Econ 126: Computational Macroeconomics
Winter 2019: Jenkins

Final Project Problem List

1 Factor Prices in a Decentralized RBC Model

In this problem, you will simulate the dynamic equilibrium of a decentralized RBC model. As usual, the model features an infinitely-lived household that chooses consumption, labor, and capital accumulation to maximize the present value of its lifetime utility. However, instead of assuming that the household produces output goods itself, we'll suppose the existence of a firm sector that pays the household in exchange for capital and labor services. A key product of this modeling approach is that it allows us to model how factor prices – i.e., the real wage and the capital rental rate – change over the business cycle.

1.1 The Model

1.1.1 Household Sector

A representative household lives for an infinite number of periods. The expected present value of lifetime utility to the household from consuming $C_0, C_1, C_2, \ldots$ and working $L_0, L_1, L_2, \ldots$ is denoted by U_0:

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left[\log(C_t) + \varphi \log(1 - L_t)\right], \tag{1}$$

where $0 < \beta < 1$ is the household's subjective discount factor and φ reflects the relative value that the household places on leisure in the utility function. E_0 denotes the expectation with respect to all information available as of date 0.

The household enters period 0 with capital $K_0 > 0$ and faces the following intertemporal budget constraint:

$$C_t + K_{t+1} = W_t L_t + R_t K_t + (1 - \delta) K_t, \tag{2}$$

where W_t is the (real) wage received per unit of labor supplied, R_t is the (real) rental rate per unit of capital, and δ is the constant rate of capital depreciation.

In period 0, the household solves:

$$\begin{aligned} \max_{C_0, L_0, K_1} \; & E_0 \sum_{t=0}^{\infty} \beta^t \left[\log(C_t) + \varphi \log(1 - L_t)\right] \\ \text{s.t.} \quad & C_t + K_{t+1} = W_t L_t + R_t K_t + (1 - \delta) K_t \end{aligned} \tag{3}$$

which, as usual, can be written as a choice of L_0 and K_1 only:

$$\max_{L_0, K_1} E_0 \sum_{t=0}^{\infty} \beta^t \left[\log(W_t L_t + R_t K_t + (1 - \delta) K_t - K_{t+1}) + \varphi \log(1 - L_t)\right] \tag{4}$$

1.1.2 Firm Sector

Competitive firms produce output Y_t according to the standard Cobb-Douglas production function:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{5}$$

where TFP A_t is stochastic:

$$\log A_{t+1} = \rho \log A_t + \epsilon_{t+1} \tag{6}$$

Each period, firms take the factor prices W_t and R_t as given and choose L_t, K_t, Y_t to maximize their profits:

$$\max_{Y_t, K_t, L_t} Y_t - W_t L_t - R_t K_t$$
$$\text{s.t.} \quad Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{7}$$

The problem can be expressed as an unconstrained problem by substituting Y_t out of the problem:

$$\max_{K_t, L_t} A_t K_t^{\alpha} L_t^{1-\alpha} - W_t L_t - R_t K_t \tag{8}$$

1.1.3 Investment

When the household chooses K_{t+1}, it implicitly chooses investment I_t which is defined by:

$$I_t = K_{t+1} - (1-\delta) K_t \tag{9}$$

1.1.4 Goods Market Clearing

In equilibrium, the quantity of goods produced Y_t has to equal the demand for those goods $C_t + I_t$:

$$Y_t = C_t + I_t \tag{10}$$

We call Equation (**??**) the goods market clearing condition and it represents the aggregate resource constraint for the economy.

1.1.5 Equilibrium

The model has 8 endogenous variables: A_t, K_t, C_t, L_t, Y_t, I_t, W_t, and R_t. Equilibrium is described by:

1. The household's first-order condition for L_t
2. The household's first-order condition for K_{t+1} (the Euler equation)

3. The firm's first-order condition for K_t
4. The firm's first-order condition for L_t
5. The production function: Equation (??)
6. The TFP evolution equation: Equation (??)
7. The capital evolution equation: Equation (??)
8. The goods market clearing equation: Equation (??)

1.1.6 Calibration

Assume the following values for the model's parameters:

Parameter	Value	Description
β	0.99	household's subjective discount factor
φ	1.7317	household preference parameter
α	0.35	Cobb-Douglas production function parameter
δ	0.025	capital depreciation rate
ρ	0.75	autocorrelation of tfp
σ	0.006	s.d. of TFP shock

1.2 Exercises

1. Include in your presentation a *brief* overview of the model and clearly explain how it differs from the centralized model of Prescott covered in class.
2. Solve for the household's first-order conditions for L_t and K_{t+1}. Include these equations in your presentation and be able to explain the intuition behind them.
3. Solve for firm's first-order condition for K_t and L_t. Include these equations in your presentation and be able to explain the intuition behind them.
4. Use Python to compute the steady state values of A_t, K_t, C_t, L_t, Y_t, I_t, W_t, and R_t.
5. Compute the impulse responses for all of the model's endogenous variables for 41 periods following a one percentage point increase in TFP in period 5. Create a set of clear, easy to read figures that depict the impulse responses and include them in your presentation. Units of plotted quantities should be percent deviations from steady state.[1] Be able to explain the intuition behind them.
6. Compute a 61 period stochastic simulation of model's endogenous variables. Set seed for the random number generator to 2019. Create a set of clear, easy to read figures that depict the stochastic simulation and include them in your presentation. Units of plotted quantities should be percent deviations from steady state. Be able to explain the intuition behind them.

[1] I.e., multiply the simulated impulse responses by 100.

7. Address this in your presentation: Which fluctuates more over the business cycle: the real wage or the real rental rate? Explain clearly how you know this.

2 Centralized RBC Model with Stochastic Government Consumption

In this problem, you will simulate the dynamic equilibrium of a centralized RBC model without labor and with stochastic government consumption. As usual, the model features an infinitely-lived household that chooses consumption and capital accumulation to maximize the present value of its lifetime utility. What's new is that we'll assume that there is a government sector that consumes a stochastic quantity of goods each period. A key product of this modeling approach is that it allows us to model how fluctuations in government consumption affect the business cycle.

2.1 The Model

2.1.1 Household Sector

A representative household lives for an infinite number of periods. The expected present value of lifetime utility to the household from consuming $C_0, C_1, C_2, \ldots$ is denoted by U_0:

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \log(C_t), \tag{11}$$

where $0 < \beta < 1$ is the household's subjective discount factor. E_0 denotes the expectation with respect to all information available as of date 0.

The household enters period 0 with capital $K_0 > 0$. Production in period t is according to a standard production function that has decreasing returns in capital K_t:

$$F(A_t, K_t) = A_t K_t^{\alpha} \tag{12}$$

where TFP A_t is stochastic:

$$\log A_{t+1} = \rho_A \log A_t + \epsilon_{t+1}^{A} \tag{13}$$

Each period the government collects a lump-sum tax T_t from the household. The household's resource constraint in each period t is therefore:

$$C_t + K_{t+1} + T_t = A_t K_t^{\alpha} + (1-\delta)K_t, \tag{14}$$

where δ is the rate of capital depreciation.

In period 0, the household solves:

$$\max_{C_0, K_1} E_0 \sum_{t=0}^{\infty} \beta^t \log(C_t)$$
$$\text{s.t.} \quad C_t + K_{t+1} + T_t = A_t K_t^{\alpha} + (1-\delta)K_t \tag{15}$$

which, as usual, can be written as a choice of K_1 only:

$$\max_{K_1} E_0 \sum_{t=0}^{\infty} \beta^t \log(A_t K_t^{\alpha} + (1-\delta)K_t - K_{t+1} - T_t) \tag{16}$$

2.1.2 Government Sector

Each period the government consumes G_t units of goods. G_t evolves according to the following process:

$$\log G_{t+1} = (1-\rho_G)\log \bar{G} + \rho_G \log G_t + \epsilon^G_{t+1} \tag{17}$$

By assumption, the government always runs a balanced budget so:

$$T_t = G_t \tag{18}$$

2.1.3 Investment and Output

When the household chooses K_{t+1}, it implicitly chooses investment I_t which is defined by:

$$I_t = K_{t+1} - (1-\delta)K_t \tag{19}$$

and output Y_t which is defined by:

$$Y_t = A_t K_t^{\alpha} \tag{20}$$

2.1.4 Goods Market Clearing

In equilibrium, the quantity of goods produced Y_t has to equal the demand for those goods $C_t + I_t + G_t$:

$$Y_t = C_t + I_t + G_t \tag{21}$$

We call Equation (**??**) the goods market clearing condition and it represents the aggregate resource constraint for the economy.

2.1.5 Equilibrium

The model has 7 endogenous variables: A_t, G_t, K_t, C_t, T_t, Y_t, I_t. Equilibrium is described by:

1. The household's first-order condition for K_{t+1} (the Euler equation)
2. The TFP evolution equation: Equation (**??**)
3. The government consumption evolution equation: Equation (**??**)
4. The government budget constraint: Equation (**??**)
5. The capital evolution equation: Equation (**??**)
6. The production function: Equation (**??**)
7. The goods market clearing equation: Equation (**??**)

2.1.6 Calibration

Assume the following values for the model's parameters:

Parameter	Value	Description
β	0.99	household's subjective discount factor
α	0.35	Cobb-Douglas production function parameter
δ	0.025	capital depreciation rate
ρ_A	0.75	autocorrelation of tfp
σ_A	0.006	s.d. of TFP shock
$\bar{G}$	–	steady state government consumption
ρ_G	0.9	autocorrelation of government consumption
σ_G	0.015	s.d. of government consumption shock

2.2 Exercises

1. Include in your presentation a *brief* overview of the model and clearly explain how it differs from the centralized model of Prescott covered in class.

2. Download two series from FRED[2]:

   – Government Consumption Expenditures and Gross Investment (Series ID: GCE)

   – Gross Domestic Product (Series ID: GCE)

   Find the average of the ratio of government consumption to GDP for the US for all dates available.[3] Report this value in your presentation.

3. Solve for the household's first-order condition for K_{t+1}. Include this equation in your presentation and be able to explain the intuition behind it.

4. Use Python to compute the steady state values of A_t, K_t, Y_t, and I_t. You will have to do this manually. Since you don't know $\bar{G}$ yet, you can't use `linearsolve` for this step.

5. Use the average ratio of government consumption to GDP for the US to *calibrate* $\bar{G}$:

   $$\bar{G} = \bar{Y} \times \left[\text{Avg. G-to-Y ratio}\right] \tag{22}$$

   Then use Python to compute the steady state values of C_t and T_t.

6. Compute the impulse responses for all of the model's endogenous variables for 41 periods following a one percentage point increase in government consumption in period 5.[4] Create a set of clear, easy to read figures that depict the impulse responses and include them in your presentation. Units of plotted quantities should be percent deviations from steady state.[5] Be able to explain the intuition behind them.

[2]https://fred.stlouisfed.org/

[3]Compute the ratio for each date *first*, then compute the average of the ratio.

[4]Note that like A_t, G_t is a state (or predetermined) variable.

[5]I.e., multiply the simulated impulse responses by 100.

3 Centralized RBC Model with Stochastic Income Taxes

In this problem, you will simulate the dynamic equilibrium of a centralized RBC model without labor and with a stochastic income tax. As usual, the model features an infinitely-lived household that chooses consumption and capital accumulation to maximize the present value of its lifetime utility. What's new is that we'll assume that there is a government sector that taxes a stochastic proportion of the household's income each period. A key product of this modeling approach is that it allows us to model how income taxes affect the business cycle.

3.1 The Model

3.1.1 Household Sector

A representative household lives for an infinite number of periods. The expected present value of lifetime utility to the household from consuming $C_0, C_1, C_2, \ldots$ is denoted by U_0:

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \log(C_t), \tag{23}$$

where $0 < \beta < 1$ is the household's subjective discount factor. E_0 denotes the expectation with respect to all information available as of date 0.

The household enters period 0 with capital $K_0 > 0$. Production in period t is according to a standard production function that has decreasing returns in capital K_t:

$$F(A_t, K_t) = A_t K_t^{\alpha} \tag{24}$$

where TFP A_t is stochastic:

$$\log A_{t+1} = \rho_A \log A_t + \epsilon_{t+1}^{A} \tag{25}$$

Each period the government collects income taxes from the household and transfers any surplus back to the household. Let T_t denote one plus the income tax rate. That is, if the tax rate in period t were 5 percent, then T_t would equal 1.05. Given this definition, the household's resource constraint in each period t is therefore:

$$C_t + K_{t+1} = (2 - T_t) A_t K_t^{\alpha} + (1 - \delta) K_t + S_t, \tag{26}$$

where δ is the rate of capital depreciation and S_t is the value of any government budget surplus transferred back to the household.[6]

In period 0, the household solves:

$$\begin{aligned} &\max_{C_0, K_1} \; E_0 \sum_{t=0}^{\infty} \beta^t \log(C_t) \\ &\text{s.t.} \quad C_t + K_{t+1} = (2 - T_t) A_t K_t^{\alpha} + (1 - \delta) K_t + S_t \end{aligned} \tag{27}$$

which, as usual, can be written as a choice of K_1 only:

$$\max_{K_1} \; E_0 \sum_{t=0}^{\infty} \beta^t \log\left[(2 - T_t) A_t K_t^{\alpha} + (1 - \delta) K_t + S_t - K_{t+1}\right] \tag{28}$$

[6] Note that $2 - T_t = 1 - [\text{tax rate}]_t$

3.1.2 Government Sector

Each period the government sets the tax rate $T_t - 1$. T_t evolves according to the following process:

$$\log T_{t+1} = (1-\rho_T)\log \bar{T} + \rho_T \log T_t + \epsilon^T_{t+1} \tag{29}$$

where $\bar{T}$ is steady state T_t. Assuming that the government doesn't consume any goods, the budget surplus S_t equals the income tax revenue:

$$S_t = (T_t - 1)A_t K_t^\alpha \tag{30}$$

3.1.3 Investment and Output

When the household chooses K_{t+1}, it implicitly chooses investment I_t which is defined by:

$$I_t = K_{t+1} - (1-\delta)K_t \tag{31}$$

and output Y_t which is defined by:

$$Y_t = A_t K_t^\alpha \tag{32}$$

3.1.4 Goods Market Clearing

In equilibrium, the quantity of goods produced Y_t has to equal the demand for those goods $C_t + I_t + G_t$:

$$Y_t = C_t + I_t \tag{33}$$

We call Equation (**??**) the goods market clearing condition and it represents the aggregate resource constraint for the economy.

3.1.5 Equilibrium

The model with stochastic taxes has 7 endogenous variables: A_t, T_t, K_t, C_t, S_t, Y_t, I_t. Equilibrium is described by:

1. The household's first-order condition for K_{t+1} (the Euler equation)
2. The TFP evolution equation: Equation (**??**)
3. The tax rate evolution equation: Equation (**??**)
4. The government budget constraint: Equation (**??**)
5. The capital evolution equation: Equation (**??**)
6. The production function: Equation (**??**)
7. The goods market clearing equation: Equation (**??**)

3.1.6 Calibration

Assume the following values for the model's parameters:

Parameter	Value	Description
β	0.99	household's subjective discount factor
α	0.35	Cobb-Douglas production function parameter
δ	0.025	capital depreciation rate
ρ_A	0.75	autocorrelation of tfp
σ_A	0.006	s.d. of TFP shock
$\bar{T}$	–	steady state income tax rate (plus 1)
ρ_T	–	autocorrelation of tax rate
σ_T	0.039	s.d. of tax rate shock

3.2 Exercises

1. Include in your presentation a *brief* overview of the model and clearly explain how it differs from the centralized model of Prescott covered in class.

2. Download two series from FRED[7]:

   – Federal government current tax receipts (Series ID: W006RC1Q027SBEA)

   – Gross Domestic Product (Series ID: GDP)

   Find the average of the ratio of federal tax receipts to GDP for the US for all dates available and use the result to calibrate $\bar{T}$:[8]

$$\bar{T} \;=\; 1 + \Big[\text{Avg. tax-to-GDP ratio}\Big] \tag{34}$$

   Report the calibrated value of $\bar{T}$ in your presentation.

3. Solve for the household's first-order condition for K_{t+1}. Include this equation in your presentation and be able to explain the intuition behind it.

4. Use Python to compute the steady state values of A_t, T_t, K_t, Y_t, C_t, I_t, and S_t.

5. Suppose that $\rho_T = 0.8$. Compute the impulse responses for all of the model's endogenous variables following a one percentage point increase in the tax rate.[9] Create a set of clear, easy to read figures that depict the impulse responses and include them in your presentation. Be able to explain the intuition behind them.

6. For $\rho_T = 0.00001$, 0.8, and 0.99, compute the impulse responses for 41 periods for all of the model's endogenous variables following a one percentage point increase in the tax rate in period 5. Plot the three computed impulse responses for each endogenous

[7] https://fred.stlouisfed.org/

[8] Compute the ratio for each date *first*, then compute the average of the ratio.

[9] Note that like A_t, T_t is a state (or predetermined) variable.

variable on a separate set of axes and include the plots in your presentation. For example, one set of axes will contain the impulse responses for consumption for $\rho_T = 0.00001$, 0.8, and 0.99. Make sure to clearly indicate which line corresponds to which value of ρ_T. Units of plotted quantities should be percent deviations from steady state.[10] Be able to explain the intuition behind them.

[10] I.e., multiply the simulated impulse responses by 100.

4 Centralized RBC Model and Interest Rates

In this problem, you will simulate the dynamic equilibrium of a centralized RBC model without labor and qith government bonds. As usual, the model features an infinitely-lived household that chooses consumption and capital accumulation to maximize the present value of its lifetime utility. What's new is that we'll assume that the household can save in two ways: by holding physical capital and by holding government bonds. A key product of this modeling approach is that it allows us to model how the (real) interest rate varies over the business cycle.

4.1 The Model

4.1.1 Household Sector

A representative household lives for an infinite number of periods. The expected present value of lifetime utility to the household from consuming $C_0, C_1, C_2, \ldots$ is denoted by U_0:

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \log(C_t), \tag{35}$$

where $0 < \beta < 1$ is the household's subjective discount factor. E_0 denotes the expectation with respect to all information available as of date 0.

The household enters period 0 with capital $K_0 > 0$. Production in period t is according to a standard production function that has decreasing returns in capital K_t:

$$F(A_t, K_t) = A_t K_t^{\alpha} \tag{36}$$

where TFP A_t is stochastic:

$$\log A_{t+1} = \rho \log A_t + \epsilon_{t+1} \tag{37}$$

Each period, the household purchases government bonds in the amount of B_{t+1} that pay gross interest R_t in period $t+1$.[11] The household's resource constraint in each period t is therefore:

$$C_t + K_{t+1} + B_{t+1} = A_t K_t^{\alpha} + (1-\delta)K_t + R_{t-1}B_t, \tag{38}$$

where δ is the rate of capital depreciation.

In period 0, the household solves:

$$\begin{aligned} &\max_{C_0, K_1, B_1} \; E_0 \sum_{t=0}^{\infty} \beta^t \log(C_t) \\ &\quad\text{s.t.} \quad C_t + K_{t+1} + B_{t+1} = A_t K_t^{\alpha} + (1-\delta)K_t + R_{t-1}B_t \end{aligned} \tag{39}$$

which, as usual, can be written as a choice of K_1 and B_1 only:

$$\max_{K_1, B_1} \; E_0 \sum_{t=0}^{\infty} \beta^t \log(A_t K_t^{\alpha} + (1-\delta)K_t + R_{t-1}B_t - K_{t+1} - B_{t+1}) \tag{40}$$

[11] In this notation, B_{t+1} is the *price* of the bonds purchased at date t and $R_t B_{t+1}$ is the *face value*.

4.1.2 Government Sector

The government neither consumes goods nor taxes the household so:

$$B_t = 0 \tag{41}$$

in all dates.

4.1.3 Investment and Output

When the household chooses K_{t+1}, it implicitly chooses investment I_t which is defined by:

$$I_t = K_{t+1} - (1-\delta)K_t \tag{42}$$

and output Y_t which is defined by:

$$Y_t = A_t K_t^\alpha \tag{43}$$

4.1.4 Goods Market Clearing

In equilibrium, the quantity of goods produced Y_t has to equal the demand for those goods $C_t + I_t$:

$$Y_t = C_t + I_t \tag{44}$$

We call Equation (**??**) the goods market clearing condition and it represents the aggregate resource constraint for the economy.

4.1.5 Equilibrium

The model has 6 endogenous variables: A_t, K_t, C_t, Y_t, I_t, R_t. Equilibrium is described by:

1. The household's first-order condition for K_{t+1} (an Euler equation)
2. The household's first-order condition for B_{t+1} (another Euler equation)
3. The TFP evolution equation: Equation (**??**)
4. The capital evolution equation: Equation (**??**)
5. The production function: Equation (**??**)
6. The goods market clearing equation: Equation (**??**)

4.1.6 Calibration

Assume the following values for the model's parameters:

Parameter	**Value**	**Description**
β	0.99	household's subjective discount factor
α	0.35	Cobb-Douglas production function parameter
δ	0.025	capital depreciation rate
ρ	0.75	autocorrelation of tfp
σ	0.006	s.d. of TFP shock

4.2 Exercises

1. Include in your presentation a *brief* overview of the model and clearly explain how it differs from the centralized model of Prescott covered in class.

2. Solve for the household's first-order condition for K_{t+1} and B_{t+1}. Include these equations in your presentation and be able to explain the intuition behind it.

3. Use Python to compute the steady state values of A_t, K_t, C_t, Y_t, I_t, and R_t.

4. Compute the impulse responses for all of the model's endogenous variables for 41 periods following a one percentage point increase in TFP in period 5. Create a set of clear, easy to read figures that depict the impulse responses and include them in your presentation. Units of plotted quantities should be percent deviations from steady state.[12] Be able to explain the intuition behind them.

5. Compute a 61 period stochastic simulation of model's endogenous variables in the presence of simultaneous shocks to TFP. Set the seed for the random number generator to 2019. Create a set of clear, easy to read figures that depict the stochastic simulation and include them in your presentation. Units of plotted quantities should be percent deviations from steady state. Be able to explain the intuition behind them.

[12] I.e., multiply the simulated impulse responses by 100.

5 The Centralized RBC Model with Different Intertemporal Elasticities of Substitution

In this problem, you will simulate the dynamic equilibrium of a decentralized RBC model. As usual, the model features an infinitely-lived household that chooses consumption, labor, and capital accumulation to maximize the present value of its lifetime utility. However, instead of assuming that the household produces output goods itself, we'll suppose the existence of a firm sector that pays the household in exchange for capital and labor services. A key product of this modeling approach is that it allows us to model how factor prices – i.e., the real wage and the capital rental rate – change over the business cycle.

5.1 The Model

5.1.1 Household Sector

A representative household lives for an infinite number of periods. The expected present value of lifetime utility to the household from consuming $C_0, C_1, C_2, \ldots$ and working $L_0, L_1, L_2, \ldots$ is denoted by U_0:

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\gamma} - 1}{1-\gamma} + \varphi \log(1 - L_t) \right], \tag{45}$$

where $0 < \beta < 1$ is the household's subjective discount factor, φ reflects the relative value that the household places on leisure in the utility function, and γ reflects how willing the household is to substitute consumption across time. Specifically, $1/\gamma$ is the household's *intertemporal elasticity of substitution* and as γ increases, the household is *less* willing to substitute consumption today for consumption tomorrow. While this particular role for γ may not be apparent from simply inspecting the utility function, the effect will be revealed in simulations. E_0 denotes the expectation with respect to all information available as of date 0.

The household enters period 0 with capital $K_0 > 0$. Production in period t is according to a standard production function that has decreasing returns in capital K_t:

$$F(A_t, K_t) = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{46}$$

where TFP A_t is stochastic:

$$\log A_{t+1} = \rho \log A_t + \epsilon_{t+1} \tag{47}$$

The household's resource constraint in each period t is:

$$C_t + K_{t+1} = A_t K_t^{\alpha} L_t^{1-\alpha} + (1-\delta) K_t, \tag{48}$$

where δ is the rate of capital depreciation.

In period 0, the household solves:

$$\max_{C_0, L_0, K_1} E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\gamma} - 1}{1-\gamma} + \varphi \log(1 - L_t) \right]$$
$$\text{s.t.} \quad C_t + K_{t+1} = A_t K_t^{\alpha} L_t^{1-\alpha} + (1-\delta) K_t \tag{49}$$

which, as usual, can be written as a choice of L_0 and K_1 only:

$$\max_{L_0,K_1} E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\left(A_t K_t^{\alpha} L_t^{1-\alpha} + (1-\delta)K_t - K_{t+1}\right)^{1-\gamma} - 1}{1-\gamma} + \varphi \log(1-L_t) \right] \tag{50}$$

5.1.2 Investment and Output

When the household chooses K_{t+1}, it implicitly chooses investment I_t which is defined by:

$$I_t = K_{t+1} - (1-\delta)K_t \tag{51}$$

and output Y_t which is defined by:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{52}$$

5.1.3 Goods Market Clearing

In equilibrium, the quantity of goods produced Y_t has to equal the demand for those goods $C_t + I_t$:

$$Y_t = C_t + I_t \tag{53}$$

We call Equation (**??**) the goods market clearing condition and it represents the aggregate resource constraint for the economy.

5.1.4 Equilibrium

The model has 6 endogenous variables: A_t, K_t, C_t, L_t, Y_t, I_t. Equilibrium is described by:

1. The household's first-order condition for K_{t+1} (the Euler equation)
2. The household's first-order condition for L_t
3. The TFP evolution equation: Equation (**??**)
4. The capital evolution equation: Equation (**??**)
5. The production function: Equation (**??**)
6. The goods market clearing equation: Equation (**??**)

5.1.5 Calibration

Assume the following values for the model's parameters:

Parameter	Value	Description
β	0.99	household's subjective discount factor
γ	–	preference parameter
φ	1.7317	preference parameter
α	0.35	Cobb-Douglas production function parameter
δ	0.025	capital depreciation rate
ρ	0.75	autocorrelation of tfp
σ	0.006	s.d. of TFP shock

5.2 Exercises

1. Include in your presentation a *brief* overview of the model and clearly explain how it differs from the centralized model of Prescott covered in class.

2. Solve for the household's first-order condition for K_{t+1}. Include this equation in your presentation and be able to explain the intuition behind it.

3. Solve for the household's first-order condition for L_t. Include this equation in your presentation and be able to explain the intuition behind it.

4. Use Python to compute the steady state values of A_t, K_t, C_t, L_t, Y_t and I_t. Note that you don't need to know the value of γ.

5. For $\gamma = 1, 2, 5, 10$, compute the impulse responses for 41 periods for all of the model's endogenous variables following a one percentage point increase in TFP in period 5. Plot the four computed impulse responses for each endogenous variable on a separate set of axes and include the plots in your presentation. For example, one set of axes will contain the impulse responses for consumption for $\gamma = 1$, 2, 5, and 10. Make sure to clearly indicate which line corresponds to which value of γ. Units of plotted quantities should be percent deviations from steady state. Be able to explain the intuition behind them.

6 An RBC Model with Variable Capital Utilization

In this problem, you will analyze the implications of variable capital utilization in a decentralized RBC model. As usual, the model features an infinitely-lived household that chooses consumption, labor, and capital accumulation to maximize the present value of its lifetime utility. But the household also chooses the *capital utilization rate*: the the share of its capital to use in production. The household faces the following new trade-off because here the depreciation rate is an increasing function of the household's capital utilization rate.

6.1 The Model

6.1.1 Household Sector

A representative household lives for an infinite number of periods. The expected present value of lifetime utility to the household from consuming $C_0, C_1, C_2, \ldots$ and working $L_0, L_1, L_2, \ldots$ is denoted by U_0:

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left[\log(C_t) + \varphi \log(1 - L_t)\right], \tag{54}$$

where $0 < \beta < 1$ is the household's subjective discount factor and φ reflects the relative value that the household places on leisure in the utility function. E_0 denotes the expectation with respect to all information available as of date 0.

The household enters period 0 with capital $K_0 > 0$ and faces the following intertemporal budget constraint:

$$C_t + K_{t+1} = A_t \left(u_t K_t\right)^{\alpha} L_t^{1-\alpha} + (1 - \delta_0 u_t^{\gamma}) K_t, \tag{55}$$

where u_t is the capital utilization rate and $0 < \delta_0 < 1$ and $\gamma > 0$ are constants.

In period 0, the household solves:

$$\begin{aligned} \max_{C_0, L_0, u_0, K_1} \quad & E_0 \sum_{t=0}^{\infty} \beta^t \left[\log(C_t) + \varphi \log(1 - L_t)\right] \\ \text{s.t.} \quad & C_t + K_{t+1} = A_t \left(u_t K_t\right)^{\alpha} L_t^{1-\alpha} + (1 - \delta_0 u_t^{\gamma}) K_t \end{aligned} \tag{56}$$

which, as usual, can be written as a choice of L_0, u_0, and K_1:

$$\max_{L_0, u_0, K_1} \quad E_0 \sum_{t=0}^{\infty} \beta^t \left[\log(A_t \left(u_t K_t\right)^{\alpha} L_t^{1-\alpha} + (1 - \delta_0 u_t^{\gamma}) K_t - K_{t+1}) + \varphi \log(1 - L_t)\right] \tag{57}$$

6.1.2 Production and Exogenous TFP

Output Y_t is determined according to the following Cobb-Douglas production function:

$$Y_t = A_t \left(u_t K_t\right)^{\alpha} L_t^{1-\alpha} \tag{58}$$

where TFP A_t is stochastic:

$$\log A_{t+1} = \rho \log A_t + \epsilon_{t+1} \tag{59}$$

6.1.3 Equilibrium

The model has 6 endogenous variables: A_t, K_t, C_t, L_t, Y_t, u_t. Equilibrium is described by:

1. The household's first-order condition for L_t
2. The household's first-order condition for u_t
3. The household's first-order condition for K_{t+1} (the Euler equation)
4. The production function: Equation (**??**)
5. The TFP evolution equation: Equation (**??**)
6. The household's budget constraint: Equation (**??**)

6.1.4 Calibration

Assume the following values for the model's parameters:

Parameter	Value	Description
β	0.99	household's subjective discount factor
φ	1.73167	household preference parameter
α	0.35	Cobb-Douglas production function parameter
δ_0	0.03115	capital depreciation rate function coefficient
γ	1.40404	capital depreciation rate function exponent
ρ	0.75	autocorrelation of TFP

6.2 Exercises

1. Download the following series from FRED[13]:

   – Capacity Utilization: Total Industry (Series ID: TCU)

   Construct a well-labeled plot of historical capacity utilization in the US. Compute the mean of capacity utilization and report that value in your presentation.

2. Include in your presentation a *brief* overview of the model and clearly explain how it differs from the centralized model of Prescott covered in class.

3. Solve for the household's first-order conditions for L_t , u_t, and K_{t+1}. Include these equations in your presentation and be able to explain the intuition behind them.

4. Use Python to compute the steady state values of A_t, K_t, u_t, C_t, L_t, Y_t.

5. (a) Compute the impulse responses for all of the model's endogenous variables for 41 periods following a one percentage point increase in TFP in period 5.

[13] https://fred.stlouisfed.org/

(b) Compute the impulse responses for a baseline model with capital utilization fixed *as a parameter* at the steady state value for u_t that you computed above. Note that this baseline model will have only 5 variables and five equilibrium conditions and should have the same steady state values for A_t, K_t, C_t, L_t, Y_t.

Plot the two computed impulse responses for each of the 6 endogenous variables on a separate set of axes and include the plots in your presentation. For example, one set of axes will contain the impulse responses for consumption for variable u_t and another for fixed u.

Units of plotted impulse responses should be percents. Create a set of clear, easy to read figures that depict the impulse responses and include them in your presentation. Units of plotted quantities should be percent deviations from steady state.[14] Be able to explain the intuition behind them.

[14] I.e., multiply the simulated impulse responses by 100.

7 An RBC Model with Capital Adjustment Costs

In this problem, you will analyze the implications of a capital adjustment cost in a decentralized RBC model. As usual, the model features an infinitely-lived household that chooses consumption, labor, and capital accumulation to maximize the present value of its lifetime utility. But the household also faces a cost for installing new capital that is increasing with respect to the magnitude of the growth rate of the capital stock.

7.1 The Model

7.1.1 Household Sector

A representative household lives for an infinite number of periods. The expected present value of lifetime utility to the household from consuming $C_0, C_1, C_2, \ldots$ and working $L_0, L_1, L_2, \ldots$ is denoted by U_0:

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left[\log(C_t) + \varphi \log(1 - L_t)\right], \tag{60}$$

where $0 < \beta < 1$ is the household's subjective discount factor and φ reflects the relative value that the household places on leisure in the utility function. E_0 denotes the expectation with respect to all information available as of date 0.

The household enters period 0 with capital $K_0 > 0$ and faces the following intertemporal budget constraint:

$$C_t + K_{t+1} = A_t K_t^{\alpha} L_t^{1-\alpha} + (1-\delta)K_t - \frac{\psi}{2}\left(\frac{K_{t+1}}{K_t} - 1\right)^2, \tag{61}$$

where $\psi \geqslant 0$ is a constant.

In period 0, the household solves:

$$\max_{C_0, L_0, K_1} \; E_0 \sum_{t=0}^{\infty} \beta^t \left[\log(C_t) + \varphi \log(1 - L_t)\right]$$

$$\text{s.t.} \quad C_t + K_{t+1} = A_t K_t^{\alpha} L_t^{1-\alpha} + (1-\delta)K_t - \frac{\psi}{2}\left(\frac{K_{t+1}}{K_t} - 1\right)^2 \tag{62}$$

which, as usual, can be written as a choice of L_0, and K_1:

$$\max_{L_0, K_1} \; E_0 \sum_{t=0}^{\infty} \beta^t \left[\log\left(A_t K_t^{\alpha} L_t^{1-\alpha} + (1-\delta)K_t - \frac{\psi}{2}\left(\frac{K_{t+1}}{K_t} - 1\right)^2 - K_{t+1}\right) + \varphi \log(1 - L_t)\right] \tag{63}$$

7.1.2 Production and Exogenous TFP

Output Y_t is determined according to the following Cobb-Douglas production function:

$$Y_t = A_t \left(u_t K_t\right)^{\alpha} L_t^{1-\alpha} \tag{64}$$

where TFP A_t is stochastic:

$$\log A_{t+1} = \rho \log A_t + \epsilon_{t+1} \tag{65}$$

7.1.3 Production and Exogenous TFP

The household's Euler condition will contain the ratios K_{t+1}/K_t and K_{t+2}/K_{t+1}. Since `linearsolve` requires variables to be dated only t or $t+1$, define a variable that G_t equals the (gross) growth rate of capital:

$$G_t = \frac{K_{t+1}}{K_t} \tag{66}$$

Then you can replace K_{t+2}/K_{t+1} with G_{t+1} when inputting the model into `linearsolve`.

7.1.4 Equilibrium

The model has 6 endogenous variables: A_t, K_t, C_t, L_t, Y_t, G_t. Equilibrium is described by:

1. The household's first-order condition for L_t
2. The household's first-order condition for K_{t+1} (the Euler equation)
3. The household's budget constraint: Equation (**??**)
4. The production function: Equation (**??**)
5. The TFP evolution equation: Equation (**??**)
6. The growth rate of capital: Equation (**??**)

7.1.5 Calibration

Assume the following values for the model's parameters:

Parameter	Value	Description
β	0.99	household's subjective discount factor
φ	1.73167	household preference parameter
α	0.35	Cobb-Douglas production function parameter
ψ	–	capital adjustment cost parameter
ρ	0.9	autocorrelation of TFP

7.2 Exercises

1. Include in your presentation a *brief* overview of the model and clearly explain how it differs from the centralized model of Prescott covered in class.
2. Solve for the household's first-order conditions for L_t and K_{t+1}. Include these equations in your presentation and be able to explain the intuition behind them.
3. Use Python to compute the steady state values of A_t, K_t, G_t, C_t, L_t, Y_t.

4. For $\psi = 0, 1, 10, 30$, compute the impulse responses for 41 periods for all of the model's endogenous variables following a one percentage point increase in TFP in period 5. Plot the four computed impulse responses for each endogenous variable on a separate set of axes and include the plots in your presentation. For example, one set of axes will contain the impulse responses for consumption for $\psi = 0$, 1, 10, and 30. Make sure to clearly indicate which line corresponds to which value of ψ. Units of plotted quantities should be percent deviations from steady state. Be able to explain the intuition behind them.

8 The Centralized RBC Model with Different Intertemporal Elasticities of Labor Supply

In this problem, you will simulate the dynamic equilibrium of a decentralized RBC model. As usual, the model features an infinitely-lived household that chooses consumption, labor, and capital accumulation to maximize the present value of its lifetime utility. You will simulated the model for different values elasticities of labor supply to see how the elasticity of labor supply affects the dynamics of the model.

8.1 The Model

8.1.1 Household Sector

A representative household lives for an infinite number of periods. The expected present value of lifetime utility to the household from consuming $C_0, C_1, C_2, \ldots$ and working $L_0, L_1, L_2, \ldots$ is denoted by U_0:

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left[\log C_t + \varphi \frac{(1-L_t)^{1-\eta} - 1}{1-\eta} \right], \tag{67}$$

where $0 < \beta < 1$ is the household's subjective discount factor, φ reflects the relative value that the household places on leisure in the utility function, and η reflects the curvature of the household's utility flow from leisure. Specifically, $1/\eta$ is the household's *elasticity of labor supply* and as η increases, the household is *less* willing to substitute leisure for consumption. While this particular role for η may not be apparent from simply inspecting the utility function, the effect will be revealed in simulations. E_0 denotes the expectation with respect to all information available as of date 0.

The household enters period 0 with capital $K_0 > 0$. Production in period t is according to a standard production function that has decreasing returns in capital K_t:

$$F(A_t, K_t) = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{68}$$

where TFP A_t is stochastic:

$$\log A_{t+1} = \rho \log A_t + \epsilon_{t+1} \tag{69}$$

The household's resource constraint in each period t is:

$$C_t + K_{t+1} = A_t K_t^{\alpha} L_t^{1-\alpha} + (1-\delta) K_t, \tag{70}$$

where δ is the rate of capital depreciation.

In period 0, the household solves:

$$\max_{C_0, L_0, K_1} E_0 \sum_{t=0}^{\infty} \beta^t \left[\log C_t + \varphi \frac{(1-L_t)^{1-\eta} - 1}{1-\eta} \right]$$
$$\text{s.t.} \quad C_t + K_{t+1} = A_t K_t^{\alpha} L_t^{1-\alpha} + (1-\delta) K_t \tag{71}$$

which, as usual, can be written as a choice of L_0 and K_1 only:

$$\max_{L_0, K_1} E_0 \sum_{t=0}^{\infty} \beta^t \left[\log \left(A_t K_t^{\alpha} L_t^{1-\alpha} + (1-\delta) K_t - K_{t+1} \right) + \varphi \frac{(1-L_t)^{1-\eta} - 1}{1-\eta} \right] \tag{72}$$

8.1.2 Investment and Output

When the household chooses K_{t+1}, it implicitly chooses investment I_t which is defined by:

$$I_t = K_{t+1} - (1-\delta)K_t \tag{73}$$

and output Y_t which is defined by:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{74}$$

8.1.3 Goods Market Clearing and Equilibrium

In equilibrium, the quantity of goods produced Y_t has to equal the demand for those goods $C_t + I_t$:

$$Y_t = C_t + I_t \tag{75}$$

We call Equation (**??**) the goods market clearing condition and it represents the aggregate resource constraint for the economy.

8.1.4 Equilibrium

The model has 6 endogenous variables: A_t, K_t, C_t, L_t, Y_t, I_t. Equilibrium is described by:

1. The household's first-order condition for K_{t+1} (the Euler equation)
2. The household's first-order condition for L_t
3. The TFP evolution equation: Equation (**??**)
4. The capital evolution equation: Equation (**??**)
5. The production function: Equation (**??**)
6. The goods market clearing equation: Equation (**??**)

8.1.5 Calibration

Assume the following values for the model's parameters:

Parameter	Value	Description
β	0.99	household's subjective discount factor
η	–	preference parameter
φ	–	preference parameter
α	0.35	Cobb-Douglas production function parameter
δ	0.025	capital depreciation rate
ρ	0.75	autocorrelation of tfp
σ	0.006	s.d. of TFP shock

8.2 Exercises

1. Include in your presentation a *brief* overview of the model and clearly explain how it differs from the centralized model of Prescott covered in class.

2. Solve for the household's first-order condition for K_{t+1}. Include this equation in your presentation and be able to explain the intuition behind it.

3. Solve for the household's first-order condition for L_t. Include this equation in your presentation and be able to explain the intuition behind it.

4. Use Python to compute the steady state values of A_t, K_t, C_t, L_t, Y_t and I_t. Note that you don't need to know the value of γ.

5. For $\eta = 1$, 2, 3, and 4:

   (a) Assume that steady state labor supply is $\bar{L} = 1/3$ and use the Euler equation along with Equations (**??**), (**??**), (**??**), a and (**??**) to solve for steady state A_t, K_t, C_t, Y_t and I_t.

   (b) Use the household's first-order condition for L_t to compute the value of the parameter φ implied by $\bar{L}$ and the current value of η. This step ensures that all simulations will be relative to the same steady state.

   (c) Compute the impulse responses for 41 periods for all of the model's endogenous variables following a one percentage point increase in TFP in period 5. Plot the four computed impulse responses for each endogenous variable on a separate set of axes and include the plots in your presentation. For example, one set of axes will contain the impulse responses for consumption for $\eta = 1$, 2, 3, and 4. Make sure to clearly indicate which line corresponds to which value of η. Units of plotted quantities should be percent deviations from steady state. Be able to explain the intuition behind them.

9 The Centralized RBC Model with Labor with Durable and Nondurable Consumption Goods

In this problem, you will simulate the dynamic equilibrium of a decentralized RBC model. As usual, the model features an infinitely-lived household that chooses consumption, labor, and capital accumulation to maximize the present value of its lifetime utility. However, the household also chooses an amount of a durable consumption good to obtain. The durable consumption good provides utility and gradually depreciates over time so it provides value over time.

9.1 The Model

9.1.1 Household Sector

A representative household lives for an infinite number of periods. The expected present value of lifetime utility to the household from consuming nondurable goods $C_0, C_1, C_2, \ldots$, consuming durable goods $D_0, D_1, D_2, \ldots$, and working $L_0, L_1, L_2, \ldots$ is denoted by U_0:

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left[\log C_t + \eta \log D_t + \varphi \log(1 - L_t)\right], \tag{76}$$

where $0 < \beta < 1$ is the household's subjective discount factor, η reflects the relative value that the household places on the utility flow from durable goods, and φ reflects the relative value that the household places on leisure in the utility function. E_0 denotes the expectation with respect to all information available as of date 0.

The household enters period 0 with capital $K_0 > 0$. Production in period t is according to a standard production function that has decreasing returns in capital K_t:

$$F(A_t, K_t) = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{77}$$

where TFP A_t is stochastic:

$$\log A_{t+1} = \rho_A \log A_t + \epsilon^A_{t+1} \tag{78}$$

The household's resource constraint in each period t is:

$$C_t + K_{t+1} + D_{t+1} = A_t K_t^{\alpha} L_t^{1-\alpha} + (1 - \delta_k) K_t + (1 - \delta_d) D_t, \tag{79}$$

where δ_k is the depreciation rate of capital and δ_d is the depreciation rate of durable consumption goods.

In period 0, the household solves:

$$\begin{aligned} \max_{C_0, L_0, K_1} \; & E_0 \sum_{t=0}^{\infty} \beta^t \left[\log C_t + \eta \log D_t + \varphi \log(1 - L_t)\right] \\ \text{s.t.} \quad & C_t + K_{t+1} + D_{t+1} = A_t K_t^{\alpha} L_t^{1-\alpha} + (1 - \delta_k) K_t + (1 - \delta_d) D_t \end{aligned} \tag{80}$$

which can be written as a choice of L_0, K_1, and D_1 only:

$$\max_{L_0, K_1} \; E_0 \sum_{t=0}^{\infty} \beta^t \left[\log\left(A_t K_t^{\alpha} L_t^{1-\alpha} + (1 - \delta_k) K_t + (1 - \delta_d) D_t - K_{t+1} - D_{t+1}\right) + \eta \log D_t + \varphi \log(1 - L_t)\right] \tag{81}$$

9.1.2 Production

Output Y_t which is defined by:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \quad (82)$$

9.1.3 Goods Market Clearing

In equilibrium, the quantity of goods produced has to equal the demand for those goods:

$$Y_t = C_t + K_{t+1} - (1-\delta_k)K_t + D_{t+1} - (1-\delta)_d)D_t \quad (83)$$

We call Equation (**??**) the goods market clearing condition and it represents the aggregate resource constraint for the economy.

9.1.4 Equilbrium

The model has 6 endogenous variables: A_t, K_t, D_t, C_t, L_t, Y_t. Equilibrium is described by:

1. The household's first-order condition for K_{t+1} (the Euler equation)
2. The household's first-order condition for D_{t+1} (another Euler equation)
3. The household's first-order condition for L_t
4. The TFP evolution equation: Equation (**??**)
5. The production function: Equation (**??**)
6. The goods market clearing equation: Equation (**??**)

9.1.5 Calibration

Assume the following values for the model's parameters:

Parameter	**Value**	**Description**
β	0.995	household's subjective discount factor
η	–	preference parameter
φ	1.85742	preference parameter
α	0.35	Cobb-Douglas production function parameter
δ_k	0.025	capital depreciation rate
δ_d	0.25	durable good depreciation rate
ρ_A	0.75	autocorrelation of tfp
σ_A	0.006	s.d. of TFP shock

9.2 Exercises

1. Download the following series from FRED[15]:

   - Personal Consumption Expenditures: Nondurable Goods (Series ID: PCND)
   - Personal Consumption Expenditures: Durable Goods (Series ID: PCDG)

   Construct a well-labeled plot of historical ratio of nondurable consumption goods to durable consumption goods for the US. Compute the mean of the ratio of nondurable to durable consumption and report that value in your presentation.[16]

2. Include in your presentation a *brief* overview of the model and clearly explain how it differs from the centralized model of Prescott covered in class.

3. Solve for the household's first-order condition for K_{t+1}. Include this equation in your presentation and be able to explain the intuition behind it.

4. Solve for the household's first-order condition for D_{t+1}. Include this equation in your presentation and be able to explain the intuition behind it.

5. Solve for the household's first-order condition for L_t. Include this equation in your presentation and be able to explain the intuition behind it.

6. Assume that in the steady state, the ratio C/D equals the average for the US that you found in part (a). Use the first-order condition for D_{t+1} to compute the value for η that would equate the C/D ratio in the model with what you found in part (a).

7. Use Python to compute the steady state values of A_t, K_t, D_t, C_t, L_t, Y_t.

8. Compute the impulse responses for all of the model's endogenous variables for 41 periods following a one percentage point increase in TFP in period 5. Create a set of clear, easy to read figures that depict the impulse responses and include them in your presentation. Units of plotted quantities should be percent deviations from steady state.[17] Be able to explain the intuition behind them.

[15] https://fred.stlouisfed.org/

[16] Compute the ratio for each date *first*, then compute the average of the ratio.

[17] I.e., multiply the simulated impulse responses by 100.

10 The Centralized RBC Model with Habit Formation in Leisure

In this problem, you will simulate the dynamic equilibrium of a decentralized RBC model. As usual, the model features an infinitely-lived household that chooses consumption, labor, and capital accumulation to maximize the present value of its lifetime utility. However, instead in this version of the RBC model, the household values not just current leisure, but current leisure relative to past leisure.

10.1 The Model

10.1.1 Household Sector

A representative household lives for an infinite number of periods. The expected present value of lifetime utility to the household from consuming $C_0, C_1, C_2, \ldots$ and working $L_0, L_1, L_2, \ldots$ is denoted by U_0:

$$U_0 \quad = \quad E_0 \sum_{t=0}^{\infty} \beta^t \left[\log C_t + \varphi \log\left(1 - L_t - h(1 - L_{t-1})\right)\right], \tag{84}$$

where $0 < \beta < 1$ is the household's subjective discount factor, φ reflects the relative value that the household places on leisure in the utility function, and h reflects the degree to which the household forms habits in its leisure activity. E_0 denotes the expectation with respect to all information available as of date 0.

The household enters period 0 with capital $K_0 > 0$. Production in period t is according to a standard production function that has decreasing returns in capital K_t:

$$F(A_t, K_t) \quad = \quad A_t K_t^{\alpha} L_t^{1-\alpha} \tag{85}$$

where TFP A_t is stochastic:

$$\log A_{t+1} \quad = \quad \rho \log A_t + \epsilon_{t+1} \tag{86}$$

The household's resource constraint in each period t is:

$$C_t + K_{t+1} \quad = \quad A_t K_t^{\alpha} L_t^{1-\alpha} + (1-\delta) K_t, \tag{87}$$

where δ is the depreciation rate of capital.

In period 0, the household solves:

$$\begin{aligned} \max_{C_0, L_0, K_1} \; & E_0 \sum_{t=0}^{\infty} \beta^t \left[\log C_t + \varphi \log\left(1 - L_t - h(1 - L_{t-1})\right)\right] \\ \text{s.t.} \quad & C_t + K_{t+1} = A_t K_t^{\alpha} L_t^{1-\alpha} + (1-\delta) K_t \end{aligned} \tag{88}$$

which can be written as a choice of L_0 and K_1 only:

$$\max_{L_0, K_1} \; E_0 \sum_{t=0}^{\infty} \beta^t \left[\log\left(A_t K_t^{\alpha} L_t^{1-\alpha} + (1-\delta)K_t - K_{t+1}\right) + \varphi \log\left(1 - L_t - h(1 - L_{t-1})\right)\right] \tag{89}$$

10.1.2 Investment, Output, and Market Clearing

When the household chooses K_{t+1}, it implicitly chooses investment I_t which is defined by:

$$I_t = K_{t+1} - (1-\delta)K_t \tag{90}$$

and output Y_t which is defined by:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{91}$$

In equilibrium, the quantity of goods produced Y_t has to equal the demand for those goods $C_t + I_t$:

$$Y_t = C_t + I_t \tag{92}$$

We call Equation (**??**) the goods market clearing condition and it represents the aggregate resource constraint for the economy.

10.1.3 An Auxiliary Variable

The household's first-order condition for L_t will contain L_{t-1}, L_t, and L_{t+1}. Since `linearsolve` requires variables to be dated only t or $t+1$, define a variable that L_t^{lag}:

$$L_{t+1}^{lag} = L_t \tag{93}$$

Note that L_t^{lag} is a state variable while L_t is not.

10.1.4 Equilbrium

The model has 7 endogenous variables: A_t, K_t, L_t, C_t, Y_t, I_t, L_t^{lag}. Equilibrium is described by:

1. The household's first-order condition for K_{t+1} (the Euler equation)
2. The household's first-order condition for L_t
3. The TFP evolution equation: Equation (**??**)
4. The capital evolution equation: Equation (**??**)
5. The production function: Equation (**??**)
6. The goods market clearing equation: Equation (**??**)
7. The auxiliary labor variable definition: Equation: (**??**)

10.1.5 Calibration

Assume the following values for the model's parameters:

Parameter	**Value**	**Description**
β	0.995	household's subjective discount factor
h	–	preference parameter
φ	–	preference parameter
α	0.35	Cobb-Douglas production function parameter
δ	0.025	capital depreciation rate
ρ_A	0.75	autocorrelation of tfp
σ_A	0.006	s.d. of TFP shock

10.2 Exercises

1. Include in your presentation a *brief* overview of the model and clearly explain how it differs from the centralized model of Prescott covered in class.

2. Solve for the household's first-order condition for K_{t+1}. Include this equation in your presentation and be able to explain the intuition behind it.

3. Solve for the household's first-order condition for L_t. Include this equation in your presentation and be able to explain the intuition behind it.

4. For $h = 0.01$, 0.1, 0.25, 0.5, and 0.75:

    (a) Assume that steady state labor supply is $\bar{L} = 1/3$ and use the Euler equation along with Equations (**??**), (**??**), (**??**), a and (**??**) to solve for steady state A_t, K_t, C_t, Y_t and I_t.

    (b) Use the household's first-order condition for L_t to compute the value of the parameter φ implied by $\bar{L}$ and the current value of h. This step ensures that all simulations will be relative to the same steady state.

    (c) Compute the impulse responses for 41 periods for all of the model's endogenous variables following a one percentage point increase in TFP in period 5. Plot the four computed impulse responses for each endogenous variable on a separate set of axes and include the plots in your presentation. For example, one set of axes will contain the impulse responses for consumption for $h = 0.01$, 0.1, 0.25, 0.5, and 0.75. Make sure to clearly indicate which line corresponds to which value of h. Units of plotted quantities should be percent deviations from steady state. Be able to explain the intuition behind them.