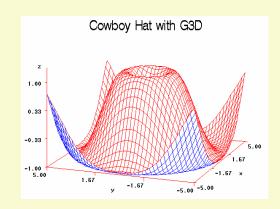
Multiple Linear Regression

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School of Dental Sciences

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by Lin Naing

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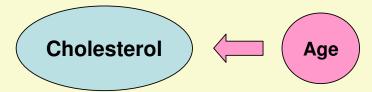
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Simple Linear Regression

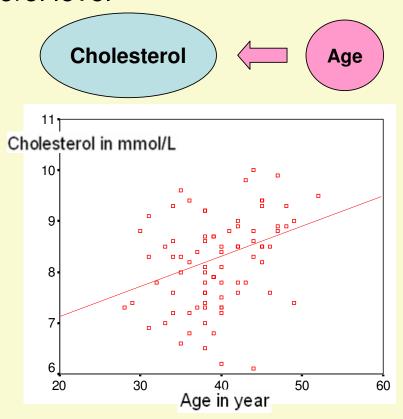
To determine the relationship between age and blood cholesterol level



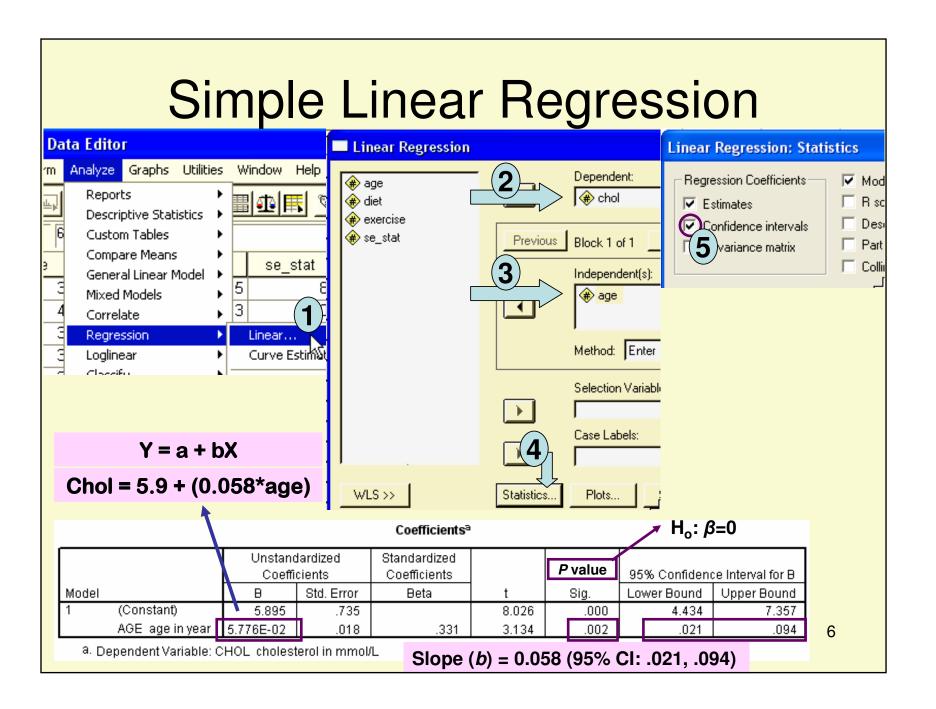
- ► Here, we may use either 'correlation analysis' or 'regression analysis', as both cholesterol and age are numerical variables.
- ► Correlation can give the strength of relationship, but regression can describe the relationship in more detail.
- ► In above example, if we decide to do <u>regression</u>, cholesterol will be our outcome (dependent) variable, because age may determine cholesterol but cholesterol cannot determine age.

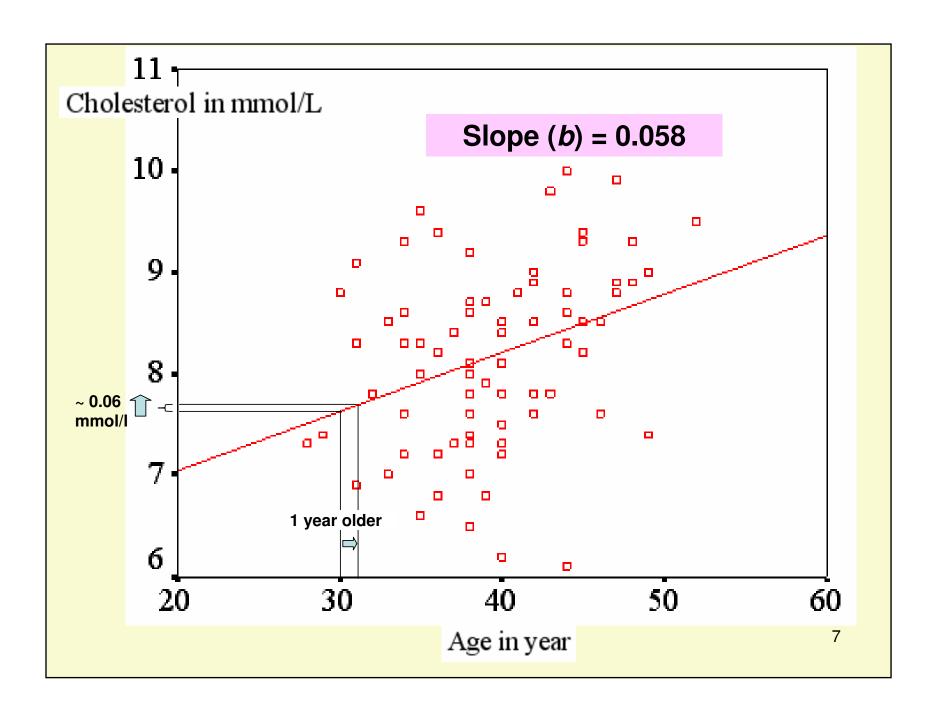
Simple Linear Regression

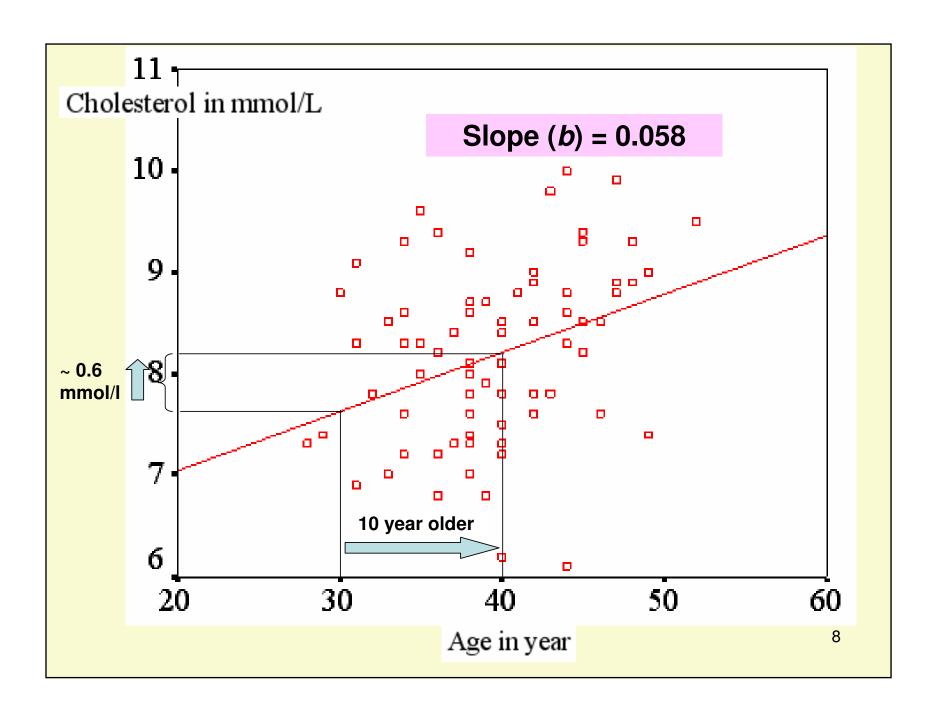
 To determine the relationship between age and blood cholesterol level



Simple Linear Regression Graphs Utilities Windo Scatterplot Gallery Define Interactive Simple Matrix 11 Cancel Bar... Cholesterol in mmol/L Help Line... Overlay Area... Pie... High-Low... Simple Scatterplot Pareto... Control... Y Axis: Boxplot... 🐞 diet Error Bar... 20 60 Age in year exercise Scatter... ⊕ se_stat 5

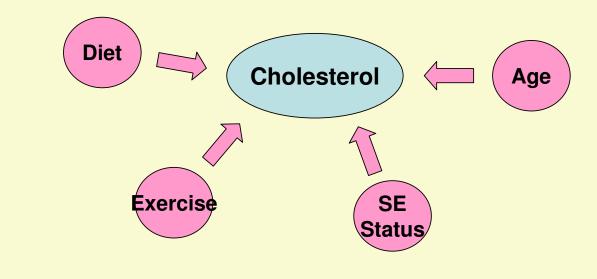




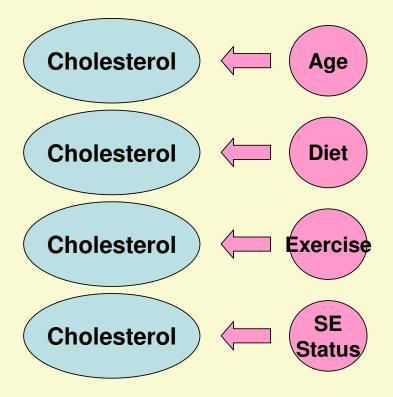


Basic Theory of MLR

 Most of the outcomes (events) are determined (influenced) by more than one factors (e.g. blood pressure, cholesterol level, etc.)

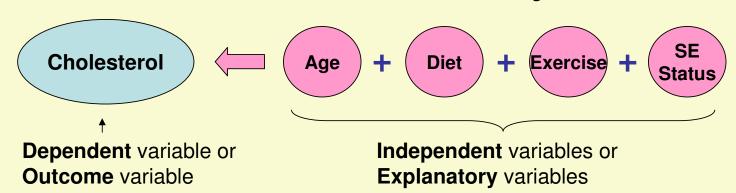


Basic Theory



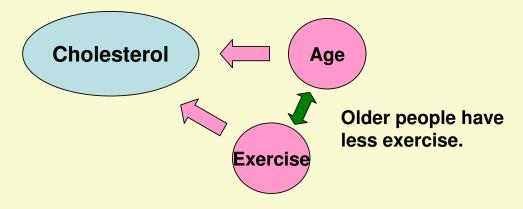
- If we look at each factor to the outcome at one time, it will not be realistic.
- We should look at the relationship of these factors to the outcome at the same time.

Basic Theory



When we look at the relation of these factors (explanatory variables) to the outcome at the same time,

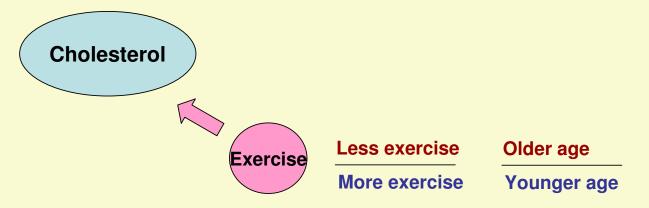
- We will obtain the "<u>independent effect</u>" of explanatory variables to outcome.
- We can also study the "<u>interaction</u>" (IA) between independent variables (Synergistic/Antagonistic IA).





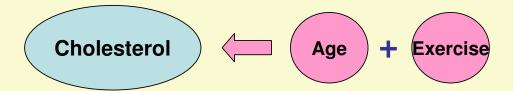
Effect that we found here, is not only the pure effect of age, but also additional effect from exercise. (Older people have less exercise – so that the relationship of being higher cholesterol among older age is exaggerated by the effect of less exercise).

In this example, the result (of the relationship between cholesterol and age) is confounded by exercise.

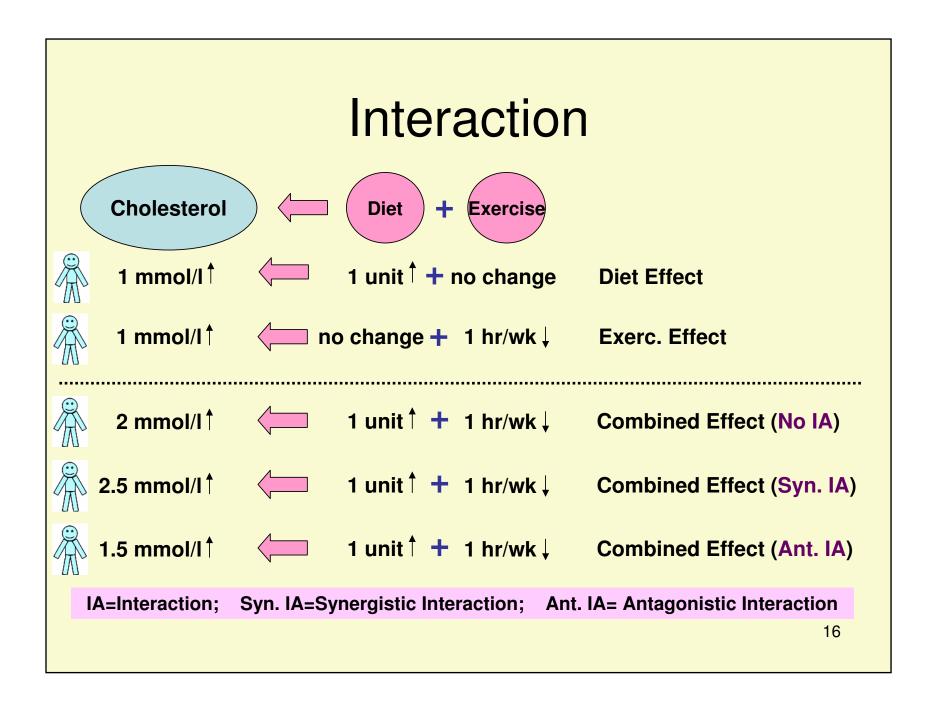


Effect that we found here, is not only the pure effect of exercise, but also additional effect from age. (Less exercise people are older people – so that the relationship of being higher cholesterol among less exercise people is exaggerated by the effect of older age).

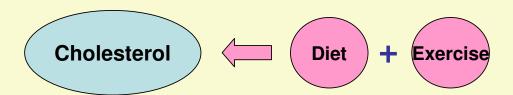
In this example, the result (of the relationship between cholesterol and exercise) is confounded by age.



But, if we subject them together in the regression model, the confounding effects were eliminated and we can get the "independent effect" of each independent variable.



Interaction



Example:

Those with higher cholesterol diet, their cholesterol level will be higher.

Say, 1 unit more in cholesterol diet score, cholesterol level will be higher for 1 mmol/L.

Those with less exercise, their cholesterol level will be higher.

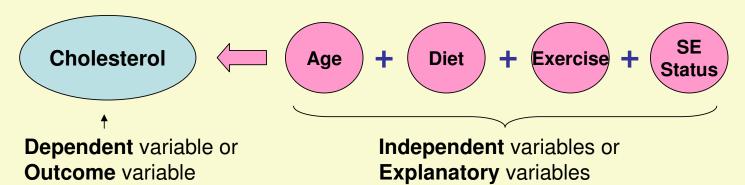
Say, 1 hour less exercise in a week, cholesterol will higher for 1 mmol/L.

It means ... for 1 unit more in cholesterol diet AND 1 hour less exercise in a week, there should be an increase in cholesterol for 2 mmol/L.

If it doesn't happen as above, but it increases for 3 mmol/L, it means that there is a <u>synergistic interaction</u> between diet and exercise.

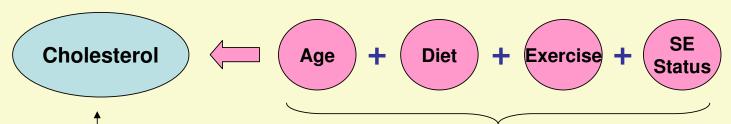
If it doesn't happen as above, but it increases only for 1.5 mmol/L, it means that there is an <u>antagonistic interaction</u> between diet and exercise.

Basic Theory



- This analysis is used for
 - Exploring associated / influencing / risk factors to outcome (exploratory study)
 - Developing prediction model (exploratory study)
 - Confirming a specific relationship (confirmatory study)

Basic Theory



Dependent variable or **Outcome** variable

Numerical

Independent variables or Explanatory variables

Numerical (MLR analysis)
Categorical or Mixed (GLR analysis)

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \dots + \beta_n X_n$$

- If the dependent variable is numerical and independent variables are numerical, it will be called <u>Multiple Linear Regression</u> (MLR) analysis.
- MLR can be with categorical independent variables, but special name is given as <u>General Linear Regression</u> analysis.

Steps in Handling MLR

Step 1: Data exploration (Descriptive Statistics)

Step 2: Scatter plots and Simple Linear Regression (SLR)

Step 3: Variable selection

⇒ Preliminary main-effect model

Step 4: Checking interaction & multicollinearity^a

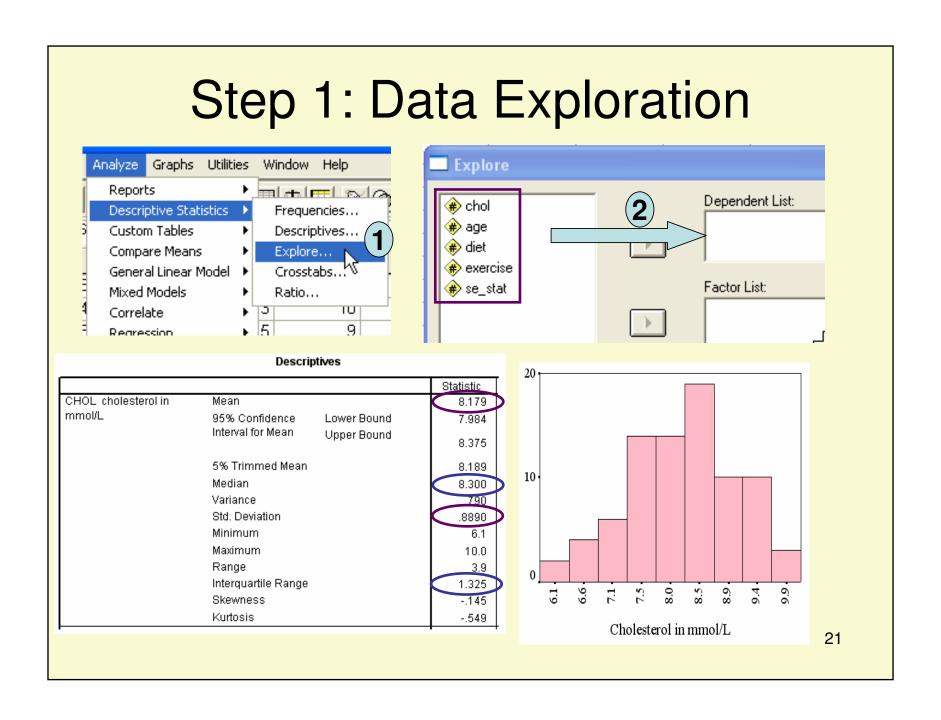
⇒ Preliminary final model

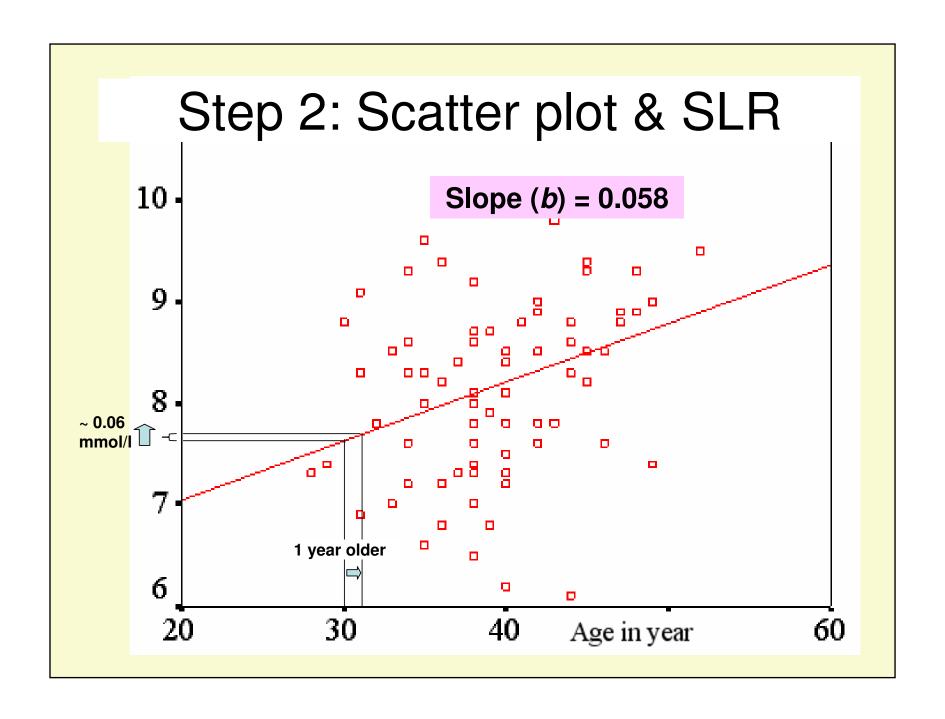
Step 5: Checking model assumptions^a

⇒ Final model

Step 6: Interpretation & data presentation

a need remedial measures if problems are detected





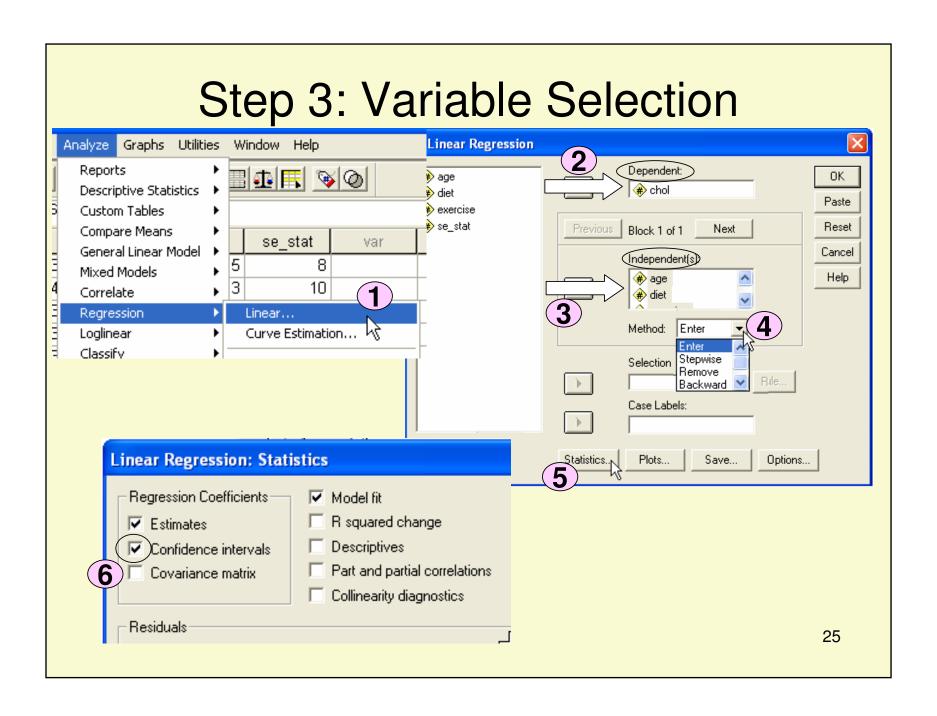
Step 2: Scatter plot & SLR

Independent Variable	SLRa						
Independent Variable	b (95%Cl)	<i>P</i> value					
Age (year) Duration of exercise (hrs/wk) Diet inventory score Socio-economic index	0.06 (0.02, 0.09) -0.62 (-0.79, -0.46) 0.45 (0.30, 0.61) 0.21 (0.17, 0.25)	.002 <.001 <.001 <.001					

Simple linear regression (Outcome as Cholesterol mmol/L)
 b = crude regression coefficient

Step 3: Variable Selection

- Automatic / Manual methods
 - Forward method
 - Backward method
 - Stepwise method
 - All possible models method
- Nowadays, as computers are faster, automatic methods can be done easily.
- In SPSS, forward, backward and stepwise can be used.
- All 3 methods should be used for this step. Take the biggest model (all selected variables should be significant) for further analysis.



Result: Stepwise

	•	-	t. Ott		- 0 0			
			Coefficients ^a					
	Unstandardized Coefficients		Standardized Coefficients			95% Confidence Interval for B		
Model	В	Std. Error	Beta	t	Sig.	Lower Bound	Upper Bound	
1 (Constant)	5.845	.241		24.264	.000	5.366	6.325	
socio-econmic status (index)	.211	.021	.748	10.069	.000	.170	.253	
2 (Constant)	7.660	.587		13.048	.000	6.492	8.829	
socio-econmic status (index)	.158	.025	.559	6.235	.000	.108	.208	
duration of exercise (hours/week)	288	.086	301	-3.352	.001	460	117	
3 (Constant)	8.593	.633		13.574	.000	7.332	9.853	
socio-econmic status (index)	1.369E-02	.052	.048	.262	.794	090	.118	
duration of exercise (hours/week)	550	.117	574	-4.688	.000	784	317	
diet inventory (higher the score, higher	.372	.120	.451	3.106	.003	.134	.610	
Y = 1	$\beta_0 + \beta_1$	$X_1 + \beta_2$	$_{2}X_{2}+\beta_{3}X_{3}$	K ₃ +	+ β _r	X _n	.516	
(hours/week)	576	.004	001	-8.057	.000	703	450	
Cholesterol =	7.297 -	- (.540*	exercise)	+ (.394	l*diet)	+ (.033*a	ige) 509	
5 (Constant)	7.297	.620		11.763	.000	6.062	8.532	
duration of exercise	540	.062	563	-8.702	.000	663	416	
(hours/week)	1					II .		
(hours/week) diet inventory (higher the score, higher cholesterol content)	.394	.052	.478	<i>P</i> va 7.527	.000	.290	.498	

Result: Forward

Coefficients^a

I I		Unstand Coeffi		Standardized Coefficients			95% Confidence Interval for	
Model		В	Std. Error	Beta	t	Sig.	Lower Bound	Upper Bound
1	(Constant)	5.845	.241		24.264	.000	5.366	6.325
	socio-econmic status (index)	.211	.021	.748	10.069	.000	.170	.253
2	(Constant)	7.660	.587		13.048	.000	6.492	8.829
	socio-econmic status (index)	.158	.025	.559	6.235	.000	.108	.208
	duration of exercise (hours/week)	288	.086	301	-3.352	.001	460	117
3	(Constant)	8.593	.633		13.574	.000	7.332	9.853
	socio-econmic status (index)	1.369E-02	.052	.048	.262	.794	090	.118
	duration of exercise (hours/week)	550	.117	574	-4.688	.000	784	317
	diet inventory (higher the score, higher cholesterol content)	.372	.120	.451	3.106	.003	.134	.610
4	(Constant)	7.151	.783		9.131	.000	5.591	8.710
	socio-econmic status (index)	1.545E-02	.050	.055	.309	.758	084	.115
	duration of exercise (hours/week)	511	.113	533	-4.519	lues	736	286
	diet inventory (higher the score, higher cholesterol content)	.363	.114	.440	3.168	.002	.135	.591
	age in year	3.285E-02	.011	.188	2.900	.005	.010	.055

a. Dependent Variable: cholesterol in mmol/L

Result: Backward

Coefficients^a

		Unstandardized Coefficients		Standardized Coefficients			95% Confidence Interval for B	
Model		В	Std. Error	Beta	t	Sig.	Lower Bound	Upper Bound
1	(Constant)	7.151	.783		9.131	.000	5.591	8.710
	age in year	3.285E-02	.011	.188	2.900	.005	.010	.055
	diet inventory (higher the score, higher cholesterol content)	.363	.114	.440	3.168	.002	.135	.591
	duration of exercise (hours/week)	511	.113	533	-4.519	.000	736	286
	socio-econmic status (index)	1.545E-02	.050	.055	.309	.758	084	.115
2	(Constant)	7.297	.620		11.763	.000	6.062	8.532
	age in year	3.281E-02	.011	.188	2.914	.005	.010	.055
	diet inventory (higher the score, higher cholesterol content)	.394	.052	.478	7.527 P vá	.000 ues	.290	.498
	duration of exercise (hours/week)	540	.062	563	-8.702	.000	663	416

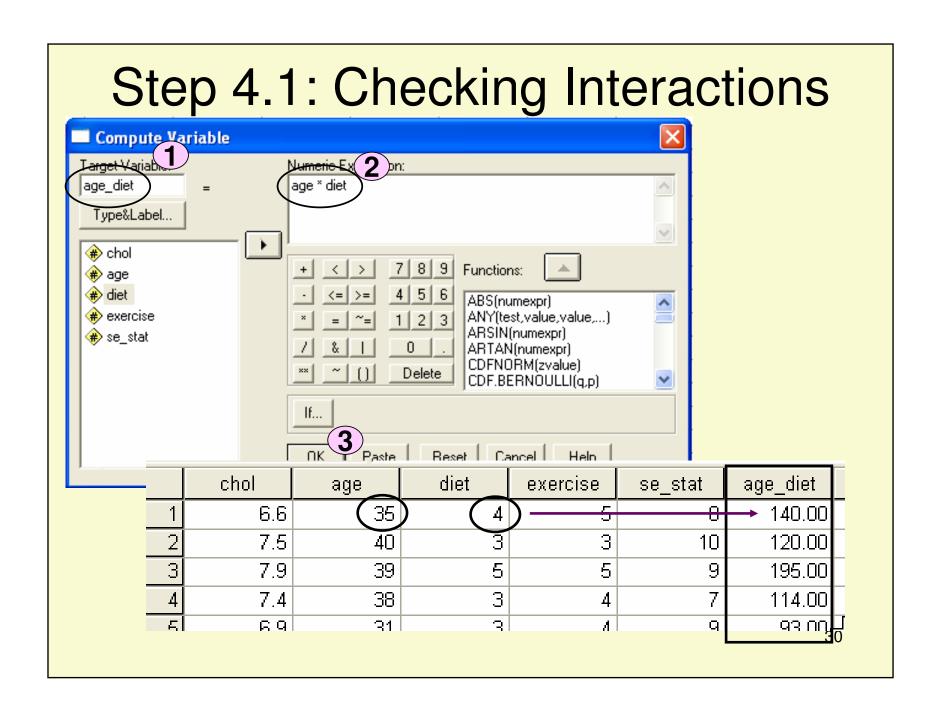
a. Dependent Variable: cholesterol in mmol/L

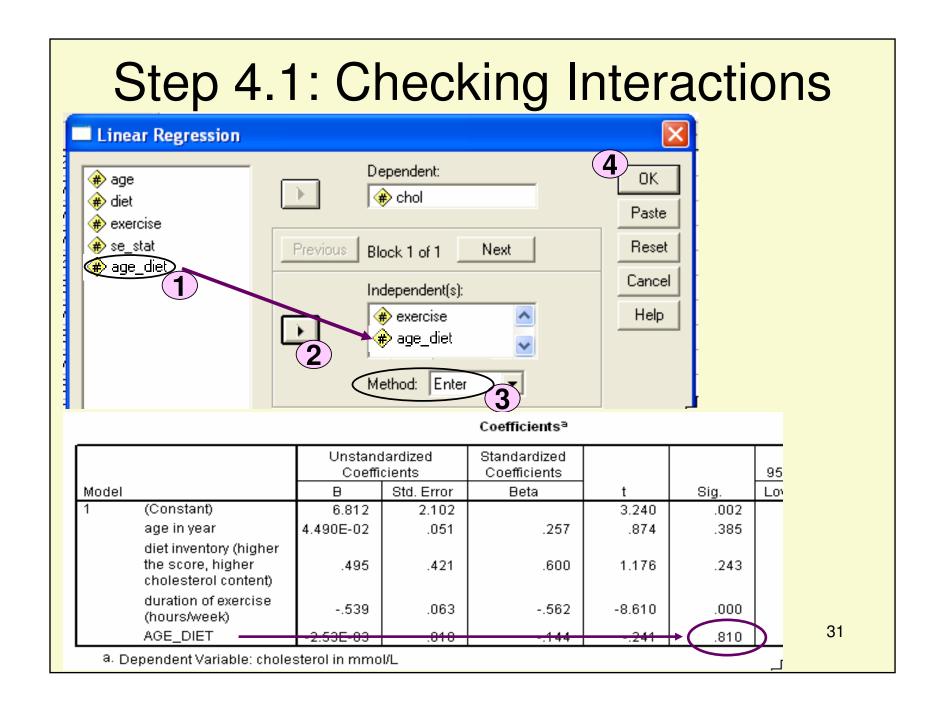
From the above 3 automatic procedures, we obtain the <u>preliminary main</u> <u>effect model</u> as:

Cholesterol = $7.297 - (.540 \times exercise) + (.394 \times diet) + (.033 \times age)$

Step 4.1: Checking Interactions

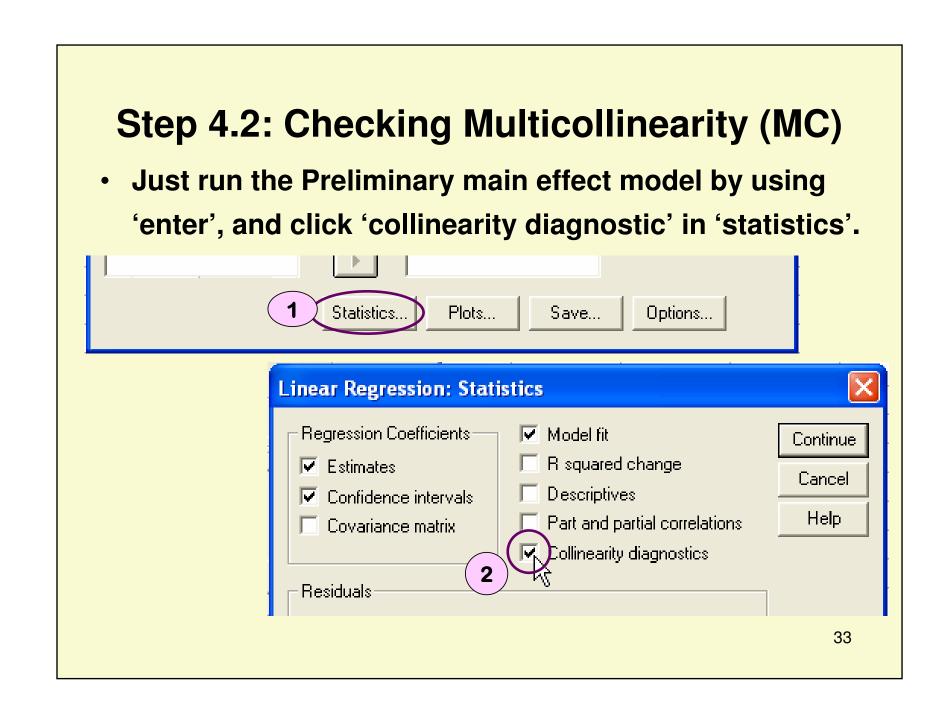
- All possible 2-ways interactions (ex*diet; ex*age; diet*age) are checked.
 - Interaction terms are calculated (Transform⇒Compute).
 - Add into the model as additional independent variable.
 - Run the model using 'enter'.
 - If an interaction term is significant (*P*<.05), it means that there is an interaction between the 2 variables. And therefore, the appropriate model is the main effect variables plus the significant interaction term.
 - Check one interaction term at a time.
- In our example data, all 3 interaction terms are not significant. It means that no interaction term should be added.





Step 4.2: Checking Multicollinearity (MC)

- If the independent variables are highly correlated, the regression model is said to be "statistically not stable".
 - P values of the involved variables are considerably larger (than what it should be).
 - The width of 95% CI of the regression coefficients are larger.
 - Appropriate variables may be rejected wrongly.
 - Therefore, statistically, it is said that 'the model is not stable'.
- We have to check the obtained model whether this kind of problem (MC) exists or not.



Step 4.2: Checking Multicollinearity (MC)

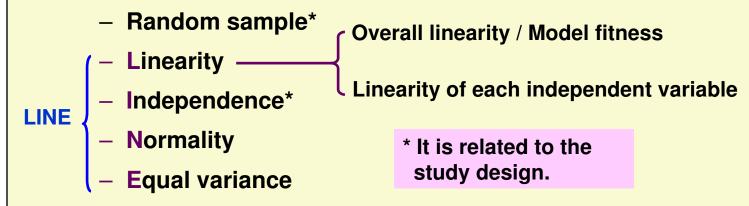
 Just run the Preliminary main effect model by using 'enter', and click 'collinearity diagnostic' in 'statistics'.

		Unstandar Coefficis		95% Confidenc	ce Interval for B	Collinearity <u>Statistics</u>				
Model		В	8	Lower Bound	Upper Bound	Tolerance	VIF			
1	(Constant)	7.297	70	6.062	8.532					
	age in year	3.281E-02	15	.010	.055	.957	1.045			
	diet inventory (higher the score, higher cholesterol content)	.394	10	.290	.498	.988	1.012			
	duration of exercise (hours/week)	540	Û	663	416	.950	1.053			
a Denendent Variable: cholesterol in mmol/L										

Look at VIF (Variance-inflation factor). VIF measures the extent of multicollinearity problem. If VIF is more than 10, the problem needs remedial measures. Consult a statistician.

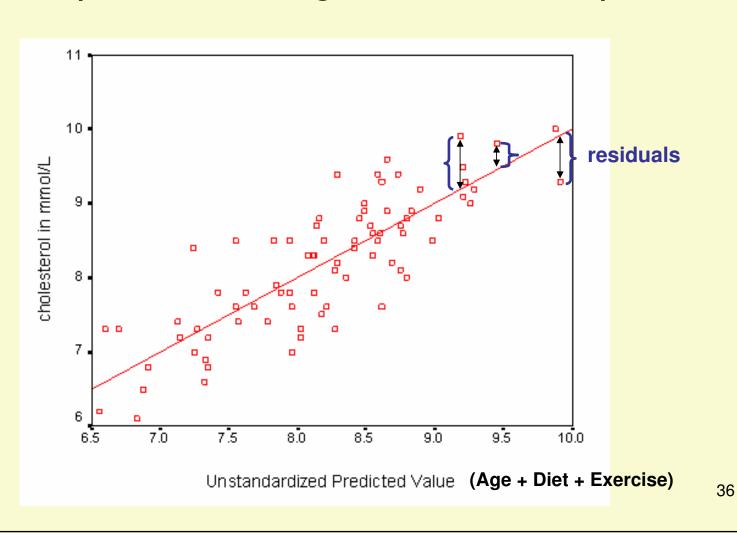
Step 5: Checking model assumptions

Assumptions are ...

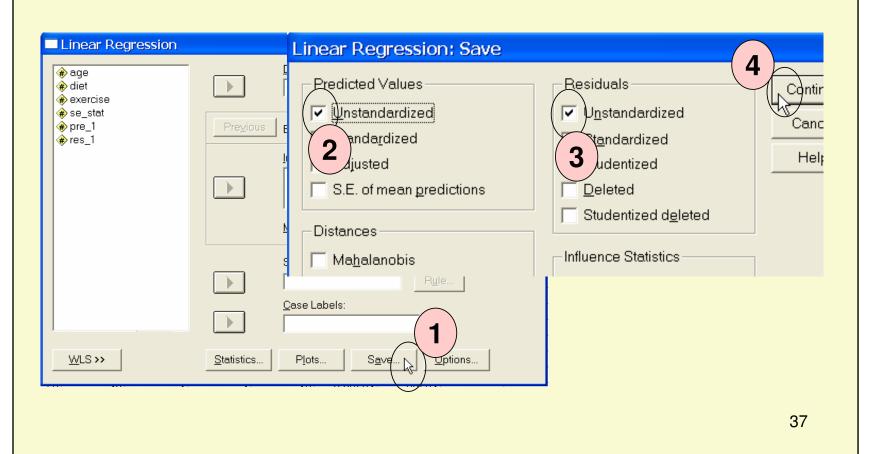


- All are performed by using residual plots.
- <u>A residual</u> means "observed value" minus "predicted value" of dependent variable.





Steps to calculate residuals ...



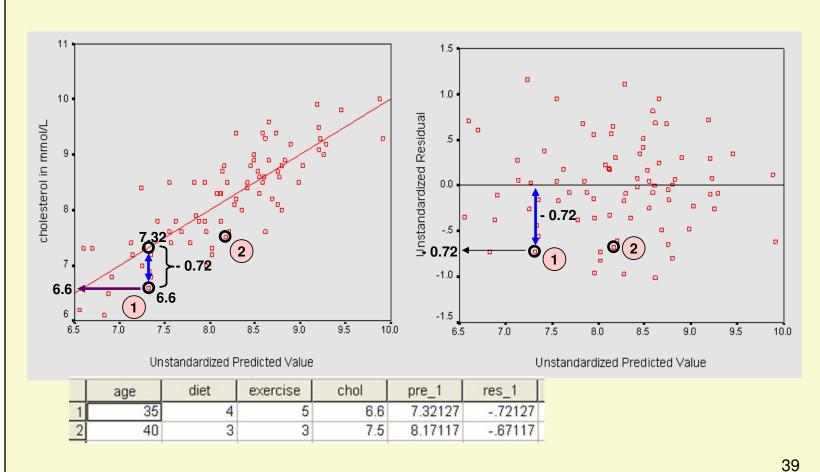
	age	diet	exercise	chol	pre_1	res_1		
1	35	4	5	6.6	7.32127	72127		
2	40	3	3	7.5	8.17117	67117		
3	39	5	5	7.9	7.84649	.05351		
4	38	3	4	7.4	7.56563	16563		
5	31	3	4	6.9	7.33597	43597		
6	31	5	4	8.3	8.12397	.17603		
- 7	38	6	5	7.6	8.20768	60768		
8	48	4	3	8.9	8.82763	.07237		
9	39	5	5	7.9	7.84649	.05351		
10	38	7	5	8.6	8.60168	00168		
		_	_					

Chol (pred.) = 7.297 – (.540*exercise) + (.394*diet) + (.033*age)

Chol (pred.) = 7.297 – (.540*5) + (.394*4) + (.033*35)

Chol (*pred.*) = 7.32

Residual = Chol (observed) - Chol (pred.) = 6.6 - 7.32 = -.72



- Assumptions are ...
 - Random sample*
 - Linearity (Overall model linearity/fitness; linearity of each numerical independent variable)

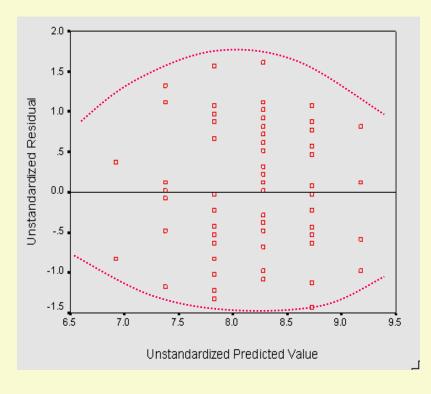
LINE

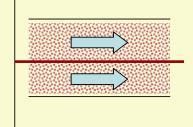
- Independence*
- Normality
- Equal variance

* It is	related	to	the
stud	y desig	ın.	

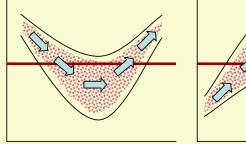
	3 types of residual plot	Assumptions
1	Scatter plot: Residuals vs Predicted	Linearity – overall fitness
		Equal variance of residuals
2	. Histogram of residuals	Normality of residuals
3	Scatter plot: Residuals vs each indep.	Linearity of each indep. Var.
	var. (numerical)	numerical

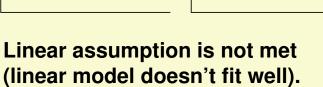
OVERALL LINEARITY

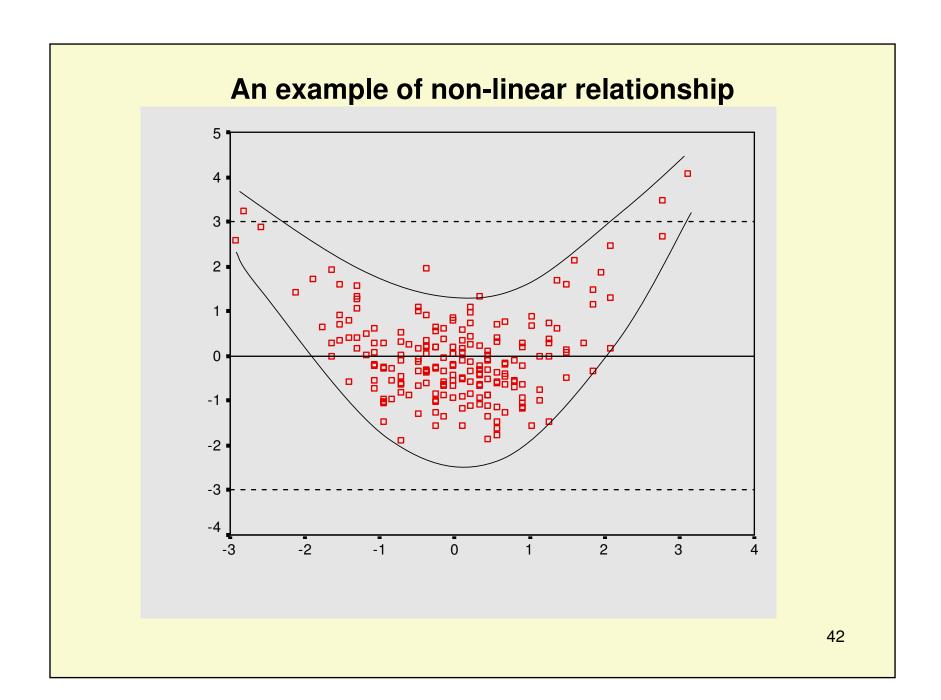


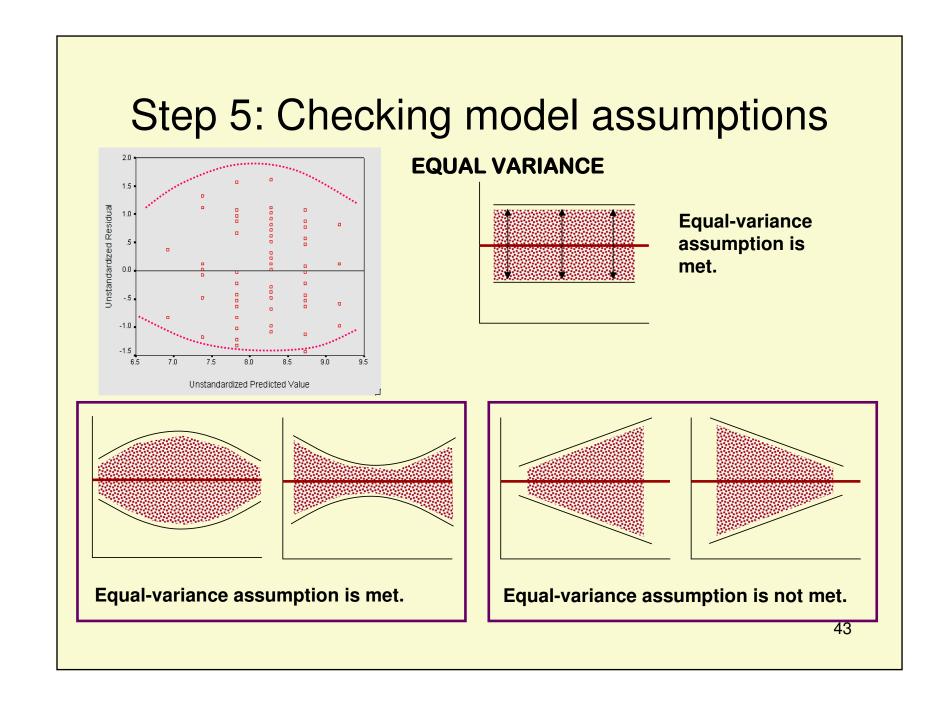


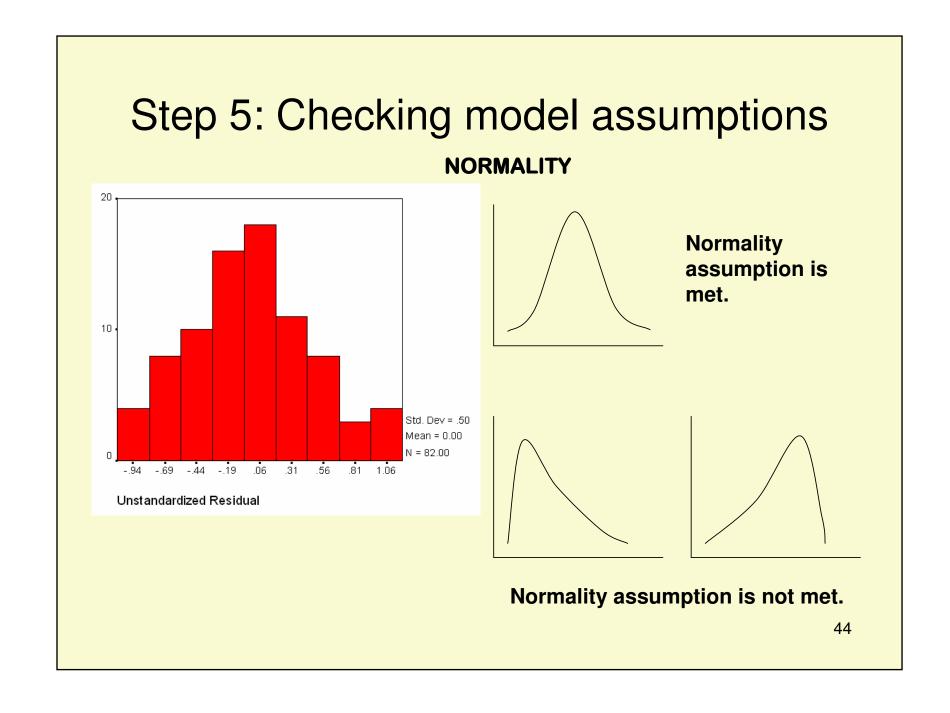
Linearity assumption is met (linear model fits well).



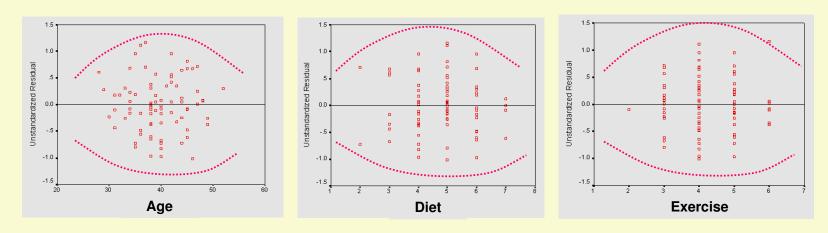








Checking linearity of each numerical independent variables



- If there is no relationship between residuals and a numerical independent variable, the relationship of the independent variable with the outcome is linear.
- In above example, all are considered linear relationship.
- If not linear, we may be need to transform data (see statistician).

Step 6: Presentation/Interpretation

Table 4: Factors associated with blood cholesterol level (mmol/L) among the study population (n=82)

Variables	SLR ^a	MLR ^b			
variables	b ^c (95% CI) <i>P</i> value	<i>Adj.b</i> ^d (95%Cl) <i>t-</i> stat. <i>P</i> value			
Age (year) Duration of exercise (hrs/wk)	0.06 (0.02, 0.09) .002 -0.62 (-0.79,-0.46) <.001	0.03 (0.01, 0.06) 2.91 .005 -0.54 (-0.66, -0.42) -8.70 <.001			
Diet inventory score Socio-economic index	0.45 (0.30, 0.61) <.001 0.21 (0.17, 0.25) <.001	0.39 (0.29, 0.50) 7.53 <.001			

^a Simple linear regression

There is no interaction between independent variables, and multicollinearity problem)

• For prediction study, it is essential to report the final model (equation).

Chol
$$(pred.) = 7.30 + (.03*age) - (.54*exercise) + (.39*diet)$$

^b Multiple linear regression (R²=0.69; The model reasonably fits well; Model assumptions are met;

^c Crude regression coefficient

d Adjusted regression coefficient

Step 6: Presentation/Interpretation

- There is a significant linear relationship between age and cholesterol level (*P*=.005). Those with 10 years older have cholesterol level higher for 0.3 mmol/L (95% CI: 0.1, 0.6 mmol/L).
- There is a significant linear relationship between duration of exercise and cholesterol level (*P*<.001). Those having 1 hr/wk less exercise have cholesterol level higher for 0.54 mmol/L (95% CI: 0.66, 0.42 mmol/L).
- There is a significant linear relationship between diet inventory index and cholesterol level (*P*<.001). Those with 1 unit more in the index, have cholesterol level higher for 0.39 mmol/L (95% CI: 0.29, 0.50 mmol/L).
- With the 3 significant variables, the model explains 69% of variation of the blood cholesterol level in the study sample. (R^2 =0.69)

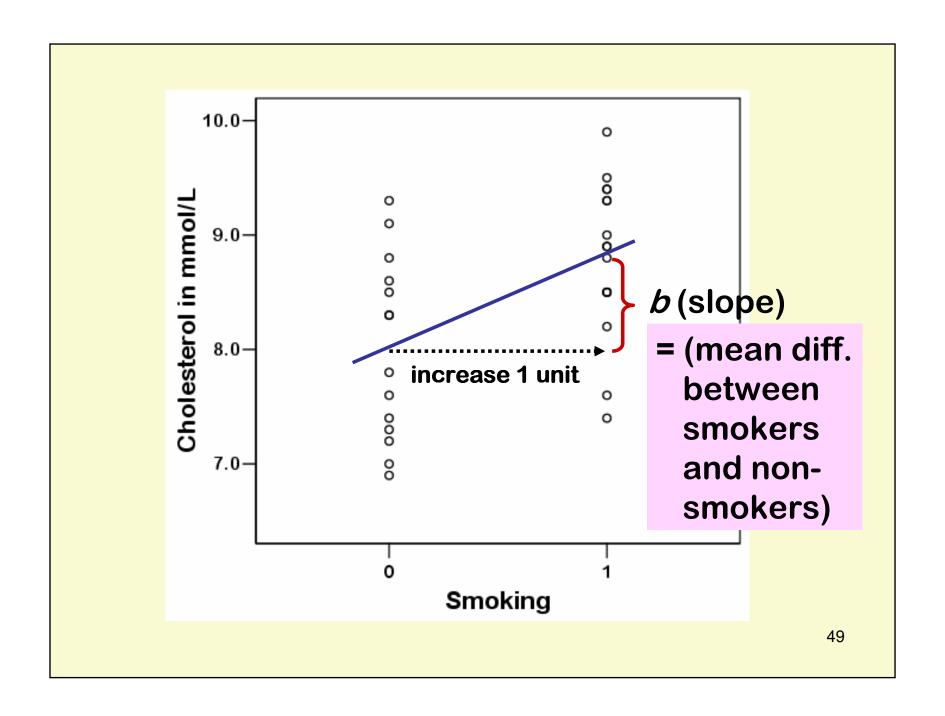
Cautions:

It should be coded (0, 1) for dichotomous variable.

Example 1: sex (male=1, female=0)
It means we are comparing male against female
(female as reference)

Example 2: smoking (smokers=1, non-smoker=0) It means we are comparing smokers against non-smoker (non-smoker as reference)

Say, outcome is cholesterol, smoking as independent var., and we got b=2.0. It means smokers will have cholesterol level higher than non-smokers for 2.0 mmol/L.



Cautions:

If you have more than 2 categories in categorical variable, we have to create **Dummy Variables**.

Example: Education level (no education=0; primary school level=1; secondary level=2)

Then, we need to create 2 dummy variables: (e.g. edu1 & edu2)

	edu1	edu2
No edu. →	0 5	0 1
Primary edu. →	1	0
Secondary edu.→	0	1

Here, reference is 'no education',

educa1 is comparing 'primary' against 'no edu', and

educa2 is comparing 'secondary' against 'no edu'.

Example 2: Education level (no education=0;

primary=1; secondary=2; tertiary=3)

Then, we need to create 3 dummy variables: (e.g. edu1 & edu2 & edu3)

	edu1	edu2	edu3
No edu. →	0 5	0 🦹	0 📉
Primary edu. →	1	0	0
Secondary edu.→	0	1	0
Tertiary edu. →	0	0	1

Cautions:

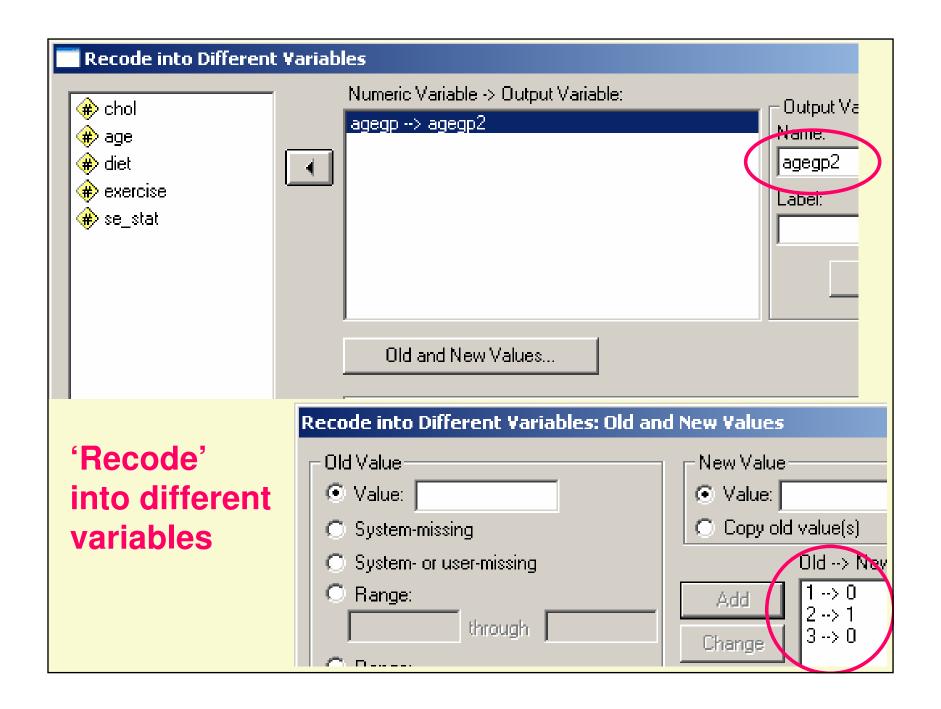
If you have more than 2 categories in categorical variable, we have to create <u>Dummy Variables</u>.

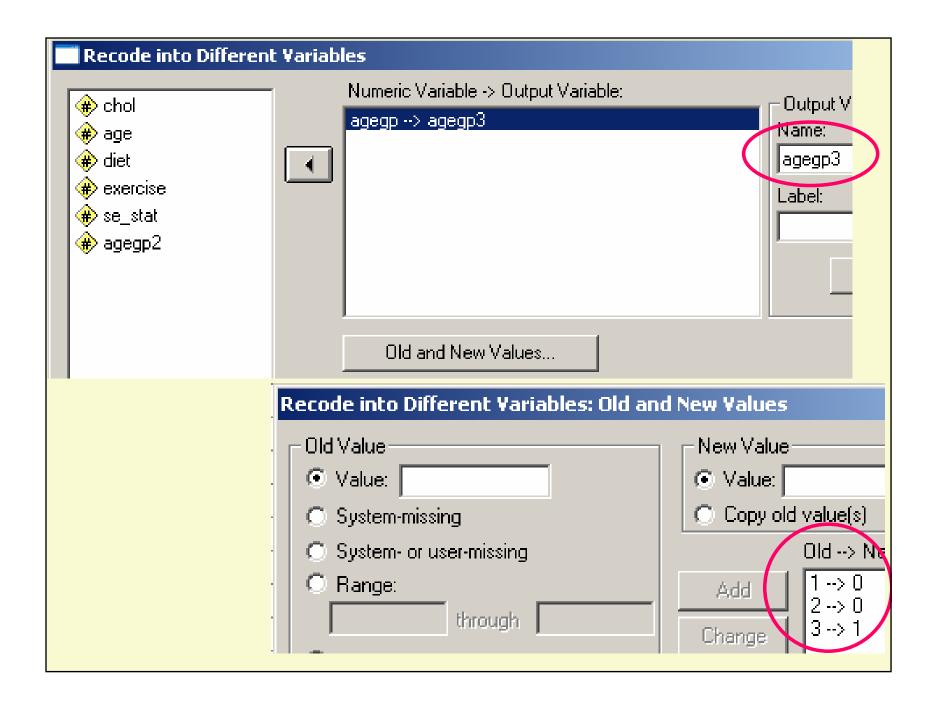
Example: Agegp: Age (<35)=1; Age (35-44)=2; Age (>=45)=3

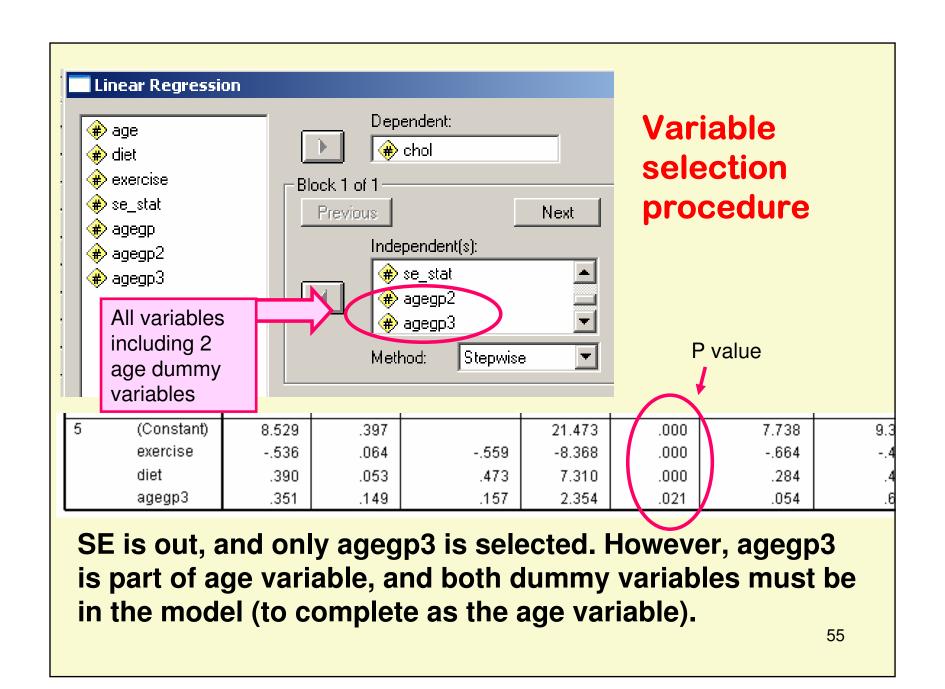
Then, we need to create 2 dummy variables: (e.g. agegp2 & agegp3)

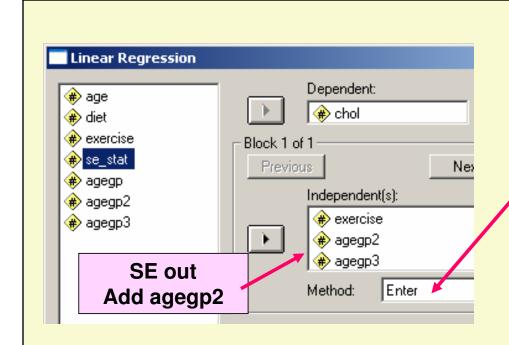


Here, reference is 'young', agegp2 is comparing 'older' against 'young', and agegp3 is comparing 'eldest' against 'young'.









We have to force agegp2 to complete as the age-group variable.

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		Unstandardized Coefficients		Standardized Coefficients			_!
Model		B	Std. Error	Beta	t	Sig.	L
1	(Constant)	8.445	.415		20.373	.000	Г
	diet	.391	.054	.474	7.302	.000	
	exercise	539	.064	562	-8.369	.000	
	agegp2	.114	.155	.062	.737	.464	
	agegp3	.439	.192	.197	2.291	.025	

a. Dependent Variable: chol

How to interpret 'b' of categorical variable?

Coefficients^a

		Unstandardized Coefficients		Standardized Coefficients			_ !
Model		В	Std. Error	Beta	t	Sig.	L
1	(Constant)	8.445	.415		20.373	.000	Γ
	diet	.391	.054	.474	7.302	.000	П
	exercise	539	.064	562	-8.369	.000	
	agegp2	.114	.155	.062	.737	.464	
	agegp3	.439	.192	.197	2.291	.025	

a. Dependent Variable: chol

There is no significant difference in cholesterol level between older age-group (35-44) and young group (<35) (*P*=0.464).

The eldest group (>=45) have significantly higher cholesterol level than the young group (<35) (P=0.025).

The eldest group (>=45) have 0.44 mmol/L higher cholesterol level than the young group (<35) (95% CI: 0.06, 0.82 mmol/L). 57