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Exercise 7.3.5:
For the following sets X, equivalence relations $\sim$, and elements x $\in X$, determine the equivalence class for the given element.

1. Let $X = \mathbb{Z}$. For a,b $\in X$,define a $\sim$b if and only if a - b $\in 3\mathbb{Z}$.
(a) Let x = 0.Find [x].
[x] = { k : k $\in 3\mathbb{Z}$ }

(b) Let x = 1.Find [x].
[x] = { k + 1 : k $\in 3\mathbb{Z}$}

(c) Let x = 2.Find [x].
[x] = {k + 2 : k $\in 3\mathbb{Z}$}

(d) Let x = 3.Find [x].
[x] = { k : k $\in 3\mathbb{Z}$}

(e) Let x = 4.Find[x].
[x] = { k + 1 : k $\in 3\mathbb{Z}$}

2. Let X = $\mathcal{P}$ ({1,2,3,4,5}).For A,B $\in$ X,define A $\sim$ B if and only if A and B have the same number of elements.

(a) Let A = { 1,2 }.Find [A].
[A] = { { 1,2 },{1,3},{1,4},{1,5},{2,3 },{2,4},{2,5},{3,4},{3,5},{4,5}}

(b) Let A = { 2,3 }.Find [A].
[A] = { { 1,2 },{1,3},{1,4},{1,5},{2,3 },{2,4},{2,5},{3,4},{3,5},{4,5}}

(c) Let A = { 1 }.Find [A].
[A] = { {1},{2},{3},{4},{5} }

(d) Let A = { 2 }.Find [A].
[A] = { {1},{2},{3},{4},{5} }

(e) Let A = { 1,2,3,4,5 }.Find[A].
[A] = {{ 1,2,3,4,5}}

3. Let X = $\mathbb{Z} \times \mathbb{N}$. Define(a,b) $\sim$ (c,d) if and only if ad = bc.

(a) Let (a,b) = (1,2).Find [(a,b)].
[(1,2)] is the equivalence class comprising all c $\in \mathbb{Z}$ and d $\in \mathbb{N}$ such that c = 2d.
[(a,b)] = {(c,d) $\in$ X: d = 2c }

(b) Let (a,b) = (2,4). Find [(a,b)].
[(1,2)] is the equivalence class comprising all c $\in \mathbb{Z}$ and d $\in \mathbb{N}$ such that d = 2c.

[(a,b)] = { (c,d) $\in$ X: d = 2c }

(c) Let (a,b) = (1,3). Find [(a,b)].

[(1,3)] is the equivalence class comprising all c $\in \mathbb{Z}$ and d $\in \mathbb{N}$ such that b = 3a.
[(a,b)] = { (c,d) $\in$ X: d = 3c }

(d) Let (a,b) = (3,9). Find [(a,b)].

[(1,3)] is the equivalence class comprising all c $\in \mathbb{Z}$ and d $\in \mathbb{N}$ such that d = 3c.
[(a,b)] = { (c,d) $\in$ X: d = 3c }

(e) Let (a,b) = (5,2). Find [(a,b)]. [(5,2)] is the equivalence class comprising all c $\in \mathbb{Z}$ and d $\in \mathbb{N}$ such that d = $\frac{2}{5}$c.
[(a,b)] = { (c,d) $\in$ X: d = $\frac{2}{5}$c }

(f) Let (a,b) = (10,4). Find [(a,b)].

[(5,2)] is the equivalence class comprising all c $\in \mathbb{Z}$ and d $\in \mathbb{N}$ such that d = $\frac{2}{5}$c.
[(a,b)] = { (c,d) $\in$ X: d = $\frac{2}{5}$ c }

4. Let X = C^1 ($\mathbb{R}$) be the set of all real-valued functions f on $\mathbb{R}$ such that f is differentiable and $f^{'}$ is continuous. Define f $\sim$ g if and only if $f^{'}$ (t) = $g^{'}$(t) for every t $\in \mathbb{R}$.
Translating f(t) by real numbers produces [f]

(a) Let f(t) = t^2 for all t $\in \mathbb{R}$.Find [f].
[f] = { g(t) = t^2 + c : c $\in \mathbb{R}$ }

(b) Let f(t) = e^2t for all t $\in \mathbb{R}$.Find [f].
[f] = { g(t) = e^2t + c : c $\in \mathbb{R}$ }

(c) Let f $\in$ X be arbitrary. Find [f].
[f] = { g(t) = f + c : c $\in \mathbb{R}$ }