The quantum computing threat to cryptography

Course 01410, Crypto I

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Assistant Professor, Cybersecurity Engineering Section, DTU Compute

After today, you will…

- …be able to put quantum computing as a disruptive technology into perspective,
- …have identified cryptographic infrastructure that is susceptible to quantum computing attacks,
- …have a basic understanding of how quantum computers can be used to attack cryptographic schemes, and
- …have a rough idea when cryptanalytically relevant quantum computers can be expected to become reality.

Intro

Quantum computers

Quantum computers

- Accelerating effort to build a quantum computer

Quantum computers

- Accelerating effort to build a quantum computer
- Major investments:

Google IBM

intel Microsoft

Quantum computers

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We need to prepare cryptography for the arrival of quantum computers!

Quantum computers

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We need to prepare cryptography for the arrival of quantum computers!

- Security against quantum attackers

Quantum computers

- Accelerating effort to build a quantum computer
- Major investments:



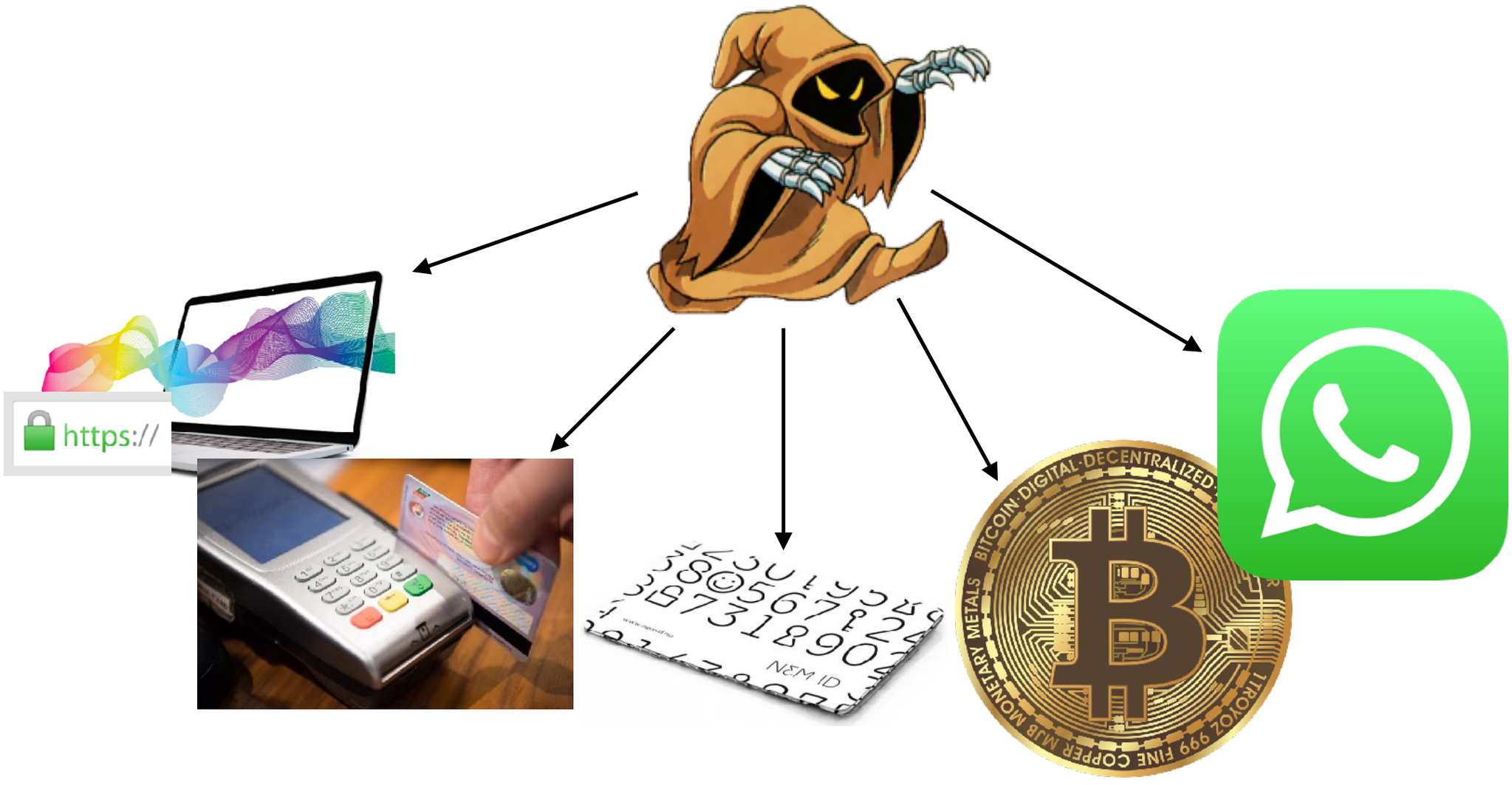




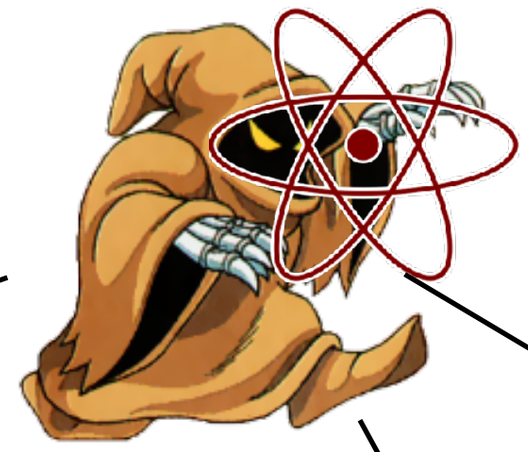
We need to prepare cryptography for the arrival of quantum computers!

- Security against quantum attackers
- Quantum cryptography

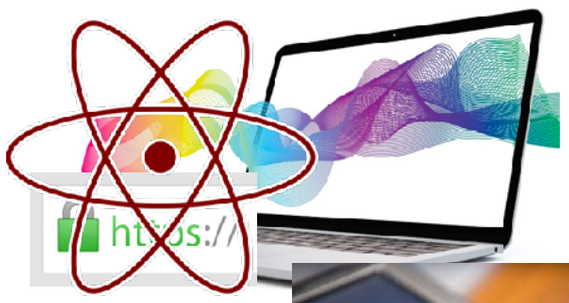
Cryptography is everywhere



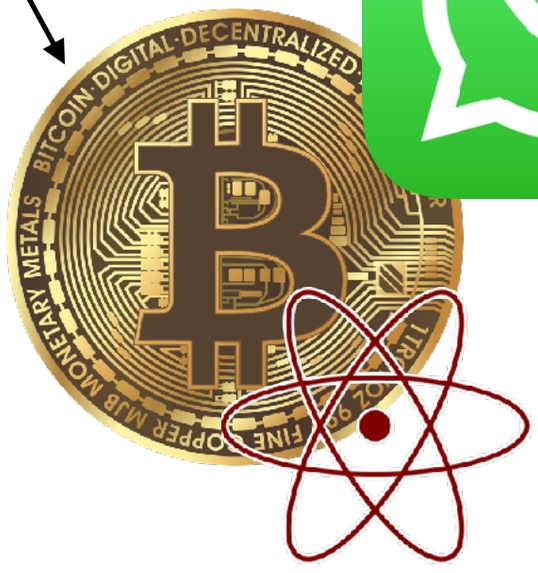
Quantum computing is changing cryptography

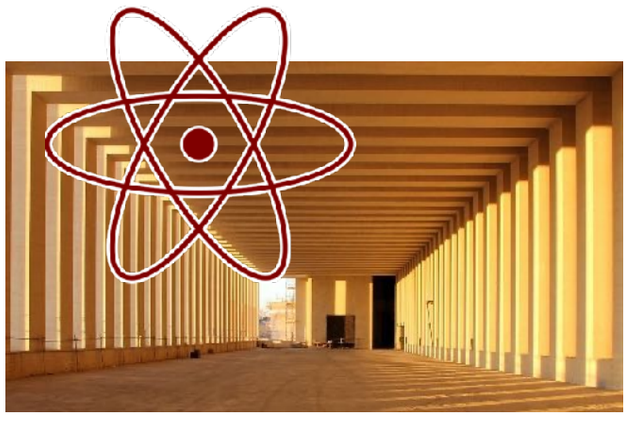


Post-quantum security



Quantum crypto

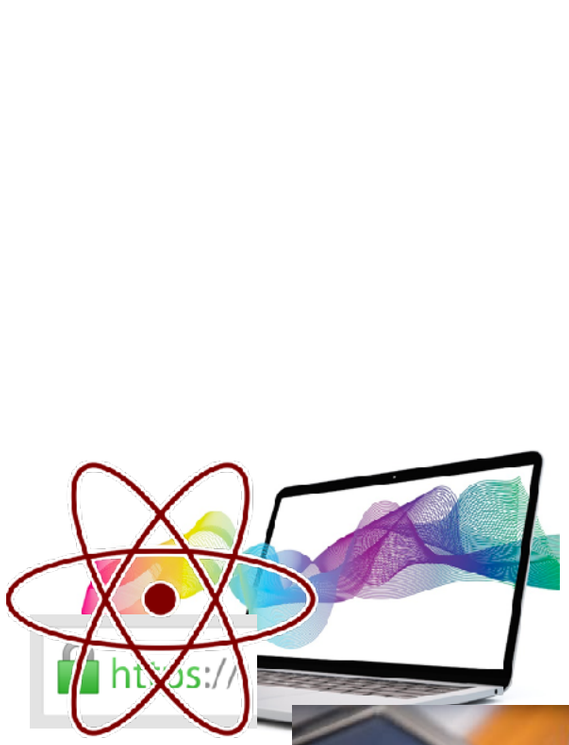




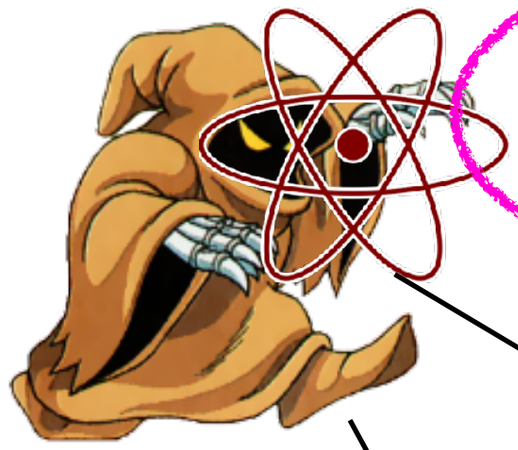
Theoretical Foundation

Quantum-enhanced crypto

Quantum computing is changing cryptography



Quantum crypto



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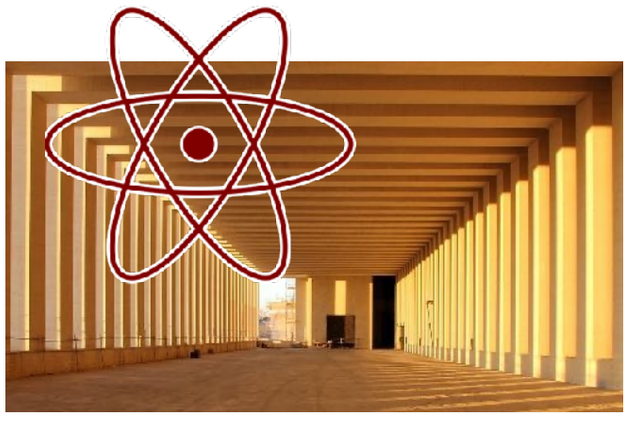

Theoretical Foundation

What is a quantum computer?

Quantum physics is everywhere

Quantum physics is everywhere

In Information technology:

- Semiconductors

Quantum physics is everywhere

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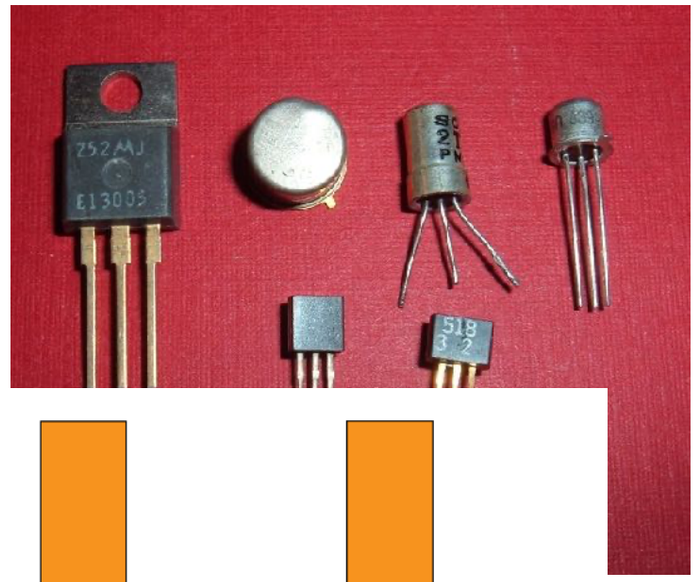
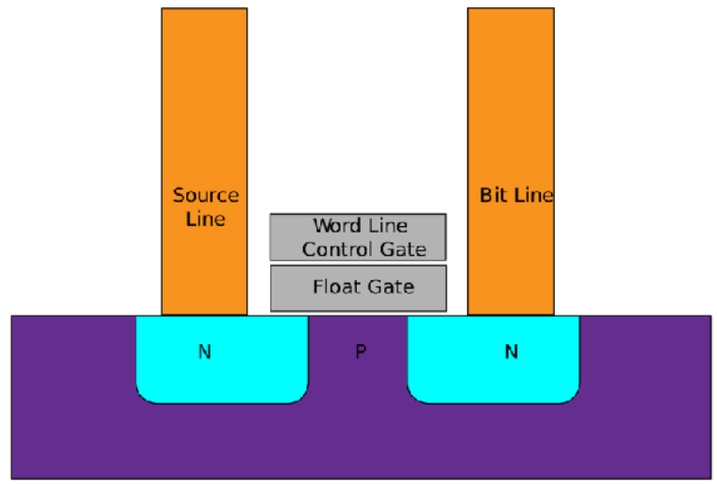
- Semiconductors
  - Transistors

Quantum physics is everywhere

In Information technology:

- Semiconductors
  - Transistors
  - Flash memory





Quantum physics is everywhere

In Information technology:

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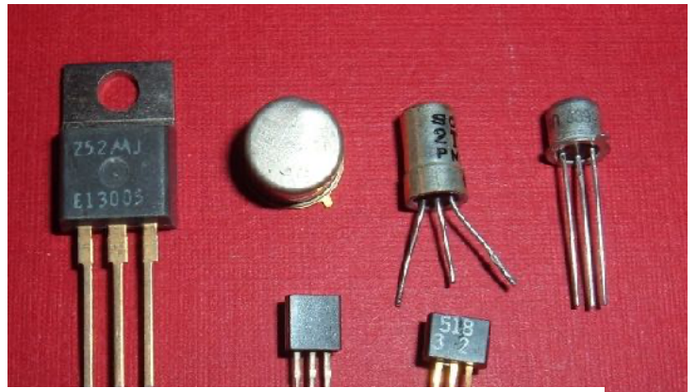
  - OLED
  - …



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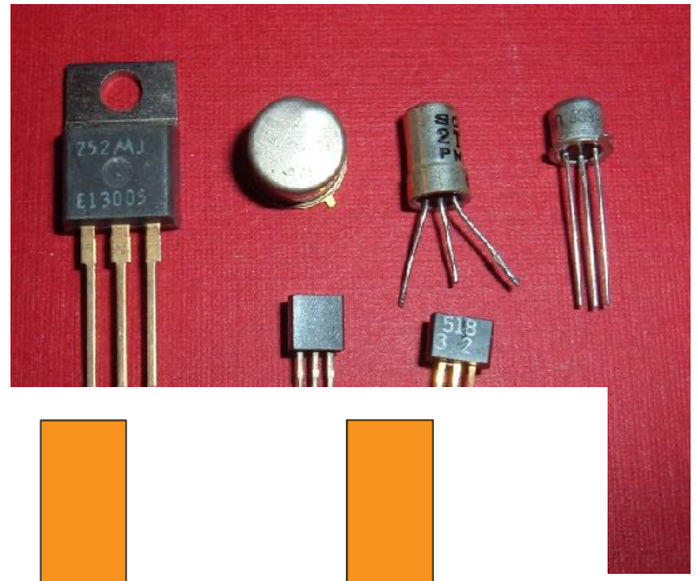

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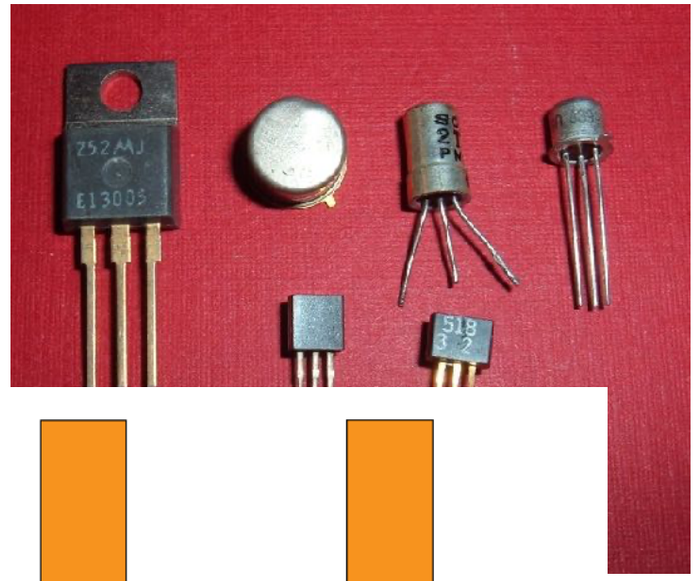
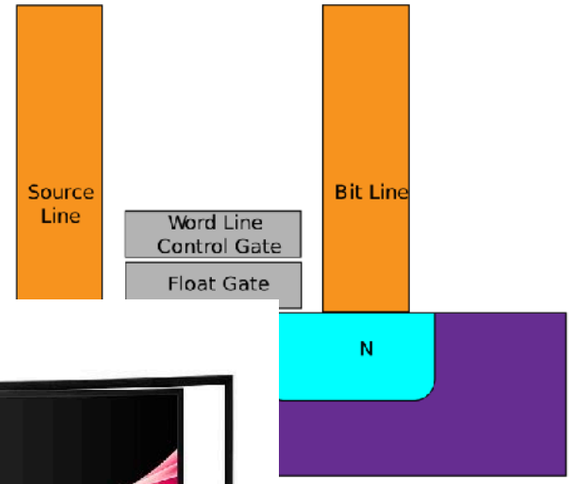
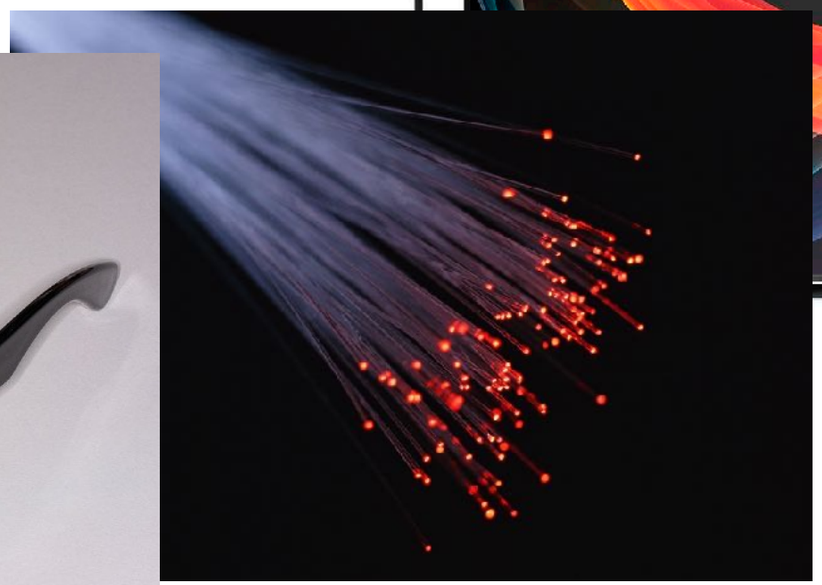
- 3D movie theatre
- …







Example: polarization of light

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Classical* property of Light: Color

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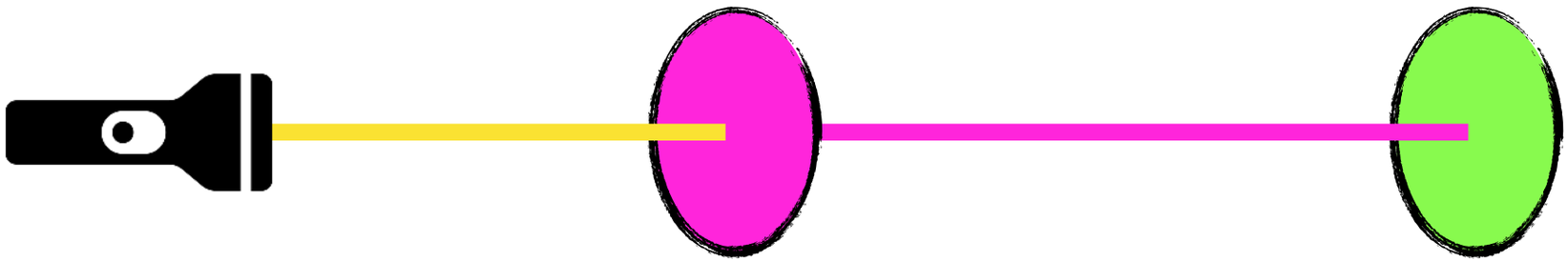

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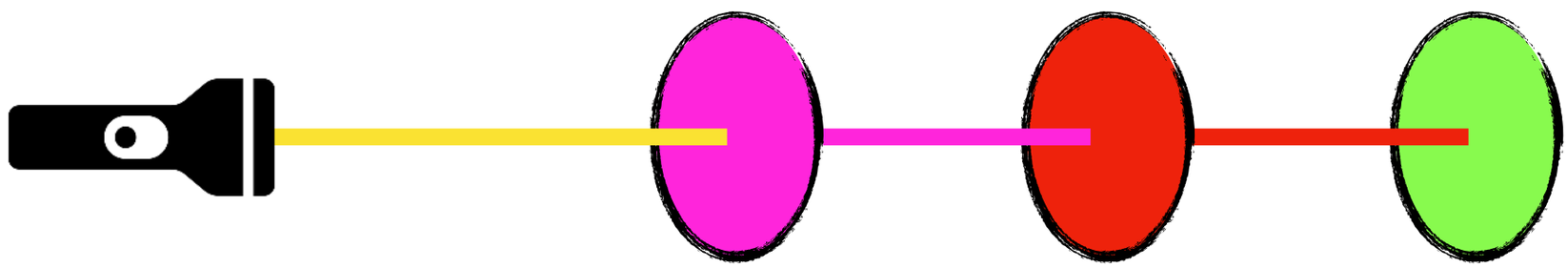

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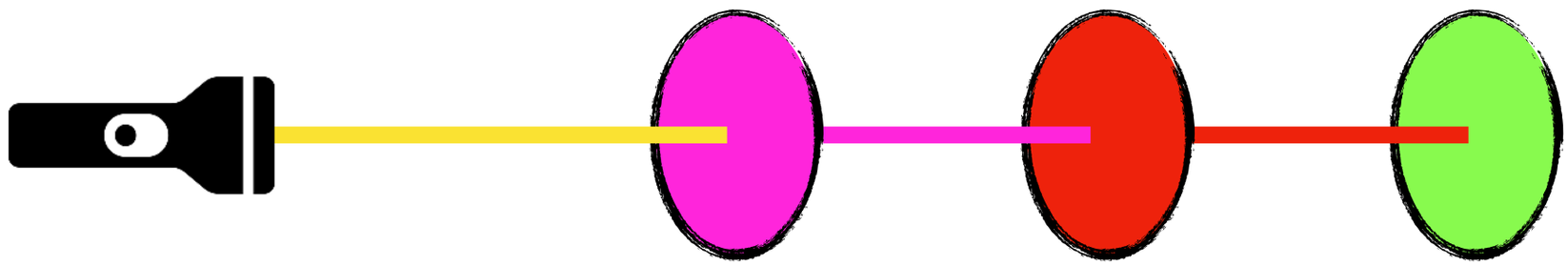

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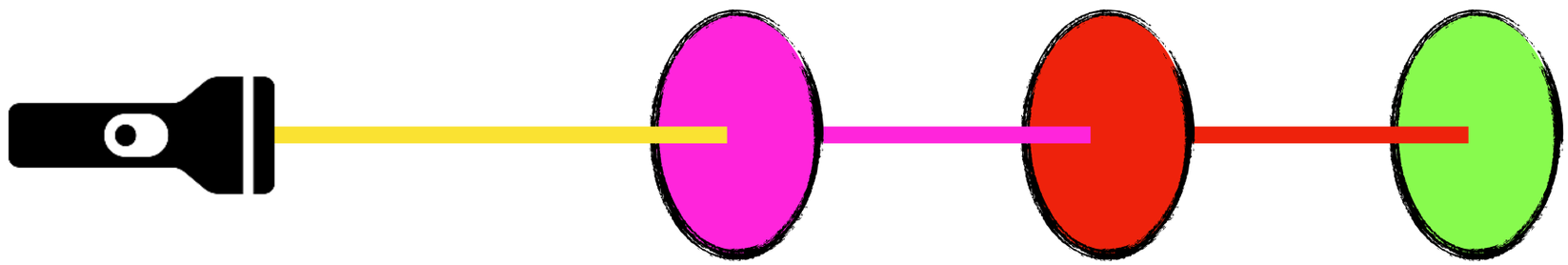

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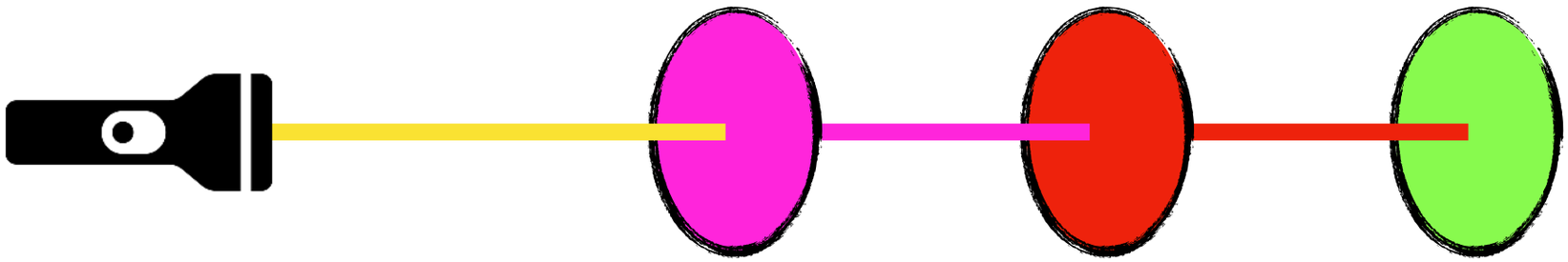

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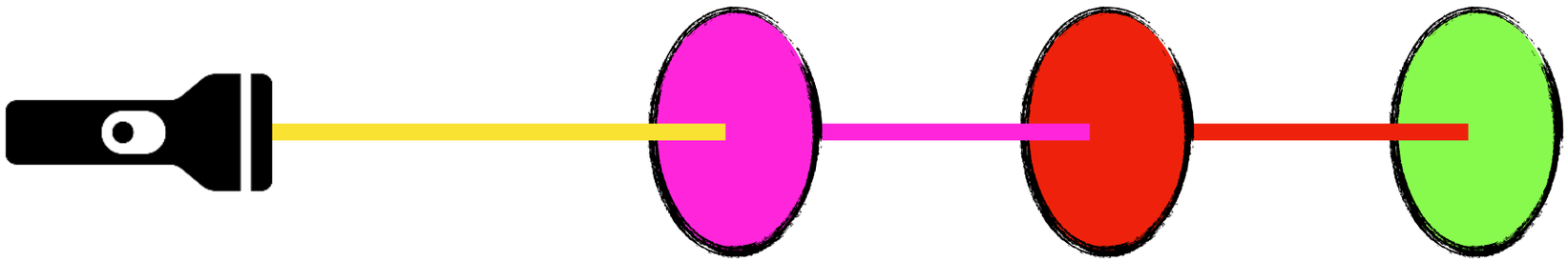

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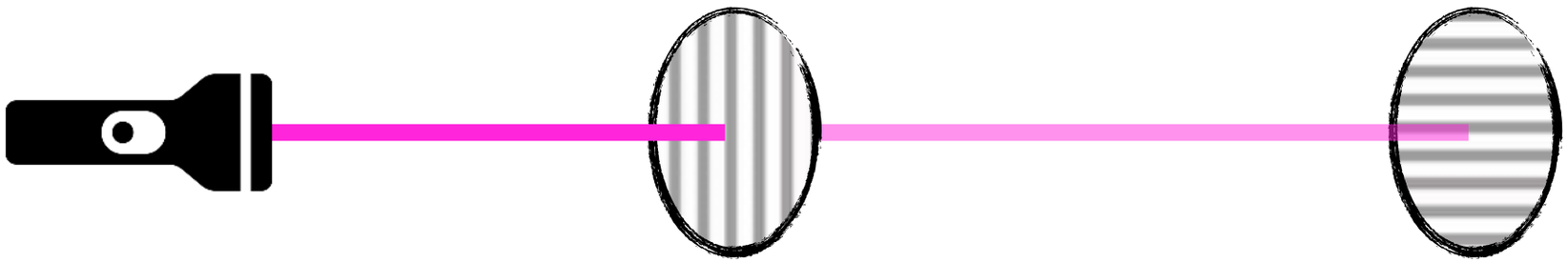

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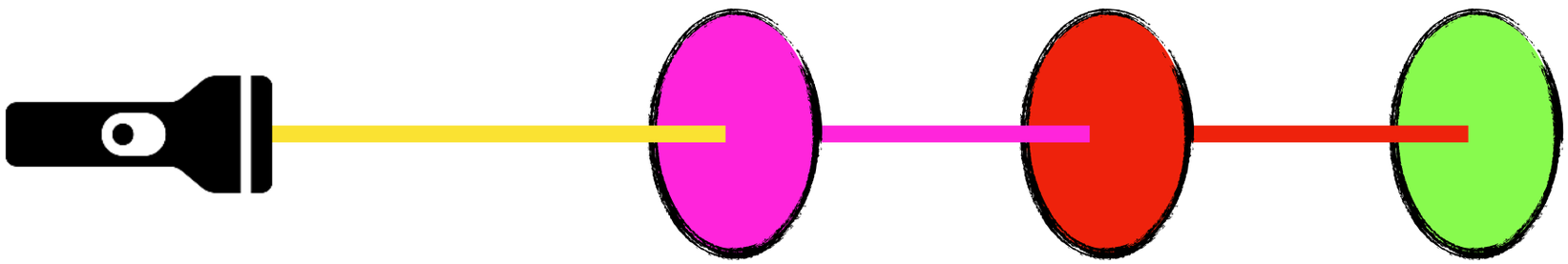

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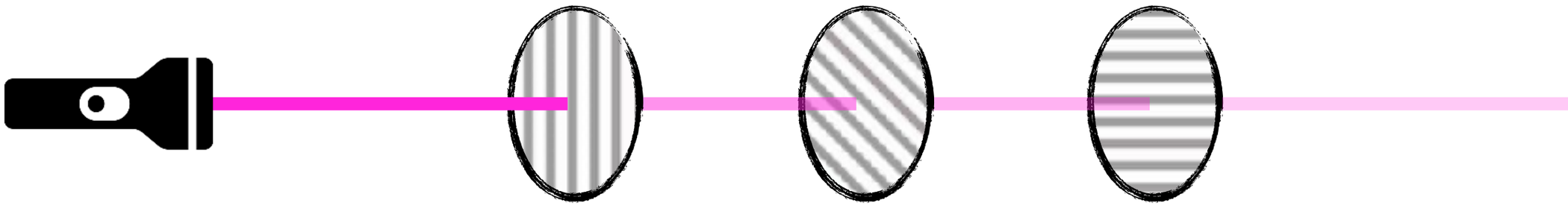

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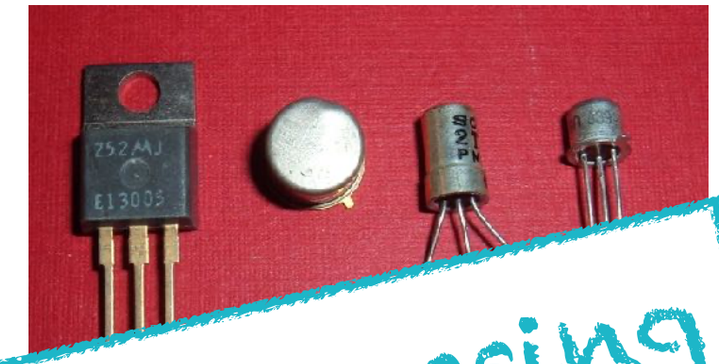

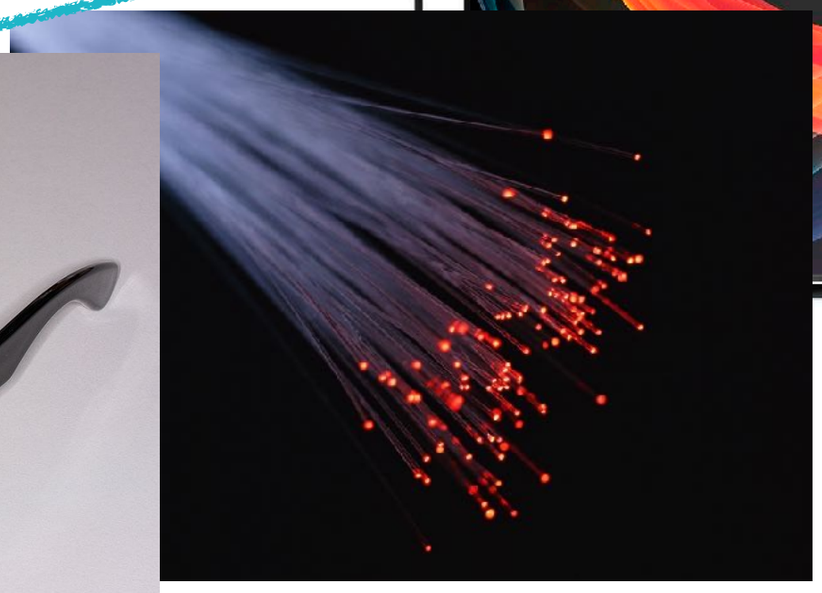

"Quantum physics is used for processing "non-quantum" Information.

Quantum computing

"Information is Physical."

Rolf Landauer

Quantum computing

"Information is Physical."

Rolf Landauer

$\Longrightarrow$ If the physics is quantum, information is as well.

What is a quantum computer?

Classical information: “Does the flashlight emit green light?”

Specified by a bit $b \in \{0,1\}$

More generally: bit strings $x \in \{0,1\}^n$

Basic operations: Logical gates

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Basic operations: “Quantum gates”⇔Unitary matrices

Physical realization

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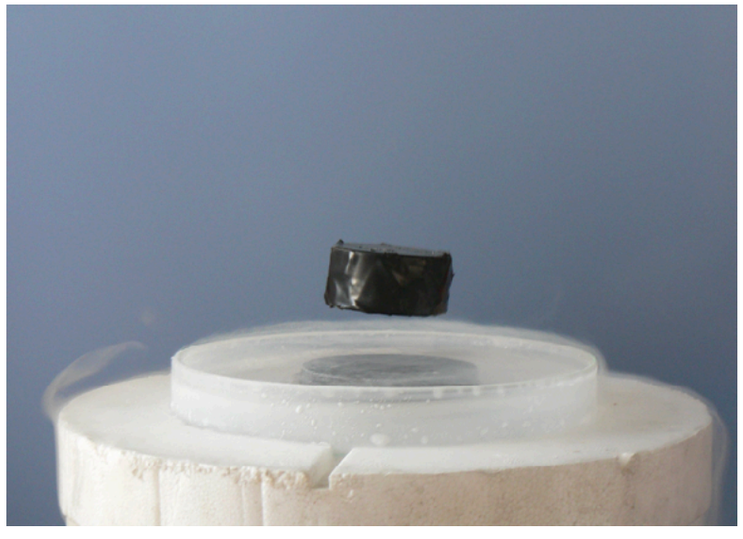
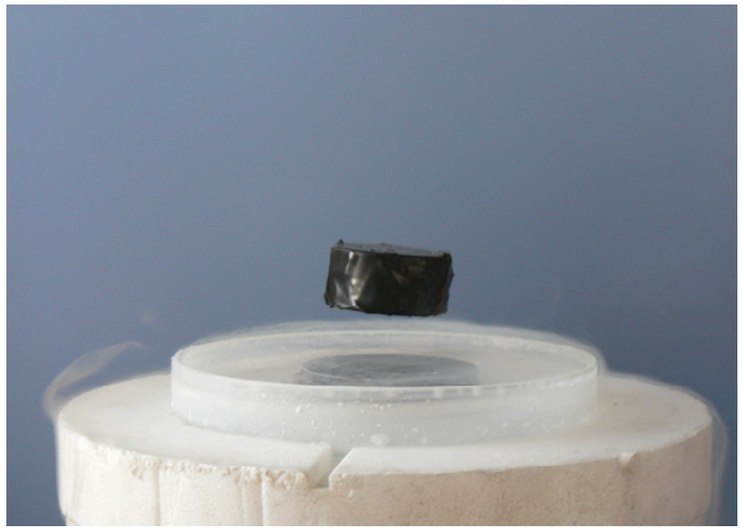
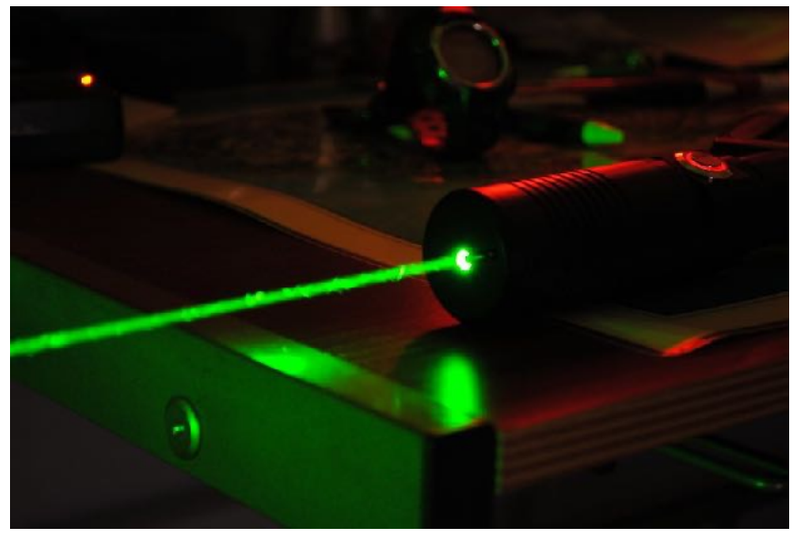


- Superconducting

Physical realization



- Superconducting
- Photonic



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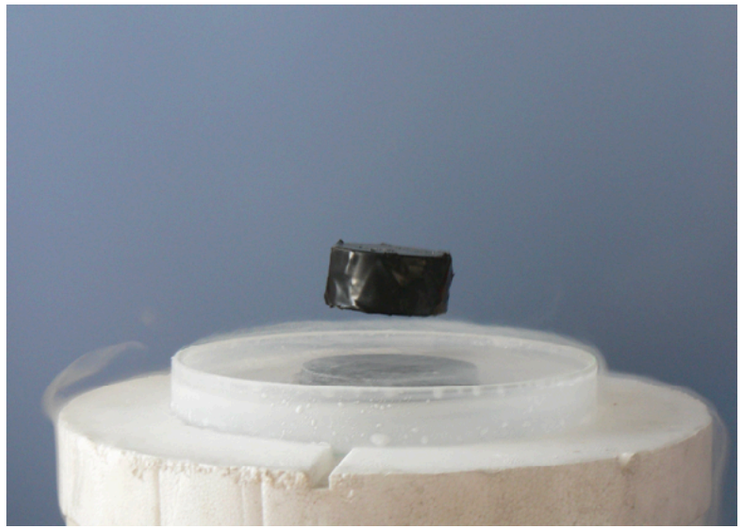
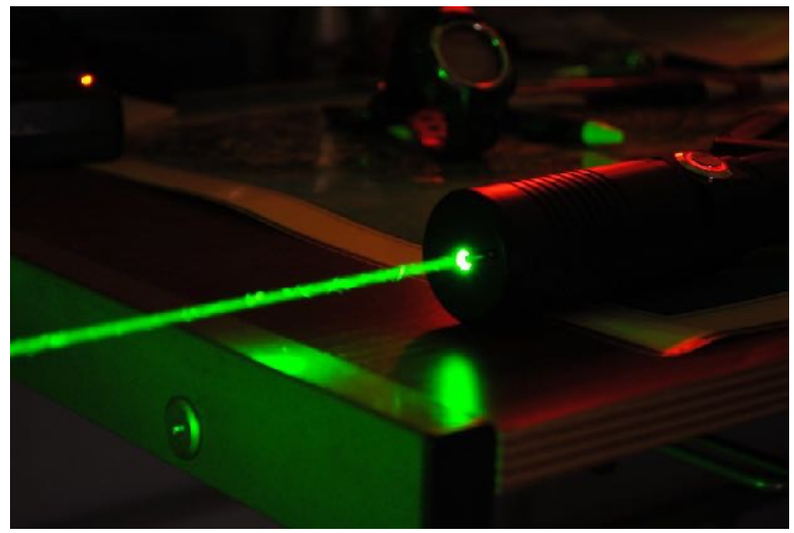
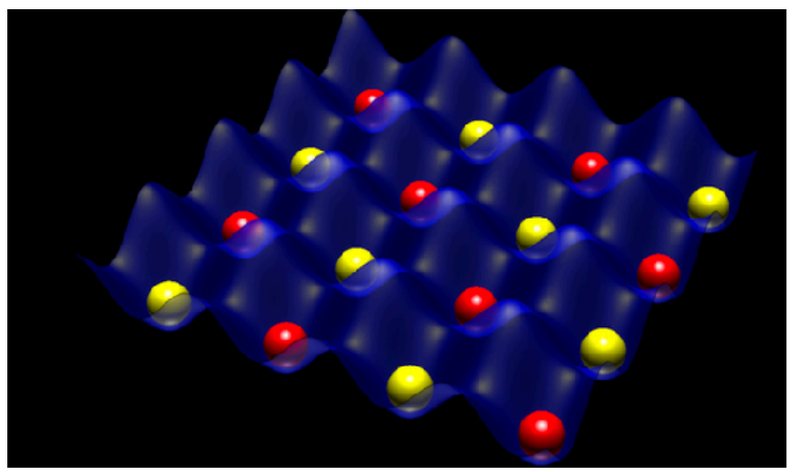
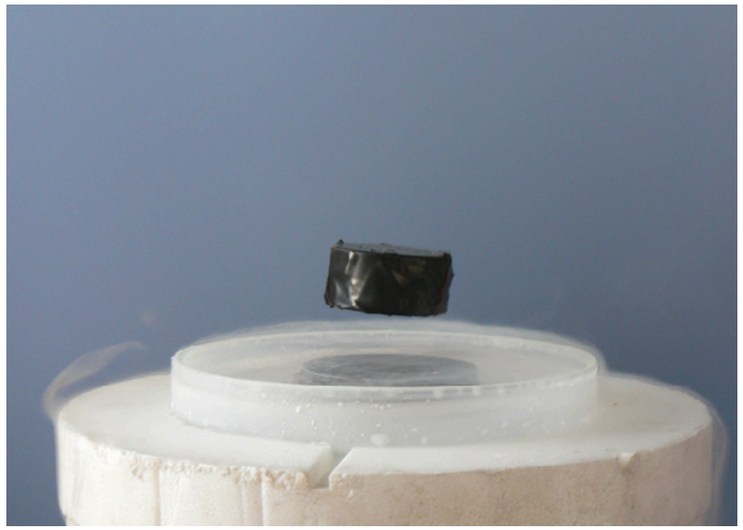


- Superconducting
- Photonic
- Cold atoms

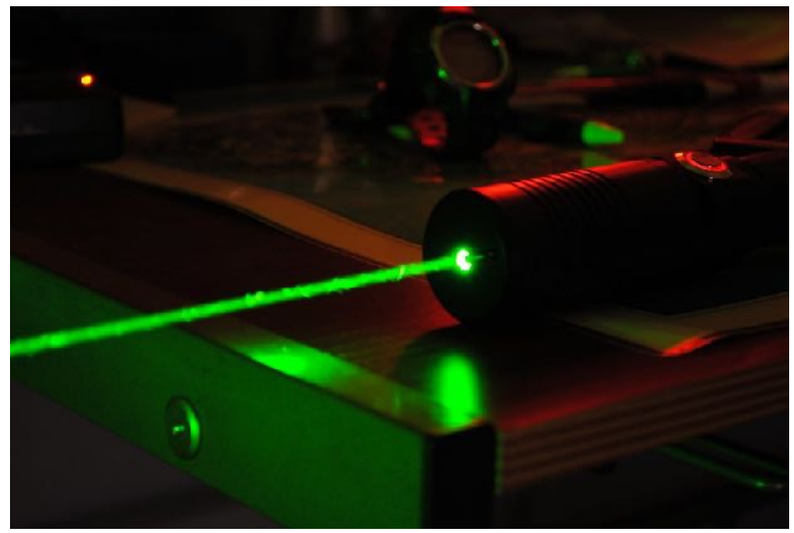


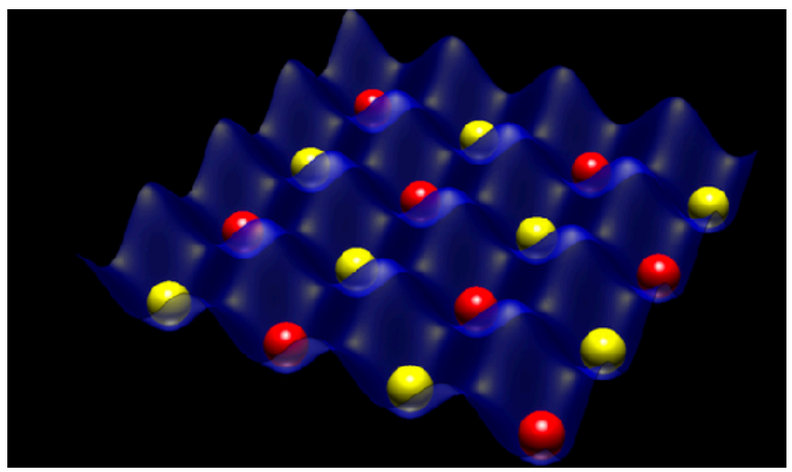


Physical realization



- Superconducting
- Photonic
- Cold atoms
- Trapped ions





But what is it good for...

Potential applications of quantum computing

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- Quantum chemistry simulation (For material science, pharmacology, battery tech…)

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- Optimization?

Potential applications of quantum computing

- Quantum chemistry simulation (For material science, pharmacology, battery tech…)
- **Breaking cryptographic schemes!**
- Optimization?

Which cryptographic schemes are quantum-vulnerable?

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- Diffie-Hellman (ordinary and elliptic curve)

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- RSA
- Diffie-Hellman (ordinary and elliptic curve)

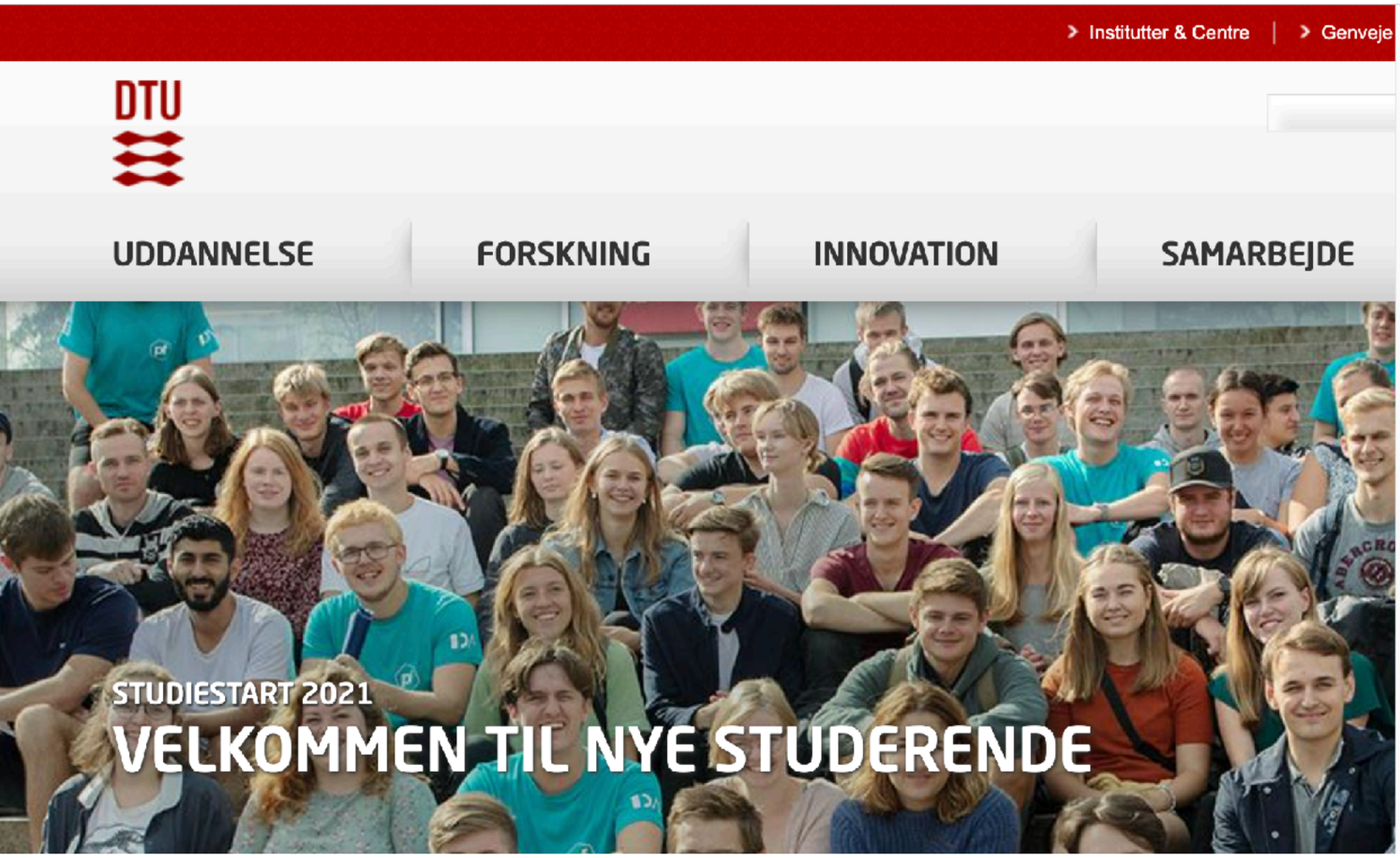
=>All* currently used public-key crypto!

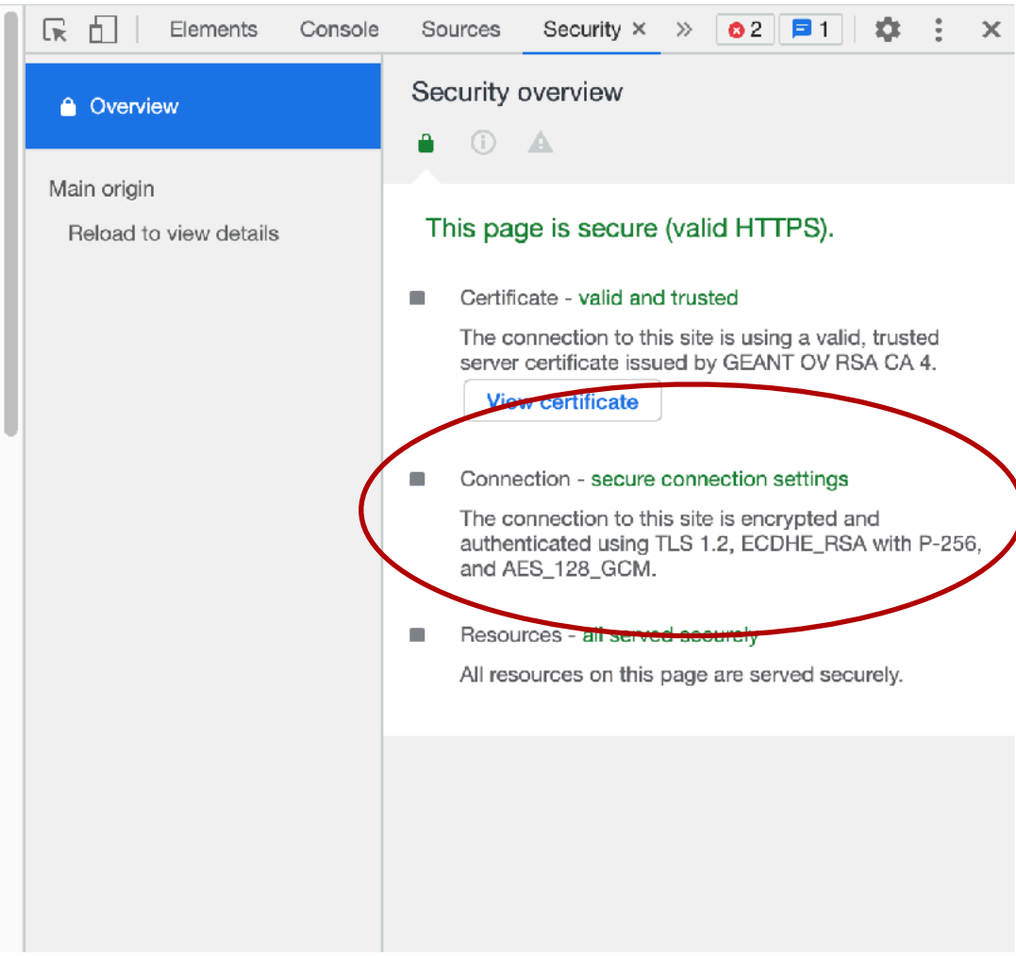
Where are the broken schemes?

Discuss in pairs and collect examples: what technology you have used today that depends on quantum-broken crypto?

Where is the broken stuff?

Example: TLS





Quantum vulnerabilities in TLS

Breakdown of TLS used by dtu.dk and Brave 1.28.106

TLS (TLS 1.2, ECDHE_RSA
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Handshake: Authenticated key exchange, ECDHE_RSA with P-256

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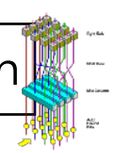
Block cipher: AES 128



AES round function

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Handshake: Authenticated key exchange, ECDHE_RSA with P-256

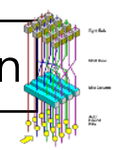
Session: Authenticated encryption with associated data, Galois Counter Mode (AES_128_GCM)

Key Exchange ECDHE (Elliptic curve Diffie Hellman) with curve P-256

Digital signature scheme: RSA hash-then-sign PKCS#1 v1.5

Block cipher: AES 128

AES round function



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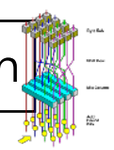
DH problem

AES round function



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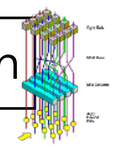

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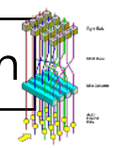

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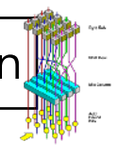

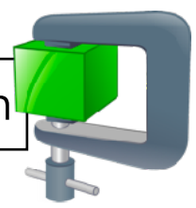

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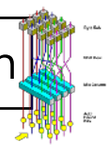

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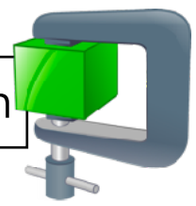

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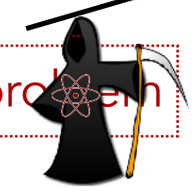

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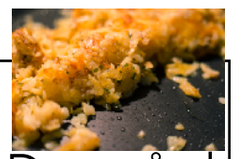
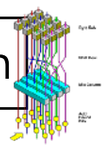


AES round function



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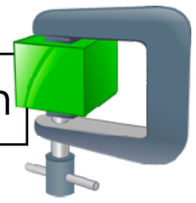

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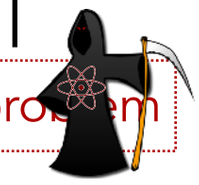

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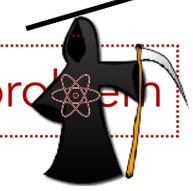

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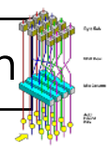

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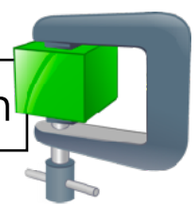

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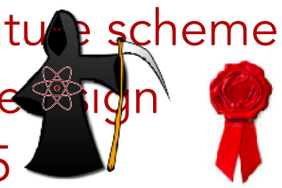

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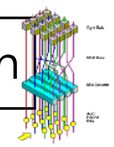
AES round function



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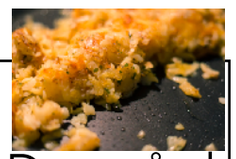

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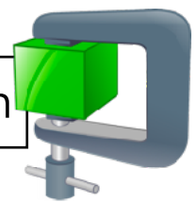

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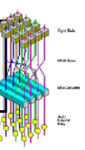

AES round function

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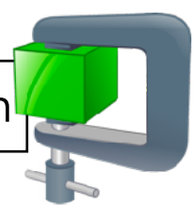

How does a quantum computer break crypto?

Example: RSA

- Public-key cryptosystem: encryption and digital signature
- Based on number theory
- Used e.g. in HTTPS, Certificates…
- Can be broken via *factoring*

Factoring

Factoring

- Multiplying is easy.

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$$7 \cdot 11$$

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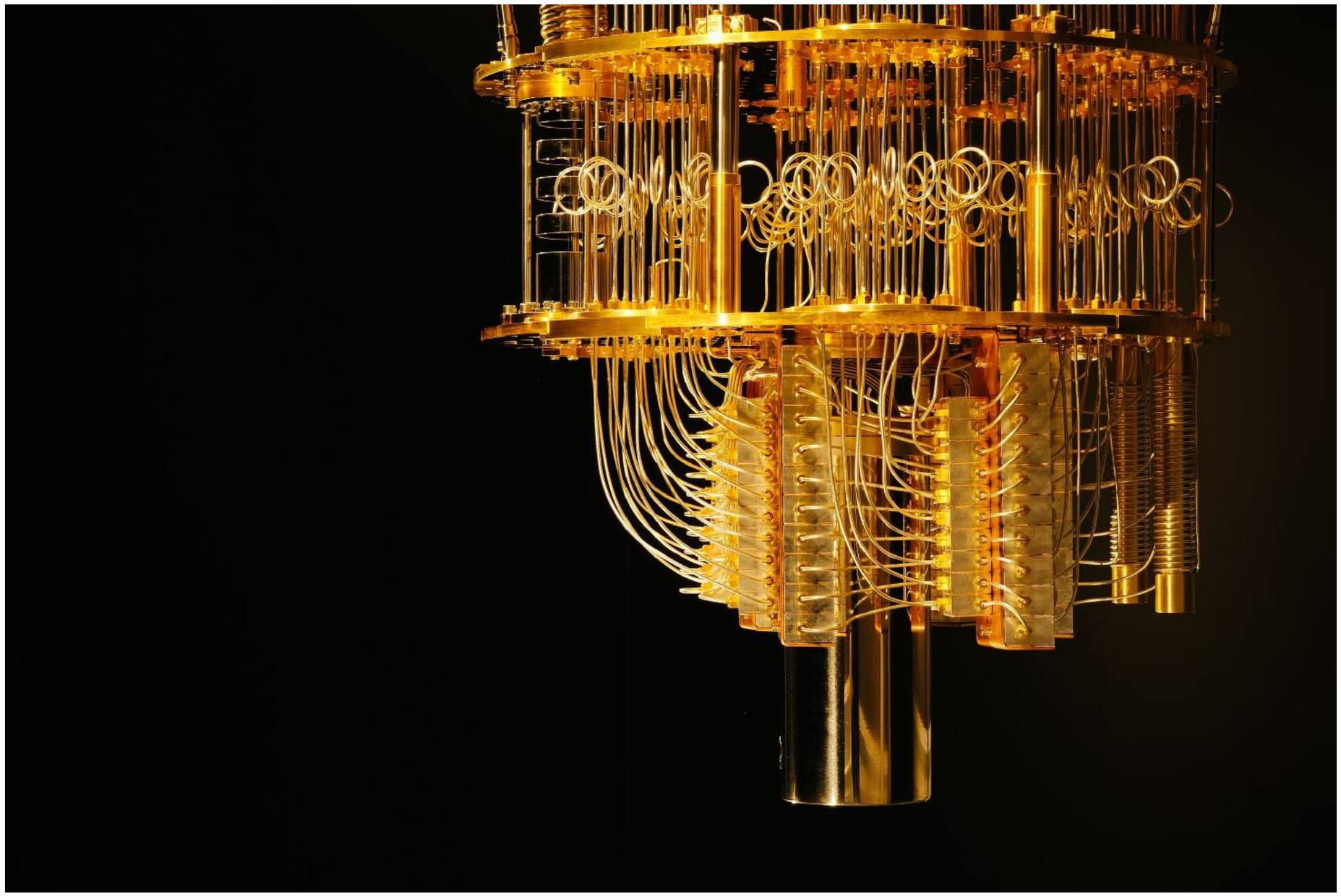
Turns out that…

…it is not hard for quantum computers.



Factoring

How a quantum computer factors integers:

Problem: given N not prime, find $q \notin \{1, N\}$ such that $q \mid N$

Algorithm:

- Pick a random a with $1 < a < N$
- Compute $\gcd(a, N)$. If it's $\neq 1$ we found a factor (Yay!)
- Otherwise, let $f(x) = a^x \mod N$, find $r < N$ with $f(x + r) = f(x)$
- hope that r is even and $-1 \neq a^{r/2} \mod N$
- Output $\gcd(a^{r/2} - 1, N)$

Factoring

How a quantum computer fa

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The period-finding problem

Period-finding via Fourier transform

- Otherwise, let $f(x) = a^x \mod N$ and find r with $f(x + r) = f(x)$????

Quantum Fourier transform

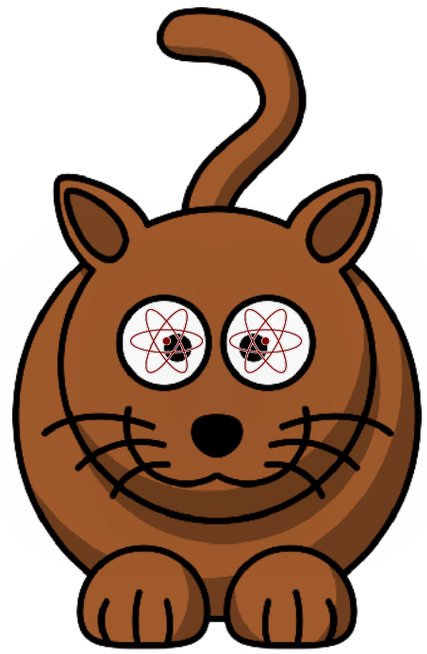
Cartoon slide (sorry!)

Quantum Fourier transform

Cartoon slide (sorry!)

Classical Computer

Quantum Computer



Quantum Fourier transform

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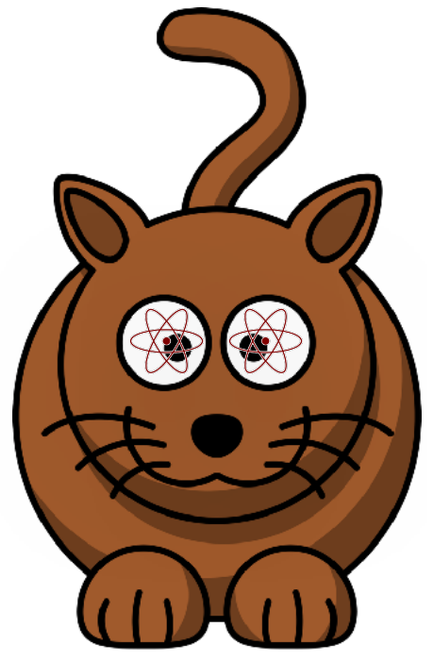
Classical Computer	Quantum Computer
Bits!	Qubits!
$x \in \{0,1\}^n$	$\lvert\psi\rangle \in \mathbb{C}^{2^n}$



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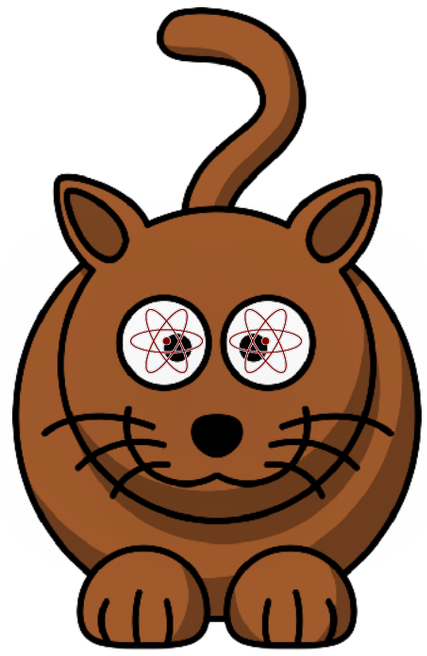
Fourier sampling in $\mathcal{O}(\ell)$.	Fourier sampling in $\mathcal{O}(\log \ell)$.



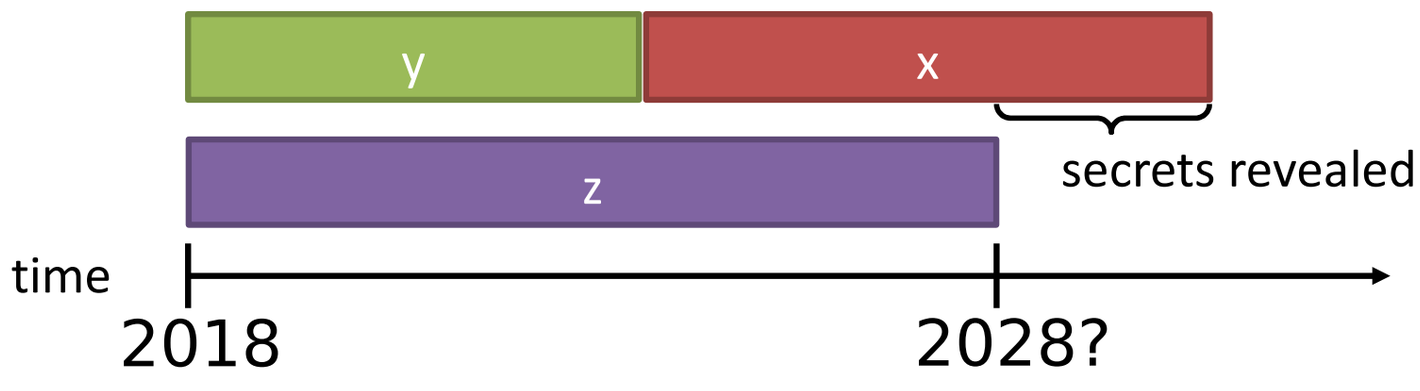
Assessing quantum attack risk

By when do we need to fix quantum-vulnerabilities?

Depends on:

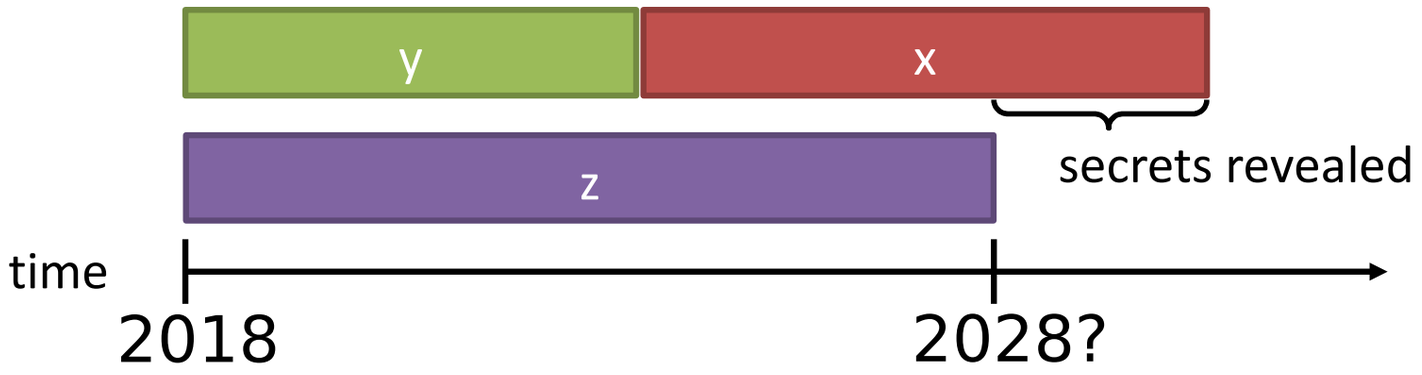
- How long do you need to keep your secrets secure? (x years)
- How much time will it take to re-tool the existing infrastructure? (y years)
- How long will it take for a large-scale quantum computer to be built? (z years)

Theorem (Mosca): If x + y > z, then worry.



Slide by Michele Mosca

By when do we need to fix quantum-vulnerabilities?



- How big is y?
- How big is x?
- How big is z? Hard to say… ⇒Exercise sheet!

Summary: quantum computing threat

- Quantum computers are fundamentally different from regular computers
- Quantum computers are currently not available
- They have *a* computing power *incomparable* to regular computers
- They are good at discovering *structure* in big mathematical objects
- Once they are built, they can be used to launch attacks against RSA and Diffie-Hellman