
Notes on GRE subject of physics

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Preface

When you find this file, probably you have looked up for some GRE textbooks for subject of physics. Then I guess you didn't find any useful material nor the Official Guide this time.

But when you login the Amazon America, I highly recommend you buying the book called ***Conquering the physics GRE*** by Yoni Kahn which will cost you 300 RMB and 1 to 2 months to get here (China).

Here, I summarized most part of the important contents and add some others. After a through reading of the book I've just mentioned and having finished 6 sample tests. I actually finished the exam in 110 minutes and left 60 mins for check, and the feeling for yesterday's questions is that they are definitely easy for us who have experienced Chinese GaoKao and over 98% of those questions I've encountered yesterday have been covered by this brief notes and the book ***Conquering the physics GRE***.

If you work hard enough, I believe you can definitely do better than me.

Best wishes for you!

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Caveat

1. The GRE only has two kinds of calculation questions, either calculable in very limited steps or crackable through some tricks. Thus, ***if you come up with a method potentially hard to calculate, it's wrong.***
2. We must look over every options, at least roughly. Don't ignore the others completely.
3. We must carry out the symbol calculations first, and then try to plug the numbers.
4. Lower our threshold for reexamine by making more marks. For example, if we read too fast about a question, we can make a mark about it and reread it afterwards.

1 Classical Mechanics

Dynamics is to predict, if you don't want to or have known a system's motion, you don't need Newton (Force).

1. Two-body one dimensional collision

$$v'_1 = \frac{m_1 - m_2}{m_1 + m_2} v_1 + \frac{2 m_2}{m_1 + m_2} v_2$$

and

$$v'_1 - v'_2 = v_1 - v_2.$$

in whichever inertial reference frame.

2. Parallel and series:

$$Q_i = C_i U \quad U \text{ is constant, parallel;}$$

$$F_i = k_i x \quad x \text{ is constant, parallel;}$$

$$I_i = G_i U \quad U \text{ is constant, parallel.}$$

Thus, we have

$$C, k \rightarrow \frac{1}{R},$$

or, C, k BingJia ChuanDao.

3. The string propagation speed in continuous medium:

$$v = \sqrt{\frac{T}{\rho}},$$

where T is the tension.

4. As long as a body is orbiting on a circular orbit surrounding a planet, it's kinetic energy is half the magnitude of its current potential energy:

$$E_k = \frac{mv^2}{2} = \frac{GMm}{2r} = \frac{1}{2} \left| -\frac{GMm}{r} \right|.$$

Also, Kepler's law:

$$T = 2\pi \frac{r^{3/2}}{\sqrt{G(M+m)}}.$$

5. The acceleration of CM is the tension.

$$F_{\text{external}} = \frac{d}{dt} \sum_i m_i v_i = \frac{d}{dt} M v_c.$$

Under special circumstances, that is same acceleration field a_{external} for all particles, the relative acceleration in the **CM** for each particle a_i will be irrelevant to such a field. Since

$$v'_i = \dot{v}_i - \dot{v}_c = (a_{\text{external}} + a_{i,\text{internal}}) - a_c = a_{i,\text{internal}}.$$

- 6.

$$\text{harmonic oscillator: } \omega = \sqrt{\frac{k}{m}};$$

$$\text{simple pendulum: } \omega = \sqrt{\frac{g}{l}};$$

$$\text{compound pendulum: } \omega = \sqrt{\frac{mgL_C}{I}},$$

where L_C is the length from the pivot to the center of mass of this whole object.

7. In natural coordinates,

$$\vec{a} = \frac{dv}{dt} \vec{e}_t + \frac{v^2}{\rho} \vec{e}_n.$$

This formula is important because it provides us a powerful approach to calculate the radius of curvature for any path.

8. Coriolis force:

$$a' = F/m + r \times \dot{\omega} + \omega^2 R + 2v' \times \omega. \quad (1.1)$$

9. When deriving the Lagrange equation, we've only used the variation for the least action, no assumption made about whether the potential is time-dependent or not. Thus, it's valid in both circumstances.

10. The condition for non-slipping rotation down along a ramp $v_{\text{rotate}} = v_{\text{translation}}$:

$$a = g \sin \vartheta - \mu g \cos \vartheta,$$

$$\mu mg \cos \vartheta R = I \frac{a}{R}.$$

Usually, we are only interested in a.

Write the second equation in another way,

$$\mu g \cos \vartheta = \frac{I}{mR^2} a.$$

Sum with the first equation, we get:

$$a = \frac{g \sin \vartheta}{1 + \frac{I}{mR^2}}. \quad (1.2)$$

As $I \rightarrow 0$, $a \rightarrow g \sin \vartheta$.

And limitation for μ :

$$\mu = \frac{\tan \vartheta}{1 + \frac{mR^2}{I}}.$$

As $I \rightarrow 0$, $\mu \rightarrow 0$, since we will have a mass point again, and the condition for it to rotate down freely is just frictionless.

11. For a plane rotation (set $\vartheta = 0$):

$$V_{\text{eff}} := \frac{l^2}{2mr^2} + U(r),$$

where

$$l = mr^2 \dot{\phi}.$$

Then,

$$\frac{d^2 r}{dt^2} = -\frac{dV_{\text{eff}}}{dr}.$$

Here, it's totally a one-dimensional question and you cannot use it to analyze the rotation energy.

Angular Momentum

With the assumption that

the direction of each interforce between two elements within a system is on the line between the two elements,

we can easily prove that

the angular momentum about a system's original point is only determined by the external torque.

From

$$r_i \times m_i a = r_i \times F_i,$$

we get

$$\frac{d}{dt} \sum_i (r_i \times m_i \frac{dr_i}{dt}) = \sum_i (r_i \times F_i^{(e)}).$$

That's where we give the definition of J and this equation is valid from gas to rigid body.

1.1 Special case: $\vec{\omega}$ not at original point

Suppose we have a system rotating with ω through its center of mass.

Let J_c represents the angular momentum in **CM**.

$$\begin{aligned} J_c &= \sum_i m_i r'_i \times v'_i \\ &= \sum_i m_i (r_i - r_c) \times (v_i - v_c) \\ &= \sum_i m_i r_i \times v_i - 2Mr_c \times v_c + \sum_i m_i r_c \times v_c \\ &= J - Mr_c \times v_c \end{aligned}$$

Or to say

$$J = J_c + r_c \times P_c.$$

Thus, we've obtained a conclusion of vital importance.

1.2 Angular momentum for a rigid body

$$\begin{aligned} J &= \sum_i (r_i \times m_i v_i) \\ &= \sum_i m_i [\omega r_i^2 - r_i (\omega \times r_i)] \\ &= \begin{pmatrix} \dots & \dots & \dots \end{pmatrix} \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \\ dW &= \tau_z d\phi_z. \end{aligned}$$

1.3 Vocabulary

Due north: exactly towards north.

2 Electromagnetics

2.1 P

The definition of electric dipole moment P

Apparently, we have

$$-\nabla P = \rho_p.$$

For an interface

$$\sigma_p = -(P_{2n} - P_{1n}). \quad (2.1)$$

2.2 D

Since

$$\nabla \cdot (\epsilon_0 E + P) = \rho_f,$$

we define

$$D = \epsilon_0 E + P. \quad (2.2)$$

In a linear system

$$D = \epsilon E. \quad (2.3)$$

The above two equations imply that if we know one of the three quantities, we know all of them.

From Eq. (2.2) we know that

$$\nabla \cdot D = \rho_f,$$

implying that

$$\sigma_f = D_{2n} - D_{1n}. \quad (2.4)$$

But what are free charges?

Usually, they are the ones on the surface of a conductor, and usually they do not exist in mediums. In such cases, the magnitude of D at both sides will be equal.

Thus when we try to calculate the bound charge σ_p on the interface of mediums, we can use the divergence of E directly due to the absence of σ_f .

Only in very special circumstances, like when the medium is capable of leaking electricity or conducting under a certain voltage, there could be a difference of D between two different mediums, and hence σ_f between them.

2.3 The R, L, C Circuit

For the sign for the three devices, they all serve to be decrease of the voltage.

1.

$$\text{Condition: } \frac{R}{2L} < \omega_0 = \frac{1}{\sqrt{LC}}$$

$$q = \frac{Q_0}{\cos \phi} e^{-\frac{1}{2} \frac{R}{L} t} \cos(w' t + \phi),$$

where

$$w' = \sqrt{w_0^2 - \left(\frac{R}{2L}\right)^2}.$$

2.

$$\text{Condition: } \frac{R}{2L} = w_0 = \frac{1}{\sqrt{LC}}$$

$$q = (A + Bt) e^{-\frac{1}{2} \frac{R}{L} t}$$

3.

$$\text{Condition: } \frac{R}{2L} > w_0 = \frac{1}{\sqrt{LC}}$$

$$q = A e^{-\frac{R}{2L} t - \gamma t} + B e^{-\frac{R}{2L} t + \gamma t}$$

where

$$\gamma = \sqrt{\left(\frac{R}{2L}\right)^2 - w_0^2}.$$

4. The general differential equation solution of for forced oscillation.

5.

$$X_C := \frac{1}{i\omega C}, \quad \phi = \pi/2$$

(we've done a derivative.)

where ω represents the angular frequency of the emf.

$$X_L := i\omega L, \quad \phi = -\pi/2.$$

6. C//(R+L) (not very important)

The resonant

$$\omega_r = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}.$$

While the solution for L//(R+C) seems not very friendly.

7. The forced RLC oscillation circuit (in series) (very important):

$$I_0 = \frac{\mathcal{E}_0}{\sqrt{R^2 + (\frac{1}{\omega C} - \omega L)^2}}.$$

$$q_0 = \frac{\mathcal{E}_0}{\omega \sqrt{R^2 + (\frac{1}{\omega C} - \omega L)^2}}.$$

The resonant angular frequency for I is

$$\omega_0 = \frac{1}{\sqrt{LC}}.$$

The resonant angular frequency for Q is:

$$\omega_r = \sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}.$$

2.4 Little knowledge points

1. Lorentz force formula always holds, regardless high speed or low:

$$F_e = qv \times B.$$

2. The current through L and the charges on C's plates are both continuous about time.

3.

$$\nabla \times B = \mu_0(J + \epsilon_0 \dot{E})$$

$$\nabla \times E = -\dot{B}$$

4.

$$dB = \frac{\mu_0}{4\pi} \frac{Idl \times e_r}{r^2}.$$

$$dB = \frac{\mu_0}{4\pi} \frac{JdV \times e_r}{r^2}.$$

5. **For both solenoid and small-inner-diameter toroid, those who only have a unilateral magnetic field):**

$$B = \mu_0 I \frac{dN}{dl} = \mu_0 In. \quad (2.5)$$

$$L := \frac{\Phi_{\text{total}}}{I} = \frac{N\Phi_B}{I},$$

which is to say:

the total magnetic flux is proportional to the current.

For both solenoid and toroid (which only have a unilateral magnetic field):

$$L = \frac{(nl)(\mu_0 InA)}{I} = \mu_0 n^2 lA. \quad (2.6)$$

6. Self-induced emf:

$$\mathcal{E}_L = -\frac{d\Phi_{\text{total}}}{dt} = -L\frac{dI}{dt}.$$

In practical use, as long as we keep in mind that L is a voltage drop along the direction of I , we can just ignore the minus sign here.

7. Deduction of energy stored in L :

For a RL circuit:

$$\mathcal{E} = L\dot{I} + IR.$$

Multiplied by I

$$\mathcal{E}I = LI\dot{I} + I^2R.$$

From **energy conservation law**, we assert that magnetic energy stored in the inductor is

$$dU_B = LI\dot{I}dt = LI dI = L\frac{dI^2}{2}.$$

8. Energy stored in capacitance:

$$U_E = \frac{Q^2}{2C}.$$

9. Mutual induction

Define

$$M_{21} := \frac{\Phi_{2,\text{total}}}{I_1} = \frac{N_2\Phi_{21}}{I_1},$$

where the Φ here is caused by the current in coil 1.

This implies **the magnetic flux in a coil is proportional to the current in the other coil.**

We now assert that

$$M_{21} = M_{12}.$$

By the same procedure done before, we get

$$\mathcal{E}_2 = -M\dot{I}_1,$$

similarly,

$$\mathcal{E}_1 = -M\dot{I}_2,$$

where we have used the assertion that $M_{21} = M_{12}$.

10.

$$\frac{V_2}{N_2} = \frac{V_1}{N_1}.$$

Using

$$P = I_1 U_1 = I_2 U_2,$$

we have

$$I_1 N_1 = I_2 N_2.$$

Thus,

$$R := \frac{V}{I} \propto N^2$$

11. Electric dipole

$$\Phi(r) = \frac{1}{4\pi\epsilon_0} \frac{\mathbf{p} \cdot \mathbf{e}_r}{r^2}.$$

12. For a disk-like capacitor, it there's a current source with current $I(t)$, then it's easy to show that the induced B seems to come from the so called "displacement current":

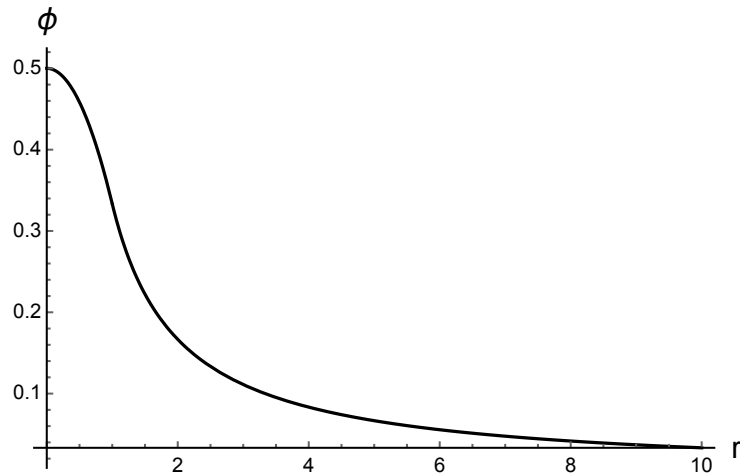
$$\dot{\mathbf{E}} = \frac{\dot{\sigma}}{\epsilon} = \frac{I(t)}{A\epsilon},$$

thus,

$$2\pi r B(r, t) = \mu\epsilon \dot{\mathbf{E}} \pi r^2 = \mu \frac{I}{A} \pi r^2 \rightarrow \mu J_D \pi r^2.$$

13. Resonant state in LCR circuit corresponds zero imaginary part for the net Z .

14. The electric potential for a uniformly charged sphere with radius $R=1$.



15. The diamagnetic material will make the magnetic permeability lower than μ_0 ; while paramagnetic material make it larger than μ_0 .

16. The magnetic lines start from N into the space and come back to S.

3 Thermodynamics

3.1 Basic concepts

Canonical ensemble: fixed particle number N , volume V and fixed T ;

Microcanonical ensemble: fixed N , V and energy E .

$$\Omega_{\text{B-M}} = \frac{N!}{\prod_l a_l!} \prod_l w_l^{a_l}. \quad (3.1)$$

$$\Omega_{\text{B-E}} = \prod_l C_{a_l+w_l-1}^{w_l-1} = \prod_l \frac{(a_l + w_l - 1)!}{a_l! (w_l - 1)!}. \quad (3.2)$$

$$\Omega_{\text{F-D}} = \prod_l C_{w_l}^{a_l} = \prod_l \frac{w_l!}{a_l! (w_l - a_l)!}. \quad (3.3)$$

3.2 Canonical ensemble

Partition function:

$$Z := \sum_i e^{-\beta E_i}.$$

Thus,

$$\langle E \rangle = \frac{\partial}{\partial(-\beta)} \ln Z.$$

$$S := -k \sum_i p_i \ln p_i = \frac{\langle E_i \rangle}{T} + k \ln Z. \quad (3.4)$$

The last equality is easy to be proved by the former definition of Z and p_i .

3.3 Entropy for ideal single-atom gases

$$\begin{aligned} Z_N &= \frac{1}{N!} \frac{V}{(2\pi)^{3N}} \int (dk)^N (dx)^N e^{-\beta E} \\ &= \frac{1}{N!} \frac{V}{(2\pi)^{3N}} \left(\int dk dx e^{-\beta \epsilon} \right)^N \\ &= \frac{1}{N!} \frac{V}{(2\pi)^{3N}} \left(\frac{8m\pi^3}{\beta h^2} \right)^{\frac{3N}{2}}, \end{aligned}$$

The $N!$, obviously, comes from the identity of those atoms.

$$\ln Z_N = N \ln V + \frac{3N}{2} \ln \frac{2\pi mk}{h^2} + \frac{3N}{2} \ln T - \ln(N!).$$

Thus,

$$\begin{aligned}
S_{\text{ideal 1-atom gas}} &= \frac{\langle E \rangle}{T} + k \ln Z_N \\
&= Nk \left(\ln \frac{V}{N} + \frac{3}{2} \ln T + \frac{5}{2} + \frac{3}{2} \ln \frac{2\pi mk}{h^2} \right) \\
&= Nk \left(\ln \frac{V}{N} + \frac{3}{2} \ln T \right) + \dots
\end{aligned} \tag{3.5}$$

3.4 Ideal 2-atom molecule

For translation kinetic energy,

$$U_t = \frac{3N}{2} kT.$$

For rotation,

$$T_r = \frac{\hbar^2}{2I} \sim 1K;$$

For vibration, we set such a oscillator corresponds to angular frequency ω , we would have

$$T_v = \frac{\hbar\omega}{k} \sim 10^3 K.$$

This means that, in the case of vibration at normal low temperature, the energy interval as kT_v/kT is so large that it cannot be represented by integral, which, otherwise, would agree with the result of equipartition theorem.

3.5 the Second Law of Thermodynamics

$$0 \geq \oint \frac{\delta Q}{T}.$$

Or

$$dS \geq \frac{\delta Q}{T}.$$

3.6 Free Expansion and Mix Entropy for ideal gas

In this case, $\delta W = 0$, and $\delta Q = 0$. Thus, $dU = 0 = TdS - PdV$. We construct a reversible process to calculate the entropy change:

$$\Delta S = \int \frac{P}{T} dV = \int \frac{nR}{V} dV = nR \ln(V_2/V_1).$$

Now, let's talk about the mix of two different ideal gases.

We can resort to the above equation to save a lot of redundancy usually encountered in formal textbooks.

Suppose they are isolated into two rooms $V_1 = V_2 = V$ by a partition and they have same

pressure P and temperature T (thus, same amount n). Then since they have no mutual force on each other, we can view them as transparent to each other. Thus it's totally equivalent to the free expansion case;

$$\Delta S_{\text{mix}} = 2nR \ln(2V/V) = 2nR \ln 2.$$

3.7 Little knowledge points

1. Jacobian variable transformation;
2. Canonical and Grand canonical distributions, if we got some time;
- 3.

$$\begin{aligned} G &= U - TS + PV; \\ dG &= -SdT + VdP. \\ F &= U - TS; \\ dF &= -SdT - PdV. \end{aligned}$$

- 4.

$$\left(\frac{\partial T}{\partial P} \right)_S = \left(\frac{\partial V}{\partial S} \right)_P.$$

As long as p and T are at the same side, no minus sign.
 PV are always diagonal; TS are always diagonal.

5. Bloch's electron mass formula:

$$\begin{aligned} v &= \nabla_k w = \frac{1}{\hbar} \nabla_k E. \\ F &= \frac{dp}{dt} = \hbar \frac{dk}{dt}. \\ a &= \frac{dv}{dt} = \frac{1}{\hbar} \frac{d}{dt} \nabla_k E = \frac{1}{\hbar} \nabla_k \frac{dE}{dt}. \end{aligned} \tag{3.6}$$

This is to say:

$$a^a = \frac{1}{\hbar} \frac{\partial}{\partial k_a} \frac{dE}{dt} = \frac{1}{\hbar} \frac{\partial}{\partial k_a} \frac{\partial E}{\partial k_\beta} \frac{dk^\beta}{dt}.$$

From Eq. 3.6, we obtain that

$$a_a = \frac{1}{\hbar^2} \frac{\partial E}{\partial k_a k_\beta} \frac{dF^\beta}{dt},$$

and we define

$$m_{a\beta} = \frac{\hbar^2}{E_{,a\beta}}.$$

6. If the chemical potential μ has a discontinuous Nth order derivative but a continuous (N-1)th order derivative, then it's called Nth order derivative.
 s and v are first derivative of μ . Thus, phase change for water, which has latent heat of phase change and different mole volumes for different phases, is of first order phase change.

The change between superconducting state and the otherwise is 2nd order phase change (discontinuous C_v).

But now, sometimes we just call the **non-first-order phase change** (continuous s and v) as **continuous phase change** and **first order phase change** with discontinuous s and v as **discontinuous phase change** (such as water).

7.

$$N = g \frac{V}{(2\pi)^3} \int_0^{k_F} 4\pi k^2 dk = g \frac{V}{(2\pi)^3} \frac{4\pi k_F^3}{3}.$$

Thus,

$$\rho = \frac{g}{(2\pi)^3} \frac{4\pi k_F^3}{3}.$$

8. For ideal solid, we view each atom as a 3-D harmonic oscillator. For each direction, it has one quadratic component for displacement and one for velocity. Thus, the total molar capacity is

$$C_v = 3R \quad (\text{Dulong-Peti principle}).$$

9. Clockwise path in T-S plane and P-V plane mean that this engine did positive work to a thing after a circle.
10. For the phase-transitional curve:

$$\frac{dp}{dT} = \frac{\Delta s}{\Delta v} = \frac{L}{T\Delta v}, \quad (3.7)$$

where L is the latent heat.

11.

$$(\delta Q)_V = C_V dT.$$

$$(\delta Q)_P = C_P dT.$$

12. **M-B, B-E and F-D all assumed negligible interaction between particles.**

Therefore, these three distribution are only applicable to thin gas state.

13. In the acoustic mode, the Na and Cl are in same direction.
 In the optical mode, they are in opposite directions (EM radiation).

14. Van der Waals function:

$$\left(P + \frac{a}{v_m^2}\right)(v_m - b) = RT,$$

where b measures the size of the particle and a measures the attraction between particles.

3.8 Vocabulary

Quasistatic process: an infinitely slow process;

Isentropic process: zero entropy change for both the system and its surroundings, which is equivalent to Reversible adiabatic process;

4 Optics

4.1 little knowledge points

1. The sign rule: If curvature center is on the right side of the interface (a convex surface with respect to the light, $r > 0$; otherwise, $r < 0$;

Concave lens: $f < 0$;

Virtual images (left to the lens): $s' < 0$

Virtual objects (right to the lens): $s < 0$

However, for the reflective mirror, the above convention may be a little bit different.

2. Define Transverse Magnification:

$$\beta = -\frac{s'}{s}.$$

3. Introduce angle magnification:

$$\gamma = m = -\frac{s}{s'}.$$

4. distance of distinct vision = 25 cm.

5. If we put a thing at the focus of a **convex lens** ($s' \rightarrow \infty$), the magnifying glass's angular magnification is:

$$M = \frac{s_0}{f} = \frac{25\text{cm}}{f},$$

where we have assumed the observed object without lens is 25cm from our eyes.

Otherwise, employ the general equation:

$$M = \frac{s_0 f - s'}{f l - s'},$$

where l is the distance between our eyes and the lens.

6. For a **microscope**, the angular magnification:

$$M = -\frac{\Delta s_0}{f_a f_e} = \frac{s_0}{f'},$$

where Δ is the distance between two convex focuses and we've define image focal length $f := -f_a f_e / \Delta$.

7. For a common focal **telescope** ($\Delta = f_o + f_e$) visual angular magnification is

$$M = -\frac{f_o}{f_e}.$$

Here the s_0 didn't show up, since we cannot put a planet right before you as comparison.

8. Q: Do those three special light path only applicable to lens?

Well, **1)** the incident light passing through F at the object part, will be parallel to light axis after refraction, no matter thin lens or a spherical surface; **2)** the same is true for a parallel incident light. **3)** As for the light passing through light center, the center for spherical surface is its curvature center and that for lens is merely its position at the light axis.

9. In which cases do we have half wave loss?

Wai (from sparse to dense) Zheng Lue (normal and grazing incident).

10. Take the index at left to be 1.

$$\frac{1}{s} + \frac{1}{s'} = \frac{n_{\text{right}} - n_{\text{left}}}{r} = \frac{n - 1}{r}, \quad (4.1)$$

where if the curvature center is at the right side (convex for the incident light), $r > 0$. If we have two interfaces placed closely, we have the second equation:

$$\frac{1}{s'} + \frac{1}{s''} = \frac{1 - n}{r_2}.$$

Then add them up:

$$\frac{1}{s} + \frac{1}{s''} = \frac{n - 1}{r_1} + \frac{1 - n}{r_2}.$$

We then have

$$\frac{1}{f} = (n - 1) \left(\frac{1}{r_1} - \frac{1}{r_2} \right).$$

11. Doppler Effect:

$$f = \frac{c + v_r}{c - v_s} f_0,$$

where v_r is the velocity of the receiver. v_r and v_s are larger than 0, if the receiver and source are approaching each other.

12. Wave speed in string:

$$v = \sqrt{\frac{\text{Tension}}{\rho}};$$

Wave speed in air:

$$v = \sqrt{\frac{K}{\rho}},$$

where $K = P\gamma = Pc_p/c_v$ is the bulk modulus.

13. Rayleigh scattering:

$$I \propto I_0 \omega^4 \alpha^6.$$

14. Brewster's angle:

$$\tan \theta_B = \frac{n_2}{n_1}.$$

15. Q: A hologram picture records what kind of information?

A: Amplitude and phase.

16. For a birefringence crystal, the **o-wave** is the "orthogonal" ordinary wave that perpendicular to the plane that optic axis and incident light mutually make, while the **e-wave** is the one in such a plane.

- 17.

$$a = \frac{\pi}{\lambda} w \sin \theta,$$

$$\gamma = \frac{\pi}{\lambda} d \sin \theta,$$

$$E = E_0 \frac{\sin(N\gamma)}{\sin \gamma} \frac{\sin a}{a}.$$

5 Quantum Mechanics

5.1 The property for 1-D wave function

1. $\langle \psi | \hat{P} | \psi \rangle$ for real stationary state's wave function is 0.

$$\langle P_a \rangle = -i\hbar \int e^{iEt/\hbar} \psi \frac{\partial}{\partial x_a} e^{-iEt/\hbar} \psi dx = -\frac{i\hbar}{2} \int \frac{\partial(\psi^2)}{\partial x_a} dx = 0$$

2. For 1-D stationary state wave function, it can always be written into real function.
3. For an 1-D stationary state, it will never be degenerate state.
4. For an 1-D stationary state, the number of nodes on x-axis for the nth level is n, where we take n=0 as ground state.

5. For an 1-D stationary state, if V is even, $\psi(x)$ will have definite parity. Furthermore, the parity of ϕ_n alternates as n increases.
6. For any 1-D state, its total energy $E > V_{\min}$.

5.2 little knowledge points

1.

$$l_p = \sqrt{\frac{G\hbar}{c^3}} \sim 10^{-35} m.$$

2. Hydrogen-like atoms:

3. Triplet has lower energy than singlet, since the spin part of triplet is symmetric and then its space part is antisymmetric and that results in a further distance for these two electrons (lower repelling potential).

4.

$$J = \frac{\hbar}{m} \text{Im}(\psi^* \nabla \psi).$$

5.

$$[AB, C] = A[B, C] + [A, C]B.$$

$$[x, p^n] = ni\hbar p^{n-1};$$

$$[x^n, p] = ni\hbar x^{n-1}.$$

6. We must be careful when calculating the orthonormal condition: make the **complex conjugation** of the coefficients.

7. For 1-D harmonic oscillator,

$$H = \hbar\omega \left(a^\dagger a + \frac{1}{2}\right).$$

$$[a, a^\dagger] = 1.$$

$$\langle T_n \rangle = \langle V_n \rangle = \frac{E_n}{2}.$$

8. δ function potential:

$$-\frac{\hbar^2}{2m} \frac{d^2 \psi(x)}{dx^2} + A\delta(x)\psi(x) = E\psi(x).$$

Making an integral from $-\epsilon$ to ϵ , we get

$$-\frac{\hbar^2}{2m}(\psi(0_+)' - \psi(0_-)') + A\psi(0) = 0.$$

Such a potential field **only has one bound state** with $E < 0$ (and to have such a state A must be negative).

9. The radial normalization:

$$\int_0^\infty |R(r)|^2 r^2 dr = 1.$$

10. Perturbation theory (Nondegeneracy): First and second order of energy correction and first order correction for wavefunction.
11. The shape of the radial wavefunction for different energy level (at least be familiar with $n=1$ and 2) of the Hydrogen atom.
12. No matter it's a potential barrier or a well, suppose its width is L and the kinetic energy in it is $k' = \sqrt{\frac{2m(E-V)}{\hbar^2}}$. As long as $k'L = n\pi$, the reflection rate will be 0, and the transmitting rate is 1. This is called transmission resonance.

5.3 First order perturbation wavefunction and second order energy correction.

6 Atomic physics

6.1 Hydrogen-like atom

$$E_n = -\frac{\mu(kQq)^2}{2\hbar^2 n^2} \propto \frac{\mu Q^2}{n^2}. \quad (6.1)$$

$$r_n = \frac{n^2 \hbar^2}{\mu(kQq)} \propto \frac{n^2}{\mu Q}. \quad (6.2)$$

$$E = -\frac{kQq}{2r_n}, \quad (6.3)$$

which is not surprise, since Bohr's model relates directly to classic dynamics. If $l \neq 0$, the wavefunction is zero at the origin for any states.

6.2 Bohr Magnetron

$$\mu_B = \frac{|e|\hbar}{m_e} > 0, \quad (6.4)$$

and

$$\mu := g\mu_B \frac{J}{\hbar}.$$

For electron's spin $g \approx -2$.

For its orbital magnetic moment $g \rightarrow -1$ as $M_{\text{nucleus}}/m_e \rightarrow \infty$.

6.3 Fine structure for hydrogenic atom (α^2 order)

1. Relativistic correction for kinetic energy for electron.

$$\Delta E \propto - \left[-\frac{3}{4n^4} + \frac{1}{n^3(l+1/2)} \right].$$

2. l-s coupling.

$$V_{ls} = \alpha \frac{j(j+1) - l(l+1) - s(s+1)}{2}.$$

where

$$\alpha \propto -\frac{1}{2nl(l+1)(l+1/2)}.$$

These two effects certainly break the degeneracy of l (both) and j (the second effect), **individually**.

But, they only break the degeneracy of j **collectively** (see below).

3. **Fine Structure energy shift** (α^2):

$$\Delta E = \frac{E_n Z^2 \alpha^2}{n} \left(\frac{1}{j+1/2} - \frac{3}{4n} \right). \quad (6.5)$$

4. **Lamb shift** (α^3): For same n and j , $nS_{1/2} > nP_{1/2}$.
5. Hyperfine structure, the smallest correction-corresponding 21 cm wavelength for Hydrogen atom, results from the spin-spin coupling between the nucleus and the electron.

6.4 Zeeman Effect

$$\Delta E = -\mu \cdot B.$$

1. Small B (**Zeeman Effect**):

$$\mu_{\text{total}} := g_s \mu_B \frac{S}{\hbar} + g_l \mu_B \frac{L}{\hbar}.$$

$$\langle \mu \rangle = \frac{(\mu_{\text{total}} \cdot J)}{J^2} J = - \left[1 + \frac{j(j+1) + s(s+1) - l(l+1)}{2j(j+1)} \right] \mu_B \frac{J}{\hbar}$$

Since the electron has negative charge, we don't have the minus sign here:

$$\Delta E = m_j |g_j| \mu_B B,$$

where

$$g_j = 1 + \frac{j(j+1) + s(s+1) - l(l+1)}{2j(j+1)}.$$

2. Large B (**Paschen-Back effect**) (easier effect)

The alone interaction of s and B or l and B is stronger than L-S coupling.

Since the electron has negative charge, again, we don't need to worry about the minus sign:

$$\Delta E = m_l \mu_B B + 2m_s \mu_B B.$$

6.5 Selection rule

6.5.1 Single electron atomic radiation (electric dipole approximation)

Since photon has spin 1, we should expect that the transition to be $|l-1\rangle, |l\rangle, |l+1\rangle$. However, we only have

$$\Delta l = \pm 1,$$

due to the absence of $s=0$ for photon. Also, we have the algebra for z-component:

$$\Delta m_l = \pm 1, 0.$$

The zero change comes from the situation when the polarization direction for the photon is along z-axis.

However, a special case is that $2s \rightarrow 1s$ does occur, for in this case the hydrogen emitted 2 photons.

Sometimes we may even consider the situation of L-S coupling.

6.5.2 L-S coupling

When electrons are close,
first,

$$\hat{s}_1 + \hat{s}_2, \hat{l}_1 + \hat{l}_2;$$

then

$$\hat{L} + \hat{S}.$$

Selection rule:

$$\Delta l_i = \pm 1;$$

$$\Delta L = \pm 1, 0;$$

$$\Delta S = 0;$$

$$\Delta J = \pm 1, 0 \text{ (but } 0 \nrightarrow 0 \text{)}.$$

What's the relationship between parity and angular momentum?

6.6 Little knowledge points

1. The measured energy radiated from atoms are always smaller than the energy level difference itself due to the recoil effect to the atom.

2. Define red shift:

$$1 + z := \frac{\lambda_{\text{obs}}}{\lambda_{\text{emit}}}.$$

3. Gas denser \rightarrow higher random collision and lattice vibration \rightarrow dephasing/ causing phase chaos \rightarrow shorter lifetime for excitement state and broader lines in the spectrum.
4. The lowest order for Stark effect is proportional to $|E|^n$?

6.7 Mossbauer effect

In the Mossbauer effect, a narrow resonance for nuclear gamma emission and absorption results from the momentum of recoil being delivered to a surrounding crystal lattice rather than to the emitting or absorbing nucleus alone. When this occurs, no gamma energy is lost to the kinetic energy of recoiling nuclei at either the emitting or absorbing end of a gamma transition: emission and absorption occur at the same energy, resulting in strong, resonant absorption.

6.8 Raman effect

The scattering photons from **molecules** are different from the incident light, arising from the vibration, rotation of molecules and independent on the incident frequency.

6.9 Periodic elements table

The image shows a standard periodic table of elements. It is color-coded by groups: Group 1 (Alkali Metals) is pink, Group 2 (Alkaline Earth Metals) is purple, Groups 3-10 (Transition Metals) are various shades of blue and green, Groups 11-12 (Post-Transition Metals) are yellow and orange, Groups 13-18 (Nonmetals and Noble Gases) are green, blue, and yellow. The table includes element symbols, names, and atomic numbers. Below the main table, there are two rows for the Lanthanide and Actinide series. At the bottom, there is a legend for the color-coded groups: Alkali Metal, Alkaline Earth, Transition Metal, Basic Metal, Semimetal, Nonmetal, Halogen, Noble Gas, Lanthanide, and Actinide.

6.10 Vocabulary

graphene: ShiMoXi

atomic kernel

nuclide: defined by containing a certain number of neutrons (A-Z) and protons (Z).

Deuteron: 2H .

endothermic reaction: reactions that needs thermal in-take

exothermic reaction

nucleus: the central part of an atom (Plural form: nuclei)

protonium: a proton and an antiproton orbiting each other.

7 Electrodynamics and Special Relativity

7.1 Black-body radiation

It's an ideal situation to talk about radiation.

$$Udk = \frac{gV}{(2\pi)^3} \hbar \omega \frac{4\pi k^2}{e^{\hbar \omega / kT} - 1} dk.$$

Thus,

$$u(\omega) \propto \frac{\omega^3}{e^{\hbar \omega / kT} - 1}. \quad (7.1)$$

Integrate it, we get

$$u \propto T^4.$$

This would easily introduce the **Stefan-Boltzmann law**.

$$j = \sigma T^4,$$

where j is the magnitude of the flux at the surface of a black body. Multiplying with the total surface we would get the total radiation power.

If we write the distribution as a function of $u(\lambda)d\lambda$ and calculate the peak wavelength, we have

$$\lambda_{\max} = \frac{2.9 \times 10^{-3} K \cdot m}{T}. \quad (7.2)$$

7.2 Little knowledge points

1. Cherenkov Radiation: In a full period (T), the charged particle moved vT while its radiation has moved cT/n , where n is the refractive index, then

$$\cos \theta_c = \frac{c}{nv}.$$

2.

$$u = \frac{\bar{u} + \beta}{1 + \bar{u}\beta}.$$

3. In insulating medium, the case is an exact analogy to that in vacuum, except that $\mu \rightarrow \mu_0$ and $\epsilon \rightarrow \epsilon_0$. In the absence of J and ρ we have

$$\nabla \cdot E = 0$$

$$\nabla \cdot H = 0$$

$$\nabla \times E = -\mu \dot{H}$$

$$\nabla \times H = \epsilon \dot{E}.$$

If we make a 4-dimension **Fourie's Transformation** we get:

$$k \cdot E = 0$$

$$k \cdot H = 0$$

$$k \times E = i\omega\mu H$$

$$k \times H = -i\omega\epsilon E,$$

where E, H are all $E(k), H(k)$.

4. In a perfect or a good conductor (large σ), there's no electromagnetic field (no E , no B).
5. For a norm incident electromagnetic wave towards a conductor, its reflective electric field E will be reversed.

6. Larmor radiation

Power P:

$$P = \frac{2}{3} \frac{1}{4\pi\epsilon_0 c^3} q^2 a^2 = \frac{1}{6\pi\epsilon_0 c^3} q^2 a^2.$$

Diapole radiation Poynting vector:

$$\langle S \rangle = \left(\frac{\mu_0}{32\pi^2 c} \right) \frac{p_0^2 \omega^4 \sin^2 \vartheta}{r^2},$$

where p_0 is the amplitude of the oscillating diapole.

7.

$$\mu_0 \epsilon_0 c^2 = 1.$$

8. Poynting Vector:

$$S := E \times H.$$

$$\langle S \rangle = \frac{1}{2} \text{Re}\{E_0 \times H_0^*\} = \frac{1}{2\mu} \text{Re}\{E_0 \times B_0^*\}.$$

Since in electromagnetic wave

$$|B| = |E/c|,$$

we have

$$\langle S \rangle = \frac{E_0^2}{2\mu c} e_k = \frac{c\epsilon E_0^2}{2} e_k = \langle \rho \rangle c,$$

where

$$\langle \rho \rangle = \frac{\epsilon \langle E^2 \rangle}{2} + \frac{\langle B^2 \rangle}{2\mu} = \epsilon \langle E^2 \rangle = \frac{\epsilon E_0^2}{2}.$$

9. In the relativistic case, the radius of the path for a charged particle in cyclotron becomes:

$$R = \frac{P}{qB},$$

where

$$P = \frac{mv}{\sqrt{1 - v^2}}.$$

10. PhotoElectric interaction

10.1 Photoelectric effect: few keV;

10.2 Compton scattering: 10 keV to few MeV;

$$\Delta \hat{n} = \frac{h}{m_e c} (1 - \cos \vartheta). \quad (7.3)$$

10.3 Pair production: Dominant process when $E_\gamma > 10$ MeV.

$$\gamma + \text{nucleus} \rightarrow \text{nucleus} + e^- + e^+.$$

8 Analog Circuit

1. N semiconductor:

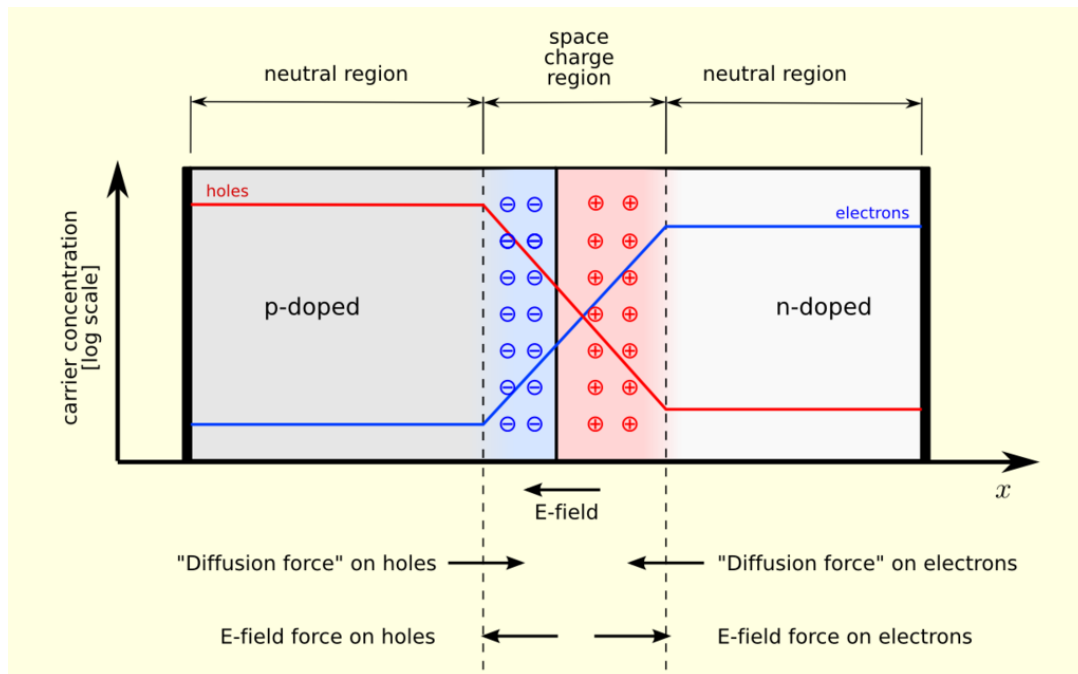
Put some **+5** valence atoms (eg: Phosphorus) into Si and thus bring an extra electron for each atom, since P only has 4 covalent bonds to unite 4 Si atoms. The **extra electrons** serve as current carriers (Negative carriers).

2. P semiconductor:

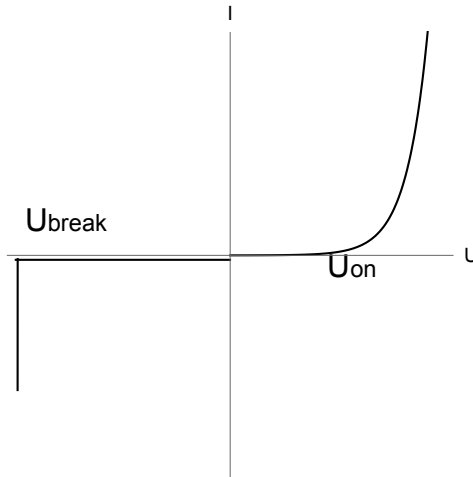
Put some **+3** valence atoms (eg: Aluminum) into Si and thus lack one electron for each atom, since Al only has 3 covalent bonds to unite 4 Si atoms. The **extra holes**, in which electrons can freely fill themselves, serve as current carriers (Positive carriers).

3. Put P and N semiconductor together we get a PN junction. Thus extra electrons in N area would spontaneously diffuse to P part; while holes from positive part would "spread" to N part, equivalently. Therefore, a higher diffusion voltage at the N side occurs.

If we apply an extra voltage to make its inherent voltage lower, we will obtain an current.



4. The voltage-ampere characteristics curve for an ordinary diode:



For a voltage-regulator tube, this is almost the same; just that its U_{break} is more stable and we call it U_Z .

5. Zener diode (It can be used as voltage-regulator tube) :

A Zener diode is a particular type of diode that, unlike a normal one, allows current to flow not only from its anode to its cathode, but also in the reverse direction, when the so-called "Zener voltage" is reached. Zener diodes have a highly doped p-n junction.

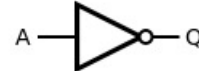
6. And gate and Or gate:



(a) AND



(b) OR



(c) NOT

$$\overline{A \cdot B} = \bar{A} + \bar{B}.$$

$$\overline{A + B} = \bar{A} \cdot \bar{B}.$$

7. Tunnel diode:

A tunnel diode or Esaki diode is a type of semiconductor that is capable of very fast operation (oscillation), well into the microwave frequency region, made possible by the use of the quantum mechanical effect called tunneling.

9 Laboratory Method

9.1 errors

If measurement is quoted with two separate uncorrelated sources of error, say statistical σ_{stat} and systematic error σ_{sys} , the total error is

$$\sigma_{\text{tot}} = \sqrt{\sigma_{\text{stat}}^2 + \sigma_{\text{sys}}^2}. \quad (9.1)$$

For a function,

$$\sigma_z^2 = \sum_i^n \left(\frac{\partial z}{\partial x_i} \right)^2 \sigma_{x_i}^2.$$

No matter $f = xy$ or $f = x/y$, surprisingly, we have:

$$\frac{\sigma_z}{z} = \sqrt{\left(\frac{\sigma_x}{x} \right)^2 + \left(\frac{\sigma_y}{y} \right)^2}.$$

Suppose we have n measurements, define **weighted averages** by error deviation:

$$X := \frac{\sum_i^n X_i / \sigma_{X_i}^2}{\sum_i^n 1 / \sigma_{X_i}^2},$$

and

$$\frac{1}{\sigma_{\text{tot}}^2} = \sum_i \frac{1}{\sigma_{X_i}^2}.$$

$$\text{Uncertainty} = \frac{\sigma}{\bar{X}_i}.$$

9.2 Poisson distribution

$$P(k) = \frac{e^{-\hat{\lambda}} \hat{\lambda}^k}{k!}.$$

$$E(k) = \hat{\lambda},$$

$$\sigma^2 = \hat{\lambda}.$$

The distribution of occurrence time after one event is

$$P(t) = \hat{\lambda} e^{-\hat{\lambda} t}.$$

9.3 Interaction for photons

Photoabsorption (Photoelectric absorption): few keV; interact with a whole atom.

Compton scattering 10 keV ~ few MeV; interact with a electron in a atom.

Pair production: If $E_\gamma > 2m_e c^2$, the electric field near a nucleus can induce the photon to produce an e^- , e^+ pair. This is dominant process at the level of larger than 10 MeV, and it's an interaction with the nucleus.

10 Common sense

1. γ ray: $10^4 \sim 10^8 eV$ ($10keV \sim 100MeV$)
X ray: $50eV \sim 1MeV$.
2. To measure low temperature: **electrical resistance thermometer** (high precision);
and **thermoelectric couple (low precision)**.
To measure high temperature: **optical pyrometer**.
3. Earth-Moon distance 384 Mm. ($Mm = 10^6$)
Sun-earth distance is 149 Gm. ($Gm = 10^9m$)
4. Valence band, conduction band, n-semiconductor, p-semiconductor.
5. $\sqrt{5} = 2.236$; $\sqrt{10} = 3.16$.
6. e^- and μ_e both have lepton number $L_e = +1$.
7. $eV \sim$ chemical reaction;
 $MeV \sim$ nuclear reaction and nuclear binding energy.
8. **In weak interaction, the strangeness certainly doesn't need to be conserved.**
9. Binding energy defined as positive) for nucleus (At least get familiar with the order of magnitude.

$$E_{\text{binding}} = aA - bA^{2/3},$$

which means that the binding is weaker at the surface.

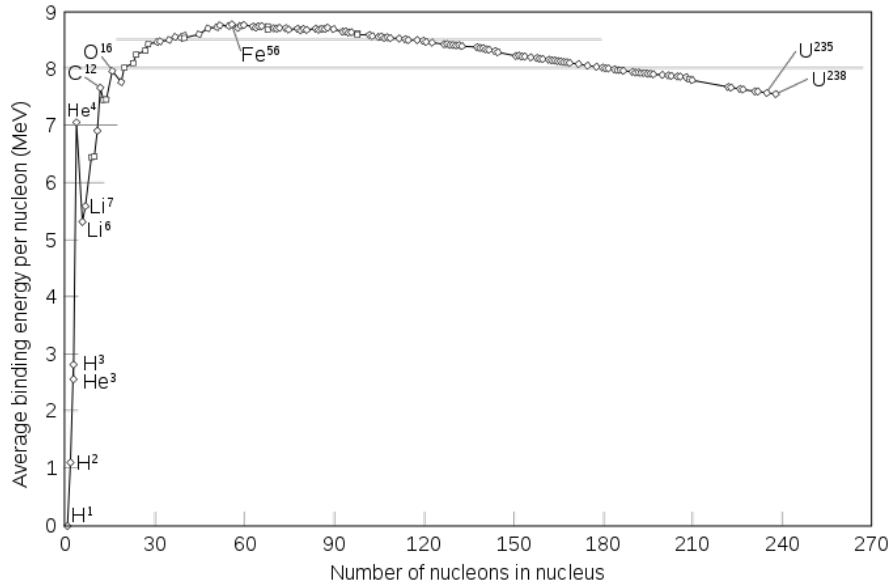


Figure 1: The order is at Mev for nuclear energy.

10. The resolving power for spectrometer:

$$r = \frac{\hbar}{\Delta\hbar},$$

where $\Delta\hbar$ is the nearest wavelength it can distinguish.

11. Cooper pair:

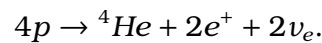
In many superconductors, the attractive interaction between electrons (necessary for pairing) is brought about indirectly by the interaction between the electrons and the vibrating crystal lattice (the phonons). The two electrons are called cooper pair and behave collectively like bosons.

12. J/ψ is composed of $c\bar{c}$.

13. **Precise** measurements have small variance (good method).

14. **Accurate** measurements are close to the true value (truth).

15. The overall reaction in our sun is:



16. If an unstable particle has more than one decay channel,

$$\frac{1}{\tau} = \sum_i \frac{1}{\tau_i}.$$

17. Particles having relativistic velocity are called minimum ionizing particles.
18. The characteristic decay lifetime for strong interaction (fastest) is 10^{-23} seconds, for electromagnetic force (slower) is $10^{-18} \sim 10^{-16}$ seconds, and weak interaction (the slowest) is $10^{-10} \sim 10^{-8}$ seconds.

10.1 Astrophysics

1. Redshift: A photon's wavelength today are longer than it was at past.

$$\hat{\lambda}_{\text{today}} = \hat{\lambda}_o \frac{a_{\text{today}}}{a_o}.$$

This can be used to measure time: For a period time z having elapsed:

$$z + 1 = \frac{\hat{\lambda}_{\text{today}}}{\hat{\lambda}_o},$$

which indicates that time for "now" is 0, time for the past is positive and future negative.

- 2.

$$T(\text{temperature}) \propto \frac{1}{D},$$

where D is the linear scale of our universe.

3. Hubble's law:

$$v \propto D.$$

4. A bound system, like our solar system, doesn't expand.
5. Big bang nucleosynthesis is responsible to the formation from H to Be;
Stellar nucleosynthesis (continue until now) is responsible for the generation from C to Fe.

6. **Dark matter:**

The introduction of dark matter arises from the insufficiency of the mass we have known in the universe to account for the centripetal force of the galaxies since they are rotating too fast. So its force form is **attraction**, obviously.

7. **Dark energy:**

The introduction of dark energy which provides a positive pressure throughout our universe (**repulsion**) is to try to provide an explanation for our continually expanding universe. This kind of "energy" is more like a cosmic constant which keeps its own magnitude despite the expansion of universe.

10.2 Lasers

Suppose there are three energy level: 1, 2, and 3.

1) Population reversion: $1 \rightarrow 3$.

2) Spontaneous emission: $3 \rightarrow 2$ (slow reaction).

3) Stimulated emission: $2 \rightarrow 1$ (fast reaction).

Chain reaction, just one photon having the ν corresponding to $(E_2 - E_1)/h$ will start such a chain reaction, and all stimulated photon will have the same phase, polarization and, of course, frequency as the first one.

1. Dye laser: Laser medium is usually organic dye dissolved in liquid. This laser does not operate at the wavelength of the ordinary visible color.
2. Solid-state laser (neodymium-YAG laser, or Ruby laser): Such laser is made of crystal or glass, the transitions are between atomic energy levels.
3. Collisional gas lasers (Helium-neon laser): Transition between energy level occurs through collision between atoms.
4. Molecular gas laser (CO_2): Laser medium is also gas, and transitions are vibrational energy levels. It's cheap and the triatomic structure gives it a rather rich vibrational spectrum.
5. Semiconductor or diode laser. Photons come from the annihilation between the electrons at the bottom of the conduction band and the holes at the top of the valence band.
6. Free-electron lasers. No atoms here. Photons are radiated by synchronous acceleration. $a \propto \sin(\omega t)$.