```
>> %% Volume of the tanks
  V=[10 5.3 11 12 4.3];

%% Inflow matrix 1 [mol/min]
b1=[51; 0; 27; 0; 0];

%% Inflow matrix 2  [M/min]=[mol/(L*min)]
b2=[51/V(1); 0; 27/V(3); 0; 0];

%% Fluid Rate of flow Matrix [L/min]
Q=[ -14    0      0    5.5    0;
      9   -9      0      0    0;
      5    0    -12      0    4;
      0  4.5     12  -16.5    0;
      0  4.5      0    1.5   -6];

%% Rate Matrix [1/min]
R=[Q(1,:)/V(1);
   Q(2,:)/V(2);
   Q(3,:)/V(3);
   Q(4,:)/V(4);
   Q(5,:)/V(5);]

R =

  Columns 1 through 4

                  -1.4                     0                     0                  0.55
      1.69811320754717     -1.69811320754717                     0                     0
     0.454545454545455                     0     -1.09090909090909                     0
                     0                 0.375                     1                -1.375
                     0      1.04651162790698                     0     0.348837209302326

  Column 5

                     0
                     0
```

```
         0.363636363636364
                         0
          -1.3953488372093

>> %% Finding the eigenvalues of R
e=eig(R)

e =

          -0.34161338483812 +                     0i
          -2.01213779895561 +     0.679814462137368i
          -2.01213779895561 -     0.679814462137368i
          -1.29674107645811 +      0.62513266343316i
          -1.29674107645811 -      0.62513266343316i

>> [V, E]=eig(R);
>> %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% Solution vectors                                                                  %
c_vector1=@(t) V(:,1).*exp(e(1)*t);                                                  %
                                                                                     %
c_vector_intermidiate=@(t) (cos(imag(e(3))*t)+(sin(imag(e(3))*t))*sqrt(-1))*V(:,3);  %
                                                                                     %
c_vector2=@(t) real(c_vector_intermidiate(t))*exp(real(e(3))*t);                     %
c_vector3=@(t) imag(c_vector_intermidiate(t))*exp(real(e(3))*t);                     %
                                                                                     %
c_vector_intermidiate2=@(t) (cos(imag(e(5))*t)+(sin(imag(e(5))*t))*sqrt(-1))*V(:,5); %
                                                                                     %
c_vector4=@(t) real(c_vector_intermidiate2(t))*exp(real(e(5))*t);                    %
c_vector5=@(t) imag(c_vector_intermidiate2(t))*exp(real(e(5))*t);                    %
%% The sum c_vector1+c_vector2+...+c_vector5 equals the homogenous part of           %
%% the analytical solution for c'(t)=R*c(t)+b2.                                      %
c_vector_h=@(t) [c_vector1(t),c_vector2(t),c_vector3(t),c_vector4(t),c_vector5(t)];
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% Analytical Solution for
%% Non-Homogeneous part
```

```
%%
c_vector_nh=lin_hom_sys(R,(-1)*b2);
%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
c_vector_nh

c_vector_nh =

          6.33389544688027
          6.33389544688027
          7.04342327150084
          6.84991568296796
          6.46290050590219

>> %% The solution for c'(t)=R*c(t)+b2 is C(t)=c_vector_h(t)*w + c_vector_nh
%% where each row of c_vector_h(t) is multiplied by an arbitrary constant
%% as is common in general solutions, by the principle of linear superposition
%% The value of each of these constants can be found applying the initial conditions.
%% in this case C(0)=0 (zero 5x1 vector), so we get a linear system:
%% -c_vector_nh=c_vector_h(0)*w ~ b3=A*w

A=c_vector_h(0)

A =

  Columns 1 through 4

        -0.287319340730301        0.0400359558124153        -0.252065825298207        -0.143573405203895
          -0.3596762484668         0.480841244776707         0.322119597274646         0.140332766676723
        -0.436480762081284         0.203658175482949       -0.0259158205576298        0.0132079479842168
        -0.552899899101974        -0.356119118760743         0.231058177678548        -0.214654634610346
        -0.540247016314873          -0.6144376403809                         0         0.729966362220564

  Column 5

        -0.165140649023423
        -0.480105383717494
```

```
         0.324571574695876
         0.132182144622021
                         0

>> b3=(-1)*c_vector_nh

b3 =

         -6.33389544688027
         -6.33389544688027
         -7.04342327150084
         -6.84991568296796
         -6.46290050590219

>> %% These are the constants that give the specific solution to the IVP
w=lin_hom_sys(A,b3);
w

w =

          16.0969656124024
         -2.58567417344085
          4.62001300762691
         0.883193573519296
          1.90174535349253

>> %% Since the 1st e'value is not complex valued the solution is e^eig1*t
eig1=real(e(1));
%% Since the 2nd & 3rd e'values are a complex-conjugate-pair
%% the result is e^eigComplex*t=e^(eig2R±eig2I)*t=e^(eig2R*t)*(cos(eig2I*t)+sin(eig2I*t)).
%% The 4th & 5th e'values are also a complex-conjugate-pair, so
%% the solutions are of the same form.
eig2R=real(e(2)); eig2I=imag(e(2));
eig3R=real(e(4)); eig3I=imag(e(4));

p11=w(1)*real(V(1,1)); %% -4.624969547513764
p12=w(2)*real(V(1,3))+w(3)*imag(V(1,3)); %% -1.268067328609109
```

```
p13=w(2)*imag(V(1,3))-w(3)*real(V(1,3)); %% 0.466793457854492
p14=w(4)*real(V(1,5))+w(5)*imag(V(1,5)); %% etc...
p15=w(4)*imag(V(1,5))-w(5)*real(V(1,5));


C1=@(t) p11.*exp(eig1*t)+exp(eig2R*t).*(p12.*cos(eig2I*t)+p13.*sin(eig2I*t))+exp(eig3R*t).*(p14.*cos(eig3I*t)+p15.*sin↙
(eig3I*t))+c_vector_nh(1);

p21=w(1)*real(V(2,1));
p22=w(2)*real(V(2,3))+w(3)*imag(V(2,3));
p23=w(2)*imag(V(2,3))-w(3)*real(V(2,3));
p24=w(4)*real(V(2,5))+w(5)*imag(V(2,5));
p25=w(4)*imag(V(2,5))-w(5)*real(V(2,5));

C2=@(t) p21.*exp(eig1*t)+exp(eig2R*t).*(p22.*cos(eig2I*t)+p23.*sin(eig2I*t))+exp(eig3R*t).*(p24.*cos(eig3I*t)+p25.*sin↙
(eig3I*t))+c_vector_nh(2);

p31=w(1)*real(V(3,1));
p32=w(2)*real(V(3,3))+w(3)*imag(V(3,3));
p33=w(2)*imag(V(3,3))-w(3)*real(V(3,3));
p34=w(4)*real(V(3,5))+w(5)*imag(V(3,5));
p35=w(4)*imag(V(3,5))-w(5)*real(V(3,5));
>>
>> V

V =

  Columns 1 through 2

        -0.287319340730301 +                    0i        0.0400359558124153 +     0.252065825298207i
          -0.3596762484668 +                    0i         0.480841244776707 -     0.322119597274646i
        -0.436480762081284 +                    0i         0.203658175482949 +    0.0259158205576298i
        -0.552899899101974 +                    0i        -0.356119118760743 -     0.231058177678548i
        -0.540247016314873 +                    0i          -0.6144376403809 +                    0i

  Columns 3 through 4
```

```
   0.0400359558124153 -     0.252065825298207i         -0.143573405203895 +     0.165140649023423i
    0.480841244776707 +     0.322119597274646i          0.140332766676723 +     0.480105383717494i
    0.203658175482949 -    0.0259158205576298i         0.0132079479842168 -     0.324571574695876i
   -0.356119118760743 +     0.231058177678548i         -0.214654634610346 -     0.132182144622021i
     -0.6144376403809 +                    0i          0.729966362220564 +                    0i

  Column 5

   -0.143573405203895 -     0.165140649023423i
    0.140332766676723 -     0.480105383717494i
   0.0132079479842168 +     0.324571574695876i
   -0.214654634610346 +     0.132182144622021i
    0.729966362220564 +                    0i

>>
```