Chapter 2

The Physics Behind PDRCs

A Passive Daytime Radiative Cooling (PDRC) device operates by absorbing a lower amount of blackbody radiation than it emits, thereby facilitating electricity-free cooling, even in daylight conditions. Consequently, one of the pivotal attributes of a PDRC device is the imperative for an absorptivity (α) as close to 0% or, conversely, a reflectivity (R) of 100% within the solar spectrum (ranging from 0.3 to 2.5 micrometers). This specification ensures that the device's surface remains entirely unaffected by solar heating during daylight hours.

To enhance the effectiveness of PDRC, it becomes essential to accurately measure and optimize this reflectivity (R) within the solar spectrum. One approach for achieving this goal is to perceive light as an electromagnetic wave, and from this perspective, derive a quantifiable means to measure R through the renowned *Fresnel equations*.

The Fresnel equations are mathematical expressions that delineate the proportion of incident energy that is either transmitted or reflected at the interface of two materials with differing refractive indices. This concept aligns precisely with our objectives, as we plan to stack plane surfaces featuring distinct reflective properties and refractive indices. This chapter serves as an exploration of the theoretical framework underpinning the derivation of R via the Fresnel Equations, delving into associated phenomena such as total internal reflection. Additionally, we explore the practical application of these principles to PDRC devices.

2.1 Fresnel Equations

Consider a light ray incident at point P upon a planar interface, leading to the generation of both reflected and refracted rays. It is noteworthy that the refractive index at the interface for both the incident and reflected rays (n_1) differs from the refractive index associated with the refracted ray (n_2). The plane of incidence lies within the x-z plane and is defined by both the surface normal and the incident ray.

In the context of each ray, the direction of wave propagation ($\vec{\mathbf{k}}$), can be established by the vector cross product of the electric field ($\vec{\mathbf{E}}$), and magnetic field ($\vec{\mathbf{B}}$) vectors, expressed as $\vec{\mathbf{E}} \times \vec{\mathbf{B}}$. This relationship can be conveniently determined using the right-hand rule, offering

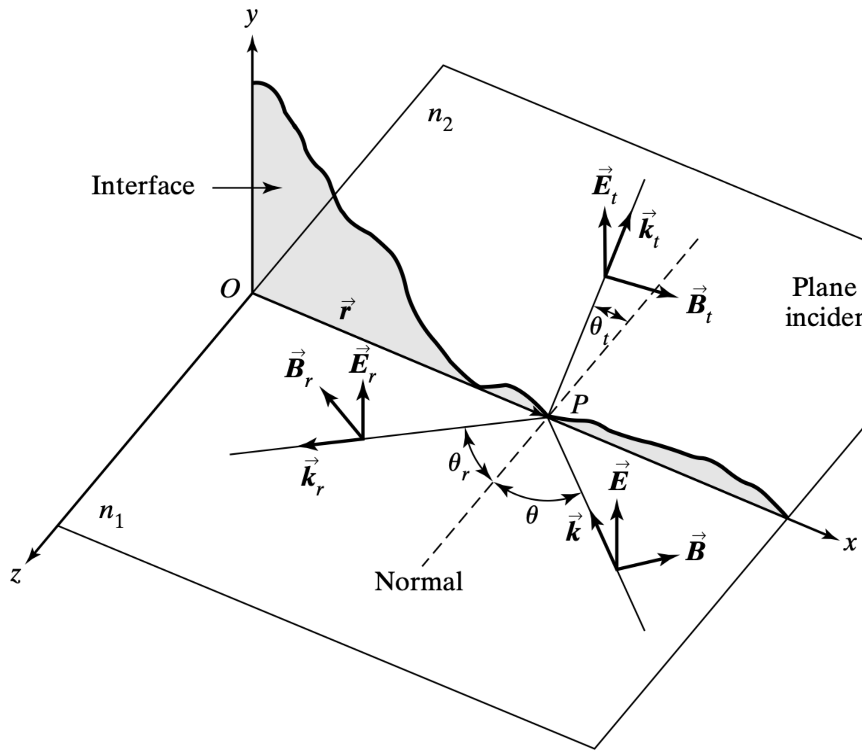

Figure 2.1: The transverse electric (TE) set-up

a valuable method for directional assessment.

Let us consider that the incident light comprises of plane harmonic waves:

$$\vec{\mathbf{E}} = \vec{\mathbf{E}_0} e^{i(\vec{\mathbf{k}} \cdot \vec{\mathbf{r}} - \omega t)} \tag{2.1}$$

In our analytical approach, we will specifically examine a linearly polarized light wave, where the electric field vector $\vec{\mathbf{E}}$ is oriented perpendicular to the plane of incidence. According to the right-hand rule, this configuration places the magnetic field vector $\vec{\mathbf{B}}$ within the plane of incidence. This particular polarization is known as the *transverse electric* (TE) mode

The reflected and transmitted waves can then also be expressed as plane harmonic wave equations:

$$\vec{\mathbf{E}_r} = \vec{\mathbf{E}_{0r}} e^{i(\vec{\mathbf{k_r}} \cdot \vec{\mathbf{r}} - \omega_r t)} \tag{2.2}$$

$$\vec{\mathbf{E}_t} = \vec{\mathbf{E}_{0t}} e^{i(\vec{\mathbf{k_t}} \cdot \vec{\mathbf{r}} - \omega_t t)} \tag{2.3}$$

At the interface where all three waves emerge simultaneously, a crucial boundary condition must be established to govern the relationship between their respective wave amplitudes. This boundary condition stipulates that the waves both incident upon and emerging from the plane of incidence should exhibit continuity and differentiability. This requirement is contingent upon the assumption that the interface is isotropic.

2.1.1 Boundary Conditions for TE Waves

In the context of our TM mode configuration, we can express the wave equations for the incident, reflected, and transmitted electric field components waves as follows:

$$\vec{\mathbf{E}_0} = E\hat{y} \qquad \vec{\mathbf{E}_{0r}} = E_r\hat{y} \qquad \vec{\mathbf{E}_{0t}} = E_t\hat{y}$$

where E, E_r, and E_t denote the complex field amplitudes corresponding to the incident, reflected, and transmitted waves, respectively. These wave equations adhere to the boundary conditions, ensuring the continuity of electric field components parallel to the interface, as in:

$$E + E_r = E_t \tag{2.4}$$

By basic trigonometry and Maxwell's equations, we can find the corresponding magnetic fields to be:

$$\vec{\mathbf{B}} = (B\cos(\theta\hat{x}) - B\sin(\theta\hat{z}))e^{i(\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}-\omega t)}$$
$$\vec{\mathbf{B}}_r = (-B_r\cos(\theta_r\hat{x}) - B_r\sin(\theta_r\hat{z}))e^{i(\vec{\mathbf{k_r}}\cdot\vec{\mathbf{r}}-\omega t)}$$
$$\vec{\mathbf{B}}_t = (B_t\cos(\theta_t\hat{x}) - B_t\sin(\theta_t\hat{z}))e^{i(\vec{\mathbf{k_t}}\cdot\vec{\mathbf{r}}-\omega t)}$$

As the magnetic field vector lies transversely to the plane of incidence, adherence to the boundary conditions necessitates the connection of parallel components of the magnetic field, as defined by:

$$B\cos(\theta) - B_r\cos(\theta) = B_t\cos(\theta_t) \tag{2.5}$$

Here, it's important to note that $\theta = \theta_r$ according to the law of reflection. Equations 2.4 and 2.5 stand as two pivotal equations arising from the boundary conditions for TE waves. These equations, instrumental in determining R, serve as a critical foundation. Nevertheless, before we delve into the calculation of R through these boundary conditions, it is imperative to demonstrate the applicability of the same procedure to the transverse magnetic case.

2.1.2 Boundary Conditions for TM Waves

Another polarization mode for electromagnetic waves is known as the *transverse magnetic* (TM) polarization. In this mode, the magnetic field vector is oriented perpendicular to the plane of incidence, while the electric field vector lies transverse to the plane of incidence. Within the framework of our TM mode configuration, we can formulate the wave equations governing the electric field components of the incident, reflected, and transmitted waves as follows:

$$\vec{\mathbf{E}} = (E\cos(\theta\hat{x}) - E\sin(\theta\hat{z}))e^{i(\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}-\omega t)}$$
$$\vec{\mathbf{E}}_r = (E_r\cos(\theta_r\hat{x}) + E_r\sin(\theta_r\hat{z}))e^{i(\vec{\mathbf{k_r}}\cdot\vec{\mathbf{r}}-\omega t)}$$
$$\vec{\mathbf{E}}_t = (E_t\cos(\theta_t\hat{x}) - E_t\sin(\theta_t\hat{z}))e^{i(\vec{\mathbf{k_t}}\cdot\vec{\mathbf{r}}-\omega t)}$$

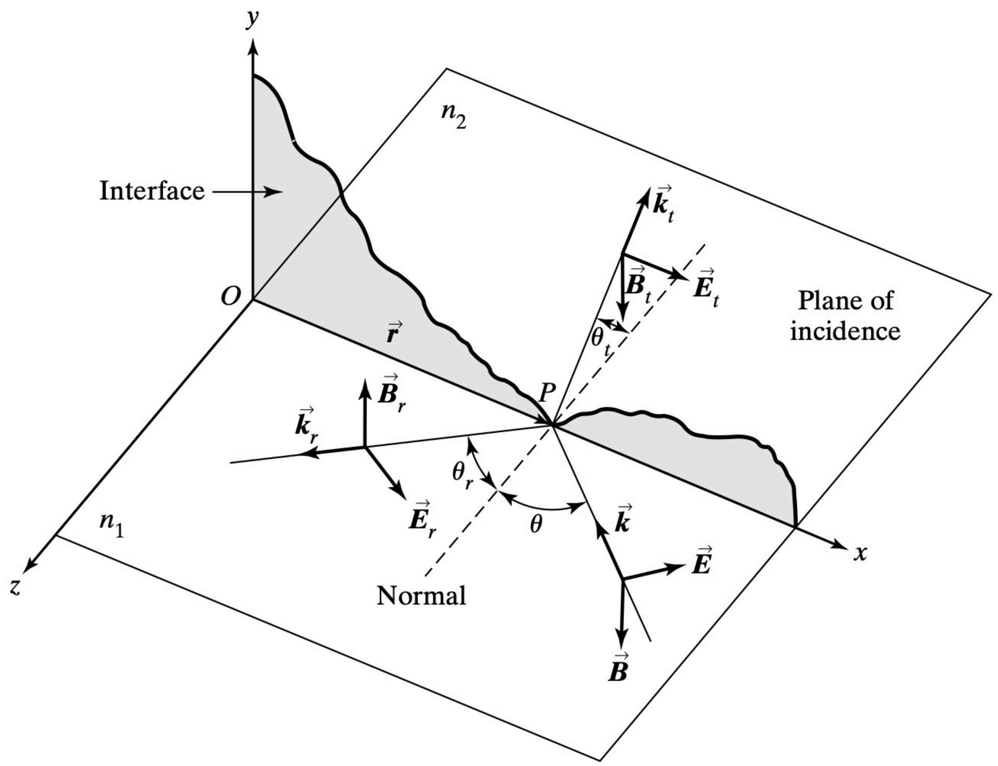

Figure 2.2: The transverse electric (TM) set-up

Consequently, the magnetic field components can be written as:

$$\vec{\mathbf{B}} = -B\hat{y}e^{i(\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}-\omega t)}$$
$$\vec{\mathbf{B}}_r = B_r\hat{y}e^{i(\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}-\omega t)}$$
$$\vec{\mathbf{B}}_t = -B_t\hat{y}e^{i(\vec{\mathbf{k_t}}\cdot\vec{\mathbf{r}}-\omega t)}$$

These equations adhere to the boundary conditions, guaranteeing the continuity of electric field components parallel to the interface, as demonstrated by:

$$-B + B_r = -B_t \tag{2.6}$$

$$Ecos(\theta) + E_r cos(\theta) = E_t cos(\theta_t) \tag{2.7}$$

2.1.3 Reflection and Transmission Coefficients

To determine the reflectance R and, consequently, the transmittance T, it is imperative to define the reflection and transmission coefficients, as follows:

$$r = \frac{E_r}{E} \tag{2.8}$$

and

$$t = \frac{E_t}{E} \tag{2.9}$$

However, it is essential to calculate these coefficients for both the TE and TM cases. Recall that the electric and magnetic field amplitudes are linked through the following relation:

$$E = \nu B = \left(\frac{c}{n}\right) B \tag{2.10}$$

We can employ 2.10 to replace every occurrence of B in the boundary conditions with its corresponding E. Initiating with the TE case, remember the two vital boundary condition equations for the TE mode:

$$TE : \begin{cases} E + E_r = E_t \\ n_1 E cos(\theta) - n_1 E_r cos(\theta) = n_2 E_t cos(\theta_t) \end{cases} \tag{2.11}$$

We can proceed to solve the system of equations above for the TE case to determine r_{TE} (by eliminating all instances of E_t). Introducing the concept of the *relative refractive index*, denoted as n and defined as $n \equiv \frac{n_2}{n_1}$, we can derive:

$$r_{TE} = \frac{E_r}{E} = \frac{cos(\theta) - ncos(\theta_t)}{cos(\theta) + ncos(\theta_t)} \tag{2.12}$$

By the law of refraction, $sin(\theta) = nsin(\theta_t)$, we can eliminate θ_t by noting that:

$$ncos(\theta_t) = n\sqrt{cos^2(\theta_t)} = n\sqrt{1 - sin^2(\theta_t)} = \sqrt{n^2 - sin^2(\theta)} \tag{2.13}$$

Finally, our r_{TE} is:

$$r_{TE} = \frac{E_r}{E} = \frac{cos(\theta) - \sqrt{n^2 - sin^2(\theta)}}{cos(\theta) + \sqrt{n^2 - sin^2(\theta)}} \tag{2.14}$$

Likewise, employing our boundary conditions for the TM case along with 2.10, we arrive at the reevaluated boundary conditions for the TM mode, which are as follows:

$$TM : \begin{cases} -n_1 E + n_1 E_r = -n_2 E_t \\ E cos(\theta) + E_r cos(\theta) = E_t cos(\theta_t) \end{cases} \tag{2.15}$$

With this revised form of the boundary condition, we can compute r_{TM} in a manner similar to our determination of r_{TE}, resulting in:

$$r_{TE} = \frac{E_r}{E} = \frac{-n^2 cos(\theta) + \sqrt{n^2 - sin^2(\theta)}}{n^2 cos(\theta) + \sqrt{n^2 - sin^2(\theta)}} \tag{2.16}$$

Similarly, if we follow the same steps we did for evaluating r_{TM} and r_{TE}, we can subsequently figure out t_{TM} and t_{TE} (we now eliminate E_r instead of E_t). We find:

$$t_{TE} = \frac{E_t}{E} = \frac{2cos(\theta)}{cos(\theta) + \sqrt{n^2 - sin^2(\theta)}} \tag{2.17}$$

$$t_{TM} = \frac{E_t}{E} = \frac{2ncos(\theta)}{n^2cos(\theta) + \sqrt{n^2 - sin^2(\theta)}} \tag{2.18}$$

Observing, for instance, the interconnection between t_{TE} and r_{TE}, it is evident that they share the same denominator, suggesting that one can be expressed in terms of the other. To establish the relationship between the two equations, one can subtract r_{TE} from t_{TE}, yielding 1. With some additional manipulation, the corresponding equation linking t_{TM} and r_{TM} can be derived. Ultimately, r can be expressed in terms of t as follows:

$$t_{TE} = 1 + r_{TE}$$
$$nt_{TM} = 1 - r_{TM}$$

The set of equations r_{TE}, r_{TM}, t_{TE}, and t_{TM} constitutes the **Fresnel Equations**. These equations provide reflection and transmission coefficients that establish the connection between incident and reflected energy field amplitudes in any linear, isotropic, and homogeneous medium. The primary objective of the Fresnel equations is to facilitate the determination of Reflectance R and Transmittance T, which will be discussed in the subsequent subsection.

2.1.4 Reflectance and Transmittance

For the sake of energy conservation, it is imperative that, at a specific boundary, the power incident upon the boundary equals the combined power of both the reflected and transmitted energy at that boundary.

$$P_i = P_r + P_t \tag{2.19}$$

We can preemptively define reflectance R and transmittance T as:

$$R = \frac{P_r}{P_i} \tag{2.20}$$

$$T = \frac{P_t}{P_i} \tag{2.21}$$

Hence, R represents the proportion of reflected power relative to incident power, and transmittance T signifies the proportion of transmitted power in relation to incident power. This implies that 2.19 conforms to the well-known unity equation, under the assumption of zero absorption:

$$R + T = 1 \tag{2.22}$$

Power can be defined as the product of the irradiance of the electromagnetic wave and the cross-sectional area through which the wave passes, as demonstrated by:

$$I_iA_i = I_rA_r + I_tA_t \tag{2.23}$$

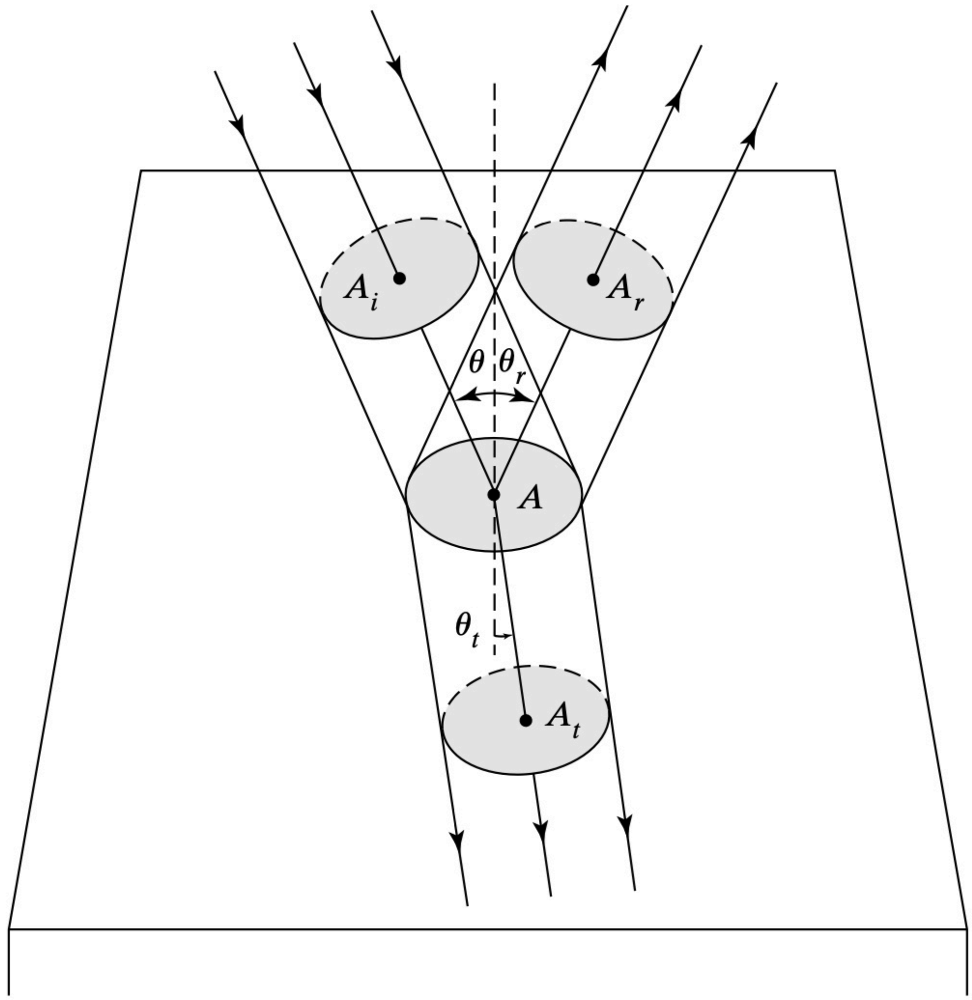

Figure 2.3: Cross sections of the incident, reflected, and transmitted beams

To ascertain A_i, A_r, and A_t, it is essential to calculate the cross-sectional area covered by the incident, reflected, and transmitted electromagnetic waves, respectively. By employing trigonometric relationships, as illustrated in the accompanying figure, we can calculate the cross-sectional areas encompassed by the incident, reflected, and transmitted waves and subsequently substitute them into the equation 2.23 to be:

$$I_i Acos(\theta) = I_r Acos(\theta) + I_t Acos(\theta_t) \tag{2.24}$$

Using the relation between irradiance and electrical field amplitude,

$$I = E_0^2(\frac{\varepsilon\nu}{2}) \tag{2.25}$$

and that $\nu_i = \nu_r$ and $\varepsilon_i = \varepsilon_r$ since they correspond to the same medium, we can rephrase the power balance equation to be:

$$E_{0i}^2 = E_{0r}^2 + E_{0t}^2 \left(\frac{\nu_t \varepsilon_t}{\nu_i \varepsilon_i}\right) \left(\frac{cos(\theta_t)}{cos(\theta)}\right) \tag{2.26}$$

$\frac{\nu_t \varepsilon_t}{\nu_i \varepsilon_i}$ is a complicated way of expressing n - the relative refractive index. Using the facts that $\mu_i = \mu_t = \mu_0$ (for nonmagnetic materials) and the relation that $\nu^2 = \frac{1}{\mu\varepsilon}$:

$$\frac{\nu_t \varepsilon_t}{\nu_i \varepsilon_i} = \frac{\nu_i^2 \mu_i}{\nu_t^2 \mu_t} \frac{\nu_t}{\nu_i} = \frac{\nu_i}{\nu_t} = n \tag{2.27}$$

Thus we can include n in the power balance equations:

$$E_{0i}^2 = E_{0r}^2 + nE_{0t}^2 \left(\frac{cos(\theta_t)}{cos(\theta)}\right) \tag{2.28}$$

Dividing through by E_{0i}^2, we get:

$$1 = r^2 + nt^2 \left(\frac{cos(\theta_t)}{cos(\theta)}\right) \tag{2.29}$$

Note that equations 2.8 and 2.9 allow us to make the substitutions for r and t above. Note that 2.29 certify that:

$$R = \frac{P_r}{P_i} = \frac{I_r}{I_i} = r^2 \tag{2.30}$$

Consequently,

$$T = nt^2 \left(\frac{cos(\theta_t)}{cos(\theta)}\right) \tag{2.31}$$

It's worth noting that T is not merely t^2, as we must consider the altered speed of the electromagnetic wave when it enters a medium with a different refractive index. This change in speed impacts the rate of energy propagation and, consequently, the power of the beam.

2.2 Multilayer films

In the development of Passive Daytime Radiative Cooling Devices (PDRCs), we intend to employ multiple stacks comprising diverse materials with distinct refractive indices, arranged in layers to enhance the overall reflection coefficient and consequently increase reflectance. It is crucial to comprehend the electromagnetic wave interaction and optimal layer stacking physics to maximize reflectance. We again need a rigorous consideration of boundary conditions as dictated by Maxwell's equations.

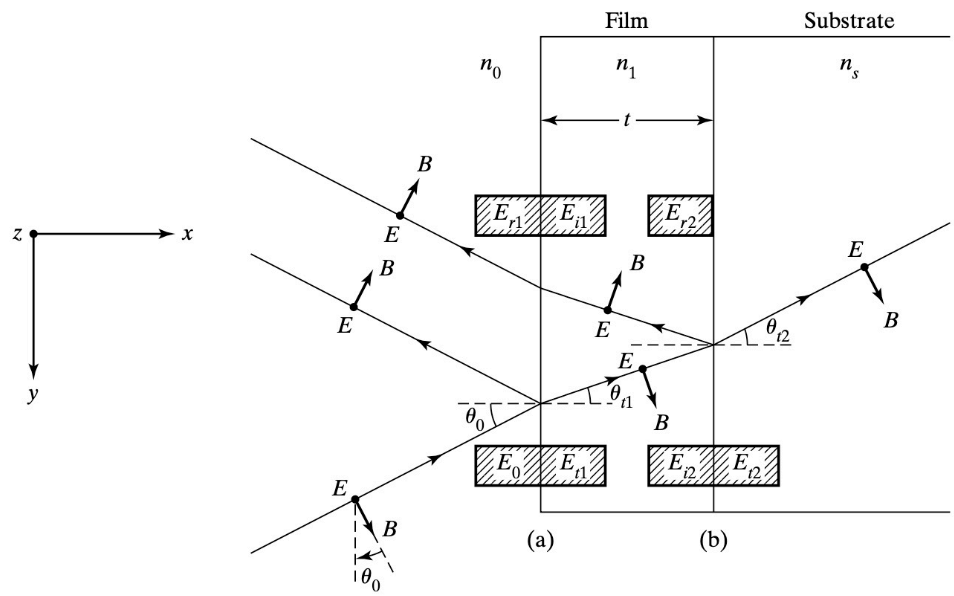

Figure 2.4: Multilayer Film Set-Up

2.2.1 Transfer Matrix

Consider the set-up above where the electric field amplitude is perpendicular to the plane of incidence (a transverse electric (TE) wave).

A TE wave propagates from air, reaching the air-film boundary. A fraction of the wave is transmitted, while another portion is reflected. The transmitted portion encounters the film-substrate interface, where part is further transmitted, and another part is reflected. This reflected component then becomes incident on the film-air interface, leading to transmission and reflection, repeating the process. This scenario can be extrapolated to involve multiple films. It is important to note that the film and substrate possess distinct refractive indices, denoted as n_1 and n_s respectively.

To capture the complex interaction between reflected and transmitted beams, we employ a systematic notation, as seen in the provided setup image. For example, E_{r1} denotes the aggregate of multiply reflected beams at interface (a), while E_{i2} signifies the sum of multiple incident beams at interface (b), directed toward the substrate.

Again, we hold true to our assumptions that the film is both homogeneous (has the same properties at every point) and isotropic (the physical properties do not differ regardless of the direction or orientation in which it is examined). Moreover, we assume further that the thickness of the film is on the order of wavelength of light.

The boundary conditions derived from Maxwell's equations stipulate that the tangential components of the electric and magnetic fields in plane waves must exhibit continuity across the interface, remaining equal on both sides. It is essential to recognize that while the electric field component is tangential to the interfaces everywhere, the orthogonal nature of the magnetic field vector to the electric field vector necessitates the determination of corresponding tangential magnetic field vectors. This endeavor aims to establish a formula that correlates the electric and magnetic fields at one interface with those at the subsequent interface.

$$E_a = E_0 + E_{r1} = E_{t1} + E_{i1} \tag{2.32}$$

$$E_b = E_{i2} + E_{r2} = E_{t2} \tag{2.33}$$

To determine the tangential magnetic field vectors, trigonometry can be applied to establish the following boundary conditions:

$$B_a = B_0 cos(\theta_0) - B_{r1} cos(\theta_0) = B_{t1} cos(\theta_{t1}) - B_{i1} cos(\theta_{t1}) \tag{2.34}$$

$$B_b = B_{i2} cos(\theta_{t1}) - B_{r2} cos(\theta_{t1}) = B_{t2} cos(\theta_{t2}) \tag{2.35}$$

We can build on equation 2.10 by expressing B in terms of E as:

$$B = n\sqrt{\varepsilon_0 \mu_0} E \tag{2.36}$$

and since $c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$, we can rewrite 2.34 and 2.35 employing 2.36.

$$B_a = \gamma_0 (E_0 - E_{r1}) = \gamma_1 (E_{t1} - E_{i1}) \tag{2.37}$$

$$B_b = \gamma_1 (E_{i2} - E_{r2}) = \gamma_s E_{t2} \tag{2.38}$$

where

$$\gamma_0 \equiv n_0 \sqrt{\varepsilon_0 \mu_0} cos(\theta_0) \tag{2.39}$$

$$\gamma_1 \equiv n_1 \sqrt{\varepsilon_0 \mu_0} cos(\theta_{t1}) \tag{2.40}$$

$$\gamma_s \equiv n_s \sqrt{\varepsilon_0 \mu_0} cos(\theta_{t2}) \tag{2.41}$$

E_{i2} differs from E_{t1} solely due to a phase difference δ arising from a single traversal of the film. The optical path length linked with one traversal is denoted as $\Delta_1 = n_1 t \cos(\theta_{t1})$. The optical path length represents the distance that light "perceives" it has covered in a medium and can be quantified in terms of the number of cycles the light beam has undergone within that specific medium. It is inherently proportional to the refractive index and the geometric length of the medium through which light is propagating.

We can express the phase difference that develops due to one traversal of the film as:

$$\delta = k_0 \Delta_1 = \left(\frac{2\pi}{\lambda_0}\right) n_1 t cos(\theta_{t1}) \tag{2.42}$$

Therefore we can express the pair electric field sums E_{i2} & E_{t1} and E_{i1} & E_{r2} as follows:

$$E_{i2} = E_{t1}e^{-i\delta}$$
$$E_{i1} = E_{r2}e^{-i\delta}$$

and consequently rephrase 2.33 and 2.38 as:

$$E_b = E_{t1}e^{-i\delta} + E_{i1}e^{i\delta} = E_{t2} \tag{2.43}$$

$$B_b = \gamma_1(E_{t1}e^{-i\delta} - E_{i1}e^{i\delta}) = \gamma_s E_{t2} \tag{2.44}$$

Looking at the middle terms, we can solve for E_{t1} and E_{i1} in terms of E_b and B_b.

$$E_{t1} = \left(\frac{\gamma_1 E_b + B_b}{2\gamma_1}\right) e^{i\delta} \tag{2.45}$$

$$E_{i1} = \left(\frac{\gamma_1 E_b - B_b}{2\gamma_1}\right) e^{-i\delta} \tag{2.46}$$

Utilizing the Euler identities $2isin(\delta) \equiv e^{i\delta} - e^{-i\delta}$ and $2cos(\delta) \equiv e^{i\delta} + e^{-i\delta}$, we can use the equations 2.45 and 2.46 and substitute them into 2.32 and 2.38 to have both:

$$E_a = E_b cos(\delta) + B_b \left(\frac{isin(\delta)}{\gamma_1}\right) \tag{2.47}$$

$$B_a = E_b(i\gamma_1 sin(\delta)) + B_b cos(\delta) \tag{2.48}$$

which we can rewrite in matrix notation as:

$$\begin{bmatrix} E_a \\ B_a \end{bmatrix} = \begin{bmatrix} cos(\delta) & \frac{isin(\delta)}{\gamma_1} \\ i\gamma_1 sin(\delta) & cos(\delta) \end{bmatrix} \begin{bmatrix} E_b \\ B_b \end{bmatrix}$$

This notation introduces the insightful 2×2 matrix that establishes a connection between the electric and magnetic fields at one interface and those at the subsequent interface. Termed the *transfer matrix*, we will refer to this significant matrix in a generic manner as:

$$M = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}$$

Expanding this framework, we can extend the consideration to the stacking of multiple films, leading to the emergence of multiple interfaces. In a more general context, for an arbitrary number N of layers:

$$\begin{bmatrix} E_a \\ B_a \end{bmatrix} = M_1 M_2 M_3 \dots M_N \begin{bmatrix} E_N \\ B_N \end{bmatrix}$$

with the multiplication of individual transfer matrices representing the entire multilayer stack in the order in which the beam encounters them.

Returning to the comprehensive form of the transfer matrix presented above, we can substitute E_a and B_a with the middle segments of equations 2.32 and 2.37. Similarly, E_b and B_b can be replaced with the rightmost segments of equations 2.43 and 2.44, resulting in:

$$\begin{bmatrix} E_0 + E_{r1} \\ \gamma_0(E_0 - E_{r1}) \end{bmatrix} = \begin{bmatrix} cos(\delta) & \frac{isin(\delta)}{\gamma_1} \\ i\gamma_1 sin(\delta) & cos(\delta) \end{bmatrix} \begin{bmatrix} E_{t2} \\ \gamma_s E_{t2} \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} E_{t2} \\ \gamma_s E_{t2} \end{bmatrix}$$

Recall $r \equiv \frac{E_{r1}}{E_0}$ and $t \equiv \frac{E_{t2}}{E_0}$ (the reflection and transmission coefficients, respectively). By matrix multiplication, we can simplify the resulting equation to obtain:

$$1 + r = m_{11}t + m_{12}\gamma_s t \tag{2.49}$$

$$\gamma_0(1 - r) = m_{21}t + m_{22}\gamma_s t \tag{2.50}$$

Solving for r by solving the system of linear equations above, we get

$$r = \frac{\gamma_0 m_{11} + \gamma_0\gamma_s m_{12} - m_{21} - \gamma_s m_{22}}{\gamma_0 m_{11} + \gamma_0\gamma_s m_{12} + m_{21} + \gamma_s m_{22}} \tag{2.51}$$

2.2.2 Antireflecting Films

To gain a deeper insight into optimizing reflectance, let's explore the characteristics conducive to antireflection. Employing a double layer of quarter-wave-thickness films allows for the attainment of minimal reflectance at a specific wavelength. We will examine the simplest scenario for normal incidence, where all angles of incidence, reflection, and refraction are zero. The corresponding transfer matrix is expressed as:

$$M = \begin{bmatrix} cos(\delta) & \frac{isin(\delta)}{\gamma_1} \\ i\gamma_1 sin(\delta) & cos(\delta) \end{bmatrix} = \begin{bmatrix} 0 & \frac{i}{\gamma_1} \\ i\gamma_1 & 0 \end{bmatrix}$$

since, at quarter-wave thickness, $t = \frac{\lambda}{4} = \frac{1}{4} \times \frac{\lambda_0}{n_1} = \frac{\lambda_0}{4n_1}$. Recall also we defined the phase difference, 2.42, to be

$$\delta = \left(\frac{2\pi}{\lambda_0}\right) n_1 t cos(\theta_{t1})$$
$$\delta = \left(\frac{2\pi}{\lambda_0}\right) n_1 t cos(0)$$
$$\delta = \left(\frac{2\pi}{\lambda_0}\right) n_1 t$$
$$\delta = \frac{\pi}{2}$$

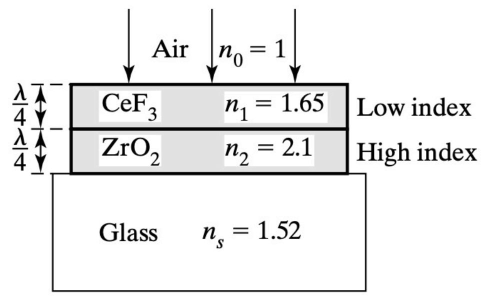

Figure 2.5: Antireflecting Double Layer

so that $cos(\delta) = 0$ and $sin(\delta) = 1$.

For two quarter-wave thickness films of similar form, the overall transfer matrix can be obtained by multiplying the individual transfer matrices:

$$M = \begin{bmatrix} -\frac{\gamma_2}{\gamma_1} & 0 \\ 0 & -\frac{\gamma_1}{\gamma_2} \end{bmatrix}$$

Equation 2.51, the reflection coefficient, then simplifies to:

$$r = \frac{\gamma_2^2\gamma_0 - \gamma_s\gamma_1^2}{\gamma_2^2\gamma_0 + \gamma_s\gamma_1^2} \tag{2.52}$$

The reflectance, R, is simply $|r|^2$ and using equations 2.39 through 2.41, we can calculate R to be

$$R = \left(\frac{n_0 n_2^2 - n_s n_1^2}{n_0 n_2^2 + n_s n_1^2}\right)^2 \tag{2.53}$$

Zero reflectance is then expected when the numerator is zero

$$n_0 n_2^2 = n_s n_1^2$$
$$\frac{n_2^2}{n_1^2} = \frac{n_s}{n_0}$$

so that

$$\frac{n_2}{n_1} = \sqrt{\frac{n_s}{n_0}} \tag{2.54}$$

For a glass substrate with a refractive index of $n_s = 1.52$ and incidence from air with $n_0 = 1$, the optimal ratio of the refractive indices for the two films should be $\frac{n_2}{n_1} = \sqrt{1.52} = 1.23$. However, achieving near-zero reflectance is challenging over the broader region of the visible spectrum.

PERSONAL NOTE - I WILL INCLUDE A FIGURE OF REFLECTANCE VERSUS WAVELENGTH ONCE I GENERATE THE STUDY ON COMSOL.

2.2.3 High-Reflectance Layers.

In the previous section, we established that to optimize for anti-reflectance, we stack layers in the order of air-low index-high index-substrate. Conversely, to optimize for high-reflectance, we follow the opposite order: air-high index-low index-substrate. A set of double layers designed to enhance reflectance is referred to as a *high-reflectance stack* or *dielectric mirror*.

PERSONAL NOTE - I WILL INCLUDE A FIGURE OF REFLECTANCE VERSUS WAVELENGTH ONCE I GENERATE THE STUDY ON COMSOL.

The transfer matrix for the two films in the order of high index-low index is

$$M_{HL} = \begin{bmatrix} 0 & \frac{i}{\gamma_H} \\ i\gamma_H & 0 \end{bmatrix} \begin{bmatrix} 0 & \frac{i}{\gamma_L} \\ i\gamma_L & 0 \end{bmatrix} = \begin{bmatrix} -\frac{\gamma_L}{\gamma_H} & 0 \\ 0 & -\frac{\gamma_H}{\gamma_L} \end{bmatrix}$$

For N similar double layers, we obtain $M = (M_{HL})^N$. In matrix form,

$$M = \begin{bmatrix} 0 & \frac{i}{\gamma_L} \\ i\gamma_L & 0 \end{bmatrix}^N = \begin{bmatrix} \left(-\frac{\gamma_L}{\gamma_H}\right)^N & 0 \\ 0 & \left(-\frac{\gamma_H}{\gamma_L}\right)^N \end{bmatrix}$$

Considering normal incidence and using equations 2.39 through 2.41,

$$\frac{\gamma_L}{\gamma_H} = \frac{n_L\sqrt{\varepsilon_0\mu_0}cos(\frac{\pi}{2})}{n_H\sqrt{\varepsilon_0\mu_0}cos(\frac{\pi}{2})}$$
$$\frac{\gamma_L}{\gamma_H} = \frac{n_L\sqrt{\varepsilon_0\mu_0}}{n_H\sqrt{\varepsilon_0\mu_0}}$$
$$\frac{\gamma_L}{\gamma_H} = \frac{n_L}{n_H}$$

Similarly, $\frac{\gamma_H}{\gamma_L} = \frac{n_H}{n_L}$

Equation 2.51, the reflection coefficient, then simplifies to:

$$r = \frac{n_0\left(\frac{-n_L}{n_H}\right)^N - n_s\left(\frac{-n_H}{n_L}\right)^N}{n_0\left(\frac{-n_L}{n_H}\right)^N + n_s\left(\frac{-n_H}{n_L}\right)^N} \tag{2.55}$$

To get R, we calculate $|r|^2$ to get

$$R = \left[\frac{\left(\frac{n_0}{n_s}\right)\left(\frac{n_L}{n_H}\right)^{2N} - 1}{\left(\frac{n_0}{n_s}\right)\left(\frac{n_L}{n_H}\right)^{2N} + 1}\right]^2 \tag{2.56}$$

PERSONAL NOTE - ONCE I FIGURE HOW TO DO SO, I SHALL INCLUDE THE STEPS IN THE DERIVATION OF 2.56.

Maximum (100%) reflectance is either achieved when:

1. When N approaches infinity.
2. When $\frac{n_L}{n_H}$ approaches zero. For example, when this ratio is 0.5 and there are 3 layers, a reflectance of 95.97 is achieved.

Since the optimal reflectance is obtained with the smallest ratio of $\frac{n_L}{n_H}$, high-reflectance stacks can be constructed using alternating layers of MgF_2 ($n_L = 1.38$) and ZnS ($n_H = 2.35$) or TiO_2 ($n_H = 240$). For example, employing alternating double layers of MgF_2 and ZnS can achieve a reflectance of 99.95% with a layer count (N) of 8.

PERSONAL NOTE - I WILL INCLUDE A FIGURE OF THE SPECTRAL REFLECTANCE OF A HIGH-LOW INDEX STACK ONCE I GENERATE IT ON COMSOL.