Chapter 3

Descriptive Measures

Section 3.1

Measures of Center

Definition 3.1

Mean of a Data Set

The **mean** of a data set is the sum of the observations divided by the number of observations.

Definition 3.2

Median of a Data Set

Arrange the data in increasing order.

- If the number of observations is odd, then the **median** is the observation exactly in the middle of the ordered list.

- If the number of observations is even, then the **median** is the mean of the two middle observations in the ordered list.

In both cases, if we let n denote the number of observations, then the median is at position $(n + 1) / 2$ in the ordered list.

Definition 3.3

Mode of a Data Set

Find the frequency of each value in the data set.

- If no value occurs more than once, then the data set has *no mode.*

- Otherwise, any value that occurs with the greatest frequency is a **mode** of the data set.

Tables 3.1, 3.2 & 3.4

Data Set I

$300	300	300	940	300
300	400	300	400	
450	800	450	1050	

Data Set II

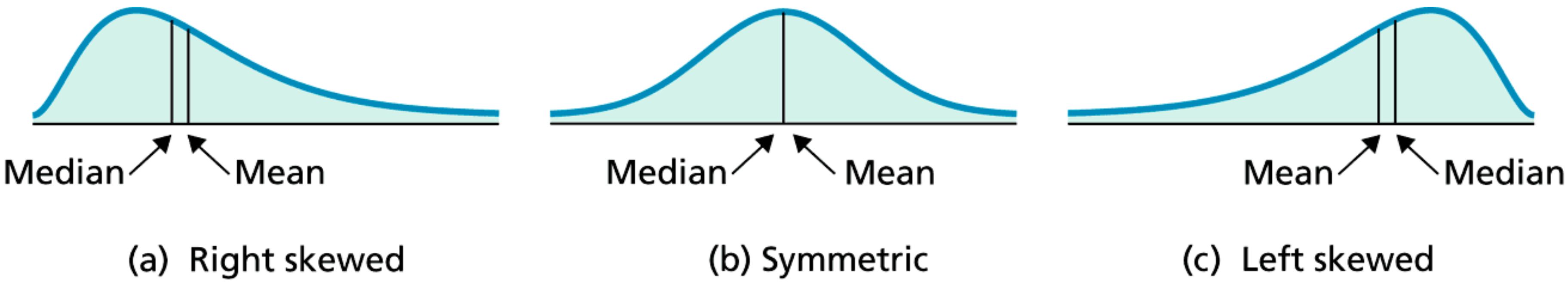
$300	300	940	450	400
400	300	300	1050	300

Means, medians, and modes of salaries in Data Set I and Data Set II

Measure of center	Definition	Data Set I	Data Set II
Mean	$\dfrac{\text{Sum of observations}}{\text{Number of observations}}$	$483.85	$474.00
Median	Middle value in ordered list	$400.00	$350.00
Mode	Most frequent value	$300.00	$300.00

Figure 3.1

Relative positions of the mean and median for
(a) right-skewed, (b) symmetric, and (c) left-skewed distributions



Definition 3.4

Sample Mean

For a variable x, the mean of the observations for a sample is called a **sample mean** and is denoted $\bar{\mathbf{x}}$. Symbolically,

$$\bar{x} = \frac{\Sigma x_i}{n},$$

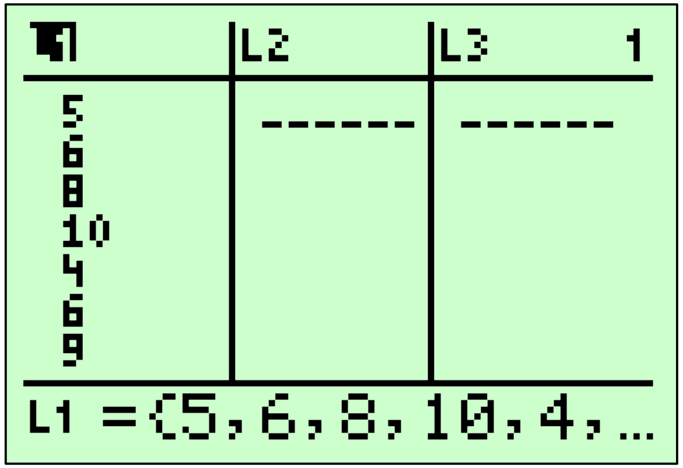
where n is the sample size.

Example: Calculating the Sample Mean (cont.)

Alternate Calculator Method

To find the sample mean on a TI-83/84 Plus calculator, follow the steps below.

- Press STAT.
- Choose option 1:Edit and press ENTER.
- Enter the data in L1.

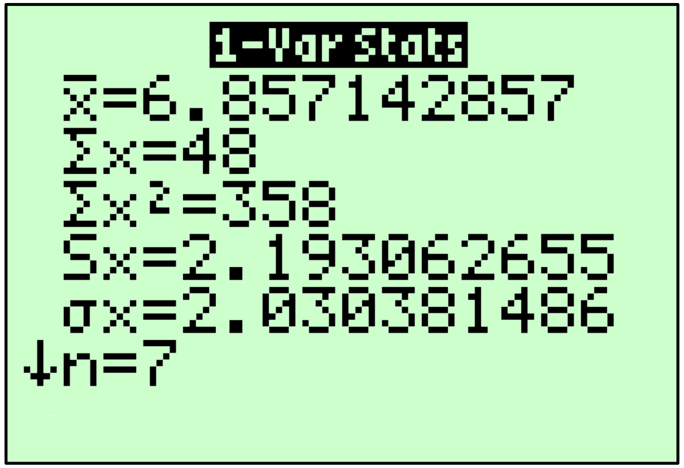


Example: Calculating the Sample Mean (cont.)

- Press STAT again.
- Choose CALC.
- Choose option 1:1-Var Stats.
- Press ENTER twice. (Note: If your data are not in L1, before pressing the ENTER second time, enter the list where your data are located, such as L3 or L5.)

Example: Calculating the Sample Mean (cont.)

The first value in the output, seen to the right, shows the value of $\bar{x} = 6.857142857$. In addition, the calculator displays many other descriptive statistics, not just the sample mean. We will use the above procedure repeatedly to find various descriptive statistics throughout this chapter.



Example: Finding the Median (cont.)

Alternate Calculator Method

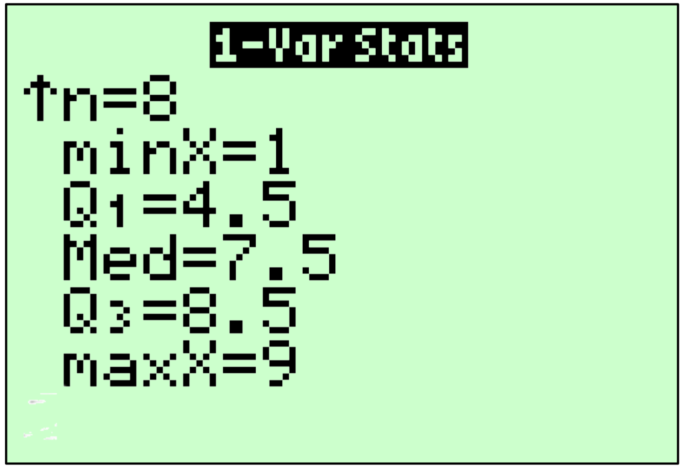
The median is one of the descriptive statistics that the TI-83/84 Plus calculator displays when you choose the 1-Var Stats option from the STAT > CALC menu. Recall from Example 3.1 that the steps to find the descriptive statistics are as follows.

Example: Finding the Median (cont.)

- Press STAT.
- Choose option 1:Edit and press ENTER.
- Enter the data in L1.
- Press STAT again.
- Choose CALC.
- Choose option 1:1-Var Stats.
- Press ENTER twice.

Example: Finding the Median (cont.)

Do you see an output value for the median? Probably not. That is because the median is actually on the second "page" of the output. Use the down arrow to scroll down to the other descriptive statistics. The one labeled "Med=7.5" is the median.

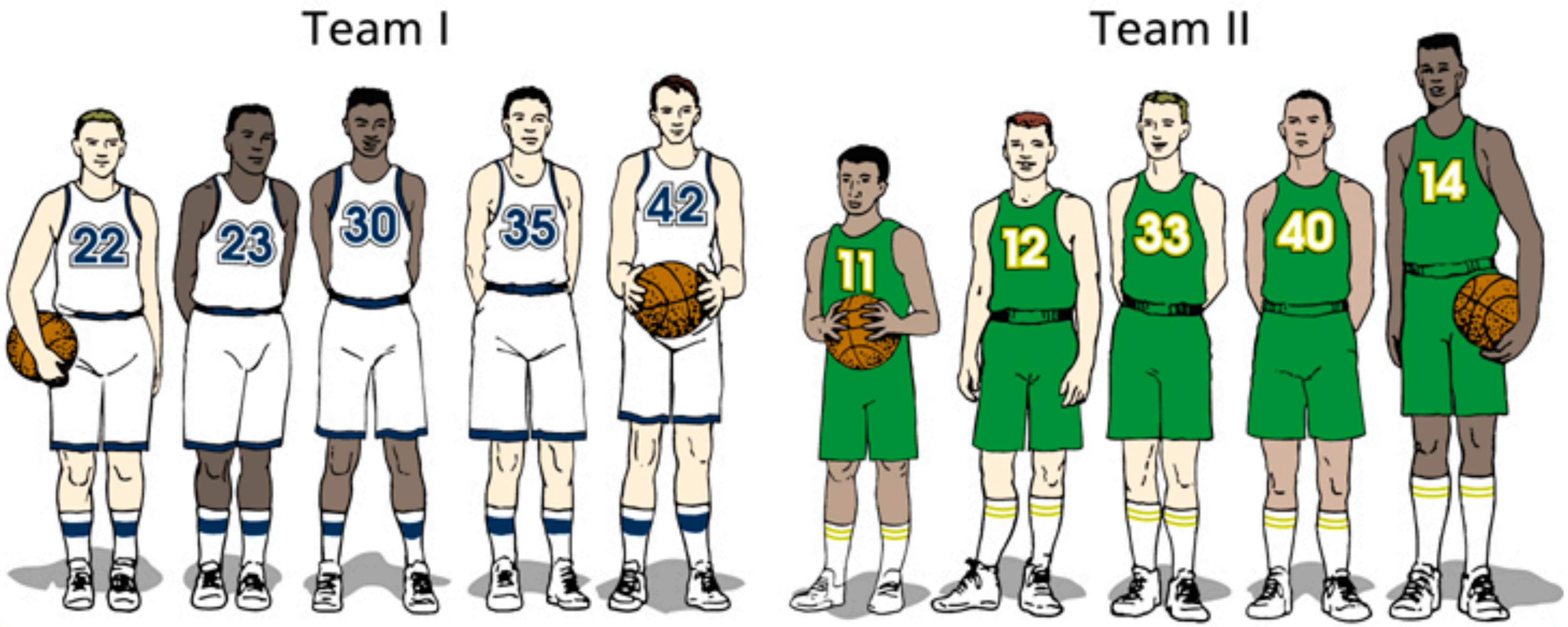


Section 3.2

Measures of Variation

Figure 3.2

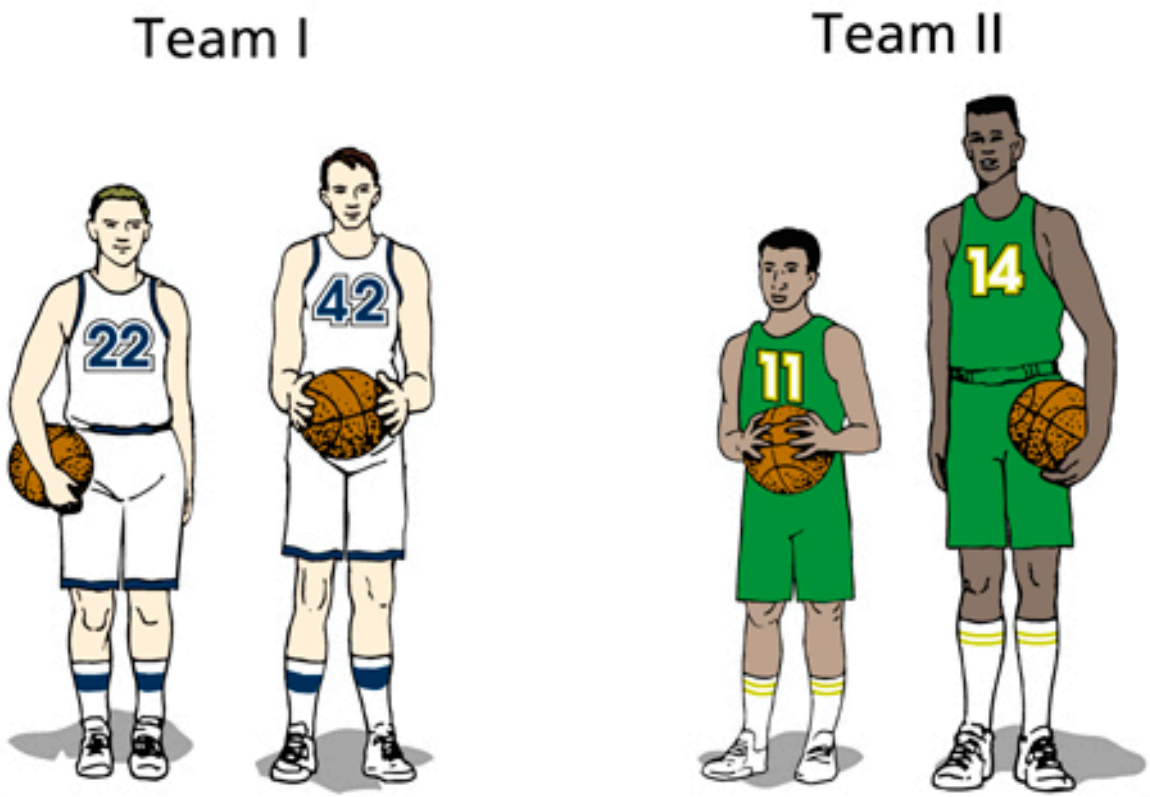
Five starting players on two basketball teams



Feet and inches	6'	6'1"	6'4"	6'4"	6'6"	5'7"	6'	6'4"	6'4"	7'
Inches	72	73	76	76	78	67	72	76	76	84

Figure 3.3

Shortest and tallest starting players on the teams



Feet and inches	6'	6'6"	5'7"	7'
Inches	72	78	67	84

Definition 3.5

Range of a Data Set

The **range** of a data set is given by the formula

$$\text{Range} = \text{Max} - \text{Min},$$

where Max and Min denote the maximum and minimum observations, respectively.

The Sample Standard Deviation

In contrast to the range, the standard deviation takes into account all the observations. It is the preferred measure of variation when the mean is used as the measure of center.

Roughly speaking, the **standard deviation** measures variation by indicating how far, on average, the observations are from the mean.

TABLE 3.6

Deviations from the mean

Height x	**Deviation from mean** $x - \bar{x}$
72	−3
73	−2
76	1
76	1
78	3

FIGURE 3.4

Observations (shown by dots) and deviations from the mean (shown by arrows)

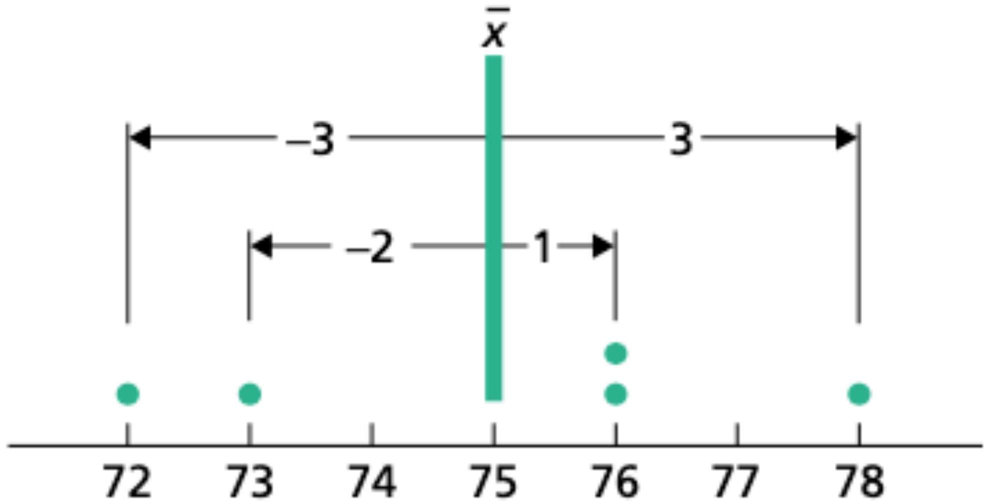


TABLE 3.7

Table for computing the sum of squared deviations for the heights of Team I

Height x	Deviation from mean $x - \bar{x}$	Squared deviation $(x - \bar{x})^2$
72	−3	9
73	−2	4
76	1	1
76	1	1
78	3	9
		24

Definition 3.6

Sample Standard Deviation

For a variable x, the standard deviation of the observations for a sample is called a **sample standard deviation.** It is denoted $\mathbf{s_x}$ or, when no confusion will arise, simply **s**. We have

$$s = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n - 1}},$$

where n is the sample size and $\bar{x}$ is the sample mean.

Formula 3.1

Computing Formula for a Sample Standard Deviation

A sample standard deviation can be computed using the formula

$$s = \sqrt{\frac{\Sigma x_i^2 - (\Sigma x_i)^2/n}{n - 1}},$$

where n is the sample size.

Key Fact 3.1

Variation and the Standard Deviation

The more variation that there is in a data set, the larger is its standard deviation.

Tables 3.10 & 3.11

Data sets that have different variation

Data Set I	41	44	45	47	47	48	51	53	58	66
Data Set II	20	37	48	48	49	50	53	61	64	70

Means and standard deviations of the data sets in Table 3.10

Data Set I	**Data Set II**
$\bar{x} = 50.0$ $s = 7.4$	$\bar{x} = 50.0$ $s = 14.2$

Example: Calculating Standard Deviation (cont.)

Alternate Calculator Method

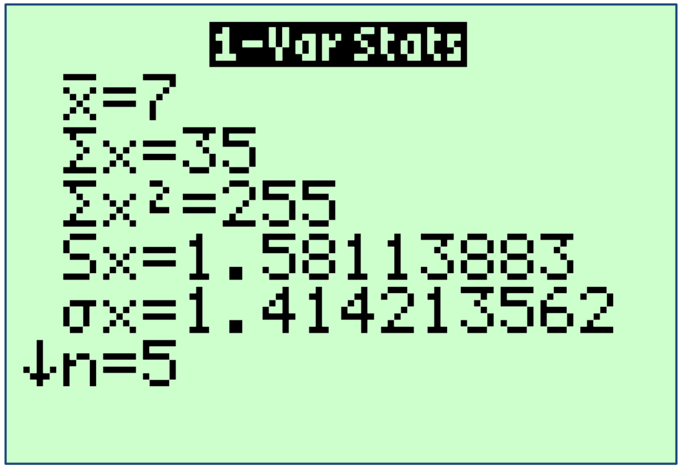
To find the sample standard deviation on a TI-83/84 Plus calculator, follow the steps below.

- Press STAT
- Choose option 1:Edit and press ENTER
- Enter the data in L1.
- Press STAT again.
- Choose CALC.
- Choose option 1:1-Var Stats.

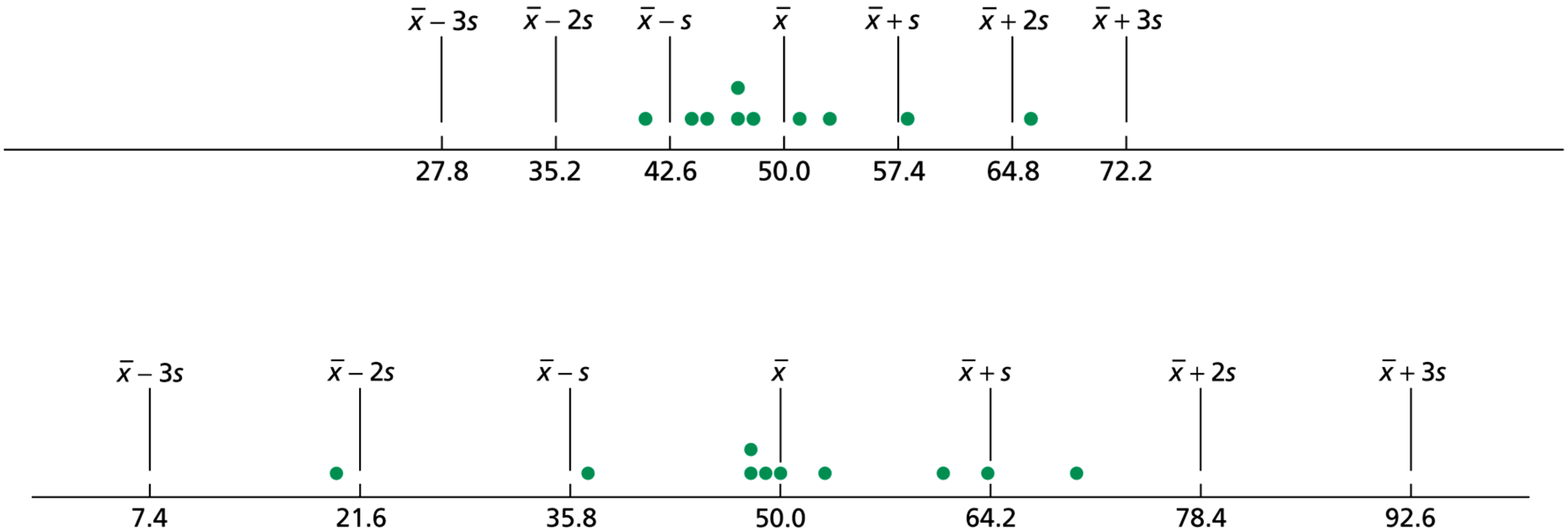
Example: Calculating Standard Deviation (cont.)

- Press ENTER twice. (Note: If your data are not in L1, before pressing ENTER the second time, enter the list where your data are located, such as L3 or L5.)

The fourth value in the output, seen in the screenshot below, gives the sample standard deviation, which is $Sx = 1.58113883 \approx 1.6$.



Figures 3.5 and 3.6



Key Fact 3.2

Three-Standard-Deviations Rule

Almost all the observations in any data set lie within three standard deviations to either side of the mean.