Chapter 2

The Physics Behind PDRCs

A Passive Daytime Radiative Cooling (PDRC) device operates by absorbing a lower amount of blackbody radiation than it emits, thereby facilitating electricity-free cooling, even in daylight conditions. Consequently, one of the pivotal attributes of a PDRC device is the imperative for an absorptivity (α) as close to 0% or, conversely, a reflectivity (R) of 100% within the solar spectrum (ranging from 0.3 to 2.5 micrometers). This specification ensures that the device's surface remains entirely unaffected by solar heating during daylight hours.

To enhance the effectiveness of PDRC, it becomes essential to accurately measure and optimize this reflectivity (R) within the solar spectrum. One approach for achieving this goal is to perceive light as an electromagnetic wave, and from this perspective, derive a quantifiable means to measure R through the renowned *Fresnel equations*.

The Fresnel equations are mathematical expressions that delineate the proportion of incident energy that is either transmitted or reflected at the interface of two materials with differing refractive indices. This concept aligns precisely with our objectives, as we plan to stack plane surfaces featuring distinct reflective properties and refractive indices. This chapter serves as an exploration of the theoretical framework underpinning the derivation of R via the Fresnel Equations, delving into associated phenomena such as total internal reflection. Additionally, we explore the practical application of these principles to PDRC devices.

2.1 Fresnel Equations

Consider a light ray incident at point P upon a planar interface, leading to the generation of both reflected and refracted rays. It is noteworthy that the refractive index at the interface for both the incident and reflected rays (n_1) differs from the refractive index associated with the refracted ray (n_2). The plane of incidence lies within the x-z plane and is defined by both the surface normal and the incident ray.

In the context of each ray, the direction of wave propagation ($\vec{\mathbf{k}}$), can be established by the vector cross product of the electric field ($\vec{\mathbf{E}}$), and magnetic field ($\vec{\mathbf{B}}$) vectors, expressed as $\vec{\mathbf{E}} \times \vec{\mathbf{B}}$. This relationship can be conveniently determined using the right-hand rule, offering

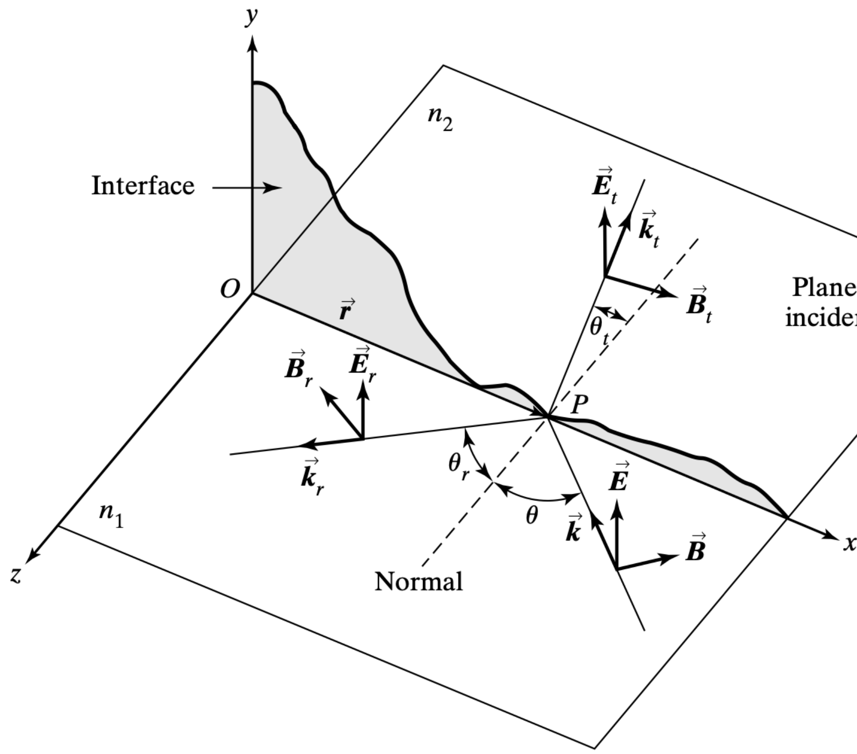

Figure 2.1: The transverse electric (TE) set-up

a valuable method for directional assessment.

Let us consider that the incident light comprises of plane harmonic waves:

$$\vec{\mathbf{E}} = \vec{\mathbf{E}_0} e^{i(\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}-\omega t)} \tag{2.1}$$

In our analytical approach, we will specifically examine a linearly polarized light wave, where the electric field vector $\vec{\mathbf{E}}$ is oriented perpendicular to the plane of incidence. According to the right-hand rule, this configuration places the magnetic field vector $\vec{\mathbf{B}}$ within the plane of incidence. This particular polarization is known as the *transverse electric* (TE) mode

The reflected and transmitted waves can then also be expressed as plane harmonic wave equations:

$$\vec{\mathbf{E}_r} = \vec{\mathbf{E}_{0r}} e^{i(\vec{\mathbf{k_r}}\cdot\vec{\mathbf{r}}-\omega_r t)} \tag{2.2}$$

$$\vec{\mathbf{E}_t} = \vec{\mathbf{E}_{0t}} e^{i(\vec{\mathbf{k_t}}\cdot\vec{\mathbf{r}}-\omega_t t)} \tag{2.3}$$

At the interface where all three waves emerge simultaneously, a crucial boundary condition must be established to govern the relationship between their respective wave amplitudes. This boundary condition stipulates that the waves both incident upon and emerging from the plane of incidence should exhibit continuity and differentiability. This requirement is contingent upon the assumption that the interface is isotropic.

2.1.1 Boundary Conditions for TE Waves

In the context of our TM mode configuration, we can express the wave equations for the incident, reflected, and transmitted electric field components waves as follows:

$$\vec{\mathbf{E}}_0 = E\hat{y} \qquad \vec{\mathbf{E}}_{0r} = E_r\hat{y} \qquad \vec{\mathbf{E}}_{0t} = E_t\hat{y}$$

where E, E_r, and E_t denote the complex field amplitudes corresponding to the incident, reflected, and transmitted waves, respectively. These wave equations adhere to the boundary conditions, ensuring the continuity of electric field components parallel to the interface, as in:

$$E + E_r = E_t \tag{2.4}$$

By basic trigonometry and Maxwell's equations, we can find the corresponding magnetic fields to be:

$$\vec{\mathbf{B}} = (B\cos(\theta\hat{x}) - B\sin(\theta\hat{z}))e^{i(\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}-\omega t)}$$
$$\vec{\mathbf{B}}_r = (-B_r\cos(\theta_r\hat{x}) - B_r\sin(\theta_r\hat{z}))e^{i(\vec{\mathbf{k_r}}\cdot\vec{\mathbf{r}}-\omega t)}$$
$$\vec{\mathbf{B}}_t = (B_t\cos(\theta_t\hat{x}) - B_t\sin(\theta_t\hat{z}))e^{i(\vec{\mathbf{k_t}}\cdot\vec{\mathbf{r}}-\omega t)}$$

As the magnetic field vector lies transversely to the plane of incidence, adherence to the boundary conditions necessitates the connection of parallel components of the magnetic field, as defined by:

$$B\cos(\theta) - B_r\cos(\theta) = B_t\cos(\theta_t) \tag{2.5}$$

Here, it's important to note that $\theta = \theta_r$ according to the law of reflection. Equations 2.4 and 2.5 stand as two pivotal equations arising from the boundary conditions for TE waves. These equations, instrumental in determining R, serve as a critical foundation. Nevertheless, before we delve into the calculation of R through these boundary conditions, it is imperative to demonstrate the applicability of the same procedure to the transverse magnetic case.

2.1.2 Boundary Conditions for TM Waves

Another polarization mode for electromagnetic waves is known as the *transverse magnetic* (TM) polarization. In this mode, the magnetic field vector is oriented perpendicular to the plane of incidence, while the electric field vector lies transverse to the plane of incidence. Within the framework of our TM mode configuration, we can formulate the wave equations governing the electric field components of the incident, reflected, and transmitted waves as follows:

$$\vec{\mathbf{E}} = (E\cos(\theta\hat{x}) - E\sin(\theta\hat{z}))e^{i(\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}-\omega t)}$$
$$\vec{\mathbf{E}}_r = (E_r\cos(\theta_r\hat{x}) + E_r\sin(\theta_r\hat{z}))e^{i(\vec{\mathbf{k_r}}\cdot\vec{\mathbf{r}}-\omega t)}$$
$$\vec{\mathbf{E}}_t = (E_t\cos(\theta_t\hat{x}) - E_t\sin(\theta_t\hat{z}))e^{i(\vec{\mathbf{k_t}}\cdot\vec{\mathbf{r}}-\omega t)}$$

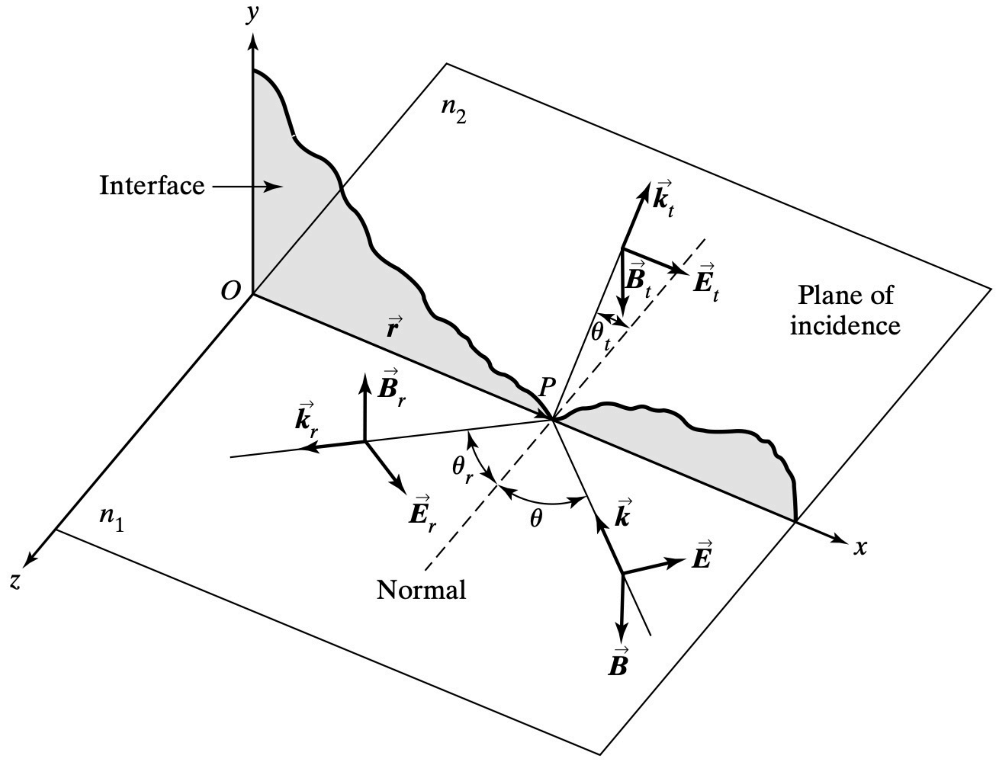

Figure 2.2: The transverse electric (TM) set-up

Consequently, the magnetic field components can be written as:

$$\vec{\mathbf{B}} = -B\hat{y}e^{i(\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}-\omega t)}$$
$$\vec{\mathbf{B}}_r = B_r\hat{y}e^{i(\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}-\omega t)}$$
$$\vec{\mathbf{B}}_t = -B_t\hat{y}e^{i(\vec{\mathbf{k_t}}\cdot\vec{\mathbf{r}}-\omega t)}$$

These equations adhere to the boundary conditions, guaranteeing the continuity of electric field components parallel to the interface, as demonstrated by:

$$-B + B_r = -B_t \tag{2.6}$$

$$Ecos(\theta) + E_rcos(\theta) = E_tcos(\theta_t) \tag{2.7}$$

2.1.3 Reflection and Transmission Coefficients

To determine the reflectance R and, consequently, the transmittance T, it is imperative to define the reflection and transmission coefficients, as follows:

$$r = \frac{E_r}{E} \tag{2.8}$$

and

$$t = \frac{E_t}{E} \tag{2.9}$$

However, it is essential to calculate these coefficients for both the TE and TM cases. Recall that the electric and magnetic field amplitudes are linked through the following relation:

$$E = \nu B = (\frac{c}{n})B \tag{2.10}$$

We can employ 2.10 to replace every occurrence of B in the boundary conditions with its corresponding E. Initiating with the TE case, remember the two vital boundary condition equations for the TE mode:

$$TE : \begin{cases} E + E_r = E_t \\ n_1 Ecos(\theta) - n_1 E_r cos(\theta) = n_2 E_t cos(\theta_t) \end{cases} \tag{2.11}$$

We can proceed to solve the system of equations above for the TE case to determine r_{TE} (by eliminating all instances of E_t). Introducing the concept of the *relative refractive index*, denoted as n and defined as $n \equiv \frac{n_2}{n_1}$, we can derive:

$$r_{TE} = \frac{E_r}{E} = \frac{cos(\theta) - ncos(\theta_t)}{cos(\theta) + ncos(\theta_t)} \tag{2.12}$$

By the law of refraction, $sin(\theta) = nsin(\theta_t)$, we can eliminate θ_t by noting that:

$$ncos(\theta_t) = n\sqrt{cos^2(\theta_t)} = n\sqrt{1 - sin^2(\theta_t)} = \sqrt{n^2 - sin^2(\theta)} \tag{2.13}$$

Finally, our r_{TE} is:

$$r_{TE} = \frac{E_r}{E} = \frac{cos(\theta) - \sqrt{n^2 - sin^2(\theta)}}{cos(\theta) + \sqrt{n^2 - sin^2(\theta)}} \tag{2.14}$$

Likewise, employing our boundary conditions for the TM case along with 2.10, we arrive at the reevaluated boundary conditions for the TM mode, which are as follows:

$$TM : \begin{cases} -n_1 E + n_1 E_r = -n_2 E_t \\ Ecos(\theta) + E_r cos(\theta) = E_t cos(\theta_t) \end{cases} \tag{2.15}$$

With this revised form of the boundary condition, we can compute r_{TM} in a manner similar to our determination of r_{TE}, resulting in:

$$r_{TE} = \frac{E_r}{E} = \frac{-n^2 cos(\theta) + \sqrt{n^2 - sin^2(\theta)}}{n^2 cos(\theta) + \sqrt{n^2 - sin^2(\theta)}} \tag{2.16}$$

Similarly, if we follow the same steps we did for evaluating r_{TM} and r_{TE}, we can subsequently figure out t_{TM} and t_{TE} (we now eliminate E_r instead of E_t). We find:

$$t_{TE} = \frac{E_t}{E} = \frac{2cos(\theta)}{cos(\theta) + \sqrt{n^2 - sin^2(\theta)}} \tag{2.17}$$

$$t_{TM} = \frac{E_t}{E} = \frac{2ncos(\theta)}{n^2cos(\theta) + \sqrt{n^2 - sin^2(\theta)}} \tag{2.18}$$

Observing, for instance, the interconnection between t_{TE} and r_{TE}, it is evident that they share the same denominator, suggesting that one can be expressed in terms of the other. To establish the relationship between the two equations, one can subtract r_{TE} from t_{TE}, yielding 1. With some additional manipulation, the corresponding equation linking t_{TM} and r_{TM} can be derived. Ultimately, r can be expressed in terms of t as follows:

$$\begin{aligned} t_{TE} &= 1 + r_{TE} \\ nt_{TM} &= 1 - r_{TM} \end{aligned}$$

The set of equations r_{TE}, r_{TM}, t_{TE}, and t_{TM} constitutes the **Fresnel Equations**. These equations provide reflection and transmission coefficients that establish the connection between incident and reflected energy field amplitudes in any linear, isotropic, and homogeneous medium. The primary objective of the Fresnel equations is to facilitate the determination of Reflectance R and Transmittance T, which will be discussed in the subsequent subsection.

2.1.4 Reflectance and Transmittance

For the sake of energy conservation, it is imperative that, at a specific boundary, the power incident upon the boundary equals the combined power of both the reflected and transmitted energy at that boundary.

$$P_i = P_r + P_t \tag{2.19}$$

We can preemptively define reflectance R and transmittance T as:

$$R = \frac{P_r}{P_i} \tag{2.20}$$

$$T = \frac{P_t}{P_i} \tag{2.21}$$

Hence, R represents the proportion of reflected power relative to incident power, and transmittance T signifies the proportion of transmitted power in relation to incident power. This implies that 2.19 conforms to the well-known unity equation, under the assumption of zero absorption:

$$R + T = 1 \tag{2.22}$$

Power can be defined as the product of the irradiance of the electromagnetic wave and the cross-sectional area through which the wave passes, as demonstrated by:

$$I_iA_i = I_rA_r + I_tA_t \tag{2.23}$$

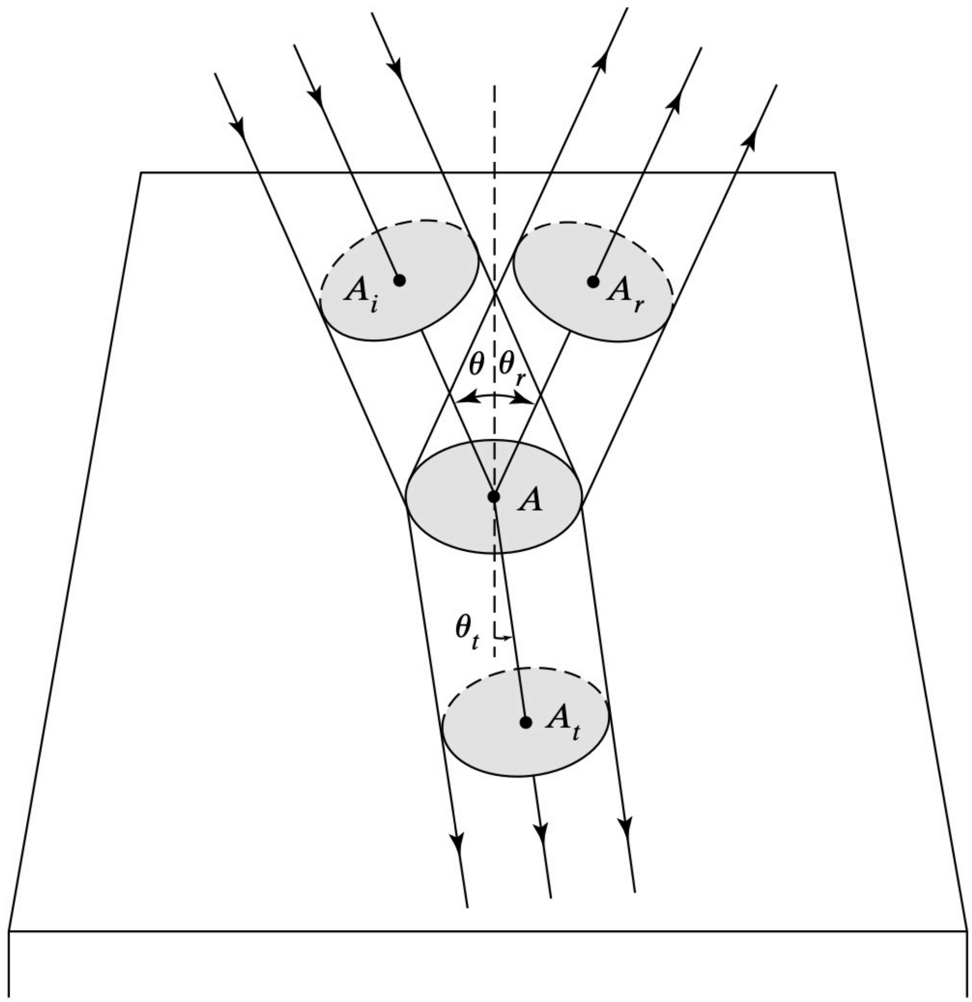

Figure 2.3: Cross sections of the incident, reflected, and transmitted beams

To ascertain A_i, A_r, and A_t, it is essential to calculate the cross-sectional area covered by the incident, reflected, and transmitted electromagnetic waves, respectively. By employing trigonometric relationships, as illustrated in the accompanying figure, we can calculate the cross-sectional areas encompassed by the incident, reflected, and transmitted waves and subsequently substitute them into the equation 2.23 to be:

$$I_i A cos(\theta) = I_r A cos(\theta) + I_t A cos(\theta_t) \tag{2.24}$$

Using the relation between irradiance and electrical field amplitude,

$$I = E_0^2 (\frac{\varepsilon \nu}{2}) \tag{2.25}$$

and that $\nu_i = \nu_r$ and $\varepsilon_i = \varepsilon_r$ since they correspond to the same medium, we can rephrase the power balance equation to be:

$$E_{0i}^2 = E_{0r}^2 + E_{0t}^2 \left(\frac{\nu_t \varepsilon_t}{\nu_i \varepsilon_i}\right) \left(\frac{cos(\theta_t)}{cos(\theta)}\right) \tag{2.26}$$

$\frac{\nu_t \varepsilon_t}{\nu_i \varepsilon_i}$ is a complicated way of expressing n - the relative refractive index. Using the facts that $\mu_i = \mu_t = \mu_0$ (for nonmagnetic materials) and the relation that $\nu^2 = \frac{1}{\mu\varepsilon}$:

$$\frac{\nu_t \varepsilon_t}{\nu_i \varepsilon_i} = \frac{\nu_i^2 \mu_i}{\nu_t^2 \mu_t} \frac{\nu_t}{\nu_i} = \frac{\nu_i}{\nu_t} = n \tag{2.27}$$

Thus we can include n in the power balance equations:

$$E_{0i}^2 = E_{0r}^2 + nE_{0t}^2 \left(\frac{cos(\theta_t)}{cos(\theta)}\right) \tag{2.28}$$

Dividing through by E_{0i}^2, we get:

$$1 = r^2 + nt^2 \left(\frac{cos(\theta_t)}{cos(\theta)}\right) \tag{2.29}$$

Note that equations 2.8 and 2.9 allow us to make the substitutions for r and t above. Note that 2.29 certify that:

$$R = \frac{P_r}{P_i} = \frac{I_r}{I_i} = r^2 \tag{2.30}$$

Consequently,

$$T = nt^2 \left(\frac{cos(\theta_t)}{cos(\theta)}\right) \tag{2.31}$$

It's worth noting that T is not merely t^2, as we must consider the altered speed of the electromagnetic wave when it enters a medium with a different refractive index. This change in speed impacts the rate of energy propagation and, consequently, the power of the beam.