Risk, Rates, Person time

Mark Otiende

Session Objectives

1. Distinction between risks and rates
2. How to calculate rates and person time
3. How to calculate and interpret confidence intervals for rates
4. How to compare two rates using rate ratio

Definitions

1. Risk

2. Rate

3. Person time

Definitions

Risk

Probability of death in five years following diagnosis of Prostate cancer is defined as the proportion of times that this would occur among a large number of men diagnosed with prostate cancer.

The probability is then said to be the **risk** of death in the five years following diagnosis of prostate cancer

Definitions

Rate

Measured in follow up studies and are the fundamental measure of the frequency of occurrence of an outcome over time.
Measures the number of new events that occur per person unit time.

e.g.
- the incidence rate of prostate cancer is 74.3/100000 men/year
- Mortality rate for children less than five years in Kilifi county is 30 per 1000 child-years

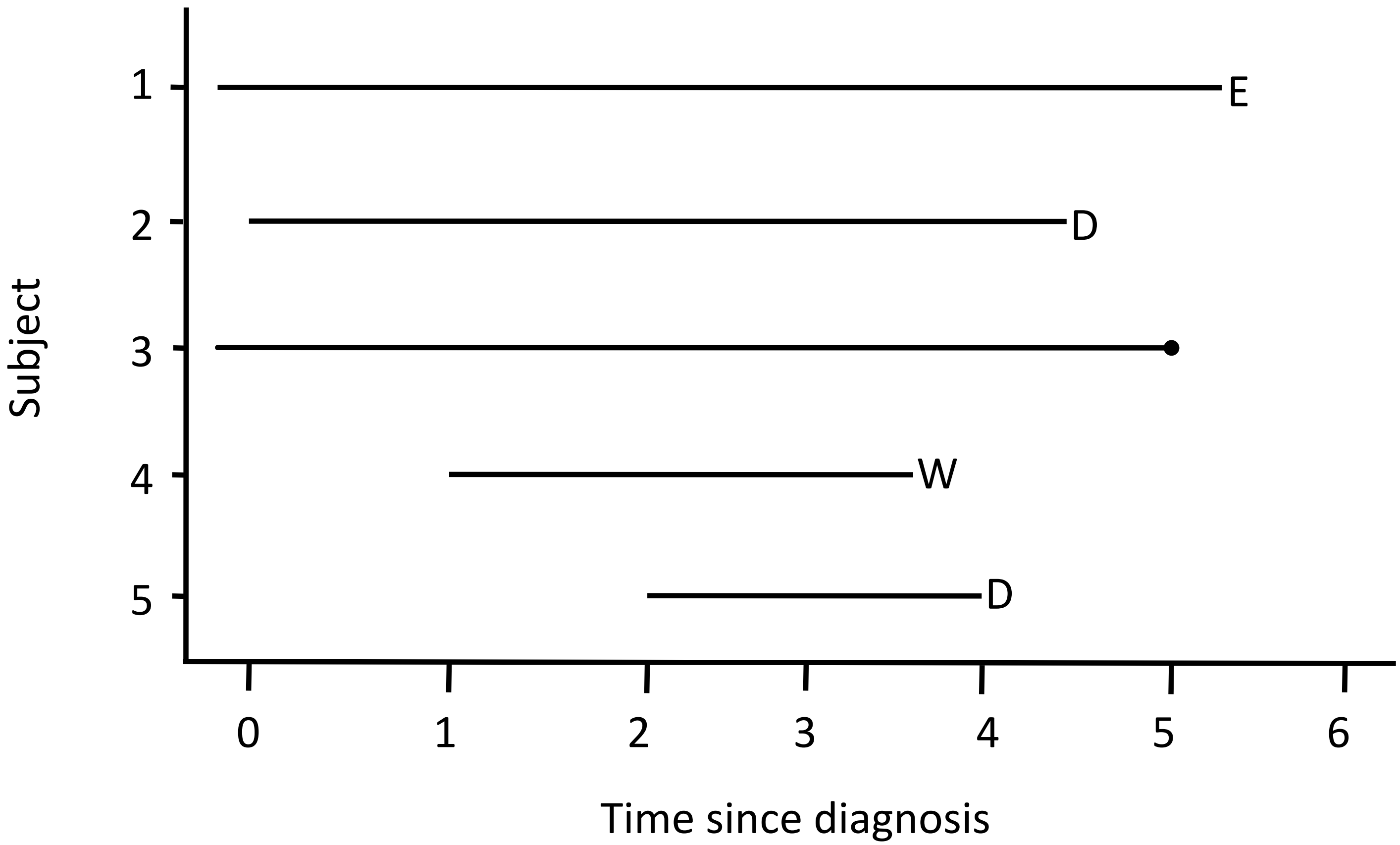
Definitions

Person time

Period of observation/follow-up-time. Starts when an individual joins a study and stops when they experience the outcome of interest, or are lost to follow-up of the follow-up period ends, whichever happens first

Definitions

Person time



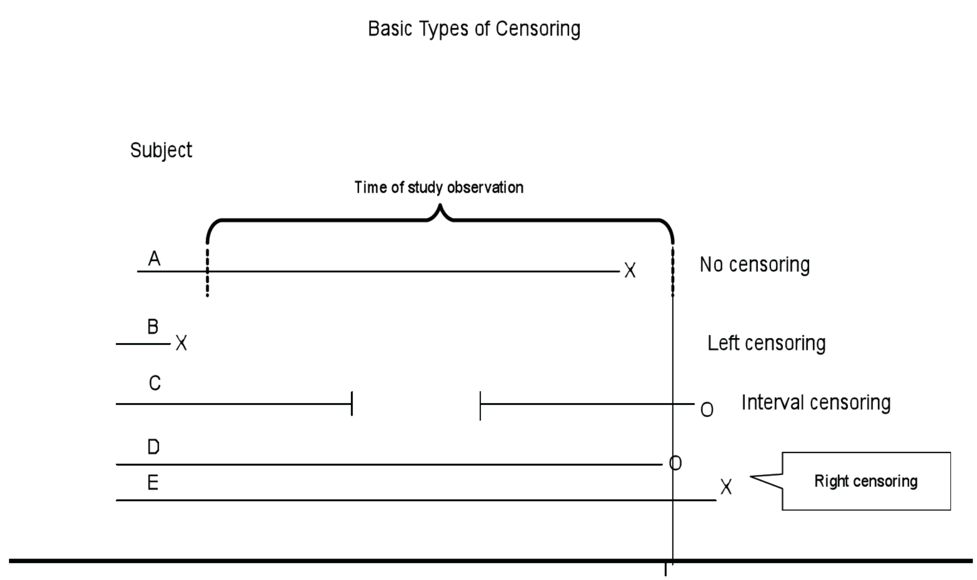
(D=died, E=emigrated, W=withdrew, ● = reached the end of follow-up without experiencing the event of interest)

Definitions

Person time

Period of observation/follow-up-time. Starts when an individual joins a study and stops when they experience the outcome of interest, or are lost to follow-up of the follow-up period ends, whichever happens first

Censoring



Basic theory of survival analysis

Computation of the Survivorship function – S(t)

Time, t_i	# at risk (n_t)	# of events (d_t)	S(t)
0	20	0	1.00
5	20	2	[1-(2/20)]*1.00=0.90
6	18	0	[1-(0/18)]*0.90=0.90
10	15	1	[1-(1/15)]*0.90=0.84
13	14	2	[1-(2/14)]*0.84=0.72

i, $$S(t_1) = 1 - r_1 = \frac{n_1 - d_1}{n_1}$$

Ii, $$S(t_j) = S(t_{(j-1)}) \times s(t_j) = s_{t_1} \times s_{t_2} \times \ldots \times s_{t_j}$$

Calculating person-time and rates

	ID	DOB	date_of_first_examination	date_exit	age_at_entry	age_at_exit	years_in_study	MI
1	151	1931-10-20	1980-05-30	1998-12-18	48.61054	67.16222	18.551674	0
2	158	1933-03-21	1981-12-02	1984-05-09	48.70089	51.13484	2.433949	1
3	658	1925-08-12	1981-10-22	1996-07-18	56.19439	70.93224	14.737850	1
4	941	1933-10-28	1982-05-29	1998-12-19	48.58316	65.14169	16.558525	0
5	1376	1935-09-19	1982-03-21	1998-11-25	46.50240	63.18412	16.681725	0
6	1467	1930-01-09	1982-07-06	1993-08-03	52.48734	63.56468	11.077343	0
7	1650	1927-11-19	1982-11-24	1998-12-31	55.01437	71.11568	16.101303	0
8	1673	1926-02-14	1983-07-03	1998-12-31	57.37988	72.87611	15.496239	0
9	1754	1921-07-21	1980-10-01	1998-12-31	59.19781	77.44559	18.247776	0
10	1765	1924-03-27	1982-12-30	1998-12-13	58.75975	74.71321	15.953457	0

Calculating person-time and rates

Person-years

$$follow\text{-}up\ time\ in\ years = (exitdate - startdate)/365.25$$

Rate is estimated from the study data by dividing the total number (d) of events observed by the total (T) of the individual person-years of observation.

$$Rate, \lambda = \frac{number\ of\ events}{total\ person\text{-}years\ of\ observation} = \frac{d}{T}$$

Confidence Interval for a rate

$$s.e.(rate) = \frac{s.e.(number\ of\ events)}{T} = \frac{\sqrt{d}}{T} = \sqrt{\frac{\lambda}{T}}$$

$$s.e.(\log rate) = \frac{1}{\sqrt{d}}$$

$$95\%\ CI\ \ for \log rate = \log rate \pm 1.96 \times \frac{1}{\sqrt{d}}$$

Confidence Interval for a rate

$$s.e.(rate) = \frac{s.e.(number\ of\ events)}{T} = \frac{\sqrt{d}}{T} = \sqrt{\frac{\lambda}{T}}$$

$$s.e.(\log rate) = \frac{1}{\sqrt{d}}$$

$$95\%\ CI\ \ for\ \log rate = \log rate \pm 1.96 \times \frac{1}{\sqrt{d}}$$

$$95\%\ CI\ \ for\ rate = \frac{rate}{\exp(1.96/\sqrt{d})} \quad to \quad rate \times \exp(1.96/\sqrt{d})$$

Confidence Interval for a rate

$$s.e.(rate) = \frac{s.e.(number\ of\ events)}{T} = \frac{\sqrt{d}}{T} = \sqrt{\frac{\lambda}{T}}$$

$$s.e.(\log rate) = \frac{1}{\sqrt{d}}$$

$$95\%\ CI\ \ for \log rate = \log rate \pm 1.96 \times \frac{1}{\sqrt{d}}$$

$$95\%\ CI\ \ for\ rate = \frac{rate}{\exp(1.96/\sqrt{d})} \quad \overbrace{to \quad rate \times \exp(1.96/\sqrt{d})}^{\text{Error Factor, EF}}$$

DeskWork

Comparing two rates

1. Rate difference
2. Rate Ratio
3. Confidence Interval for rate ratios

Rate Difference

Difference in rates of occurrence between two groups (exposed/non-exposed) in the population.

$$\text{rate difference} = \lambda_1 - \lambda_0$$

$$s.e.(rate\ difference) = \sqrt{\left(\frac{d_1}{T_1^2} + \frac{d_0}{T_0^2}\right)}$$

Rate Ratio

$$\text{Rate ratio} = \frac{\text{rate in exposed}}{\text{rate in unexposed}} = \frac{\lambda_1}{\lambda_0}$$

$$s.e\ of\ \log(rate\ ratio) = \sqrt{(1/d_1 + 1/d_0)}$$

Confidence Intervals for Rate Ratios

$$95\%\ CI = rate\ ratio / EF \quad to \quad rate\ ratio \times EF$$

$$EF = \exp\left[1.96 \times s.e.of \log(rate\ ratio)\right]$$

Z-test for the rate ratio

$$z = \frac{\log(rate\ ratio)}{s.e.of\ \log(rate\ ratio)}$$

Example

Practical

Review of Practical