Deterministic Time Series Models

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Introduction

- Macroeconomic data evolve over time and there is good evidence that:
    1. Historical events matter for the current economy
    2. Current events will matter for the future economy

- We model the *dynamic* nature of the economy using *time series* models

- A **time series** model specifies how a variable or collection of variables are determined as a function of time.

Introduction

- A **time series** model specifies how a variable or collection of variables are determined as a function of time.

- Time series models can be:

  1. **Deterministic**: non-random and purely mechanical
  2. **Stochastic**: Well-defined mathematical structure, but with random elements

- Deterministic models are well-suited for modeling long-run aspects of the economy

- Stochastic models are ideal for modeling business cycle fluctuations because the consensus is that cycles are caused by unpredictable disturbances

Discrete Versus Continuous Time

- Let y denote a variable that takes on the value y_t at date t.
- If t takes on values from a countable sequence, e.g., $t \in [0, 1, 2, \ldots)$, then y_t is a **discrete time** variable.
- Otherwise, if t takes on values from an uncountable sequence, e.g., $t \in [0, \infty)$, then y_t is a **continuous time** variable.
- We will focus exclusively on discrete time models.

First-Order Difference Equations

- Suppose that the variable y_t is determined by a linear function of y_{t-1} and some other exogenously given variable w_t:

$$y_t = \rho y_{t-1} + w_t, \tag{1}$$

where ρ is some constant.

- Equation (1) is an example of a **linear first-order difference equation**.

Examples

Example: Compounding Interest

- Suppose that you have an initial balance of b_0 dollars in an account that pays an interest rate i per compounding period.
- Your period t balance depends on your period $t-1$ balance:

$$b_t = (1+i)\, b_{t-1}. \tag{2}$$

- Equation (2) is a linear first-order difference equation in the same form as Equation (1). You can see this by setting $y_t = b_t$, $\rho = 1+i$, and $w_t = 0$ in Equation (1).

Examples

Example: Physical Capital Accumulation

- In the Solow growth model, the law of motion for physical capital is:

$$K_{t+1} = I_t + (1-\delta)K_t, \tag{3}$$

  where K_t is the capital stock in period t, δ is the rate of capital depreciation, and I_t is investment in new capital

- Treating investment I_t as exogenous, Equation (3) is a linear first-order difference equation in the same form as Equation (1). You can see this by setting $y_t = K_{t+1}$, $\rho = 1-\delta$, and $w_t = I_t$ in Equation (1).