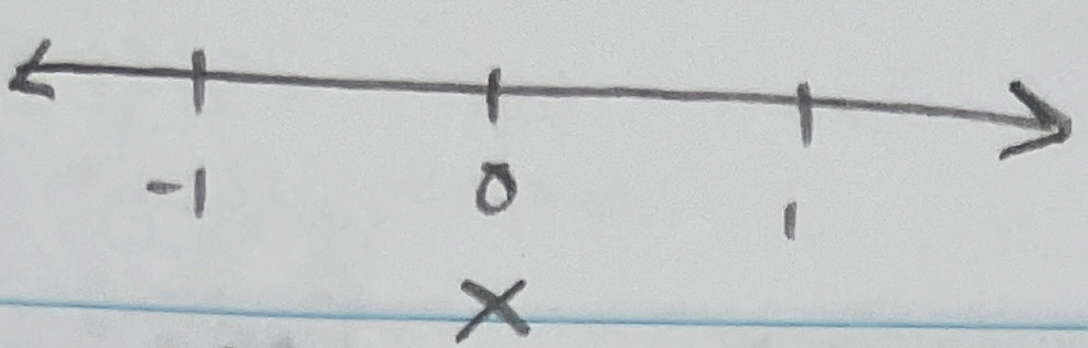
D is number of Drunks at Greenleaf



- Step is Δx long, each step is Δt time, step frequency is $1/\Delta t$
  - each step is uncorrelated, non-interacting

- At $t=0$, all drunks at $x=0$
  $t=1$, half are $+1$, half are -1
  (mean displacement $=0$)

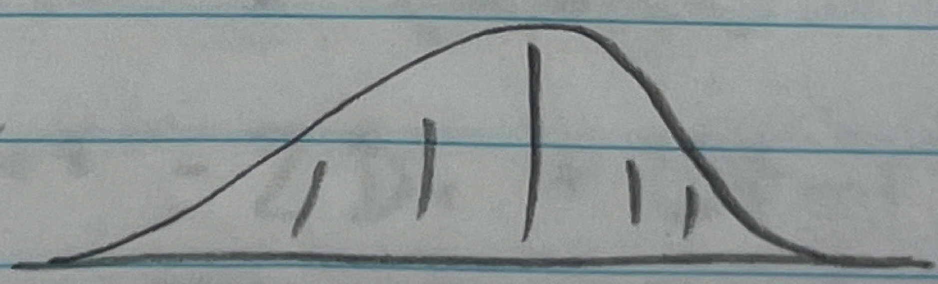
- RMS → how far is average step

$$\sqrt{\frac{\Sigma d^2}{D}} = \sqrt{\frac{\frac{D}{2}(+1)^2 + \frac{D}{2}(-1)^2}{D}} = 1$$

- At $t=2$, 4 possibilities $(+2, -2, 0, 0)$
  mean $=0$
  RMS $=\sqrt{2}$

At $t=3$, 8 posi. mean $=0$
RMS $=\sqrt{3}$

Eventually it'll be



Normal distribution

*How many steps? $N = tf$ $\quad f = 1/\Delta t$

$\bar{x}$ = RMS displacement $= \Delta x\sqrt{N} = \sqrt{(\Delta x)^2 N} = \sqrt{(\Delta x)^2 tf}$

Let's find a nice way to write $\bar{x}$

$K = \frac{1}{2}(\Delta x)^2 f$ $\left[\frac{m^2}{s}\right]$ then $\bar{x} = \sqrt{2Kt}$

Ok, Let's define a "drunk flux"

Drunks at i go to $i+1$ and $i-1$

So flux across $i+\frac{1}{2}$ (midpoint of gradient) $= q_{i+\frac{1}{2}} = \frac{-1}{\Delta t}\left(\frac{1}{2}D_{i+1} - \frac{1}{2}D_i\right)$

$$* \quad q_{i+\frac{1}{2}} = \frac{-\Delta x}{2\Delta t}\left(\frac{D_{i+1} - D_i}{\Delta x}\right) \quad *$$

$\downarrow$ AKA $q = -K\frac{\partial D}{\partial x}$

$K = \frac{\Delta x}{2\Delta t}$

"drunk conductivity"

Let's set some boundary conditions

out − in = production − storage

production = 0 (bars closed)

$$* \quad q_{i+\frac{1}{2}} - q_{i-\frac{1}{2}} = 0 - \frac{\Delta D_i}{\Delta t} \quad *$$

Substitute drunk flux rule w/ continuity

$$\frac{\Delta D_i}{\Delta t} = \left[\frac{-\Delta x}{2\Delta t}\left(\frac{D_{i+1} - D_i}{\Delta x}\right) + \left(\frac{\Delta x}{2\Delta t}\right)\left(\frac{D_i - D_{i-1}}{\Delta x}\right)\right]$$

$$= \frac{\Delta x}{2\Delta t}\left(\frac{D_{i+1} - 2D_i + D_{i-1}}{\Delta x}\right)$$

multiply each side by $\frac{\Delta x}{\Delta x}$ ...

$$\frac{\Delta D_i}{\Delta t} = \frac{(\Delta x)^2}{2\Delta t}\left[\frac{D_{i+1} - 2D_i + D_{i-1}}{(\Delta x)^2}\right]$$

$$\frac{\partial D}{\partial t} = \frac{1}{2}(\Delta x)f * \frac{\partial^2 D}{\partial x^2}$$

$$= K$$

Hooray! Displacement $= \sqrt{2Kt}$ $\quad \bar{x} = \sqrt{cKt}$

Laplace's Eqn $\frac{\partial^2 T}{\partial x^2} = 0$ * see note

Steady state heat eqn or $\frac{\partial T}{\partial t} = \Delta T$

<u>1st derivative</u> $\frac{\partial T}{\partial x}_{i+\frac{1}{2}} = \frac{T_{i+1} - T_i}{\Delta x}$ $\frac{\partial T}{\partial t} = K \frac{\partial^2 T}{\partial x^2}$

= 0 in steady-state

$\partial x_{i+\frac{1}{2}}$ is midpoint of temp gradient

RHS is difference between two gradients

<u>2nd deriv</u>

$$\left.\frac{\partial^2 T}{\partial x^2}\right|_i = \frac{\left.\frac{\partial T}{\partial x}\right|_{i+\frac{1}{2}} - \left.\frac{\partial T}{\partial x}\right|_{i-\frac{1}{2}}}{\Delta x}$$

$$= \frac{\frac{T_{i+1} - T_i}{\Delta x} - \frac{T_i - T_{i-1}}{\Delta x}}{\Delta x}$$

$$= \frac{T_{i+1} - 2T_i + T_{i-1}}{(\Delta x)^2}$$

Physical interpretation of the equation

Informally, the Laplacian operator Δ gives the difference between the average value of a function in the neighborhood of a point, and its value at that point. Thus, if *u* is the temperature, Δu tells (and by how much) the material surrounding each point is hotter or colder, on the average, than the material at that point.

By the second law of thermodynamics, heat will flow from hotter bodies to adjacent colder bodies, in proportion to the difference of temperature and of the thermal conductivity of the material between them. When heat flows into (respectively, out of) a material, its temperature increases (respectively, decreases), in proportion to the amount of heat divided by the amount (mass) of material, with a proportionality factor called the specific heat capacity of the material.

By the combination of these observations, the heat equation says the rate u_{dot} at which the material at a point will heat up (or cool down) is proportional to how much hotter (or cooler) the surrounding material is. The coefficient α in the equation takes into account the thermal conductivity, specific heat, and density of the material.

out − in = ~~production~~ 0 − storage

$$q_{i+\frac{1}{2}} - q_{i-\frac{1}{2}} = 0 - \frac{\Delta D_i}{\Delta t} \qquad i-1 \quad i \quad i+1$$

$$\frac{\Delta D_i}{\Delta t} = \left(\frac{-\Delta x}{2\Delta t}\right)\left(\frac{D_{i+1} - D_i}{\Delta x}\right) - \left(\frac{\Delta x}{2\Delta t}\right)\left(\frac{D_i - D_{i-1}}{\Delta x}\right)$$

$$* \frac{\Delta x}{\Delta x} \qquad \frac{\Delta D_i}{\Delta t} = \frac{\Delta x}{2\Delta t}\left(\frac{D_{i+1} - 2D_i + D_{i-1}}{\Delta x}\right) \; * \frac{\Delta x}{\Delta x}$$

$$* \; \frac{\Delta D_i}{\Delta t} = \frac{(\Delta x)^2}{2\Delta t}\left[\frac{D_{i+1} - 2D_i + D_{i-1}}{(\Delta x)^2}\right]$$

$$\boxed{\frac{\partial D}{\partial t} = K\frac{\partial^2 D}{\partial x^2}}$$

$$\frac{\partial^2 T}{\partial x^2} = 0 \qquad \frac{\partial T}{\partial t} = \Delta T$$

1st deriv

$$\frac{\partial T}{\partial x_{i+\frac{1}{2}}} = \frac{T_{i+1} - T_i}{\Delta x}$$

2nd deriv

$$\left.\frac{\partial^2 T}{\partial x^2}\right|_i = \frac{\left.\frac{\partial T}{\partial x}\right|_{i+\frac{1}{2}} - \left.\frac{\partial T}{\partial x}\right|_{i-\frac{1}{2}}}{\Delta x}$$

flux out — flux in

$$= \frac{\frac{T_{i+1} - T_i}{\Delta x} - \frac{T_i - T_{i-1}}{\Delta x}}{\Delta x}$$

$$= \boxed{\frac{T_{i+1} - 2T_i + T_{i-1}}{(\Delta x)^2}}$$

Files
main
Go to file
0902
0909
0916
0923
0930
1007
1014
1028
1104
Cook and Bow...
Geochem Geo...
Slingerland_Ku...
alta_stock.ipynb
ftcs.ipynb
capstone
.gitignore
README.md
55°F
Mostly clear