ECE 450 - Homework #13

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1 ECE 450 - Homework #13

1.0.1 Package Imports

```
In [1]: import numpy as np
        import seaborn as sns
        import pandas as pd
        import matplotlib.pyplot as plt
        from scipy import signal as sig
        from control import margin, tf
        import warnings
        warnings.filterwarnings('ignore')
```

1.1 Generic function to convolve any number of equations

```
In [9]: def convolve_all(values):
            temp_conv = values[0]
            if len(values) > 1:
                for next_val in values[1:]:
                    temp_conv = np.convolve(temp_conv, next_val)

            return temp_conv
```

1.2 Generic function shift a given filter to a new center frequency

```
In [11]: def shift_filter(num, den, new_center):
             new_num = np.pad(num, (len(den) - len(num), 0), 'constant')
             new_den = den

             for i in range(len(new_den)):
                 new_num[i] *= (new_center ** i)
                 new_den[i] *= (new_center ** i)

             return new_num, new_den
```

1.2.1 Generic function to plot the responses of a system

```
In [2]: # Color list for multiple lines on each subplot
        colors = ["red", "blue", "green", "gray", "purple", "orange"]
```

```
step_size = 0.005

# Generic Function to create a plot
def create_plot(x, y, xLabel=["X-Values"], yLabel=["Y-Values"],
                title=[("Plot", )], num_rows=1, size=(18, 14), logx=False):
    plt.figure(figsize=size, dpi=300)
    for c, (x_vals, y_vals, x_labels, y_labels, titles) in enumerate(zip(x, y, xLabel, y
        for c2, (y_v, t) in enumerate(zip(y_vals, titles)):
            plt.subplot(num_rows, 1, c + 1)
            # Add a plot to the subplot, use transparency so they can both be seen
            plt.plot(x_vals, y_v, label=t, color=colors[c2], alpha=0.70)
            plt.ylabel(y_labels)
            plt.xlabel(x_labels)
            plt.grid(True)
            plt.legend(loc='lower right')
            if logx:
                plt.xscale("log")

    plt.show()
```

1.2.2 Generic function to generate the $|H|$ and ϕ values of a $H(z)$ function

```
In [3]: def z_plot(num, den, T):
            phi = np.arange(0.001, np.pi, T)
            angles = angles = [np.exp(complex(0, angle)) for angle in phi]

            # Loop through all angles, calculate that angles H(z)
            h_z = []
            for z in angles:
                num_sum, den_sum = 0, 0
                for z_pow, num_val in enumerate(num):
                    num_sum += num_val * z ** (len(num) - z_pow)
                for z_pow, den_val in enumerate(den):
                    den_sum += den_val * z ** (len(den) - z_pow)
                h_z.append(num_sum / den_sum)

            return np.multiply(180 / np.pi, phi), 20 * np.log10(h_z)
```

9.1.3, 9.2.1, 9.3.1

1.3 Problem 9.1.3

2nd order monotonic monotonic band pass filter with a pass band between 500 and 550 $\frac{rad}{s}$. The sampling time is 0.1 milliseconds.

Start with $n = 1$, a bandwidth $W = 50$, and a center frequency of $\omega_c = 525$.

$$H_{lp} = \frac{1}{s+1}$$

Scale up this low-pass filter:

$$G_0(s) = H_{lp}(\frac{s}{50}) = \frac{50}{s+50}$$

Now, transform this to a bandpass filter:

$$G(s) = G_0(\frac{s^2+525^2}{s}) = \frac{50s}{s^2+50s+275,625}$$

Now, to convert this function to a z-equivalent transfer function:

$$\alpha = \frac{50}{2} = 25, \omega^2 = 275625 - (\frac{50}{2})^2 = 275000, K_2 = 50, K_1 = \frac{-50*25}{\sqrt{275000}} = -2.38$$

$$H(s) = K_1\frac{\omega}{(s+\alpha)^2+\omega^2} + K_2\frac{s+\alpha}{(s+\alpha)^2+\omega^2}$$

$$H(s) = -2.38\frac{524.4}{(s+25)^2+275000} + 50\frac{s+25}{(s+25)^2+275000}$$

$$H(t) = (-2.38e^{-25t}sin(25t) + 50e^{-25t}cos(25t)) \cdot u(t)$$

$$H(z) = -2.38T\frac{e^{-25T}sin(25T)z}{z^2-2e^{-25T}cos(25T)z+e^{-50T}} + 50T\frac{z^2-e^{-25T}cos(25T)z}{z^2-2e^{-25T}cos(25T)z+e^{-50T}}$$

Resulting in the following Z-domain transfer function:

$$H(z) = \frac{z^2 0.005 + z0.005}{z^2 - z1.995 + 1}$$

1.4 Problem 9.2.1

The below section of code gives my the $H(s)$ function for the third order butterworth filter:

```
In [18]: scipy.signal.TransferFunction([50, 0], [1, 50, 275625], dt=0.1 * 10 ** -3)

Out[18]: TransferFunctionDiscrete(
         array([50.,  0.]),
         array([1.00000e+00, 5.00000e+01, 2.75625e+05]),
         dt: 0.0001
         )
```

```
In [14]: n = 3

         pole_list = []
         for m in range(1, n + 1):
             phi = np.pi / 2 + np.pi / 2 * (2 * m - 1) / n
             pole_list.append(complex(np.cos(phi), np.sin(phi)))

         num = [1]
         den = convolve_all([[1, -pole] for pole in pole_list])
         final_num, final_den = shift_filter(num, den, 10 ** 3)

         print ("Num: ", num, "\nDen: ", np.real(den))
```

```
Num:  [1]
Den:  [1.e+00 2.e+03 2.e+06 1.e+09]
```

$$H(s) = \frac{1}{s^3 + s^2 2 \cdot 10^3 + s2 \cdot 10^6 + 10^9}$$

Which can be converted to:

$$H(z) = \frac{1}{z^3 + z^2 2 \cdot 10^3 + z2 \cdot 10^6 + 10^9}$$

1.5 Problem 9.3.1

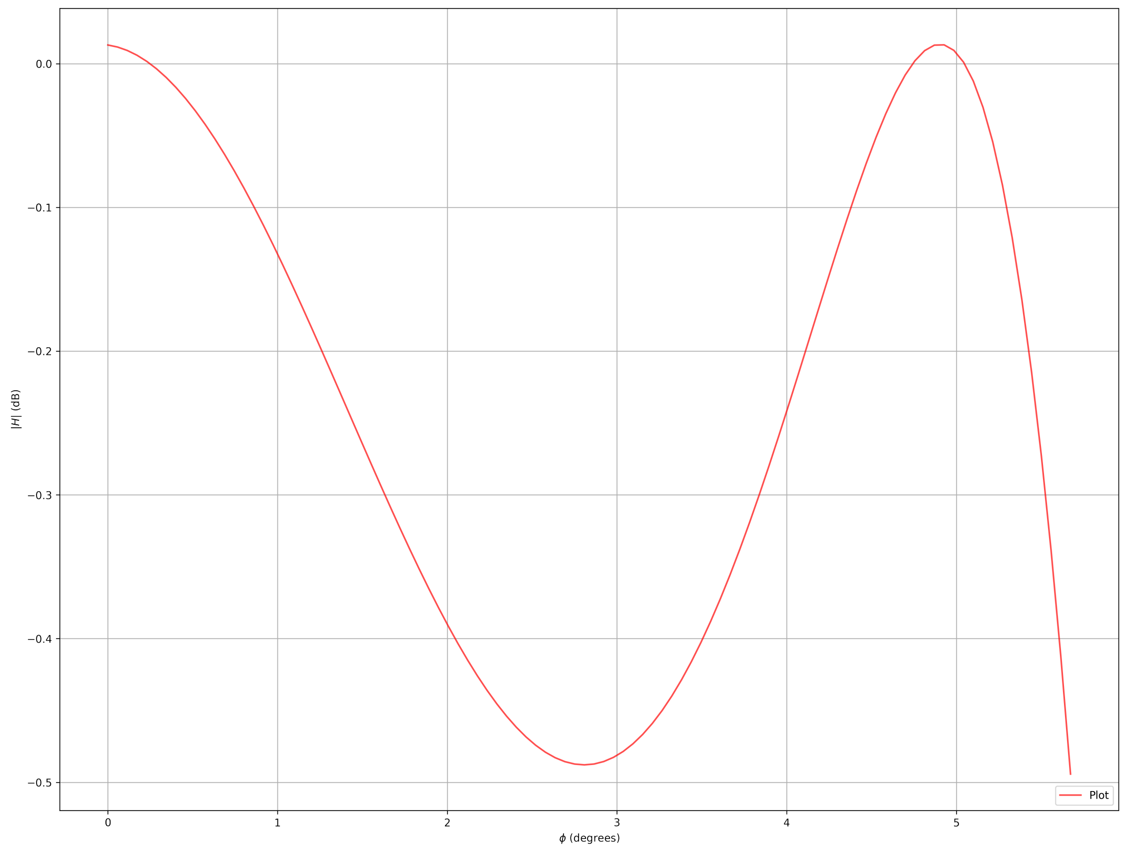
```
In [49]: T = 0.1

         phi = np.arange(0, 2 * np.pi, T / 100)
         NN = len(phi)
         H = []

         for n in range(1, NN):
             z = np.exp(complex(0, phi[n]))
             s = (2 / T) * (z - 1) / (z + 1)
             H.append(0.716 / ((s + 0.626) * (s * s + 0.626 * s + 1.142)))

         phi = np.multiply(180 / np.pi, phi[:-1])[:int(len(phi)/2)][:100]
         H = np.multiply(20, np.log10(H[:len(phi)]))[:100]

         create_plot([phi], [(H, )], ["$\phi$ (degrees)"], ["$|H|$ (dB)"])
```



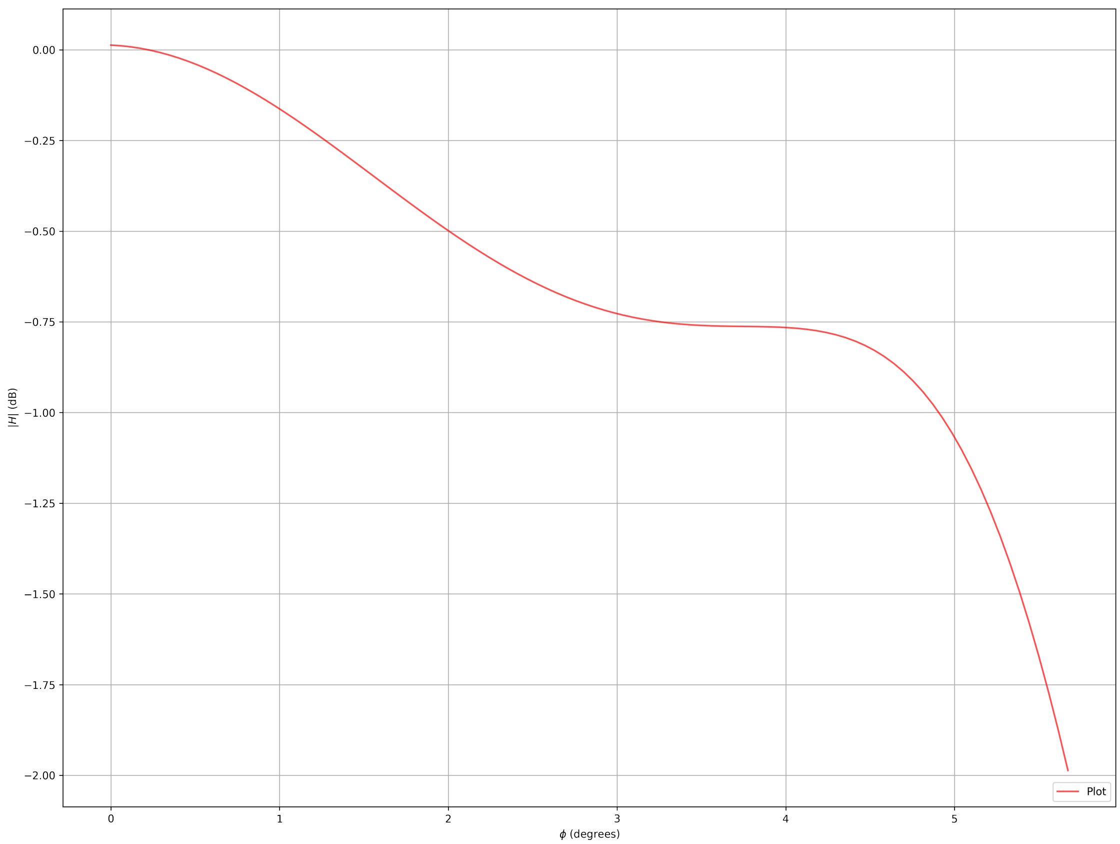
In [48]:
```
T = 0.1

phi = np.arange(0, 2 * np.pi, T / 100)
NN = len(phi)
H = []

for n in range(1, NN):
    z = np.exp(complex(0, phi[n]))
    s = (1 - 1 / z) / T
    H.append(0.716 / ((s + 0.626) * (s * s + 0.626 * s + 1.142)))

phi = np.multiply(180 / np.pi, phi[:-1])[:int(len(phi)/2)][:100]
H = np.multiply(20, np.log10(H[:len(phi)]))[:100]

create_plot([phi], [(H, )], ["$\phi$ (degrees)"], ["$|H|$ (dB)"])
```



As can be seen, the backward rectangular approximation does not meet the specification of the starting 3rd order Chevy-chase filter. The approximated $H(z)$ is drastically different.