

$$\begin{aligned} \max_{L_0, K_1} E_0 \sum_{t=0}^{\infty} \beta^t & [\log(A_t K_t^\alpha L_t^{1-\alpha} + (1 - \delta)K_t - K_{t+1}) \\ & + \varphi \log(1 - L_t)] \end{aligned} \quad (8)$$

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- The first-order condition with respect to  $K_1$  is:

$$\frac{1}{C_0} = \beta E_0 \left[ \frac{\alpha A_1 K_1^{\alpha-1} L_1^{1-\alpha} + 1 - \delta}{C_1} \right] \quad (10)$$

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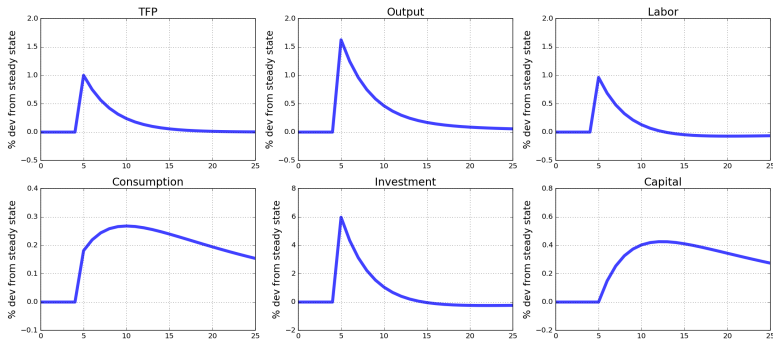
# Centralized RBC Model without Labor

- Computing numeric values for consumption and capital is not trivial. Recall the Euler equation:

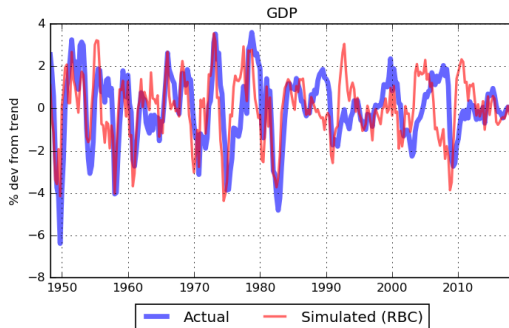
$$\frac{1}{C_t} = \beta E_t \left[ \frac{\alpha A_{t+1} K_{t+1}^{\alpha-1} L_{t+1}^{1-\alpha} + 1 - \delta}{C_{t+1}} \right] \quad (15)$$

- Consumption at date  $t$  depends on the *expectation* of consumption at date  $t + 1$  which in turn depends on the expectation of consumption at date  $t + 2$  and so on.
- Solving the problem requires numerical methods like those employed in the `linearsolve` Python package.

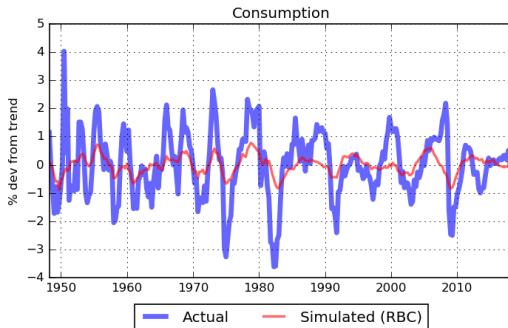
**Figure 1: Kydland and Prescott RBC model with labor.** Impulse responses to a one percent shock to TFP in period 5.



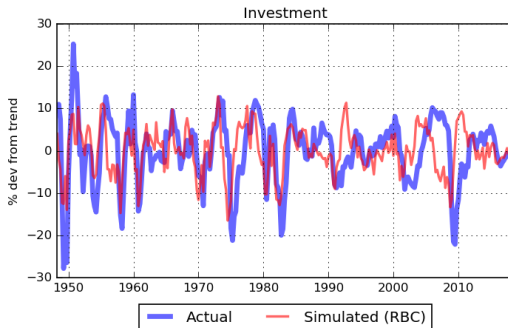
**Figure 2: GDP.** Actual and simulated data from Prescott's RBC model for the US from April 1948 to July 2018. Source: FRED.



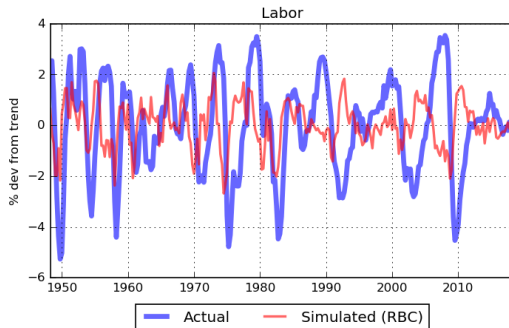
**Figure 3: Consumption.** Actual and simulated data from Prescott's RBC model for the US from April 1948 to July 2018. Source: FRED.



**Figure 4: Investment.** Actual and simulated data from Prescott's RBC model for the US from April 1948 to July 2018. Source: FRED.



**Figure 5: Labor.** Actual and simulated data from Prescott's RBC model for the US from April 1948 to July 2018. Source: FRED.





**Table 1: Standard deviations.** Actual data and data simulated from RBC models with and without labor. Units are percent deviations from trend. Source: FRED.

	Actual	RBC (w/o labor)	RBC (w/ labor)
Output	1.62	0.94	1.52
Consumption	1.16	0.22	0.32
Investment	7.50	3.39	5.56
Labor	1.89	—	0.90

**Table 2: Correlations with GDP.** Actual data and simulated data. Source: FRED.

	Actual	RBC (w/o labor)	RBC (w/ labor)
Consumption	0.79	0.62	0.59
Investment	0.85	0.99	0.99
Labor	0.87	—	0.98

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- Criticisms
  - ① Attributes *all* business cycle fluctuations to TFP shocks
  - ② Attributes *all* labor fluctuations to *voluntary* changes in labor supply.

**Prescott, Edward C.**, "Theory Ahead of Business Cycle Measurement," *Federal Reserve Bank of Minneapolis Quarterly Review*, Fall 1986, 10 (4).