Quiz #2

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> Your daily commute is distributed normally with mean 10 minutes and standard deviation 2 minutes if there is no convention downtown. However, conventions are scheduled for roughly 1 in 5 days, and your commute time is distributed normally with mean 15 minutes and standard deviation 3 minutes if there is a convention. Let Y be your commute time and $\theta = 1$ indicate there is a convention and $\theta = 0$ if there is no convention.

The following conditional probabilities are interpreted from the problem description.

$$f(y|\theta = 0) \sim N(\mu = 10, \sigma = 2) \tag{1}$$

$$f(y|\theta = 1) \sim N(\mu = 15, \sigma = 3) \tag{2}$$

$$f(\theta = 1) = \frac{1}{5} \tag{3}$$

1.1 a

> Give an expression for the prior distribution of θ

Since θ is either 1 or 0, it can be expressed as a Bernoulli Random Variable.

$$f(\theta) = \frac{1}{5}^{\theta} \frac{4}{5}^{1-\theta} \tag{4}$$

1.2 b

> Give an expression for the likelihood function. i.e. the PDF of Y given θ

Since θ is a Bernoulli random variable and the parameters for (2) assume $\theta = 1$ whereas (1) assumes $\theta = 0$, then

$$f(y|\theta) \sim N(\mu = 10 + 5\theta, \sigma = \theta + 2) \tag{5}$$

Thus the likelihood function can be described as the PDF of (5)

$$\begin{aligned} f(y|\theta) =& \frac{1}{(\theta+2)\sqrt{2\pi}} exp(-\frac{(x-(10+5\theta))^2}{2(\theta+2)^2}) \\ =& \frac{1}{(\theta+2)\sqrt{2\pi}} exp(-\frac{(x-10-5\theta)^2}{2(\theta+2)^2}) \end{aligned} \tag{6}$$

1.3 c

> Give an expression for the probability there was a convention downtown given that your commute time was Y = 16 minutes.

$$\begin{aligned} f(\theta|Y=16) =& \frac{f(Y=16|\theta)f(\theta)}{f(Y=16|\theta=1)\times f(\theta=1) + f(Y=16|\theta=0)\times f(\theta=0)} \\ =& \frac{\frac{1}{(\theta+2)\sqrt{2\pi}} exp(-\frac{(16-10-5\theta)^2}{2(\theta+2)^2}) \cdot \frac{1}{5}^{\theta} \frac{4}{5}^{1-\theta}}{\frac{1}{2\sqrt{2\pi}} exp(-\frac{6^2}{2\times 2^2})\frac{4}{5} + \frac{1}{2\sqrt{2\pi}} exp(-\frac{6^2}{2\times 2^2})\frac{4}{5}} \\ =& \frac{1}{(\theta+2)\sqrt{2\pi}} exp(-\frac{(6-5\theta)^2}{2(\theta+2)^2}) \cdot \frac{1}{5}^{\theta} \frac{4}{5}^{1-\theta} \cdot \frac{1}{0.02693} \end{aligned} \tag{7}$$

1.4 d

> The answer to (c) is a probability assigned to θ. Given we have observed $Y = 16$, is θ a random variable? In what sense is the answer to (c) a meaningful probability? (Answer this from a Bayesian perspective in 50 words or less)

$\theta|Y = 16$ is *not* a random variable. If Y is fixed, θ becomes a constant which can be plugged into (c) to produce a probability. The result of (c) is meaningful because it quantifies the parametric uncertainty for whether or not a convention is occurring.