ECE 462 - Homework #4

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1 Problem 3.1.1

Prove $F\{x(t)e^{i\omega_0 t}\} = X(\omega + \omega_0)$

To begin, the Fourier transform is as follows:

$$F(\omega) = \mathscr{F}\{f(t)\} = \int_{-\infty}^{\infty} f(t)e^{i\omega t}dt$$

Now replace $f(t)$ with $f(t)e^{i\omega_0 t}$:

$$F(\omega) = \mathscr{F}\{f(t)e^{i\omega_0 t}\} = \int_{-\infty}^{\infty} f(t)e^{i\omega t}e^{i\omega_0 t}dt$$

Which simplifies to:

$$F(\omega) = \mathscr{F}\{f(t)e^{i\omega_0 t}\} = \int_{-\infty}^{\infty} f(t)e^{i(\omega+\omega_0)t}dt$$

Thus, the original Fourier transform can be applied:

$$\mathscr{F}\{f(t)e^{i\omega_0 t}\} = F(\omega + \omega_0)$$

2 Problem 3.1.3

$$\lim_{\alpha \to 0} \frac{2\alpha}{\alpha^2 + (\omega - \omega^2)^2} = 2\pi\delta(\omega - \omega_0)$$

To begin, take the left-hand of the equation into the time-domain:

$$\mathscr{F}^{-1}\{\frac{2\alpha}{\alpha^2 + (\omega - \omega^2)^2}\} = e^{-\alpha|t|}e^{-i\omega_0 t}$$

$$\lim_{\alpha \to 0}(e^{-\alpha|t|}e^{-i\omega_0 t}) = e^{-i\omega_0 t}$$

This results in the left-hand side being: $e^{-i\omega_0 t}$. The Fourier transform of this proves the initial equation:

$$\mathscr{F}\{e^{-i\omega_0 t}\} = 2\pi\delta(\omega - \omega_0) = \lim_{\alpha \to 0} \frac{2\alpha}{\alpha^2 + (\omega - \omega^2)^2}$$

3 Problem 3.1.5

$$\mathscr{F}\{\Psi(x)\} = e^{\frac{-\alpha^2(k-k_0)^2}{2}}$$

4 Problem 3.2.1

$$DOS = \sum_{n=1}^{\infty} \delta(E - n^2)$$

5 Problem 3.4.1

The particle is transmitted through the barrier in functions of the eigenstates. We can plainly see there are periodic dips in what energies are transferred through the dual-potential well.

6 Problem 3.4.2

We could instead solve these types of problems by sending in a large range of input energy states and measuring the output. On the other hand, we could also send the equivalent of an *impulse* response - but I am not sure what the quantum equivalent of this would be.