Classical Analysis of Rates

Phillip Ayieko

Objectives

- Explain and compare the rates between two treatment groups
- Understand weaknesses of "classical analysis" of rates - move to Poisson model

Rate

- Number of events per unit time in a given number of people
- For any group of people it is calculated as:

$$\frac{numberofobservedevents}{totaltimeforwhichpeoplewereobserved}$$

- Difference between risk and rate:
    - Risk: probability of experiencing an event
    - Rate: probability of experiencing an event per unit time

Rate

- Rates are applied to individuals
- They can only be measured from a group of participants
- Key assumption: events occur uniformly over time i.e. rate is constant
- Example: if 5 events occur in 100 men over 2 years, what is the event rate?
    - No. of events: 5
    - Total time of follow-up: $100 men x 2 years = 200 person - years$
    - $Eventrate = 5/200 = 0.025 events per person - year$

Rate Ratio

- Its the ratio of two rates
- Example: if 5 events occur in 100 men over 2 years, and 10 events occur in 100 women over 5 years, what is the rate ratio comparing event rates in men vs. women?
  - Event rate for men $= 5/200 = 0.025$ events per person-year
  - Event rate for women $= 10/500 = 0.02$ events per person
  - Rate ratio comparing men to women $= 0.025/0.02 = 1.25$
  - Interpretation: the event rate in men is 1.25 times that in women
- Length of follow-up is different for men and women, but rate ratio is a standardised measure for different lengths of follow-up. It has no units.

Rate ratio

- Comparing 2 groups with respect to exposure status:

$$RATEratio = \frac{RatioinExposed}{Rateinunexposed}$$

- Alternative ways of expressing this:
  - $Rateinexposed = rateinunexposedxrateratio$
  - $Log(rateinexposed) = log(rateinunexposed) + log(rateratio)$
- If there is more than one exposure factor:
  - $Log(rateinexposed) = log(rateinunexposed) + log(rateratioforexposure1) + log(rateratioforexposure2)$ e.t.c.
    - Which is the Poisson model
- A Poisson model is given by: $log(I) = Baseline + A + B$
- Exponentiate result to get the rate

Using R

- Using creditcards data - 'Classical' analysis of rates
  - Compute the rate of group 1 (income)
  - Compute the rate of group 2 (credit)
  - Compute the rate ratio = use of credit card given income
- With poisson model - The offset variable serves to normalize the fitted cell means per some space, grouping or time interval in order to model the rates.

```
##
## Call:
## glm(formula = CrCards ~ income + offset(lcases), family = p
##     data = data)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -1.6907  -0.9329  -0.5675   0.2186   2.1681
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept) -2.386586   0.399655  -5.972 2.35e-09 ***
## income       0.020758   0.005165   4.019 5.84e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 '
##
## (Dispersion parameter for poisson family taken to be 1)
##
```

Weaknesses of 'Classical' analysis of rates

- The assumption of constant rate ratio (ranges between -inf to inf) = not true
  - The rate ratio takes zero to infinity
- We use log transform as an alternative
  - exponentiate back the results
- Many simultaneous rates (combined effect of different exposures) cannot be analyzed "classically"
- Cant tell weather the variable is a significant variable

Poisson regression Model for rate data

- Assumptions for Poisson
  - Log of disease rate changes linearly with equal increment increases in exposure variable
  - **Change in the rate from combined effects of different exposures or risk factors are multiplicative**
  - At each level of covariates the number of cases has variance equal to mean
  - Observations are independent

THANKS