# FREE SPAN ANALYSIS FOR SUBSEA GAS EXPORT PIPELINE

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This Mathcad sheet calculates the maximum allowable free span lengths along the pipeline route for onset of in-line and cross flow vortex induced vibrations (VIVs), as well as Euler buckling. The methodology is that presented in DnV RP-F105.
This Mathcad sheet also calculates the maximum allowable free span length to avoid local buckling. The method is based on limit state design code DNVGL ST-F101 and is driven by load controlled criteria.

The maximum allowable free span lengths are determined for 4 load cases:

1. Installed with the pipe empty (defined as depressurised, pipe empty)

2. Installation with the pipe flooded (defined as depressurised , pipe flooded)

3. Hydrotest prior to operation (defined as system pressure test where the pipe is flooded);

4. Pipe in operation (under design pressure)

For load case 4 (operation), the pipe is considered operating on seabed (not trenched or buried).The wave water particle kinematics are determined based on the linear Airy wave theory. The Morison equation is used to derive the hydrodynamic forces of drag and inertia.For determining the maximum allowable free span lengths to avoid in-line and cross-flow VIVs, the significant wave height along with the wave peak period and associated seabed current are used. For determining the maximum allowable free span length to avoid local buckling, the maximum wave height along with the wave maximum period and associated seabed current are used.

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## 1. INPUT DATA

Enter only data highlighted. Other data have already been defined or automatically calculated below.

Define units: $C := 1 \cdot K$ $\quad g := 9.81 \cdot m \cdot s^{-2}$ $\quad °C := 1 \cdot C$

### 1.1 Pipe Data

Pipe outer diameter: $OD := 356.1 \cdot mm$

Pipe wall thickness: $wt := 19.3 \cdot mm$

Specify material type:
(Select from the list on the right the pipe material: CMn is for carbon manganese (carbon steel );
DSS is for 22Cr duplex and 25Cr super duplex
Note: This is used for temperature derating, see below)

Material :=
- CMn
- DSS

Fabrication process:
(Select from the list the pipe fabrication process: "Seamless", "UO & TRB & ERW", or "UOE")

Process :=
- Seamless
- UO & TRB & ERW
- UOE

Pipe Young's modulus of elasticity:

$$E := \begin{cases} 2.07 \cdot 10^5 \cdot MPa & \text{if Material = "CMn"} \\ 2.00 \cdot 10^5 \cdot MPa & \text{if Material = "DSS"} \end{cases}$$

$$E = 207000 \cdot MPa$$

Pipe material density:

$$\rho_{st} := \begin{cases} 7850 \cdot \frac{kg}{m^3} & \text{if Material = "CMn"} \\ 7700 \cdot \frac{kg}{m^3} & \text{if Material = "DSS"} \end{cases}$$

$$\rho_{st} = 7850 \frac{kg}{m^3}$$

Corrosion and insulation coating thickness: $t_{coat} := 3 \cdot mm$

Coating equivalent density: $\rho_{coat} := 944 \cdot \frac{kg}{m^3}$

Note: For multi-layer coating, the equivalent coating density can be approximated by

$$\frac{coat_{thick1} \cdot density_1 + coat_{thick2} \cdot density_2 + .......}{thick_{totalsum}}$$

Pipe specified minimum yield strength: $SMYS := 448 \cdot MPa$

Pipe specified minimum tensile strength: $SMTS := 530.MPa$

Pipe temperature (for installation)

$T_{inst} := 4 \cdot °C$

Pipe design temperature (for operation):

$T_{oper} := 78 \cdot °C$

Seawater ambient temperature at seabed

$T_{amb} := 4 \cdot °C$

Pipe material Poisson's ratio

$$\nu := \begin{cases} 0.3 & \text{if } Material = \text{"CMn"} \\ 0.32 & \text{if } Material = \text{"DSS"} \end{cases}$$

$\nu = 0.3$

Pipe coefficient of thermal expansion:

$$\alpha_{pipe} := \begin{cases} 1.17 \cdot 10^{-5} \cdot C^{-1} & \text{if } Material = \text{"CMn"} \\ 1.13 \cdot 10^{-5} \cdot C^{-1} & \text{if } Material = \text{"DSS"} \end{cases}$$

$\alpha_{pipe} = 1.17 \times 10^{-5} \cdot C^{-1}$

Is material subject to supplementary requirement U?
Select "Yes" or "No".

Supp_req_U := "No"

Note: For design purpose, conservatively select "No"

Derating for yield strength & tensile strength (DNVGLSTF101)

**Note:** For installation where temperature is ambient, the material strengths is not derated. Derating the material should only apply for operation load case where operating temperature is higher than 50 deg C for carbon steel and 20 deg C for duplex steel

Derating for yield and tensile strength:

For operation:

$$f_{yCMn\_oper} := \begin{cases} 0 & \text{if } T_{oper} \le 50 \cdot °C \land Material = \text{"CMn"} \\ \left[\left(\dfrac{30 \cdot MPa}{50 \cdot °C}\right) \cdot \left(T_{oper} - 50 \cdot °C\right)\right] & \text{if } 50 \cdot °C < T_{oper} < 100 \cdot °C \land Material = \text{"CMn"} \\ \left[30 \cdot MPa + \left(T_{oper} - 100 \cdot °C\right) \cdot \left(\dfrac{20 \cdot MPa}{50 \cdot °C}\right)\right] & \text{otherwise} \end{cases}$$

$f_{yCMn\_oper} = 16.8 \cdot MPa$

$$f_{yDSS\_oper} := \begin{cases} 0 & \text{if } T_{oper} \le 20 \cdot °C \\ \left[\dfrac{40 \cdot MPa}{30 \cdot °C}\left(T_{oper} - 20 \cdot °C\right)\right] & \text{if } 20 \cdot °C < T_{oper} < 50 \cdot °C \\ \left[40 \cdot MPa + \left(T_{oper} - 50 \cdot °C\right) \cdot \dfrac{MPa}{°C}\right] & \text{otherwise} \end{cases}$$

$f_{yDSS\_oper} = 68 \cdot MPa$

For installation: $f_{yCMn\_inst} := 0$ $f_{yDSS\_inst} := 0$

## 1 .2 Environmental Data

Maximum water depth

$d_{max} := 1457 \cdot m$

1-year negative storm surge:

$Surge_{neg} := -0 \cdot m$

Min Water depth at LAT: $LAT := 1400 \cdot m$

Extreme minimum water depth: $d_{min} := LAT + Surge_{neg}$ $\quad d_{min} = 1400\,m$

Density of sea water $\rho_{water} := 1025 \cdot \frac{kg}{m^3}$

sea water kinematic viscosity
(at 4degC) $\nu_k := 1.5 \cdot 10^{-6} \cdot \frac{m^2}{sec}$

**Note:** The significant wave height and wave peak period are used to derive the maximum span lengths to prevent in-line and cross-flow vortex induced vibrations. The maximum wave height and wave maximum period are used to derive the maximum allowable span length to avoid local buckling.

Design wave for installation phase

Main swell 1-year significant wave height $H_{1s\_swell1} := 2.60 \cdot m$

Second swell 1-year significant wave height $H_{1s\_swell2} := 1.15 \cdot m$

Wind sea 1-year significant wave height $H_{1s\_wind} := 2.05 \cdot m$

1-year equivalent significant wave height $H_{s\_inst} := \sqrt{(H_{1s\_swell1})^2 + (H_{1s\_swell2})^2 + (H_{1s\_wind})^2}$

$$H_{s\_inst} = 3.505\,m$$

**Note:** The use of the 1-year value of the second swell significant wave height is conservative. For joint extreme conditions the 95% percentile value of the second swell significant wave height is recommended to be used

1-year wave peak period $T_{p\_inst} := 15.9 \cdot sec$

1-year maximum wave height $H_{max\_inst} := 2 \cdot H_{s\_inst}$ $\quad H_{max\_inst} = 7.01\,m$

1-year maximum wave period $T_{max\_inst} := T_{p\_inst}$ $\quad T_{max\_inst} = 15.9\,s$

Design wave for operation phase

Main swell 100-year significant wave height $H_{100s\_swell1} := 3.60 \cdot m$

Second swell 100-year significant wave height $H_{100s\_swell2} := 1.55 \cdot m$

Wind sea 100-year significant wave height $H_{100s\_wind} := 2.75 \cdot m$

100-year equivalent significant wave height

$$H_{s\_oper} := \sqrt{(H_{100s\_swell1})^2 + (H_{100s\_swell2})^2 + (H_{100s\_wind})^2}$$

$$H_{s\_oper} = 4.788\,m$$

**Note:** The use of the 100-year value of the second swell significant wave height is conservative. For joint extreme conditions the 95% percentile value of the second swell significant wave height is recommended to be us.

100-year wave peak period: $T_{p\_oper} := 17.5 \cdot sec$

**Note**: Conservatively, the peak period associated with the main swell is considered for the equivalent significant wave height.

100-year maximum wave height: $H_{max\_oper} := 2 \cdot H_{s\_oper}$ $\quad H_{max\_oper} = 9.576\,m$

100-year maximum wave period $T_{max\_oper} := T_{p\_oper}$ $\quad T_{max\_oper} = 17.5\,s$

Design seabed condition for installation and operation case

**Note:** The seabed current is given at 9m above seabed. The 7th power law is used to calculate the current nearer seabed (1m).

1-year current at 9m above seabed: $u_{1cur\_9m} := 0.2 \cdot \frac{m}{sec}$

10-year current at 9m above seabed: $u_{10cur\_9m} := 0.26 \cdot \frac{m}{sec}$

Installation (1-year) and operation (10-year) current at 1m above seabed:

$$u(u_{cur}) := u_{cur} \cdot \left(\frac{1 \cdot m}{9 \cdot m}\right)^{\frac{1}{7}}$$

Installation current: $u_{cur\_inst} := u(u_{1cur\_9m})$ $\quad u_{cur\_inst} = 0.146\,\frac{m}{s}$

Operation current: $u_{cur\_oper} := u(u_{10cur\_9m})$ $\quad u_{cur\_oper} = 0.19\,\frac{m}{s}$

Steady near seabed current reference height: $Z_{ref} := 1 \cdot m$

**Note:** The hydrodynamic inertia coefficient is defined below (after determination of added mass coefficient). The hydrodynamic drag coefficient is defined below as function of Reynolds number:

## 1.3 Installation, Hydrotest and Operation Data

Content density (pipe in operation) $\rho_{oper} := 200 \cdot \frac{kg}{m^3}$

Design pressure at MSL: $P_{msl} := 215 \cdot bar$

Design pressure at seabed: $P_d := P_{msl} + \rho_{oper} \cdot g \cdot d_{max}$ $P_d = 243.586 \cdot bar$

Internal pressure when installed empty: $P_{inst} := 0.bar$

Internal pressure when installed flooded: $P_{flood} := \rho_{water} \cdot g \cdot d_{max}$ $P_{flood} = 146.505 \cdot bar$

Internal pressure during hydrotest: $P_{hydrotest} := 1.10 \cdot 1.05 \cdot P_{msl} + \rho_{oper} \cdot g \cdot d_{max}$

$P_{hydrotest} = 276.911 \cdot bar$

**Note:** The coefficient shown above (1.10*1.05) for the strength hydrotest pressure are from the pipeline design DNVGLSTF-F101 .Conservatively, the maximum water depth is used to derive the hydrotest pressure and pressure in the pipeline when flooded and unpressurised.

Effective residual lay tension: $H_{lay} := 0.kN$
(from installation)

**Note:** Ignoring residual lay tension after installation is conservative

## 1.4 Soil & Geotechnical Data

Seabed soil roughness parameter: $Z_o := 5 \cdot 10^{-6} \cdot m$ (silt,CLAY)
(DNVRPF105)

Dynamic vertical stiffness $C_V := 1400 \cdot \frac{kN}{m^{2.5}}$ (soft,CLAY)
(DNVRPF105)

Dynamic lateral stiffness $C_L := 1200 \cdot \frac{kN}{m^{2.5}}$ (soft,CLAY)
(DNVRPF105)

Soil poisson's ratio $\upsilon_s := 0.45$ (soft,CLAY)

Trench depth $\Delta := 0.0 \cdot m$

**Note:** Trench depth is used to determine the relative trench depth (Δ/OD)

Seabed gap $e := 2 \cdot OD$

**Note:** this is used to calculate seabed gap ratio (e/OD).

**Note:** This is used to calculate seabed gap ratio (e/ODtot). Conservatively, a value of e/ODtot more than 0.8 is selected

## 1.5 Design Factors

Load cases counter: $i := 0 .. 3$

**Note:** 4 Load cases are analysed:

1. Hydrotest (system pressure test) -pipe is flooded
2. Installation - pipe is flooded (contingency)
3. Installation - pipe is empty (basecase)
4. Operation (pipe with operational content).

## DNV RP-F105 factors

Structural damping: $\zeta_{str} := 0.005$

Soil damping: $\zeta_{soil} := 0.01$

Hydrodynamic damping: $\zeta_h := 0.00$

Total modal damping: $\zeta_T := \zeta_{str} + \zeta_{soil} + \zeta_h$ $\zeta_T = 0.015$

Safety factor for screening - In Line $\gamma_{IL} := 1.4$

Safety Factor on Screening - Cross Flow $\gamma_{CF} := 1.4$

Safety factor on stress amplitude: $\gamma_s := 1.3$

Safety Factor on Stability Parameter: $\gamma_k := 1.0$

Safety factor on onset value for In-Line VIV $\gamma_{onIL} := 1.1$

Safety Factor on Onset Value for Cross Flow VIV $\gamma_{onCF} := 1.2$

Safety factor for natural frequency: $\gamma_f := 1.1$

Maximum fabrication factor:

$$\alpha_{fab} := \left| \begin{array}{l} 1.0 \;\; \text{if Process} = \text{"Seamless"} \\ 0.93 \;\; \text{if Process} = \text{"UO \& TRB \& ERW"} \\ 0.85 \;\; \text{if Process} = \text{"UOE"} \end{array} \right.$$

$\alpha_{fab} = 1$

Characteristic Material Strength Factor:

$$\alpha_u := \left| \begin{array}{l} 1.0 \;\; \text{if Supp\_req\_U} = \text{"No"} \\ 0.96 \;\; \text{if Supp\_req\_U} = \text{"No"} \end{array} \right.$$

$\alpha_u = 0.96$

Characteristic material strength factor for pressure test: $\alpha_{utest} := 1.0$

Material resistance factor (ULS) $\gamma_m := 1.15$

Safety class resistance factor

$$\gamma_{sc} := \begin{pmatrix} 1.046 \\ 1.04 \\ 1.04 \\ 1.308 \end{pmatrix}$$

Functional load effect factor: $\gamma_F := 1.1$

Environmental load factor $\gamma_E := 1.3$

Condition Load Effect Factor $\gamma_c := \begin{pmatrix} 1.00 \\ 1.07 \\ 1.07 \\ 1.00 \end{pmatrix}$

**Note:** For simultaneously system pressure test on an uneven seabed, the resulting condition factor will be 1.07*0.93=1

## 1.6 Boundary Conditions

Three boundary conditions are presented in DNV-RP-F105. They are:
- Pinned - Pinned
- Fixed - Fixed
- Single Span on Seabed

**Pinned-Pinned : 1** **Fixed - Fixed: 2** **Single Span: 3**

$BC := 1$

Input BC as above

$$C_1 := \begin{cases} 1.57 & \text{if } BC = 1 \\ 3.56 & \text{if } BC = 2 \\ 3.56 & \text{if } BC = 3 \end{cases} \qquad C_1 = 1.57$$

$$C_2 := \begin{cases} 1.00 & \text{if } BC = 1 \\ 4.00 & \text{if } BC = 2 \\ 4.00 & \text{if } BC = 3 \end{cases} \qquad C_2 = 1$$

$$C_3 := \begin{cases} 0.80 & \text{if } BC = 1 \\ 0.20 & \text{if } BC = 2 \\ 0.40 & \text{if } BC = 3 \end{cases} \qquad C_3 = 0.8$$

$$C_4 := \begin{cases} 4.93 & \text{if } BC = 1 \\ 14.10 & \text{if } BC = 2 \\ 8.60 & \text{if } BC = 3 \end{cases} \qquad C_4 = 4.93$$

$$C_5 := \begin{cases} \frac{1}{8} & \text{if } BC = 1 \\ \frac{1}{12} & \text{if } BC = 2 \\ \frac{1}{24} & \text{if } BC = 3 \end{cases} \qquad C_5 = 0.125$$

$$C_6 := \begin{cases} \frac{5}{384} & \text{if } BC = 1 \\ \frac{1}{384} & \text{if } BC = 2 \\ \frac{1}{384} & \text{if } BC = 3 \end{cases} \qquad C_6 = 0.013$$

**Note:** In most analyses, the pinned-pinned and fixed-fixed boundary conditions are used to determine the lower and upper limits of the allowable span respectively. Therefore, if a particular span is identified which is at some distance from the next one and could be classified as free span, it would be conservative to treat its boundary as Pinned-Pinned.

# 2. PRELIMINARY CALCULATIONS

## 2.1 General Calculations

Pipe overall outer diameter $OD_{tot} := OD + 2 \cdot t_{coat}$ $OD_{tot} = 362.1 \cdot mm$

Pipe internal diameter

$$ID := OD - 2\cdot wt$$

$$ID = 317.5\cdot mm$$

Steel cross sectional area

$$A_{st} := \frac{\pi}{4}\cdot\left(OD^2 - ID^2\right)$$

$$A_{st} = 20421.106\cdot mm^2$$

Pipeline steel unit weight

$$W_{st} := \frac{\pi}{4}\cdot\left[\left(OD^2 - ID^2\right)\cdot\rho_{st}\right]\cdot g$$

$$W_{st} = 1572.599\cdot\frac{N}{m}$$

Coating area

$$A_{coat} := \frac{\pi}{4}\cdot\left(OD_{tot}{}^2 - OD^2\right)$$

$$A_{coat} = 3384.438\cdot mm^2$$

Coating unit weight

$$W_{coat} := \rho_{coat}\cdot A_{coat}\cdot g$$

$$W_{coat} = 31.342\cdot\frac{N}{m}$$

Buoyancy

$$B_Y := \rho_{water}\cdot g\cdot\frac{\pi}{4}\cdot\left(OD_{tot}{}^2\right)$$

$$B_Y = 1035.475\cdot\frac{N}{m}$$

Internal cross sectional area

$$A_{int} := \frac{\pi}{4}\cdot ID^2$$

$$A_{int} = 79173.044\cdot mm^2$$

Contents unit weight (seawater filled)

$$W_{sea} := A_{int}\cdot\rho_{water}\cdot g$$

$$W_{sea} = 796.105\cdot\frac{N}{m}$$

Contents unit weight (during operation)

$$W_{oper} := A_{int}\cdot\rho_{oper}\cdot g$$

$$W_{oper} = 155.338\cdot\frac{N}{m}$$

Submerged weight of pipe
(seawater filled)

$$W_{s\_flood} := \left(W_{st} + W_{coat} + W_{sea} - B_Y\right)$$

$$W_{s\_flood} = 1364.57\cdot\frac{N}{m}$$

Submerged weight of pipe
(during operation)

$$W_{s\_oper} := \left(W_{st} + W_{coat} + W_{oper} - B_Y\right)$$

$$W_{s\_flood} = 1364.57\cdot\frac{N}{m}$$

Weight of pipe in air
(seawater filled)

$$W_{a\_flood} := \left(W_{st} + W_{coat} + W_{sea}\right)$$

$$W_{a\_flood} = 2400.046\cdot\frac{N}{m}$$

Weight of pipe in air
(operational content filled)

$$W_{a\_oper} := \left(W_{st} + W_{coat} + W_{oper}\right)$$

$$W_{a\_oper} = 1759.278\cdot\frac{N}{m}$$

Submerged weight of pipe (empty)

$$W_{s\_empty} := \left(W_{st} + W_{coat} - B_Y\right)$$

$$W_{s\_empty} = 568.465 \cdot \frac{N}{m}$$

Weight of pipe in air (empty)

$$W_{a\_empty} := \left(W_{st} + W_{coat}\right)$$

$$W_{a\_empty} = 1603.941 \cdot \frac{N}{m}$$

Ratio of pipe mass to displaced water

$$\rho_s := \frac{W_{a\_empty}}{\left[\frac{\pi}{4} \cdot \left(OD_{tot}{}^2\right)\right]}$$

$$\rho_s = 15575.479 \cdot \frac{N}{m^3}$$

**Note:** It is understood that the pipe mass ratio to displaced water does neither include added mass nor the mass of the content fluid (i.e. consider mass of pipe empty in air)

Specific mass ratio defined as pipe mass (not including the added mass) and the displaced water

$$\frac{\rho_s}{\rho_{water} \cdot g} = 1.549$$

The dynamic soil stiffness expression is valid for

$$1.2 < \frac{\rho_s}{\rho_{water} \cdot g} < 2.0$$

Dynamic soil stiffness:

Lateral soil stiffness

$$K_L := C_L \cdot \left(1 + \upsilon_s\right) \cdot \left[\frac{2}{3} \cdot \left(\frac{\rho_s}{\rho_{water} \cdot g}\right) + \frac{1}{3}\right] \cdot \sqrt{OD_{tot}}$$

$$K_L = 1.43 \cdot MPa$$

Vertical soil stiffness

$$K_V := \frac{C_V}{1 - \upsilon_s} \cdot \left[\frac{2}{3} \cdot \left(\frac{\rho_s}{\rho_{water} \cdot g}\right) + \frac{1}{3}\right] \cdot \sqrt{OD_{tot}}$$

$$K_V = 2.092 \cdot MPa$$

Second moment of area

$$I := \frac{\pi}{64} \cdot \left(OD^4 - ID^4\right)$$

$$I = 2.905 \times 10^{-4} \cdot m^4$$

Minimum external pressure

$$P_{exmin} := \rho_{water} \cdot g \cdot d_{min}$$

$$P_{exmin} = 140.774 \cdot bar$$

Characteristic yield strength (System pressure test)

$$f_{ytest\_inst} := \left(SMYS - f_{yCMn\_inst}\right) \cdot \alpha_{utest}$$

$$f_{ytest\_inst} = 448 \cdot MPa$$

Characteristic tensile strength
(System pressure test)

$$f_{utest\_inst} := \left(SMTS - f_{yCMn\_inst}\right)\cdot\alpha_{utest}$$

$$f_{utest\_inst} = 530\cdot MPa$$

Characteristic yield strength (Installation)
(No material de-rating)

$$f_{yinst} := SMYS\cdot\alpha_u \qquad f_{yinst} = 430.08\cdot MPa$$

Characteristic tensile strength (Installation)
(No material de-rating)

$$f_{uinst} := SMTS\cdot\alpha_u \qquad f_{uinst} = 508.8\cdot MPa$$

Characteristic yield strength (Operation)
(material de-rating)

$$f_{yoper} := \left(SMYS - f_{yCMn\_oper}\right)\cdot\alpha_u$$

$$f_{yoper} = 413.952\cdot MPa$$

Characteristic tensile strength (Operation)
(material de-rating)

$$f_{uoper} := \left(SMTS - f_{yCMn\_oper}\right)\cdot\alpha_u$$

$$f_{uoper} = 492.672\cdot MPa$$

## 2.2 Effective Axial Force

The pipeline is installed at ambient temperatures. The system pressure test is conducted at ambient temperatures, i.e.. there is no thermal induced compressive force. However, the pressure test induces an axial compressive stress due to Poisson's effect. The residual lay tension is conservatively neglected.For operation (load case 4), the differential between ambient (installation temperature) and design temperature is accounted for.

Installation and hydrotest temperature difference

$$\Delta T_{inst} := T_{inst} - T_{amb}$$

$$\Delta T_{inst} = 0\cdot C$$

Operation temperature difference

$$\Delta T_{oper} := T_{oper} - T_{amb}$$

$$\Delta T_{oper} = 74\cdot C$$

Temperature gradient for the 4 load cases:

$$\Delta T := \begin{pmatrix} \Delta T_{inst} \\ \Delta T_{inst} \\ \Delta T_{inst} \\ \Delta T_{oper} \end{pmatrix} \qquad \Delta T_i = \begin{array}{|c|}\hline 0 \\ \hline 0 \\ \hline 0 \\ \hline 74 \\ \hline \end{array}\cdot C$$

Local pressure (for the 4 load cases)

$$P_l := \begin{pmatrix} P_{inst} \\ P_{flood} \\ P_{hydrotest} \\ P_d \end{pmatrix} \qquad P_{l_i} = \begin{array}{|c|}\hline 0 \\ \hline 146.505 \\ \hline 276.911 \\ \hline 243.586 \\ \hline \end{array}\cdot bar$$

Internal pressure difference relative to as laid (at minimum water depth)

$$\Delta P := P_l - P_{inst}$$

$\Delta P_i =$

| | |
|---|---|
| 0 | $\cdot bar$ |
| 146.505 | |
| 276.911 | |
| 243.586 | |

Effective axial force (pipe considered thin walled)

$$S_{eff} := H_{lay} - \Delta P \cdot A_{int} \cdot (1 - 2 \cdot \nu) - A_{st} \cdot E \cdot \Delta T \cdot \alpha_{pipe}$$

$S_{eff_i} =$

| | |
|---|---|
| 0 | $\cdot kN$ |
| -463.97 | |
| -876.957 | |
| -4431.302 | |

**Note:** The pipe is in compression if the above effective force is negative.

## 2.3 Hydrodynamic Loading

Note: The significant wave height and associated wave peak period are used to derive the hydrodynamic loading for the VIV maximum allowable free span lengths (DNVGLSTF101). The maximum wave height and wave maximum period are used to derive the hydrodynamic loading for the ULS check (local buckling check) (DNVGLSTF101).
For both conditions, Airy linear wave theory is used to determine the wave kinematics in terms of velocity and acceleration applied to the pipe

Load cases counter: $j := 0..3$

Note: 4 environmental cases are considered

1. Significant wave loading (installation: 1-year wave + 1-year current)
2. Maximum wave loading (installation: 1-year wave + 1-year current)
3. Significant wave loading (operation 100-year wave + 10-year current)
4. Maximum wave loading (operation 100-year wave + 10-year current)

$$H := \begin{pmatrix} H_{s\_inst} \\ H_{max\_inst} \\ H_{s\_oper} \\ H_{max\_oper} \end{pmatrix} \qquad T := \begin{pmatrix} T_{p\_inst} \\ T_{max\_inst} \\ T_{p\_oper} \\ T_{max\_oper} \end{pmatrix}$$

1-year significant wave height & period

1-year maximum wave height & period

100-year significant wave height & period

100-year maximum wave height & period

$H_j =$

| | |
|---|---|
| 3.505 | m |
| 7.01 | |
| 4.788 | |
| 9.576 | |

$T_j =$

| | |
|---|---|
| 15.9 | s |
| 15.9 | |
| 17.5 | |
| 17.5 | |

## 1. Determine wave length

Wavelength 1st guess value $\lambda := 150 \cdot m$

Given

$$\lambda = \frac{g \cdot (T_j)^2}{2 \cdot \pi} \cdot \tanh\left(\frac{2 \cdot \pi \cdot d_{min}}{\lambda}\right)$$

$\lambda(j) := \text{Find}(\lambda)$ $\qquad \lambda_j := \lambda(j)$

$\lambda_j =$

| | $\lambda_j$ (m) |
|---|---|
| (for 1-year significant wave height & period) | 394.715 |
| (for 1-year maximum wave height & period) | 394.715 |
| (for 100-year significant wave height & period) | 478.151 |
| (for 100-year maximum wave height & period) | 478.151 |

## 2. Calculate wave particle velocity & acceleration

**Note:** The linear wave theory (Airy) is used to compute the wave kinematics at the seabed.
It should be mentioned that at this very deep water depth, the wave velocity and acceleration will be negligible.The use of Stokes wave theory will also yield the same results.

The maximum wave water particle velocity and acceleration are:

Maximum wave water particle velocity
(at top of pipe)

$$U_{w_j} := \frac{\pi \cdot H_j}{T_j} \cdot \frac{\cosh\left[\frac{2 \cdot \pi}{\lambda_j} \cdot (OD_{tot})\right]}{\sinh\left(2 \cdot \pi \cdot \frac{d_{min}}{\lambda_j}\right)}$$

$U_{w_j} =$

| $U_{w_j}$ ($\frac{m}{s}$) |
|---|
| 0 |
| 0 |
| 0 |
| 0 |

Maximum wave water particle acceleration
(at top of pipe)

$$A_{w_j} := \frac{2 \cdot \pi^2 \cdot H_j}{(T_j)^2} \cdot \frac{\cosh\left[\frac{2 \cdot \pi}{\lambda_j} \cdot (OD_{tot})\right]}{\sinh\left(2 \cdot \pi \cdot \frac{d_{min}}{\lambda_j}\right)}$$

$A_{w_j} =$

| $A_{w_j}$ ($\frac{m}{s^2}$) |
|---|
| 0 |
| 0 |
| 0 |
| 0 |

## 3. Current velocity at pipe

**Note:** The determination of the unfactored boundary layer average current velocity at the pipe is from DNV RP-F105

$$U_{cpipe}(u_{cur}) := u_{cur} \cdot \left( \frac{\ln\left(\frac{OD_{tot}}{m}\right) - \ln\left(\frac{Z_o}{m}\right)}{\ln\left(\frac{Z_{ref}}{m}\right) - \ln\left(\frac{Z_o}{m}\right)} \right)$$

1-year current $U_{cpipe\_inst} := U_{cpipe}(u_{cur\_inst})$ $U_{cpipe\_inst} = 0.134 \frac{m}{s}$

10-year current $U_{cpipe\_oper} := U_{cpipe}(u_{cur\_oper})$ $U_{cpipe\_oper} = 0.174 \frac{m}{s}$

$$U_{cpipe} := \begin{pmatrix} U_{cpipe\_inst} \\ U_{cpipe\_inst} \\ U_{cpipe\_oper} \\ U_{cpipe\_oper} \end{pmatrix}$$

$U_{cpipe_j} =$

| $U_{cpipe_j}$ (m/s) |
|---|
| 0.134 |
| 0.134 |
| 0.174 |
| 0.174 |

## 4. Determine hydrodynamic force applied to pipe

Maximum combined wave and current velocity $U_j := U_{cpipe_j} + U_{w_j}$

$U_j =$

| $U_j$ (m/s) |
|---|
| 0.134 |
| 0.134 |
| 0.174 |
| 0.174 |

Current flow velocity ratio (for significant wave)

For installation
$$\alpha_{inst} := \frac{U_{cpipe_0}}{U_{cpipe_0} + U_{w_0}}$$
$\alpha_{inst} = 1$

For operation
$$\alpha_{oper} := \frac{U_{cpipe_2}}{U_{cpipe_2} + U_{w_2}}$$
$\alpha_{oper} = 1$

**Flow regime**

For installation

$$Flow\_regime_{inst} := \begin{cases} \text{"Wave dominant-In-line VIV is negligible-Response model recommended for cross-flow VIV"} & \text{if } \alpha_{inst} < 0.5 \\ \text{"Wave dominant-In-line VIV mitigated by waves-Cross-flow VIV:Response model recommended"} & \text{if } 0.5 < \alpha_{inst} < 0.8 \\ \text{"Current dominant-Response model recommended for both in-line and cross-flow VIVs"} & \text{otherwise} \end{cases}$$

$Flow\_regime_{inst} =$ "Current dominant-Response model recommended for both in-line and cross-flow VIVs"

For operation

$$Flow\_regime_{oper} := \begin{cases} \text{"Wave dominant-In-line VIV is negligible-Response model recommended for cross-flow VIV"} & \text{if } \alpha_{oper} < 0.5 \\ \text{"Wave dominant-In-line VIV mitigated by waves-Cross-flow VIV:Response model recommended"} & \text{if } 0.5 < \alpha_{oper} < 0.8 \\ \text{"Current dominant-Response model recommended for both in-line and cross-flow VIVs"} & \text{otherwise} \end{cases}$$

$Flow\_regime_{oper} =$ "Current dominant-Response model recommended for both in-line and cross-flow VIVs"

Reynolds Number

$$R_{e_j} := \frac{U_j \cdot OD_{tot}}{v_k}$$

$R_{e_j} =$

| |
|---|
| 32337.785 |
| 32337.785 |
| 42039.125 |
| 42039.129 |

Keulegan-Carpenter number

$$KC_j := \frac{U_{w_j} \cdot T_j}{OD_{tot}}$$

$KC_j =$

| |
|---|
| 0 |
| 0 |
| 0 |
| 0 |

Hydrodynamic forces of drag and inertia

**Note:** The determination of the hydrodynamic forces of drag and inertia is based on Morison's equation

Seabed gap ratio

$$\frac{e}{OD_{tot}} = 1.967$$

Added mass coefficient

$$C_a := \begin{cases} 0.68 + \dfrac{1.6}{1 + 5\cdot\left(\dfrac{e}{OD_{tot}}\right)} & \text{if } \dfrac{e}{OD_{tot}} < 0.8 \\ 1 & \text{otherwise} \end{cases}$$

$C_a = 1$

Added mass

$$M_{add} := C_a \cdot \frac{\pi}{4} \cdot \left(OD_{tot}\right)^2 \cdot \rho_{water}$$

$$M_{add} = 105.553 \, \frac{kg}{m}$$

Pipe mass (flooded)

$$M_{pipe\_flood} := \frac{W_{a\_flood}}{g}$$

$$M_{pipe\_flood} = 244.653 \, \frac{kg}{m}$$

Pipe mass (empty)

$$M_{pipe\_empty} := \frac{W_{a\_empty}}{g}$$

$$M_{pipe\_empty} = 163.501 \, \frac{kg}{m}$$

Pipe mass (in operation)

$$M_{pipe\_oper} := \frac{W_{a\_oper}}{g}$$

$$M_{pipe\_oper} = 179.335 \, \frac{kg}{m}$$

Effective mass (pipe flooded)

$$M_{e\_flood} := M_{pipe\_flood} + M_{add}$$

$$M_{e\_flood} = 350.206 \, \frac{kg}{m}$$

Effective mass (pipe empty)

$$M_{e\_empty} := M_{pipe\_empty} + M_{add}$$

$$M_{e\_empty} = 269.054 \, \frac{kg}{m}$$

Effective mass
(pipe in operation)

$$M_{e\_oper} := M_{pipe\_oper} + M_{add}$$

$$M_{e\_oper} = 284.888 \, \frac{kg}{m}$$

Drag coefficient
(for both the significant and maximum wave conditions)

$$C_D := \begin{pmatrix} 1.2 \\ 1.2 \\ 1.2 \\ 1.2 \end{pmatrix}$$

Inertia coefficient

$$C_M := C_a + 1 \qquad C_M = 2$$

The combined steady current and wave velocities over one wave period (phase angle θ =0-360°) is given as:

$$t := 0, 1 .. 360$$

$$\theta_t := t \cdot \frac{\pi}{360}$$

Combined velocity

$$U_{c_{t,j}} := U_{cpipe_j} + U_{w_j} \cdot \cos(\theta_t)$$

Wave acceleration over one period

$$A_{wacc_{t,j}} := A_{w_j} \cdot \sin(\theta_t)$$

The resulting functions for the hydrodynamic forces are:

Inertia force at pipeline

$$F_{I_{t,j}} := \frac{\pi}{4} \cdot \rho_{water} \cdot C_M \cdot (OD_{tot})^2 \cdot A_{wacc_{t,j}}$$

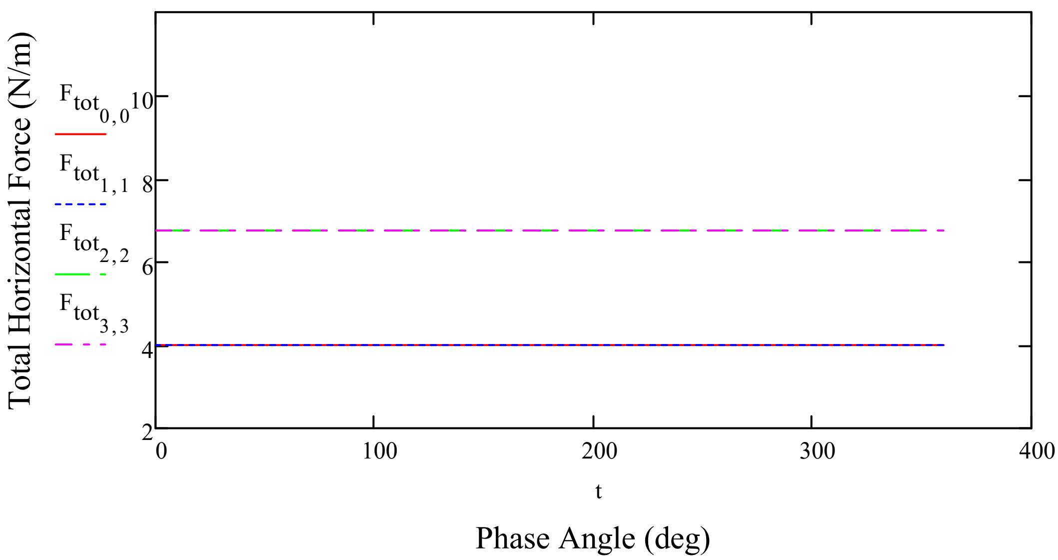
Drag force at pipeline

$$F_{D_{t,j}} := 0.5 \cdot \rho_{water} \cdot C_{D_j} \cdot (OD_{tot}) \cdot (U_{c_{t,j}}) \cdot \left|U_{c_{t,j}}\right|$$

Total horizontal hydrodynamic force
(sum of drag & inertia forces)

$$F_{tot_{t,j}} := F_{D_{t,j}} + F_{I_{t,j}}$$

TOTAL HORIZONTAL DRAG FORCE AND INERTIA




Define the critical phase angle that coincides with the maximum horizontal force.

$$f(Pangle) := 0.5 \cdot \rho_{water} \cdot C_D \cdot (OD_{tot}) \cdot (U_{cpipe} + U_w \cdot \cos(Pangle)) \cdot \left|U_{cpipe} + U_w \cdot \cos(Pangle)\right| \ \ldots$$
$$+ \frac{\pi}{4} \cdot \rho_{water} \cdot C_M \cdot (OD_{tot})^2 \cdot A_w \cdot \sin(Pangle)$$

First guess at phase angle for maximum required submerged weight in each seastate

$$\theta_{Pangle_j} := \left(\frac{\pi}{180} \cdot 345\right)$$

Given

$$0 < \theta_{Pangle} < 2 \cdot \pi$$

$$\text{Phase} := \text{Maximize}\left(f, \theta_{\text{Pangle}}\right) \qquad \theta_{\text{Pangle}} := \text{Phase}\cdot\frac{180}{\pi}$$

$$\theta_{\text{Pangle}} = \begin{pmatrix} 346 \\ 346 \\ 368 \\ 368 \end{pmatrix}\cdot\text{rad}$$

Rounding phase to nearest degree

$$\theta_{\text{Pangle}_j} := \text{round}\left(\theta_{\text{Pangle}_j}, 0\right)$$

$\theta_{\text{Pangle}_j} =$

| |
|---|
| 346 |
| 346 |
| 368 |
| 368 |

**Determine maximum load (at critical phase angle)**

Wave induced velocities

$U_{w_j}\cdot\cos\left(\theta_{\text{Pangle}_j}\right) =$

| | |
|---|---|
| 0 | $\frac{m}{s}$ |
| 0 | |
| -0 | |
| -0 | |

Total velocities

$$U_{\text{cPangle}_{j,j}} := U_{\text{cpipe}_j} + U_{w_j}\cdot\cos\left(\theta_{\text{Pangle}_j}\right)$$

$U_{\text{cPangle}_{j,j}} =$

| | |
|---|---|
| 0.134 | $\frac{m}{s}$ |
| 0.134 | |
| 0.174 | |
| 0.174 | |

Wave accelerations

$$A_{\text{waccPangle}_{j,j}} := A_{w_j}\cdot\sin\left(\theta_{\text{Pangle}_j}\right)$$

$A_{\text{waccPangle}_{j,j}} =$

| | |
|---|---|
| 0 | $\frac{m}{s^2}$ |
| 0 | |
| -0 | |
| -0 | |

Inertia force applied to pipeline

$$F_{\text{IPangle}_{j,j}} := \frac{\pi}{4}\cdot\rho_{\text{water}}\cdot C_M\cdot\left(OD_{\text{tot}}\right)^2\cdot A_{\text{waccPangle}_{j,j}}$$

$F_{\text{IPangle}_{j,j}} =$

| |
|---|
| 0 |
| 0 |
| -0 |
| -0 |

$\cdot \frac{N}{m}$

Drag force at pipeline

$$F_{DPangle_{j,j}} := 0.5 \cdot \rho_{water} \cdot C_{D_j} \cdot (OD_{tot}) \cdot (U_{cPangle_{j,j}}) \cdot |U_{cPangle_{j,j}}|$$

$F_{DPangle_{j,j}} =$

| |
|---|
| 3.996 |
| 3.996 |
| 6.754 |
| 6.754 |

$\cdot \frac{N}{m}$

$$F_{tot_{t,j}} := F_{D_{t,j}} + F_{I_{t,j}}$$

Total horizontal hydrodynamic force
(sum of drag & inertia forces)

$$F_{totPangle_{j,j}} := F_{DPangle_{j,j}} + F_{IPangle_{j,j}}$$

$F_{totPangle_{j,j}} =$

| |
|---|
| 3.996 |
| 3.996 |
| 6.754 |
| 6.754 |

$\cdot \frac{N}{m}$

Maximum lateral load
(this is the dynamic load)

$$F_{latph_j} := F_{totPangle_{j,j}}$$

$F_{latph_j} =$

| |
|---|
| 3.996 |
| 3.996 |
| 6.754 |
| 6.754 |

$\cdot \frac{N}{m}$

## 3. ALLOWABLE FREE SPAN LENGTH CALCULATIONS

3.1 Maximum Allowable Free Span Length -Local Buckling (Load controlled criteria)-(DNVGLSTF101)

**Note:** The local buckling span limitation is calculated according to Section 5-D505, Ref. 2 - Combined Loading Criteria (load controlled condition, internal overpressure).
4 load cases are analysed:
1. Hydrotest (system pressure test) -pipe is flooded
2. Installation - pipe is flooded (unpressurised)
3. Installation - pipe is empty (unpressurised, basecase)
4. Operation (system pressure) -pipe with operation content.

Derated yield and tensile stress

$$f_y := \begin{pmatrix} f_{yinst} \\ f_{yinst} \\ f_{ytest\_inst} \\ f_{yoper} \end{pmatrix} \qquad f_u := \begin{pmatrix} f_{utest\_inst} \\ f_{uinst} \\ f_{utest\_inst} \\ f_{uoper} \end{pmatrix}$$

$P_{l_i} =$

| |
|---|
| 0 |
| 146.505 |
| 276.911 |
| 243.586 |

$\cdot \mathrm{bar}$

Define pipe submerged weight for the above 4 conditions

$$W_s := \begin{pmatrix} W_{s\_empty} \\ W_{s\_flood} \\ W_{s\_flood} \\ W_{s\_oper} \end{pmatrix}$$

$W_{s_i} =$

| |
|---|
| 568.465 |
| 1364.57 |
| 1364.57 |
| 723.803 |

$\cdot \frac{N}{m}$

Wall thickness

$$t_2 := wt \qquad t_2 = 0.019 \cdot m$$

Plastic moment resistance

$$M_{p_i} := f_{y_i} \cdot (OD - t_2)^2 \cdot t_2$$

$M_{p_i} =$

| |
|---|
| 941.566 |
| 941.566 |
| 980.798 |
| 906.257 |

$\cdot kN \cdot m$

Characteristic plastic axial force resistance

$$S_{p_i} := f_{y_i} \cdot \pi (OD - t_2) \cdot t_2$$

$S_{p_i} =$

| |
|---|
| 8782.709 |
| 8782.709 |
| 9148.656 |
| 8453.358 |

$\cdot kN$

Pressure containment (min of yield limit state& burst limit state)

$$P_{b\_2_i} := \left(\frac{2 \cdot t_2}{OD - t_2}\right) \cdot \min\left(f_{y_i}, \frac{f_{u_i}}{1.15}\right) \cdot \frac{2}{\sqrt{3}}$$

$P_{b\_2_i} =$

| |
|---|
| 56.916 |
| 56.916 |
| 59.287 |
| 54.782 |

$\cdot MPa$

Normalised pressure

$$q_{h_i} := \frac{P_{l_i}}{P_{b\_2_i} \cdot \frac{2}{\sqrt{3}}}$$

$q_{h_i} =$

| |
|---|
| 0 |
| 0.223 |
| 0.404 |
| 0.385 |

$$\frac{OD}{t_2} = 18.451$$

$$\beta := \begin{cases} 0.5 & \text{if } \frac{OD}{t_2} < 15 \\ \frac{60 - \left(\frac{OD}{t_2}\right)}{90} & \text{if } 15 \le \frac{OD}{t_2} \le 60 \\ 0 & \text{if } \left(\frac{OD}{t_2}\right) > 60 \end{cases}$$

$\beta = 0.462$

Flow stress parameter

$$\alpha_{c_i} := (1 - \beta) + \beta \cdot \frac{f_{u_i}}{f_{y_i}}$$

$\alpha_{c_i} =$

| |
|---|
| 1.107 |
| 1.084 |
| 1.084 |
| 1.088 |

$$\alpha_{p_i} := \begin{cases} (1 - \beta) & \text{if } \frac{P_{l_i}}{P_{b\_2_i}} < 0.7 \\ 1 - 3 \cdot \beta \cdot \left(1 - \frac{P_{l_i}}{P_{b\_2_i}}\right) & \text{if } \frac{P_{l_i}}{P_{b\_2_i}} \ge 0.7 \end{cases}$$

$\alpha_{p_i} =$

| |
|---|
| 0.538 |
| 0.538 |
| 0.538 |
| 0.538 |

Considering the free span as Pinned-Pinned for the Ultimate Limit State (ULS), the following boundary conditions are used. The apparent (visual) span length is used for the ULS check.

The functional moment is taken as the static moment due to self weight only.

First guess at free span apparent length $L := 10 \cdot m$

Euler Buckling load

$$P_E := \frac{\pi^2 \cdot E \cdot I \cdot C_2}{L^2}$$

Estimate of static bending moment

$$M_{static} := C_5 \cdot \frac{W_s \cdot L^2}{1 + \frac{S_{eff}}{P_E}}$$

$$M_F := M_{static}$$

$$M_F = \begin{pmatrix} 7.106 \\ 18.504 \\ 20.014 \\ 35.708 \end{pmatrix} \cdot kN \cdot m$$

The environmental moment is taken as the maximum dynamic moment (maximum wave) due to the lateral load from hydrodynamic forces

For load cases 1, 2 and 3 (during installation)

$$M_{dyn\_inst_i} := C_5 \cdot \frac{F_{latph_1} \cdot L^2}{1 + \frac{S_{eff_i}}{P_E}}$$

$M_{dyn\_inst_i} =$ $\cdot kN \cdot m$

| |
|---|
| 0.05 |
| 0.054 |
| 0.059 |
| 0.197 |

For load case 4 (operation)

$$M_{dyn\_oper} := C_5 \cdot \frac{F_{latph_3} \cdot L^2}{1 + \frac{S_{eff_3}}{P_E}}$$

$$M_{dyn\_oper} = 0.333 \cdot kN \cdot m$$

Environmental Bending Moment (installation)

$$M_{E\_inst} := M_{dyn\_inst}$$

$$M_{E\_inst} = \begin{pmatrix} 0.05 \\ 0.054 \\ 0.059 \\ 0.197 \end{pmatrix} \cdot kN \cdot m$$

Environmental Bending Moment (operation)

$$M_{E\_oper} := M_{dyn\_oper}$$

$$M_{E\_oper} = 0.333 \cdot kN \cdot m$$

Functional axial force
(Effective axial force)

$$S_{F_i} := S_{eff_i}$$

Note: A dynamic amplification factor of 1.3 is assumed

$$S_{E_i} := 0.3 \cdot S_{F_i}$$

$S_{E_i} =$

| |
|---|
| 0 |
| -139.191 |
| -263.087 |
| -1329.391 |

$\cdot kN$

Design Bending Moment (installation)

$$M_{d\_inst_i} := M_{F_i} \cdot \gamma_F \cdot \gamma_{c_i} + M_{E\_inst_i} \cdot \gamma_E$$

$$M_{d\_inst} = \begin{pmatrix} 7.881 \\ 21.849 \\ 23.633 \\ 39.536 \end{pmatrix} \cdot kN \cdot m$$

**<u>Note:</u>** Ignore the last value (row 4) as it is associated with operation load case

Design Bending Moment (operation)

$$M_{d\_oper} := M_{F_3} \cdot \gamma_F \cdot \gamma_{c_3} + M_{E\_oper} \cdot \gamma_E$$

$$M_{d\_oper} = 39.712 \cdot kN \cdot m$$

Design Effective Axial Force

$$S_{d_i} := S_{F_i} \cdot \gamma_F \cdot \gamma_{c_i} + S_{E_i} \cdot \gamma_E$$

$$S_d = \begin{pmatrix} 0 \\ -727.041 \\ -1374.191 \\ -6602.64 \end{pmatrix} \cdot kN$$

**<u>Note:</u>** The first 3 rows are for installation load cases (load case 1 to 3) and the last value (row 4) as for operation load case(load case 4).

Local Buckling Criteria

$$\left[\gamma_{sc} \cdot \gamma_m \cdot \frac{M_{d_i}}{\alpha_{c_i} \cdot M_{p_i}} + \left(\frac{\gamma_{sc} \cdot \gamma_m \cdot S_{d_i}}{\alpha_{c_i} \cdot S_{p_i}}\right)^2\right]^2 + \left[\frac{\alpha_{p_i} \cdot \left(P_{l_i} - P_{exmin}\right)}{\alpha_{c_i} \cdot P_{b\_2_i} \cdot \left(\frac{2}{\sqrt{3}}\right)}\right]^2 \le 1.0$$

0

Defining in terms of the apparent length and solving for the maximum apparent (visual) span length.

Note: The user may need to vary the guess value for the length top aid solution convergence.

Note: The hydrodynamic force is that derived from the maximum wave height.

**<u>1. Load case 1: Unpressurized (pipe empty)</u>**

First guess for L

$L := 10 \cdot m$

Given

$$\left[\left[\gamma_{sc} \cdot \gamma_m \cdot \frac{\left[\left(C_5 \cdot \frac{W_{s_0} \cdot L^2}{1 + \frac{S_{eff_0}}{\frac{\pi^2 \cdot E \cdot I \cdot C_2}{L^2}}} \cdot \gamma_F \cdot \gamma_{c_0}\right) + \left(C_5 \cdot \frac{F_{latph_1} \cdot L^2}{1 + \frac{S_{eff_0}}{\frac{\pi^2 \cdot E \cdot I \cdot C_2}{L^2}}} \cdot \gamma_E\right)\right]}{\alpha_{c_0} \cdot M_{p_0}} + \left(\frac{\gamma_{sc} \cdot \gamma_m \cdot S_{d_0}}{\alpha_{c_0} \cdot S_{p_0}}\right)^2\right]^2 + \left[\frac{\alpha_{p_0} \cdot \left(P_{l_0} - P_{exmin}\right)}{\alpha_{c_0} \cdot P_{b\_2_0} \cdot \left(\frac{2}{\sqrt{3}}\right)}\right]^2\right] = 1$$

Unpressurised Empty Maximum Allowable Span Length $\quad L_{span\_empty} := \text{Minerr}(L)$

$L_{span\_empty} = 100.148\,m$

$ERR = 0.421$

## 2. Load case 2: Pipe unpressurised, pipe is flooded

$L := 30 \cdot m$

Given

$$\left[\left[\gamma_{sc} \cdot \gamma_m \cdot \frac{\left[\left(C_5 \cdot \frac{W_{s_1} \cdot L^2}{1 + \frac{S_{eff_1}}{\frac{\pi^2 \cdot E \cdot I \cdot C_2}{L^2}}} \cdot \gamma_F \cdot \gamma_{c_1}\right) + \left(C_5 \cdot \frac{F_{latph_1} \cdot L^2}{1 + \frac{S_{eff_1}}{\frac{\pi^2 \cdot E \cdot I \cdot C_2}{L^2}}} \cdot \gamma_E\right)\right]}{\alpha_{c_1} \cdot M_{p_1}} + \left(\frac{\gamma_{sc} \cdot \gamma_m \cdot S_{d_1}}{\alpha_{c_1} \cdot S_{p_1}}\right)^2\right]^2 + \left[\frac{\alpha_{p_1} \cdot \left(P_{l_1} - P_{exmin}\right)}{\alpha_{c_1} \cdot P_{b\_2_1} \cdot \left(\frac{2}{\sqrt{3}}\right)}\right]^2\right] = 1$$

Unpressurised (Flooded) Maximum Allowable Span Length $\quad L_{span\_flood} := \text{Minerr}(L)$

$ERR = 0.43 \qquad L_{span\_flood} = 30.961\,m$

## 3. Load case 3: System pressure (hydrotest), pipe is flooded

$L := 10 \cdot m$

Given

$$\left[\gamma_{sc}\cdot\gamma_m\cdot\frac{\left[\left(C_5\cdot\frac{\frac{W_{s_2}\cdot L^2}{1+\frac{S_{eff_2}}{\frac{\pi^2\cdot E\cdot I\cdot C_2}{L^2}}}}{}\cdot\gamma_F\cdot\gamma_{c_2}\right)+\left(C_5\cdot\frac{F_{latph_1}\cdot L^2}{1+\frac{S_{eff_2}}{\frac{\pi^2\cdot E\cdot I\cdot C_2}{L^2}}}\cdot\gamma_E\right)\right]}{\alpha_{c_2}\cdot M_{p_2}}+\left(\frac{\gamma_{sc}\cdot\gamma_m\cdot S_{d_2}}{\alpha_{c_2}\cdot S_{p_2}}\right)^2\right]^2+\left[\frac{\alpha_{p_2}\cdot\left(P_{l_2}-P_{exmin}\right)}{\alpha_{c_2}\cdot P_{b\_2_2}\cdot\left(\frac{2}{\sqrt{3}}\right)}\right]^2 = 1$$

$L_{span\_hyd} := \text{Minerr}(L)$

System Pressure Test Maximum Allowable Span Length

$ERR = 0.437$ $L_{span\_hyd} = 24.001\ m$

## 4. Load case 4: Pipe in operation

$L := 10\cdot m$

Given

$$\left[\gamma_{sc}\cdot\gamma_m\cdot\frac{\left[\left(C_5\cdot\frac{W_{s_3}\cdot L^2}{1+\frac{S_{eff_3}}{\frac{\pi^2\cdot E\cdot I\cdot C_2}{L^2}}}\cdot\gamma_F\cdot\gamma_{c_3}\right)+\left(C_5\cdot\frac{F_{latph_1}\cdot L^2}{1+\frac{S_{eff_3}}{\frac{\pi^2\cdot E\cdot I\cdot C_2}{L^2}}}\cdot\gamma_E\right)\right]}{\alpha_{c_3}\cdot M_{p_3}}+\left(\frac{\gamma_{sc}\cdot\gamma_m\cdot S_{d_3}}{\alpha_{c_3}\cdot S_{p_3}}\right)^2\right]^2+\left[\frac{\alpha_{p_3}\cdot\left(P_{l_3}-P_{exmin}\right)}{\alpha_{c_3}\cdot P_{b\_2_3}\cdot\left(\frac{2}{\sqrt{3}}\right)}\right]^2 = 1$$

Operation Maximum Allowable Span Length

$L_{span\_operation} := \text{Minerr}(L)$

$L_{span\_operation} = 10.245\ m$

$ERR = 0.822$

## 3.2 Maximum Allowable Free Span Length - Global Buckling (Euler Bar Buckling)

Note: Global buckling is checked using the Euler bar buckling criteria with the bar buckling axial force (compression).

Euler buckling load $P_E := \left|S_{eff}\right|$

Guess values

$L_{eff0} := 10\cdot m$ $L_{eff1} := 10\cdot m$ $L_{eff2} := 10\cdot m$ $L_{eff3} := 10\cdot m$

Given

Unpressurised -pipe empty $\left|S_{eff_0}\right| = \dfrac{\pi^2 \cdot E \cdot I \cdot C_2}{L_{eff0}^{\;2}}$

Unpressurised -pipe flooded $\left|S_{eff_1}\right| = \dfrac{\pi^2 \cdot E \cdot I \cdot C_2}{L_{eff1}^{\;2}}$

System pressure test load case $\left|S_{eff_2}\right| = \dfrac{\pi^2 \cdot E \cdot I \cdot C_2}{L_{eff2}^{\;2}}$

operation

$$\left|S_{eff_3}\right| = \frac{\pi^2 \cdot E \cdot I \cdot C_2}{L_{eff3}^{\;2}}$$

$$\begin{pmatrix} L_{span\_globuck\_empty} \\ L_{span\_globuck\_flood} \\ L_{span\_globuck\_hyd} \\ L_{span\_globuck\_oper} \end{pmatrix} := \text{Find}\left(L_{eff0}, L_{eff1}, L_{eff2}, L_{eff3}\right)$$

$$\begin{pmatrix} L_{span\_globuck\_empty} \\ L_{span\_globuck\_flood} \\ L_{span\_globuck\_hyd} \\ L_{span\_globuck\_oper} \end{pmatrix} = \begin{pmatrix} 70504029005578530 \\ 35.766 \\ 26.015 \\ 11.573 \end{pmatrix} m$$

Unpressurised -pipe empty $L_{span\_globuck\_empty} = 70504029005578530\ m$ (Pipe does not buckle)

Unpressurised -pipe flooded $L_{span\_globuck\_flood} = 35.766\ m$

System pressure test load case $L_{span\_globuck\_hyd} = 26.015\ m$

Pipe in operation $L_{span\_globuck\_oper} = 11.573\ m$

## 3.3 Maximum Allowable Free Span Length -In-Line VIVs

Note: The span lengths are determined for the system pressure test and installation cases using the 1-year significant wave height and 100-year significant wave height for operation load case

Stability parameter (pipe flooded)

$$K_{s\_flood} := \frac{4 \cdot \pi \cdot M_{e\_flood} \cdot \zeta_T}{\rho_{water} \cdot OD_{tot}^{\;2}} \qquad K_{s\_flood} = 0.491$$

Stability parameter (pipe empty)

$$K_{s\_empty} := \frac{4 \cdot \pi \cdot M_{e\_empty} \cdot \zeta_T}{\rho_{water} \cdot OD_{tot}^{\;2}} \qquad K_{s\_empty} = 0.377$$

Stability parameter (pipe with operational content)

$$K_{s\_oper} := \frac{4 \cdot \pi \cdot M_{e\_oper} \cdot \zeta_T}{\rho_{water} \cdot OD_{tot}^{\,2}}$$

$K_{s\_oper} = 0.4$

Stability including safety factor on damping

$$K_{sd\_flood} := \frac{K_{s\_flood}}{\gamma_k}$$

$K_{sd\_flood} = 0.491$

Stability including safety factor on damping

$$K_{sd\_empty} := \frac{K_{s\_empty}}{\gamma_k}$$

$K_{sd\_empty} = 0.377$

Stability including safety factor on damping

$$K_{sd\_oper} := \frac{K_{s\_oper}}{\gamma_k}$$

$K_{sd\_oper} = 0.4$

Reduced velocity for onset of In-line vortex shedding (pipe flooded)

$$V_{R\_IL\_onset\_flood} := \begin{cases} \dfrac{1.0}{\gamma_{onIL}} & \text{if } K_{sd\_flood} < 0.4 \\ \dfrac{0.6 + K_{sd\_flood}}{\gamma_{onIL}} & \text{if } 0.4 \le K_{sd\_flood} \le 1.6 \\ \dfrac{2.2}{\gamma_{onIL}} & \text{otherwise} \end{cases}$$

$V_{R\_IL\_onset\_flood} = 0.992$

Reduced velocity for onset of In-line vortex shedding (pipe empty)

$$V_{R\_IL\_onset\_empty} := \begin{cases} \dfrac{1.0}{\gamma_{onIL}} & \text{if } K_{sd\_empty} < 0.4 \\ \dfrac{0.6 + K_{sd\_empty}}{\gamma_{onIL}} & \text{if } 0.4 \le K_{sd\_empty} \le 1.6 \\ \dfrac{2.2}{\gamma_{onIL}} & \text{otherwise} \end{cases}$$

$V_{R\_IL\_onset\_empty} = 0.909$

Reduced velocity for onset of In-line vortex shedding (pipe in operation)

$$V_{R\_IL\_onset\_oper} := \begin{cases} \dfrac{1.0}{\gamma_{onIL}} & \text{if } K_{sd\_oper} < 0.4 \\ \dfrac{0.6 + K_{sd\_oper}}{\gamma_{onIL}} & \text{if } 0.4 \le K_{sd\_oper} \le 1.6 \\ \dfrac{2.2}{\gamma_{onIL}} & \text{otherwise} \end{cases}$$

$V_{R\_IL\_onset\_oper} = 0.909$

Reduced velocity (pipe flooded)

$$f_{0\_VIV\_onset\_flood} := \frac{U_{cpipe_0} + U_{w_0}}{V_{R\_IL\_onset\_flood} \cdot OD_{tot}}$$

$f_{0\_VIV\_onset\_flood} = 0.373 \cdot s^{-1}$

Reduced velocity (pipe empty)

$$f_{0\_VIV\_onset\_empty} := \frac{U_{cpipe_0} + U_{w_0}}{V_{R\_IL\_onset\_flood} \cdot OD_{tot}}$$

$f_{0\_VIV\_onset\_empty} = 0.373 \cdot s^{-1}$

Reduced velocity (operation)

$$f_{0\_VIV\_onset\_oper} := \frac{U_{cpipe_2} + U_{w_2}}{V_{R\_IL\_onset\_flood} \cdot OD_{tot}}$$

$f_{0\_VIV\_onset\_oper} = 0.485 \cdot s^{-1}$

Rearrange to determine the VIV onset frequency

For installation

$$\alpha\_inst := \begin{cases} 0.6 & \text{if } \alpha_{inst} < 0.6 \\ \alpha_{inst} & \text{otherwise} \end{cases}$$

$\alpha\_inst = 1$

For operation

$$\alpha\_oper := \begin{cases} 0.6 & \text{if } \alpha_{oper} < 0.6 \\ \alpha_{oper} & \text{otherwise} \end{cases}$$

$\alpha\_oper = 1$

VIV screening criteria equation

$$\left|S_{eff_1}\right| = \frac{\pi^2 \cdot E \cdot I \cdot C_2}{L_{eff}^2}$$

$$f_{0\_IL} = \left(\frac{U_{cpipe}}{V_{R\_IL\_onset} \cdot OD_{tot} \cdot \alpha\_inst}\right) \cdot \left[1 - \left(\frac{\frac{L}{OD_{tot}}}{250}\right)\right] \cdot \gamma_{IL}$$

In-line VIV onset frequency w/t safety factor

$$f_0 = f_{0\_IL} \cdot \gamma_F$$

The pipeline free span is determined such that the natural frequency is higher than the VIV on-set frequency, hence avoiding any VIV induced response/fatigue damage

The pipeline free span natural frequency is approximated .The free span is considered as a single span on the seabed. This is considered to realistically reflect what may occur during installation, i.e.. no multiple spans or interaction.

Static deflection ignored for in-line

$$\delta := 0 \cdot m$$

Euler Buckling load

$$P_E = \frac{\pi^2 \cdot E \cdot I \cdot C_2}{L_{eff}^2}$$

Fundamental Natural Frequency

$$f_0 = C_1 \cdot \sqrt{\frac{E \cdot I}{M_e \cdot L_{eff}^4} \cdot \left[1 + \frac{S_{eff}}{P_E} + C_3 \cdot \left(\frac{\delta}{OD_{tot}}\right)^2\right]}$$

Equating fundamental natural frequency to that obtained from screening criteria:

1. Unpressurised (pipe empty)

Assume first guess value

$$L_{eff} := 10 \cdot m$$

Given

$$C_1 \cdot \sqrt{\frac{E \cdot I}{M_{e\_empty} \cdot L_{eff}^4} \cdot \left[1 + \frac{S_{eff_0}}{\frac{\pi^2 \cdot E \cdot I \cdot C_2}{L_{eff}^2}} + C_3 \cdot \left(\frac{\delta}{OD_{tot}}\right)^2\right]} = \frac{U_{cpipe_0}}{V_{R\_IL\_onset\_empty} \cdot OD_{tot} \cdot \alpha\_inst} \cdot \left[1 - \frac{\left(\frac{L_{eff}}{OD_{tot}}\right)}{250}\right] \cdot \gamma_{IL} \cdot \gamma_F$$

$L_{eff\_IL_0} := \text{Minerr}(L_{eff})$

$L_{eff\_IL_0} = 54.725\,m$  $ERR = 0$

## 2. Unpressurised (pipe flooded)

### Assume first guess value

$L_{eff} := 10 \cdot m$

Given

$$C_1 \cdot \sqrt{\frac{E \cdot I}{M_{e\_flood} \cdot L_{eff}^4} \cdot \left[1 + \frac{S_{eff_1}}{\frac{\pi^2 \cdot E \cdot I \cdot C_2}{L_{eff}^2}} + C_3 \cdot \left(\frac{\delta}{OD_{tot}}\right)^2\right]} = \frac{U_{cpipe_1}}{V_{R\_IL\_onset\_flood} \cdot OD_{tot} \cdot \alpha\_inst} \cdot \left[1 - \frac{\left(\frac{L_{eff}}{OD_{tot}}\right)}{250}\right] \cdot \gamma_{IL} \cdot \gamma_F$$

$L_{eff\_IL_1} := \text{Minerr}(L_{eff})$

$L_{eff\_IL_1} = 30.189\,m$  $ERR = 0$

## 3. Hydrotest (pipe flooded)

$L_{eff} := 10 \cdot m$

Given

$$C_1 \cdot \sqrt{\frac{E \cdot I}{M_{e\_flood} \cdot L_{eff}^4} \cdot \left[1 + \frac{S_{eff_2}}{\frac{\pi^2 \cdot E \cdot I \cdot C_2}{L_{eff}^2}} + C_3 \cdot \left(\frac{\delta}{OD_{tot}}\right)^2\right]} = \frac{U_{cpipe_2}}{V_{R\_IL\_onset\_flood} \cdot OD_{tot} \cdot \alpha\_inst} \cdot \left[1 - \frac{\left(\frac{L_{eff}}{OD_{tot}}\right)}{250}\right] \cdot \gamma_{IL} \cdot \gamma_F$$

$L_{eff\_IL_2} := \text{Minerr}(L_{eff})$

$L_{eff\_IL_2} = 23.136\,m$  $ERR = 0$

## 4. Pipe in operation

### guess value

$L_{eff} := 10 \cdot m$

Given

$$C_1 \cdot \sqrt{\frac{E \cdot I}{M_{e\_oper} \cdot L_{eff}^4} \cdot \left[1 + \frac{S_{eff_3}}{\frac{\pi^2 \cdot E \cdot I \cdot C_2}{L_{eff}^2}} + C_3 \cdot \left(\frac{\delta}{OD_{tot}}\right)^2\right]} = \frac{U_{cpipe_3}}{V_{R\_IL\_onset\_oper} \cdot OD_{tot} \cdot \alpha\_oper} \cdot \left[1 - \frac{\left(\frac{L_{eff}}{OD_{tot}}\right)}{250}\right] \cdot \gamma_{IL} \cdot \gamma_F$$

$$L_{eff\_IL_3} := \mathrm{Minerr}\left(L_{eff}\right)$$

$L_{eff\_IL_3} = 11.475\,\mathrm{m}$ ERR = 0

Assume first guess value $L := L_{eff\_IL_0}$ $L_{eff} := L_{eff\_IL_0}$

Given

$$L = \begin{cases} \dfrac{L_{eff} \cdot \left(-0.066 \cdot \log\left(\frac{K_L \cdot L^4}{E \cdot I}\right)^2 + 1.02 \cdot \log\left(\frac{K_L \cdot L^4}{E \cdot I}\right) + 0.63\right)}{4.73} & \text{if } \log\left(\frac{K_L \cdot L^4}{E \cdot I}\right) \geq 2.7 \\ \dfrac{L_{eff} \cdot \left(0.036 \cdot \log\left(\frac{K_L \cdot L^4}{E \cdot I}\right)^2 + 0.61 \cdot \log\left(\frac{K_L \cdot L^4}{E \cdot I}\right) + 1.0\right)}{4.73} & \text{if } \log\left(\frac{K_L \cdot L^4}{E \cdot I}\right) < 2.7 \end{cases}$$

$$L_{app\_IL_0} := \mathrm{Minerr}(L)$$

$L_{app\_IL_0} = 47.553\,\mathrm{m}$ ERR = 0

Assume first guess value $L := L_{eff\_IL_1}$ $L_{eff} := L_{eff\_IL_1}$

Given

$$L = \begin{cases} \dfrac{L_{eff} \cdot \left(-0.066 \cdot \log\left(\frac{K_L \cdot L^4}{E \cdot I}\right)^2 + 1.02 \cdot \log\left(\frac{K_L \cdot L^4}{E \cdot I}\right) + 0.63\right)}{4.73} & \text{if } \log\left(\frac{K_L \cdot L^4}{E \cdot I}\right) \geq 2.7 \\ \dfrac{L_{eff} \cdot \left(0.036 \cdot \log\left(\frac{K_L \cdot L^4}{E \cdot I}\right)^2 + 0.61 \cdot \log\left(\frac{K_L \cdot L^4}{E \cdot I}\right) + 1.0\right)}{4.73} & \text{if } \log\left(\frac{K_L \cdot L^4}{E \cdot I}\right) < 2.7 \end{cases}$$

$$L_{app\_IL_1} := \mathrm{Minerr}(L)$$

$L_{app\_IL_1} = 22.67\,m$ $\quad$ $ERR = 0$

Assume first guess value $\quad L := L_{eff\_IL_2} \quad L_{eff} := L_{eff\_IL_2}$

Given

$$L = \begin{cases} \dfrac{L_{eff}\cdot\left(-0.066\cdot\log\left(\dfrac{K_L\cdot L^4}{E\cdot I}\right)^2 + 1.02\cdot\log\left(\dfrac{K_L\cdot L^4}{E\cdot I}\right) + 0.63\right)}{4.73} & \text{if } \log\left(\dfrac{K_L\cdot L^4}{E\cdot I}\right) \geq 2.7 \\ \dfrac{L_{eff}\cdot\left(0.036\cdot\log\left(\dfrac{K_L\cdot L^4}{E\cdot I}\right)^2 + 0.61\cdot\log\left(\dfrac{K_L\cdot L^4}{E\cdot I}\right) + 1.0\right)}{4.73} & \text{if } \log\left(\dfrac{K_L\cdot L^4}{E\cdot I}\right) < 2.7 \end{cases}$$

$$L_{app\_IL_2} := \text{Minerr}(L)$$

$L_{app\_IL_2} = 15.586\,m$ $\quad$ $ERR = 0$

Assume first guess value $\quad L := L_{eff\_IL_3} \quad L_{eff} := L_{eff\_IL_3}$

Given

$$L = \begin{cases} \dfrac{L_{eff}\cdot\left(-0.066\cdot\log\left(\dfrac{K_L\cdot L^4}{E\cdot I}\right)^2 + 1.02\cdot\log\left(\dfrac{K_L\cdot L^4}{E\cdot I}\right) + 0.63\right)}{4.73} & \text{if } \log\left(\dfrac{K_L\cdot L^4}{E\cdot I}\right) \geq 2.7 \\ \dfrac{L_{eff}\cdot\left(0.036\cdot\log\left(\dfrac{K_L\cdot L^4}{E\cdot I}\right)^2 + 0.61\cdot\log\left(\dfrac{K_L\cdot L^4}{E\cdot I}\right) + 1.0\right)}{4.73} & \text{if } \log\left(\dfrac{K_L\cdot L^4}{E\cdot I}\right) < 2.7 \end{cases}$$

$$L_{app\_IL_3} := \text{Minerr}(L)$$

$L_{app\_IL_3} = 2.577\,m$ $\quad$ $ERR = 0.12$

In-line VIV (Visual Span Length)

$$L_{span\_IL_i} := \begin{cases} L_{eff\_IL_i} & \text{if } BC = 1 \\ L_{eff\_IL_i} & \text{if } BC = 2 \\ L_{app\_IL_i} & \text{if } BC = 3 \end{cases}$$

$L_{span\_IL_i} =$

$\boxed{54.725}\ m$

| |
|---|
| 30.189 |
| 23.136 |
| 11.475 |

## 3.4 Maximum Allowable Free Span Length -Cross-Flow VIVs

Note: The span lengths are determined for the system pressure test and installation cases using the 1-year significant wave height and 100-year significant wave height for operation load case.Response model is used to determine the vortex shedding onset for cross-flow vibrations.

Correction factor for seabed proximity

$$\psi_{proxi\_onset} := \begin{cases} \frac{1}{5}\cdot\left(4 + 1.25\cdot\frac{e}{OD_{tot}}\right) & \text{if } \left(\frac{e}{OD_{tot}}\right) < 0.8 \\ 1 & \text{otherwise} \end{cases}$$

$\psi_{proxi\_onset} = 1$

Correction factor for trench effect
(conservative to assume no trench effect on pipe)

$$\psi_{trench\_onset} := 1 + 0.5\cdot\frac{\Delta}{OD_{tot}}$$

$\psi_{trench\_onset} = 1$

Reduced velocity for onset of Cross-flow
vortex shedding onset

$$V_{R\_CF\_onset} := \frac{3\cdot\psi_{proxi\_onset}\cdot\psi_{trench\_onset}}{\gamma_{onCF}}$$

$V_{R\_CF\_onset} = 2.5$

Cross-flow VIV onset frequency (installation)

$$f_{0\_CF\_onset\_inst} := \frac{\gamma_{CF}\cdot\left(U_{cpipe_0} + U_{w_0}\right)}{V_{R\_CF\_onset}\cdot OD_{tot}}$$

$f_{0\_CF\_onset\_inst} = 0.207\cdot s^{-1}$

Cross-flow VIV onset frequency (operation)

$$f_{0\_CF\_onset\_oper} := \frac{\gamma_{CF} \cdot \left(U_{cpipe_2} + U_{w_2}\right)}{V_{R\_CF\_onset} \cdot OD_{tot}}$$

$$f_{0\_CF\_onset\_oper} = 0.269 \frac{1}{s}$$

Cross-flow VIV onset frequency w/t safety factor (installation)

$$f_{0\_CF\_inst} := f_{0\_CF\_onset\_inst} \cdot \gamma_F$$

$$f_{0\_CF\_inst} = 0.228 \cdot s^{-1}$$

Cross-flow VIV onset frequency w/t safety factor (operation)

$$f_{0\_CF\_oper} := f_{0\_CF\_onset\_oper} \cdot \gamma_F$$

$$f_{0\_CF\_oper} = 0.296 \cdot s^{-1}$$

The pipeline free span is determined such that the natural frequency is higher than the VIV on-set frequency, hence avoiding any VIV induced response/fatigue damage (DNVRPF105)

The pipeline free span natural frequency is approximated in DNVRPF105. The free span is considered as a single span on the seabed. This is considered to realistically reflect what may occur during installation, i.e. no multiple spans or interaction

Euler buckling load

$$P_E = \frac{\pi^2 \cdot E \cdot I \cdot C_2}{L_{eff}^{\ 2}}$$

Static deflection

$$\delta = C_6 \cdot \frac{W_s \cdot L_{eff\_CF}^{\ 4}}{E \cdot I} \cdot \frac{1}{1 + \dfrac{S_{eff_i}}{\dfrac{\pi^2 \cdot E \cdot I \cdot C_2}{L_{eff\_CF}^{\ 2}}}}$$

Fundamental natural frequency

$$f_0 = C_1 \cdot \sqrt{\frac{E \cdot I}{M_e \cdot L_{eff\_CF}^4} \cdot \left[1 + \frac{S_{eff}}{P_E} + C_3 \cdot \left(\frac{\delta_i}{OD_{tot}}\right)^2\right]}$$

Effective mass (pipe flooded & empty)

$$M_e := \begin{pmatrix} M_{e\_empty} \\ M_{e\_flood} \\ M_{e\_flood} \\ M_{e\_oper} \end{pmatrix}$$

$M_{e_i} =$

| | |
|---|---|
| 269.054 | kg/m |
| 350.206 | |
| 350.206 | |
| 284.888 | |

Defining the criteria in terms of the effective span length:

Installation (load cases 1, 2 and 3)

First guess at free span effective length $\qquad L_{eff\_CF} := 10 \cdot m$

Given

$$C_1 \cdot \sqrt{\frac{E \cdot I}{M_{e_i} \cdot L_{eff\_CF}^4} \cdot \left[1 + \frac{S_{eff_i}}{\frac{\pi^2 \cdot E \cdot I \cdot C_2}{L_{eff\_CF}^2}} + C_3 \cdot \left(\frac{C_6 \cdot \frac{W_s \cdot L_{eff\_CF}^4}{E \cdot I} \cdot \frac{1}{1 + \frac{S_{eff_i}}{\frac{\pi^2 \cdot E \cdot I \cdot C_2}{L_{eff\_CF}^2}}}}{OD_{tot}}\right)^2\right]} = f_{0\_CF\_inst}$$

$$L_{eff\_CF\_inst}(i) := \mathrm{Minerr}(L_{eff\_CF}) \qquad L_{eff\_CF\_inst_i} := L_{eff\_CF\_inst}(i)$$

$L_{eff\_CF\_inst_i} =$

| | |
|---|---|
| 36.175 | m |
| 26.152 | |
| 21.802 | |
| 11.902 | |

**Operation (load case 4))**

Guess value

$L_{eff\_CF} := 10 \cdot m$

Given

$$C_1 \cdot \sqrt{\frac{E \cdot I}{M_{e_3} \cdot L_{eff\_CF}^{\;4}} \cdot \left[1 + \frac{S_{eff_3}}{\frac{\pi^2 \cdot E \cdot I \cdot C_2}{L_{eff\_CF}^{\;2}}} + C_3 \cdot \left(\frac{C_6 \cdot \frac{W_s \cdot L_{eff\_CF}^{\;4}}{E \cdot I} \cdot \frac{1}{1 + \frac{S_{eff_3}}{\frac{\pi^2 \cdot E \cdot I \cdot C_2}{L_{eff\_CF}^{\;2}}}}}{OD_{tot}}\right)^2\right]} = f_{0\_CF\_inst}$$

$$L_{eff\_CF\_oper} := \mathrm{Minerr}(L_{eff\_CF})$$

$L_{eff\_CF\_oper} = 11.902\,m$ $\quad$ $ERR = 1.552$

Static deflection (installation)

$$\delta_{inst_i} := C_6 \cdot \frac{W_{s_i} \cdot \left(L_{eff\_CF\_inst_i}\right)^4}{E \cdot I} \cdot \frac{1}{1 + \frac{S_{eff_i}}{\frac{\pi^2 \cdot E \cdot I \cdot C_2}{\left(L_{eff\_CF\_inst_i}\right)^2}}}$$

$\delta_{inst_i} =$

| | |
|---|---|
| 0.211 | m |
| 0.297 | |
| 0.224 | |
| -0.055 | |

Static deflection (operation)

$$\delta_{oper} := C_6 \cdot \frac{W_{s_3} \cdot \left(L_{eff\_CF\_oper}\right)^4}{E \cdot I} \cdot \frac{1}{1 + \frac{S_{eff_3}}{\frac{\pi^2 \cdot E \cdot I \cdot C_2}{\left(L_{eff\_CF\_oper}\right)^2}}}$$

$\delta_{oper} = -0.055\,m$

The relationship between the effective span length and apparent (visual) span length is defined in DNVRPF105.The definition for β is substituted to solve for the apparent length.

Assume first guess value $\quad L := L_{eff\_CF\_inst_0} \quad L_{eff} := L_{eff\_CF\_inst_0}$

Given

$$L = \begin{cases} \dfrac{L_{eff}\cdot\left(-0.066\cdot\log\left(\dfrac{K_L\cdot L^4}{E\cdot I}\right)^2 + 1.02\cdot\log\left(\dfrac{K_L\cdot L^4}{E\cdot I}\right) + 0.63\right)}{4.73} & \text{if } \log\left(\dfrac{K_L\cdot L^4}{E\cdot I}\right) \ge 2.7 \\ \dfrac{L_{eff}\cdot\left(0.036\cdot\log\left(\dfrac{K_L\cdot L^4}{E\cdot I}\right)^2 + 0.61\cdot\log\left(\dfrac{K_L\cdot L^4}{E\cdot I}\right) + 1.0\right)}{4.73} & \text{if } \log\left(\dfrac{K_L\cdot L^4}{E\cdot I}\right) < 2.7 \end{cases}$$

$L_{app\_CF\_inst_0} := \text{Minerr}(L)$

$L_{app\_CF\_inst} = (28.707)\ m$ $\qquad ERR = 0$

Assume first guess value $\quad L := L_{eff\_CF\_inst_1} \quad L_{eff} := L_{eff\_CF\_inst_1}$

Given

$$L = \begin{cases} \dfrac{L_{eff}\cdot\left(-0.066\cdot\log\left(\dfrac{K_L\cdot L^4}{E\cdot I}\right)^2 + 1.02\cdot\log\left(\dfrac{K_L\cdot L^4}{E\cdot I}\right) + 0.63\right)}{4.73} & \text{if } \log\left(\dfrac{K_L\cdot L^4}{E\cdot I}\right) \ge 2.7 \\ \dfrac{L_{eff}\cdot\left(0.036\cdot\log\left(\dfrac{K_L\cdot L^4}{E\cdot I}\right)^2 + 0.61\cdot\log\left(\dfrac{K_L\cdot L^4}{E\cdot I}\right) + 1.0\right)}{4.73} & \text{if } \log\left(\dfrac{K_L\cdot L^4}{E\cdot I}\right) < 2.7 \end{cases}$$

$L_{app\_CF\_inst_1} := \text{Minerr}(L)$ $\qquad L_{app\_CF\_inst_1} = 18.614\ m$ $\qquad ERR = 0$

Assume first guess value $\quad L := L_{eff\_CF\_inst_2} \quad L_{eff} := L_{eff\_CF\_inst_2}$

Given

$$L = \begin{cases} \dfrac{L_{eff}\cdot\left(-0.066\cdot\log\left(\dfrac{K_L\cdot L^4}{E\cdot I}\right)^2 + 1.02\cdot\log\left(\dfrac{K_L\cdot L^4}{E\cdot I}\right) + 0.63\right)}{4.73} & \text{if } \log\left(\dfrac{K_L\cdot L^4}{E\cdot I}\right) \ge 2.7 \\ \dfrac{L_{eff}\cdot\left(0.036\cdot\log\left(\dfrac{K_L\cdot L^4}{E\cdot I}\right)^2 + 0.61\cdot\log\left(\dfrac{K_L\cdot L^4}{E\cdot I}\right) + 1.0\right)}{4.73} & \text{if } \log\left(\dfrac{K_L\cdot L^4}{E\cdot I}\right) < 2.7 \end{cases}$$

$L_{app\_CF\_inst_2} := \mathrm{Minerr}(L)$

$L_{app\_CF\_inst_2} = 14.244\,m$

$ERR = 0$

Assume first guess value

$L := 20 \cdot m$ $\quad L_{eff} := 20 \cdot m$

Given

$$L = \begin{cases} \dfrac{L_{eff} \cdot \left(-0.066 \cdot \log\left(\dfrac{K_L \cdot L^4}{E \cdot I}\right)^2 + 1.02 \cdot \log\left(\dfrac{K_L \cdot L^4}{E \cdot I}\right) + 0.63\right)}{4.73} & \text{if } \log\left(\dfrac{K_L \cdot L^4}{E \cdot I}\right) \geq 2.7 \\ \dfrac{L_{eff} \cdot \left(0.036 \cdot \log\left(\dfrac{K_L \cdot L^4}{E \cdot I}\right)^2 + 0.61 \cdot \log\left(\dfrac{K_L \cdot L^4}{E \cdot I}\right) + 1.0\right)}{4.73} & \text{if } \log\left(\dfrac{K_L \cdot L^4}{E \cdot I}\right) < 2.7 \end{cases}$$

$L_{app\_CF\_oper} := \mathrm{Minerr}(L)$

$L_{app\_CF\_oper} = 12.423\,m$ $\quad ERR = 0$

Cross-Flow VIV (Visual Span Length)

Installation

$$L_{span\_CF\_inst_i} := \begin{cases} L_{eff\_CF\_inst_i} & \text{if } BC = 1 \\ L_{eff\_CF\_inst_i} & \text{if } BC = 2 \\ L_{app\_CF\_inst_i} & \text{if } BC = 3 \end{cases}$$

$$L_{span\_CF\_inst} = \begin{pmatrix} 36.175 \\ 26.152 \\ 21.802 \\ 11.902 \end{pmatrix} m$$

Operation

$$L_{span\_CF\_oper} := \begin{cases} L_{eff\_CF\_oper} & \text{if } BC = 1 \\ L_{eff\_CF\_oper} & \text{if } BC = 2 \\ L_{app\_CF\_oper} & \text{if } BC = 3 \end{cases}$$

$L_{span\_CF\_oper} = 11.902\,m$

## 4. SUMMARY OF RESULTS

The maximum allowable free spans have been calculated for global, local buckling criteria and onset of in-line and cross-flow vibrations. 4 load cases have been considered:
1. Installed with the pipe empty (defined as depressurised, pipe empty); and;
2. Installation with the pipe flooded (defined as depressurised , pipe flooded);
3. Hydrotest prior to operation (defined as system pressure test where the pipe is flooded);
4. Pipe in operation (under design pressure).

For load case 4 (pipe in operation), the pipe is considered operating on seabed (not trenched or buried).
For load cases 1, 2 and 3, the 1-year return wave and 1-year return current are used to derive the hydrodynamic forces at the pipe.
For load case 4, the 100-year return wave and 10-year return current are used to derive the hydrodynamic forces at the pipe.
For determining the maximum allowable free span lengths to avoid in-line and cross-flow VIVs, the significant wave height along with the wave peak period and associated seabed current are used. For determining the maximum allowable free span length to avoid local buckling, the maximum wave height along with the wave maximum period and associated seabed current are used.

### 1. Local Buckling Condition

Load case 1 -Pipe installed Empty $L_{span\_empty} = 100.148\ m$

Load case 2 -Pipe installed Flooded $L_{span\_flood} = 30.961\ m$

Load case 3 - Hydrotest $L_{span\_hyd} = 24.001\ m$

Load case 4 -Pipe in Operation $L_{span\_operation} = 10.245\ m$

### 2. Glocal Buckling Condition

Load case 1 -Pipe installed Empty $L_{span\_globuck\_empty} = 70504029005578530\ m$ (pipe does not buckle)

Load case 2 -Pipe installed Flooded $L_{span\_globuck\_flood} = 35.766\ m$

Load case 3 -Hydrotest $L_{span\_globuck\_hyd} = 26.015\ m$

Load case 4 -Pipe in Operation $L_{span\_globuck\_oper} = 11.573\ m$

### 3. In-line VIV Condition

Load case 1 -Pipe installed Empty $L_{span\_IL_0} = 54.725\ m$

Load case 2 -Pipe installed Flooded $L_{span\_IL_1} = 30.189\ m$

Load case 3 -Hydrotest $L_{span\_IL_2} = 23.136\ m$

Load case 4 -Pipe in Operation $L_{span\_IL_3} = 11.475\ m$

### 4. Cross-flow VIV Condition

Load case 1 -Pipe installed Empty $L_{span\_CF\_inst_0} = 36.175\ m$

Load case 2 -Pipe installed Flooded $L_{span\_CF\_inst_1} = 26.152\,m$

Load case 3 -Hydrotest $L_{span\_CF\_inst_2} = 21.802\,m$

Load case 4 -Pipe in Operation $L_{span\_CF\_oper} = 11.902\,m$

$$Summary\_Allowable\_span\_length := \begin{pmatrix} \text{"LOADCASES"} & \text{"As-laid"} & \text{"Flooded"} & \text{"Hydrotest"} & \text{"Operation"} \\ \text{"Local Buckling"} & L_{span\_empty} & L_{span\_flood} & L_{span\_hyd} & L_{span\_operation} \\ \text{"Global Buckling"} & \text{"No onset"} & L_{span\_globuck\_flood} & L_{span\_globuck\_hyd} & L_{span\_globuck\_oper} \\ \text{"In-line VIV"} & L_{span\_IL_0} & L_{span\_IL_1} & L_{span\_IL_2} & L_{span\_IL_3} \\ \text{"Cross-flow VIV"} & L_{span\_CF\_inst_0} & L_{span\_CF\_inst_1} & L_{span\_CF\_inst_2} & L_{span\_CF\_oper} \end{pmatrix}$$

$$Summary\_Allowable\_span\_length = \begin{pmatrix} \text{"LOADCASES"} & \text{"As-laid"} & \text{"Flooded"} & \text{"Hydrotest"} & \text{"Operation"} \\ \text{"Local Buckling"} & 100.148 & 30.961 & 24.001 & 10.245 \\ \text{"Global Buckling"} & \text{"No onset"} & 35.766 & 26.015 & 11.573 \\ \text{"In-line VIV"} & 54.725 & 30.189 & 23.136 & 11.475 \\ \text{"Cross-flow VIV"} & 36.175 & 26.152 & 21.802 & 11.902 \end{pmatrix} m$$

**Note:** For α less than 0.5 in-line VIV is negligible. For α between 0.5 to 0.8 in-line VIV is mitigated due to the presence of waves.

$\alpha_{inst} = 1$ $\alpha_{oper} = 1$

$$L\_span\_max\_0 := \begin{cases} \min\left(L_{span\_empty}, L_{span\_globuck\_empty}, L_{span\_IL_0}, L_{span\_CF\_inst_0}\right) & \text{if } \alpha_{inst} > 0.8 \\ \min\left(L_{span\_empty}, L_{span\_globuck\_empty}, L_{span\_CF\_inst_0}\right) & \text{otherwise} \end{cases}$$

$L\_span\_max\_0 = 36.175\,m$ Unpressurized, Empty

$$L\_span\_max\_1 := \begin{cases} \min\left(L_{span\_flood}, L_{span\_globuck\_flood}, L_{span\_IL_1}, L_{span\_CF\_inst_1}\right) & \text{if } \alpha_{inst} > 0.8 \\ \min\left(L_{span\_flood}, L_{span\_globuck\_flood}, L_{span\_CF\_inst_1}\right) & \text{otherwise} \end{cases}$$

$L\_span\_max\_1 = 26.152\,m$ Unpressurized, Flooded

$$L\_span\_max\_2 := \begin{cases} \min\left(L_{span\_hyd}, L_{span\_globuck\_hyd}, L_{span\_IL_2}, L_{span\_CF\_inst_2}\right) & \text{if } \alpha_{inst} > 0.8 \\ \min\left(L_{span\_hyd}, L_{span\_globuck\_hyd}, L_{span\_CF\_inst_2}\right) & \text{otherwise} \end{cases}$$

$L\_span\_max\_2 = 21.802\,m$

$$L\_span\_max\_3 := \begin{cases} \min\left(L_{span\_operation}, L_{span\_globuck\_oper}, L_{span\_IL_3}, L_{span\_CF\_oper}\right) & \text{if } \alpha_{inst} > 0.8 \\ \min\left(L_{span\_operation}, L_{span\_globuck\_oper}, L_{span\_CF\_oper}\right) & \text{otherwise} \end{cases}$$

$L\_span\_max\_3 = 10.245\,m$ Pipe in operation

$$Allowable\_span\_length := \begin{pmatrix} \text{""} & \text{"As-laid"} & \text{"Flooded"} & \text{"Hydrotest"} & \text{"Operation"} \\ \text{"Gas Export line"} & L\_span\_max\_0 & L\_span\_max\_1 & L\_span\_max\_2 & L\_span\_max\_3 \end{pmatrix}$$

$$Allowable\_span\_length = \begin{pmatrix} \text{""} & \text{"As-laid"} & \text{"Flooded"} & \text{"Hydrotest"} & \text{"Operation"} \\ \text{"Gas Export line"} & 36.175 & 26.152 & 21.802 & 10.245 \end{pmatrix} m$$

## Maximum Allowable Free span length at installation (regardless of load case)

$$L\_span\_max\_inst := \min(L\_span\_max\_0, L\_span\_max\_1, L\_span\_max\_2)$$

$L\_span\_max\_inst = 21.802\,m$

## Maximum Allowable Free span length at operation

$$L\_span\_max\_oper := L\_span\_max\_3$$

$L\_span\_max\_oper = 10.245\,m$