

## Course 02402 Introduction to Statistics

### Lecture 13: Summary of the course

DTU Compute  
Technical University of Denmark  
2800 Lyngby – Denmark

### Overview

- 1 Chapter 1: Simple graphics and summary statistics
- 2 Chapter 2: Discrete distributions
- 3 Chapter 2: Continuous distributions
- 4 Chapter 3: One-sample confidence intervals
- 5 Chapter 3: One-sample hypothesis testing
- 6 Chapter 3: Two-sample statistics
- 7 Chapter 4: Statistics by simulation
- 8 Chapter 5: Simple linear regression analysis
- 9 Chapter 6: Multiple linear regression analysis
- 10 Chapter 7: Inference for proportions
- 11 Chapter 8: One-way analysis of variance
- 12 Chapter 8: Two-way analysis of variance

### Overview - the 12 lectures

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### Chapter 1: Simple graphics and summary statistics

- Look at data as it is (descriptive statistics).
- Summary statistics:
  - Mean  $\bar{x}$
  - Standard deviation  $s$ , variance  $s^2$
  - Median, upper and lower quartiles
- Simple graphics:
  - Scatter plot ( $xy$  plot)
  - Histogram, cumulative distribution
  - Box plots, bar charts, pie charts

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## Chapter 2: Discrete distributions

- General concepts:
  - Definition of a stochastic variable
  - Density function
  - Distribution function
  - Mean and variance
- Specific distributions:
  - The binomial distribution
  - The hypergeometric distribution
  - The Poisson distribution

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## Chapter 2: Continuous distributions

- General concepts:
  - Density function, distribution function
  - Mean, variance
  - Calculation rules for stochastic variables
- Specific distributions:
  - Normal
  - Log-normal, uniform, exponential

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## Chapter 3: One-sample confidence intervals

- General concepts
  - Estimation, confidence intervals
  - Population and a random sample
  - Sampling distributions ( $t$  and  $\chi^2$ )
  - Central Limit Theorem (CLT)
- Specific methods, one sample:
  - Confidence intervals for the mean
  - Confidence intervals for the variance (and standard deviation)

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## Chapter 3: One-sample hypothesis testing

- General concepts:
  - Hypotheses, p-value, significance level
  - Type I and Type II error, power
- Specific methods, one sample:
  - $t$ -test for the mean
  - Sample size needed for desired power
  - Normal QQ-plot

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## Chapter 3: Two samples

- Specific methods, two samples:
  - Test and confidence interval for the mean difference ( $t$ -test)
- Specific methods, two PAIRED samples:
  - "Take difference"  $\Rightarrow$  "One sample"
- Planning for precision and/or power
  - One-sample confidence interval: sample size needed for desired precision
  - One-sample hypothesis test: sample size needed for desired power (or other combinations)
  - Two-sample hypothesis test: sample size needed for desired power (or other combinations)

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## Chapter 4: Statistics by simulation

- Introduction to simulation
- Error propagation rules
- Bootstrapping
  - Parametric
  - Non-parametric
  - Confidence intervals (and hence also hypothesis testing)
- Specific situations (4 versions of confidence intervals):
  - One-sample and two-sample data
  - Parametric and non-parametric

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## Chapter 5: Simple linear regression analysis

- Two quantitative variables,  $x$  and  $y$ .
- Calculating least squares line.
- Inference for a simple linear regression model:
  - Statistical model:  $Y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$
  - Confidence intervals and tests for  $\beta_0$  and  $\beta_1$ .
  - Confidence interval for the expected line.
  - Prediction interval.
- $r$  and  $r^2$ 
  - $r$  describes the strength of a linear relation.
  - $r^2$  expresses the proportion of the  $y$  variability explained by the linear relation.

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## Chapter 6: Multiple linear regression analysis

- Many quantitative variables,  $x_1, \dots, x_p$  and  $y$ .
- Calculating least squares fit
- Inference for a multiple linear regression model
  - Statistical model:  $Y_i = \beta_0 + \beta_1 x_{1,i} + \dots + \beta_p x_{p,i} + \varepsilon_i$
  - Confidence intervals and tests for  $\beta_i$ ,  $i = 1, \dots, p$ .
  - Confidence interval for the expected fit.
  - Prediction interval.
- $r^2$  expresses the proportion of the  $y$  variability explained by the linear relation.

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## Chapter 7: Inference for Proportions

- Specific methods, one, two and  $k > 2$  samples
  - Binary/categorical response
- Estimation and confidence intervals for proportions
  - Large sample vs. small sample methods
- Hypotheses for one proportion
- Hypotheses for two proportions
- Analysis of contingency tables ( $\chi^2$ -test) (all expected  $> 5$ )

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## Chapter 8: One-way analysis of variance

- Specific methods,  $k$  INDEPENDENT samples
- One-way analysis of variance
  - Compares the means of the groups
  - ANOVA-table:  $SST = SS(Tr) + SSE$
  - $F$ -test
  - Post hoc test: Pairwise  $t$ -test with/without Bonferroni correction.

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## Chapter 8: Two-way Analysis of Variance

- Block design - two-way analysis of variance
- ANOVA-table:  $SST = SS(Tr) + SS(BI) + SSE$ 
  - $SST$ ,  $SS(Tr)$  and  $SS(BI)$  calculated like for one-way ANOVA
  - $SSE = SST - SS(Tr) - SS(BI)$
- $F$ -test
- Post hoc test: Pairwise  $t$ -test with/without Bonferroni correction.