

# CSCI 310 : Advanced Algorithms

Analysis of Algorithm: Fundamentals

# Organizational

- ▶ Teaching Assistant: Sophia Frankel
- ▶ Online Office Hours: Monday 12:00-1:30pm and Wednesday, 10:00-11:30am.
- ▶ Please email her if you are going to meet her during the time.
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## Recollection from Last Lecture

An *algorithm* is a sequence of unambiguous instructions for solving a problem. i.e for obtaining a required output for any legitimate input in a finite amount of time.

Proposed Improvement: Replace legitimate input with **finite input**.

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# Reasons to study Algorithms

- ▶ How to design algorithms?
- ▶ How to analyze algorithm efficiency?

# Analysis of Algorithms

- ▶ Problems
  - ▶ Correctness
  - ▶ Time Efficiency
  - ▶ Space Efficiency
  - ▶ Optimality
- ▶ Approaches - Theoretical and Empirical Analysis.

# Correctness

There are two main ways to verify if an algorithm solves a given problem:

- ▶ Experimental (by testing): the algorithm is executed for a several instances of the input data
- ▶ Formal (by proving): it is proved that the algorithm produces the right answer for any input data
- ▶ In practice: testing, informed by formal methods

# Efficiency- Measures

- ▶ Time Complexity - Clock time, Processor cycle, Operations Count
- ▶ Space Complexity - Byte of Memory



## Example : Sequential Search

- ▶ Input: a list of items and a key.  
Output: the item matching the specified key  
Think: Names and grade records
- ▶ Algorithm: Check each name in order through the entire list of names. Return either the matching record or not present
- ▶ What impacts the speed most?

# Input Size

- ▶ Does measuring strictly input size tell us everything?
  - ▶ How does this map to speed?
  - ▶ How does this compare across algorithms?
- ▶ Input size impacts speed, doesn't directly determine speed.

# Basic Operations

- ▶ Basic operation count determines speed
  - ▶ Exact formula e.g.,  $C(n) = n(n - 1)/2$
- ▶ Formula indicating order of growth with specific multiplicative constant:
  - ▶ e.g.,  $C(n) \approx 0 : 5n^2$
- ▶ Formula indicating order of growth with unknown multiplicative constant:
  - ▶ e.g.,  $C(n) \approx cn^2$

# Orders of growth

- ▶ The relation of basic operations to size of input determines how efficient an algorithm performs as  $n$  grows
- ▶ Basically: does increasing the input size increase the operation count a LOT or a little?

Orders of growth-  $n$ ,  $\log n$ ....

# Basic Analysis Framework

- ▶ Efficiency based on input size and basic operation count
- ▶ This framework applies to both speed and storage analysis
- ▶ One assumption we are missing

## Case-wise efficiency

- ▶ Best case: what is the absolute fastest efficiency for a good input size  $n$ ?
- ▶ Average case: out of several executions, what is the average efficiency?
- ▶ Worst case: what is the absolute slowest efficiency for a bad input of size  $n$ ?
- ▶ Amortized efficiency: does repeating the algorithm many times reduce the per execution speed?

## Example : Sequential Search

- ▶ Best case: 1.
- ▶ Average case:  $\frac{n}{2}$ ?
- ▶ Worst case:  $n$

# Summary

- ▶ Time and storage are most critical
- ▶ We measure efficiency in basic operation counts with respect to the input size
- ▶ Always consider: how fast does the speed/storage requirement grow (i.e. order of growth)?
- ▶ Different qualities of inputs affect the efficiency.



## Next time

- ▶ Levitin Chapter 2.2
- ▶ Homework:  
Chapter 2.1: 5,6, 7 and 9 Remember the hints in the back of the book.
- ▶ Due Date not set. After we meet next lecture, we will discuss.