

Quiz #2

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Your daily commute is distributed normally with mean 10 minutes and standard deviation 2 minutes if there is no convention downtown. However, conventions are scheduled for roughly 1 in 5 days, and your commute time is distributed normally with mean 15 minutes and standard deviation 3 minutes if there is a convention. Let Y be your commute time and $\theta = 1$ indicate there is a convention and $\theta = 0$ if there is no convention.

The following conditional probabilities are interpreted from the problem description.

$$f(y|\theta = 0) \sim N(\mu = 10, \sigma = 2) \quad (1)$$

$$f(y|\theta = 1) \sim N(\mu = 15, \sigma = 3) \quad (2)$$

$$f(\theta = 1) = \frac{1}{5} \quad (3)$$

1.1 a

Give an expression for the prior distribution of θ

Since θ is either 1 or 0, it can be expressed as a Bernoulli Random Variable.

$$f(\theta) = \frac{1}{5} \frac{4^{1-\theta}}{5} \quad (4)$$

1.2 b

Give an expression for the likelihood function. i.e. the PDF of Y given θ

Since θ is a Bernoulli random variable and the parameters for (2) assume $\theta = 1$ whereas (1) assumes $\theta = 0$, then

$$f(y|\theta) \sim N(\mu = 10 + 5\theta, \sigma = \theta + 2) \quad (5)$$

Thus the likelihood function can be described as the PDF of (5)

$$\begin{aligned} f(y|\theta) &= \frac{1}{(\theta + 2)\sqrt{2\pi}} \exp\left(-\frac{(x - (10 + 5\theta))^2}{2(\theta + 2)^2}\right) \\ &= \frac{1}{(\theta + 2)\sqrt{2\pi}} \exp\left(-\frac{(x - 10 - 5\theta)^2}{2(\theta + 2)^2}\right) \end{aligned} \quad (6)$$

1.3 c

Give an expression for the probability there was a convention downtown given that your commute time was $Y = 16$ minutes.

$$\begin{aligned} f(\theta|Y = 16) &= \frac{f(Y = 16|\theta)f(\theta)}{f(Y = 16|\theta = 1) \times f(\theta = 1) + f(Y = 16|\theta = 0) \times f(\theta = 0)} \\ &= \frac{\frac{1}{(\theta+2)\sqrt{2\pi}} \exp\left(-\frac{(16-10-5\theta)^2}{2(\theta+2)^2}\right) \cdot \frac{1}{5} \frac{4^{1-\theta}}{5}}{\frac{1}{2\sqrt{2\pi}} \exp\left(-\frac{6^2}{2 \times 2^2}\right) \frac{4}{5} + \frac{1}{2\sqrt{2\pi}} \exp\left(-\frac{6^2}{2 \times 2^2}\right) \frac{4}{5}} \\ &= \frac{1}{(\theta + 2)\sqrt{2\pi}} \exp\left(-\frac{(6 - 5\theta)^2}{2(\theta + 2)^2}\right) \cdot \frac{1}{5} \frac{4^{1-\theta}}{5} \cdot \frac{1}{0.02693} \end{aligned} \quad (7)$$

1.4 d

The answer to (c) is a probability assigned to θ . Given we have observed $Y = 16$, is θ a random variable? In what sense is the answer to (c) a meaningful probability? (Answer this from a Bayesian perspective in 50 words or less)

$\theta|Y = 16$ is *not* a random variable. If Y is fixed, θ becomes a constant which can be plugged into (c) to produce a probability. The result of (c) would be meaningful if, taking into consideration prior knowledge about convention frequency, contingency plans regarding the convention need to be made for traffic or otherwise.