ST203: R for Data Science and Statistics

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Assignment 3 – 2021

- Do all questions: only one randomly chosen question will be marked.
- Upload your script file via Moodle before 23:59 on Friday 26 November.
- You may include your code and your *commented* answers in the same script file.
- You may submit either an R script (‘.R‘) or an R Markdown file (‘.Rmd‘).
- Place your name and student number on the first line of your R script or in the YAML header in your R Markdown file.

Question 1

Type in the function `qroots` given in class. Investigate what happens to this version of qroots when $a = 0$. Fix the program so it gives the correct answer in this case.

Question 2

a) Write and test a function which generates the first n Fibonacci numbers.

b) Construct a sequence of ratios of the form f_n/f_{n-1}, $n = 2, \ldots, 30$ where f_n is the nth Fibonacci number. Does the sequence appear to be converging? Check by plotting the values of f_n/f_{n-1} versus n.

c) Compute the golden ratio $(1 + \sqrt{5})/2$. Is the sequence converging to this ratio?

Question 3

Use the `matrix`, `seq` and `rep` functions to construct the following 5x5 Hankel matrix.

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 8 & 9 \end{bmatrix}$$

Convert the code into a function (with a single argument, `n`) which can be used to construct matrices of dimension $n \times n$ which have the same pattern. Use the function to output 10×10 and 12×12 Hankel matrices.

Question 4

a) Construct a matrix X whose first column contains the numbers 1,2,3,4 and whose second column contains the squares of the numbers in the first column.

b) Calculate $X^\top X$ and its inverse. Verify the result (i.e. show that the matrix multiplied by its inverse is the identity) using `crossprod`.

c) Calculate $XX^\top$ and its inverse. Why is there a problem? Check the determinant using the function `det` and explain.

Question 5

Suppose you have a loan of 1,000 euro. This is subject to an annual interest rate of 11%, compounded monthly. (This means a debt of d is $d(1 + 0.11/12)$ after a month). You make monthly repayments of 12 euro. Use a `while` loop to aid you with the following questions:

a) How long does it take to clear the debt (in months)?

b) Draw a plot showing the amount owed every month.

c) How much total interest is paid in the end?