

Course 02402 Introduction to Statistics

Lecture 4: Confidence intervals

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Overview

- 1 Intro and example
- 2 Distribution of the sample mean
 - The t -distribution
- 3 Confidence interval (CI) for μ
 - Example: Heights
- 4 The language of statistics and the formal framework
- 5 Non-normal data, the Central Limit Theorem (CLT)
- 6 Formal interpretation of the CI
- 7 CI for variance σ^2 and standard deviation σ

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Example: Heights

Sample, $n = 10$:

168 161 167 179 184 166 198 187 191 179

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Sample mean and standard deviation:

$$\bar{x} = 178$$

$$s = 12.21$$

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168 161 167 179 184 166 198 187 191 179

Sample mean and standard deviation:

$$\bar{x} = 178$$

$$s = 12.21$$

Estimate population mean and standard deviation:

$$\hat{\mu} = 178$$

$$\hat{\sigma} = 12.21$$

Example: Heights

Sample, $n = 10$:

168 161 167 179 184 166 198 187 191 179

Sample mean and standard deviation:

$$\bar{x} = 178$$

$$s = 12.21$$

Estimate population mean and standard deviation:

$$\hat{\mu} = 178$$

$$\hat{\sigma} = 12.21$$

NEW: Confidence interval for μ :

$$178 \pm 2.26 \cdot \frac{12.21}{\sqrt{10}} = [169.3; 186.7]$$

NEW: Confidence interval for σ :

$$[8.4; 22.3]$$

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(Empirical) distribution of sample mean

```
# 'True' mean and standard deviation
mu <- 178
sigma <- 12

# Sample size
n <- 10

# Simulate normal distributed  $X_i$  for  $n = 10$ 
x <- rnorm(n = n, mean = mu, sd = sigma)
x

# Empirical density
hist(x, prob = TRUE, col = 'blue')
# Compute sample mean
mean(x)

# Repeat the simulated sampling many times (100 samples)
mat <- replicate(100, rnorm(n = n, mean = mu, sd = sigma))

# Compute the sample mean for each sample
xbar <- apply(mat, 2, mean)
xbar

# See the distribution of the sample means
hist(xbar, prob = TRUE, col = 'blue')
# Empirical mean and variance of sample means
mean(xbar)
var(xbar)
```

Theorem 3.3: Distribution of the sample mean of i.i.d. normal random variables

The distribution of \bar{X}

Assume that X_1, \dots, X_n are independent and identically distributed (*i.i.d.*) normal random variables, $X_i \sim N(\mu, \sigma^2), i = 1, \dots, n$, then:

$$\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

Mean, variance and 'normality' follow from 'rules':

The mean of \bar{X} (Theorem 2.56):

$$E(\bar{X}) = \frac{1}{n} \sum_{i=1}^n E(X_i) = \frac{1}{n} \sum_{i=1}^n \mu = \frac{1}{n} n\mu = \mu$$

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The variance of \bar{X} (Theorem 2.56):

$$\text{Var}(\bar{X}) = \frac{1}{n^2} \sum_{i=1}^n \text{Var}(X_i) = \frac{1}{n^2} \sum_{i=1}^n \sigma^2 = \frac{1}{n^2} n\sigma^2 = \frac{\sigma^2}{n}$$

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The 'normality' of \bar{X} (Theorem 2.40):

By this theorem, the distribution of \bar{X} is a normal distribution with mean μ and variance σ^2/n as specified above.

Distribution of the error $\bar{X} - \mu$

The standard deviation of \bar{X} :

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$

Distribution of the error $\bar{X} - \mu$

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The standard deviation of $\bar{X} - \mu$:

$$\sigma_{(\bar{X}-\mu)} = \frac{\sigma}{\sqrt{n}}$$

Standardized version of the above, Theorem 3.4

Distribution of the standardized sample mean (or standardized error):

Assume that X_1, \dots, X_n are i.i.d. normal random variables, $X_i \sim N(\mu, \sigma^2)$ for $i = 1, \dots, n$, then:

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1^2)$$

That is, the standardized sample mean Z follows a standard normal distribution.

Practical problem (and solution)

How do we use the results from the previous slides to say something about μ ...
... when the 'true', unknown, population standard deviation σ enters into all the formulas?

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Obvious solution:

Use the estimate s instead of σ in formulas.

Practical problem (and solution)

How do we use the results from the previous slides to say something about μ ...
... when the 'true', unknown, population standard deviation σ enters into all the formulas?

Obvious solution:

Use the estimate s instead of σ in formulas.

BUT:

Then, we need new theory! (There is also uncertainty linked to s .)

Theorem 3.5, a more applicable extension of the above

The t -distribution takes the uncertainty of s into account:

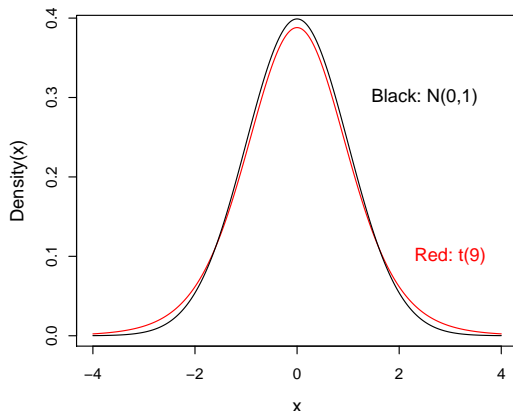
Assume that X_1, \dots, X_n are i.i.d. normal distributed random variables, where $X_i \sim N(\mu, \sigma^2)$ for $i = 1, \dots, n$, then:

$$T = \frac{\bar{X} - \mu}{S/\sqrt{n}} \sim t(n-1)$$

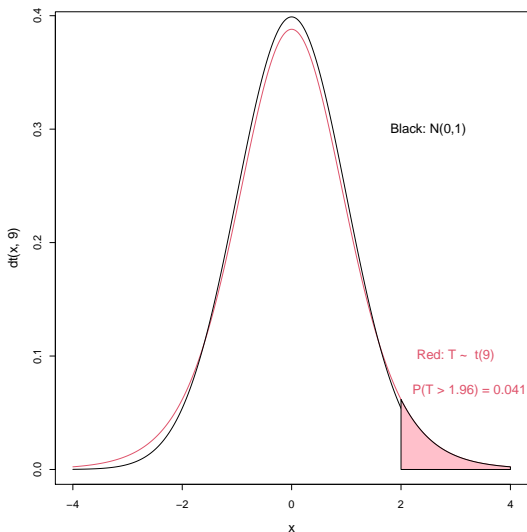
where $t(n-1)$ is the t -distribution with $n-1$ degrees of freedom.

The t -distribution with 9 degrees of freedom ($n = 10$)

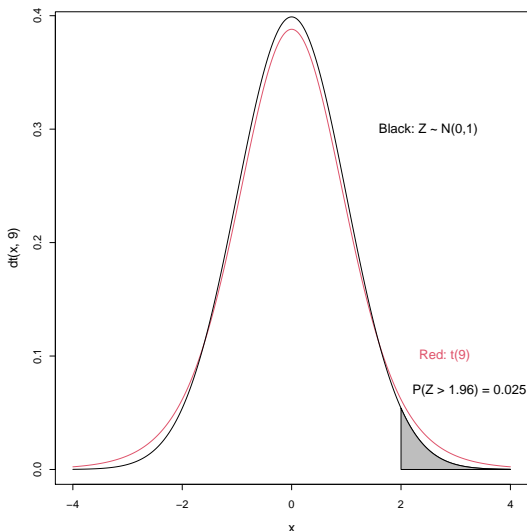
```
x <- seq(-4, 4, by = 0.01)
plot(x, dt(x, df = 9), type = "l", col = "red", ylab = "Density(x)")
lines(x, dnorm(x), type = "l")
text(2.5, 0.3, "Black: N(0,1)")
text(3, 0.1, "Red: t(9)", col = "red")
```



The t -distribution with 9 degrees of freedom and standard normal distribution



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Method 3.9: One-sample Confidence Interval (CI) for μ

Use the correct t -distribution to construct the confidence interval:

For a sample x_1, \dots, x_n the $100(1 - \alpha)\%$ confidence interval is given by:

$$\bar{x} \pm t_{1-\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

where $t_{1-\alpha/2}$ is the $100(1 - \alpha/2)\%$ quantile from the t -distribution with $n - 1$ degrees of freedom.

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where $t_{1-\alpha/2}$ is the $100(1 - \alpha/2)\%$ quantile from the t -distribution with $n - 1$ degrees of freedom.

Most commonly using $\alpha = 0.05$:

The most commonly used is the 95% confidence interval:

$$\bar{x} \pm t_{0.975} \cdot \frac{s}{\sqrt{n}}$$

Example: Heights, 95% CI

```
# 0.975 quantile for the t(9) distribution (n = 10):  
qt(0.975, df = 9)
```

Gives the result $t_{0.975} = 2.26$.

Now, we can recognize the already given result

$$178 \pm 2.26 \cdot \frac{12.21}{\sqrt{10}}$$

which is

$$178 \pm 8.74 = [169.3, 186.7].$$

Example: Heights, 99% CI

```
# 0.995 quantile for the t(9) distribution (n = 10):  
qt(0.995, df = 9)
```

Gives the result $t_{0.995} = 3.25$.

In this case,

$$178 \pm 3.25 \cdot \frac{12.21}{\sqrt{10}}$$

giving

$$178 \pm 12.55 = [165.5; 190.5]$$

An R function for computing these CI (and more):

```
# Data
x <- c(168, 161, 167, 179, 184, 166, 198, 187, 191, 179)

# 99% CI for mu
t.test(x, conf.level = 0.99)

##
##  One Sample t-test
##
## data:  x
## t = 46, df = 9, p-value = 5e-12
## alternative hypothesis: true mean is not equal to 0
## 99 percent confidence interval:
##  165.5 190.5
## sample estimates:
## mean of x
##      178
```

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The formal framework for *statistical inference*

From eNote, Chapter 1:

- An *observational unit* is the single entity/level at which information is sought (e.g. a person). (**Observationsenhed**)
- The *statistical population* consists of all possible “measurements” on each possible *observational unit*. (**Population**)
- The *sample* from a statistical population is the actual set of data collected. (**Stikprøve**)

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Language and concepts:

- μ and σ are parameters describing the population.
- \bar{x} is the *estimate* of μ (specific realization).
- \bar{X} is the *estimator* of μ (now seen as a random variable).
- The word '*statistic(s)*' is used for both.

The formal framework for *statistical inference* - Example

From eNote, Chapter 1. Example: Heights

We measured the heights of 10 randomly selected students.

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The sample:

The 10 specific numbers (heights): x_1, \dots, x_{10} .

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The population:

The heights for all people in Denmark.

The formal framework for *statistical inference* - Example

From eNote, Chapter 1. Example: Heights

We measured the heights of 10 randomly selected students.

The sample:

The 10 specific numbers (heights): x_1, \dots, x_{10} .

The population:

The heights for all people in Denmark.

Observational unit:

A person.

Statistical inference = Learning from data

Learning from data:

Learning about parameters of distributions that describe populations.

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How to ensure this:

For example, by making sure that the sample is taken completely at random.

Random Sampling

Definition 3.12:

- A random sample from an (infinite) population: A set of observations X_1, X_2, \dots, X_n constitutes a random sample of size n from the infinite population $f(x)$ if:
 - ① Each X_i is a random variable whose distribution is given by $f(x)$.
 - ② These n random variables are independent.

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 - ② These n random variables are independent.

What does that mean?

- ① All observations must come from the same population.
- ② They cannot share any information with each other (e.g., shouldn't be related).

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Theorem 3.14: The Central Limit Theorem (CLT)

"No matter the distribution of X_i ", the distribution of the mean of i.i.d. random variables approaches a normal distribution:

Let \bar{X} be the mean of a random sample of size n taken from a population with mean μ and variance σ^2 . Then

$$Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$$

is a random variable whose distribution function approaches that of the standard normal distribution, $N(0, 1^2)$, as $n \rightarrow \infty$.

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is a random variable whose distribution function approaches that of the standard normal distribution, $N(0, 1^2)$, as $n \rightarrow \infty$.

Hence, if n is large enough, we can assume (approximately) that:

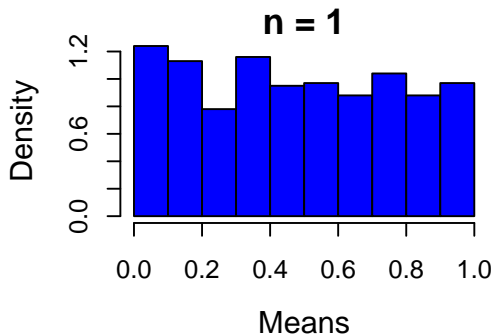
$$\frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim N(0, 1^2)$$

CLT example: Mean of uniformly distributed observations

```
n <- 1 # Sample size
k <- 1000 # No. of samples (i.e. no. of means to be computed)

# Simulations from U(0,1)-distribution (k = 1000 samples, each of size n = 1)
u <- matrix(runif(k*n), ncol = n)

# Empirical density of means
hist(apply(u, 1, mean), col = "blue", main = "n = 1", xlab = "Means", prob = TRUE)
```

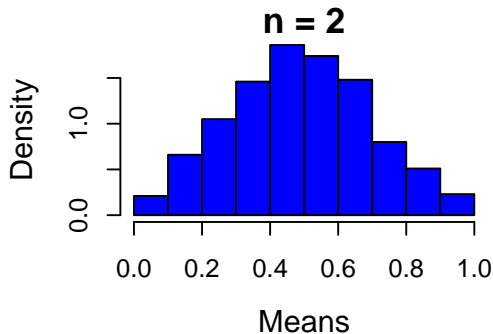


CLT example: Mean of uniformly distributed observations

```
n <- 2 # Sample size
k <- 1000 # No. of samples (i.e. no. of means to be computed)

# Simulations from U(0,1)-distribution (k = 1000 samples, each of size n = 2)
u <- matrix(runif(k*n), ncol = n)

# Empirical density of means
hist(apply(u, 1, mean), col = "blue", main = "n = 2", xlab = "Means", xlim = c(0,1), prob = TRUE)
```

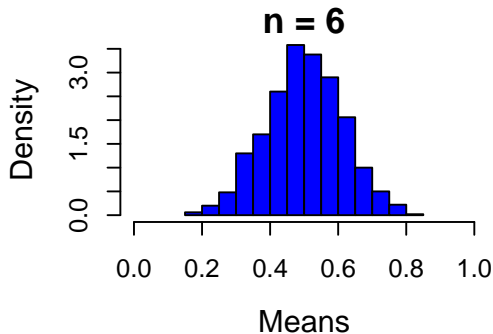


CLT example: Mean of uniformly distributed observations

```
n <- 6 # Sample size
k <- 1000 # No. of samples (i.e. no. of means to be computed)

# Simulations from U(0,1)-distribution (k = 1000 samples, each of size n = 6)
u <- matrix(runif(k*n), ncol = n)

# Empirical density of means
hist(apply(u, 1, mean), col = "blue", main = "n = 6", xlab = "Means", xlim = c(0,1), prob = TRUE)
```

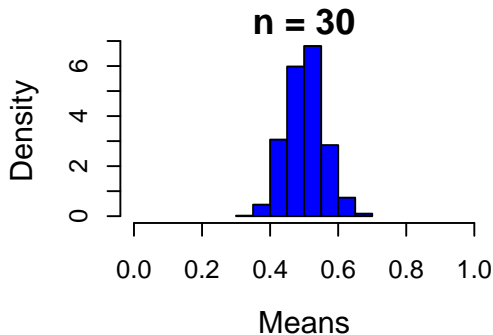


CLT example: Mean of uniformly distributed observations

```
n <- 30 # Sample size
k <- 1000 # No. of samples (i.e. no. of means to be computed)

# Simulations from U(0,1)-distribution (k = 1000 samples, each of size n = 30)
u <- matrix(runif(k*n), ncol = n)

# Empirical density of means
hist(apply(u, 1, mean), col = "blue", main = "n = 30", xlab = "Means", xlim = c(0,1), prob = TRUE)
```



Consequence of the CLT:

Our CI-method also works for non-normal data:

We can use the confidence-interval based on the t -distribution in basically any situation, as long as n is large enough.

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Our CI-method also works for non-normal data:

We can use the confidence-interval based on the t -distribution in basically any situation, as long as n is large enough.

When is n "large enough"?

Actually difficult to say exactly, BUT:

- Rule of thumb: $n \geq 30$
- Even for smaller n the approach can be (almost) valid for non-normal data.

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'Repeated sampling' interpretation

In the long run, we catch the true value in 95% of cases (95% CI):

The confidence interval will vary in both width (s) and position (\bar{x}) if the study is repeated.

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More formally expressed (Theorem 3.5):

$$P\left(\frac{|\bar{X} - \mu|}{S/\sqrt{n}} < t_{0.975}\right) = 0.95$$

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In the long run, we catch the true value in 95% of cases (95% CI):

The confidence interval will vary in both width (s) and position (\bar{x}) if the study is repeated.

More formally expressed (Theorem 3.5):

$$P\left(\frac{|\bar{X} - \mu|}{S/\sqrt{n}} < t_{0.975}\right) = 0.95$$

Which is equivalent to:

$$P\left(\bar{X} - t_{0.975} \frac{S}{\sqrt{n}} < \mu < \bar{X} + t_{0.975} \frac{S}{\sqrt{n}}\right) = 0.95$$

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Motivating Example

Production of tablets

In the production of tablets, an active matter is mixed with a powder and then the mixture is formed to tablets. It is important that the mixture is homogenous, so that each tablet has the same strength.

We consider a mixture (of the active matter and powder) from where a large amount of tablets is to be produced.

We seek to produce the mixtures (and the final tablets) so that the mean content of the active matter is 1 mg/g with the smallest variance as possible. A random sample is collected where the amount of active matter is measured. It is assumed that all the measurements follow a normal distribution with the unit mg/g.

The sampling distribution of the variance estimator, Theorem 2.81

Assume i.i.d. normal distributed variables, $X_i \sim N(\mu, \sigma^2)$ for $i = 1, \dots, n$.

Variance estimators behaves like a χ^2 -distribution:

Let

$$S^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2$$

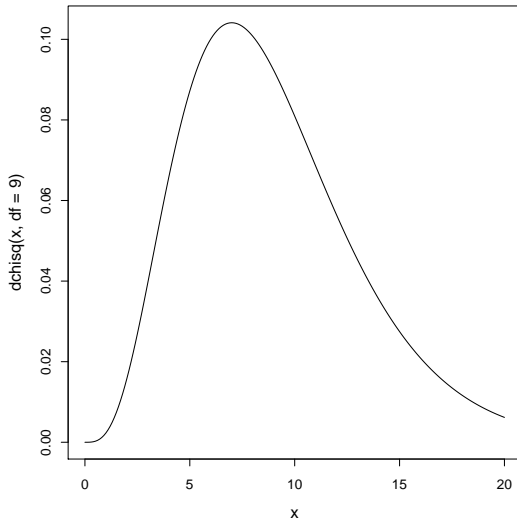
then:

$$\chi^2 = \frac{(n-1)S^2}{\sigma^2}$$

is a stochastic variable following the χ^2 -distribution with $\nu = n - 1$ degrees of freedom.

χ^2 -distribution with $\nu = 9$ degrees of freedom

```
x <- seq(0, 20, by = 0.1)
plot(x, dchisq(x, df = 9), type = "l")
```



Method 3.19: Confidence interval for the variance and standard deviation

Assume i.i.d. normal distributed variables, $X_i \sim N(\mu, \sigma^2)$ for $i = 1, \dots, n$.

The variance:

A $100(1 - \alpha)\%$ confidence interval for the variance σ^2 is:

$$\left[\frac{(n-1)s^2}{\chi_{1-\alpha/2}^2}; \frac{(n-1)s^2}{\chi_{\alpha/2}^2} \right]$$

where the quantiles come from a χ^2 -distribution with $\nu = n - 1$ degrees of freedom.

Method 3.19: Confidence interval for the variance and standard deviation

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$$\left[\frac{(n-1)s^2}{\chi_{1-\alpha/2}^2}; \frac{(n-1)s^2}{\chi_{\alpha/2}^2} \right]$$

where the quantiles come from a χ^2 -distribution with $\nu = n - 1$ degrees of freedom.

The standard deviation:

A $100(1 - \alpha)\%$ confidence interval for the standard deviation σ is:

$$\left[\sqrt{\frac{(n-1)s^2}{\chi_{1-\alpha/2}^2}}; \sqrt{\frac{(n-1)s^2}{\chi_{\alpha/2}^2}} \right]$$

Example

Data:

A random sample with $n = 20$ tablets is taken and from this we get:

$$\hat{\mu} = \bar{x} = 1.01, \hat{\sigma}^2 = s^2 = 0.07^2$$

Example

Data:

A random sample with $n = 20$ tablets is taken and from this we get:

$$\hat{\mu} = \bar{x} = 1.01, \hat{\sigma}^2 = s^2 = 0.07^2$$

95% confidence interval for the variance - we need the χ^2 -quantiles (19 degrees of freedom):

$$\chi_{0.025}^2 = 8.9065, \chi_{0.975}^2 = 32.8523$$

```
qchisq(c(0.025, 0.975), df = 19)
```

```
[1] 8.907 32.852
```

Example

So the confidence interval for the variance σ^2 becomes:

$$\left[\frac{19 \cdot 0.07^2}{32.85}; \frac{19 \cdot 0.07^2}{8.907} \right] = [0.002834; 0.01045]$$

Example

So the confidence interval for the variance σ^2 becomes:

$$\left[\frac{19 \cdot 0.07^2}{32.85}; \frac{19 \cdot 0.07^2}{8.907} \right] = [0.002834; 0.01045]$$

and the confidence interval for the standard deviation σ becomes:

$$\left[\sqrt{0.002834}; \sqrt{0.01045} \right] = [0.053; 0.102]$$

Example: Heights

We need the χ^2 -quantiles with $\nu = 9$ degrees of freedom:

$$\chi_{0.025}^2 = 2.700389, \chi_{0.975}^2 = 19.022768$$

```
qchisq(c(0.025, 0.975), df = 9)
```

```
[1] 2.70 19.02
```

Example: Heights

We need the χ^2 -quantiles with $\nu = 9$ degrees of freedom:

$$\chi_{0.025}^2 = 2.700389, \chi_{0.975}^2 = 19.022768$$

```
qchisq(c(0.025, 0.975), df = 9)
```

```
[1] 2.70 19.02
```

So the confidence interval for the height standard deviation σ becomes:

$$\left[\sqrt{\frac{9 \cdot 12.21^2}{19.022768}}; \sqrt{\frac{9 \cdot 12.21^2}{2.700389}} \right] = [8.4; 22.3]$$

Example: Heights

Sample, $n = 10$:

168 161 167 179 184 166 198 187 191 179

Sample mean and standard deviation:

$$\bar{x} = 178$$

$$s = 12.21$$

Estimate population mean and standard deviation:

$$\hat{\mu} = 178$$

$$\hat{\sigma} = 12.21$$

NEW: **Confidence interval, μ :**

$$178 \pm 2.26 \cdot \frac{12.21}{\sqrt{10}} = [169.3; 186.7]$$

NEW: **Confidence interval, σ :**

$$[8.4; 22.3]$$

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- 5 Non-normal data, the Central Limit Theorem (CLT)
- 6 Formal interpretation of the CI
- 7 CI for variance σ^2 and standard deviation σ