Homework #2

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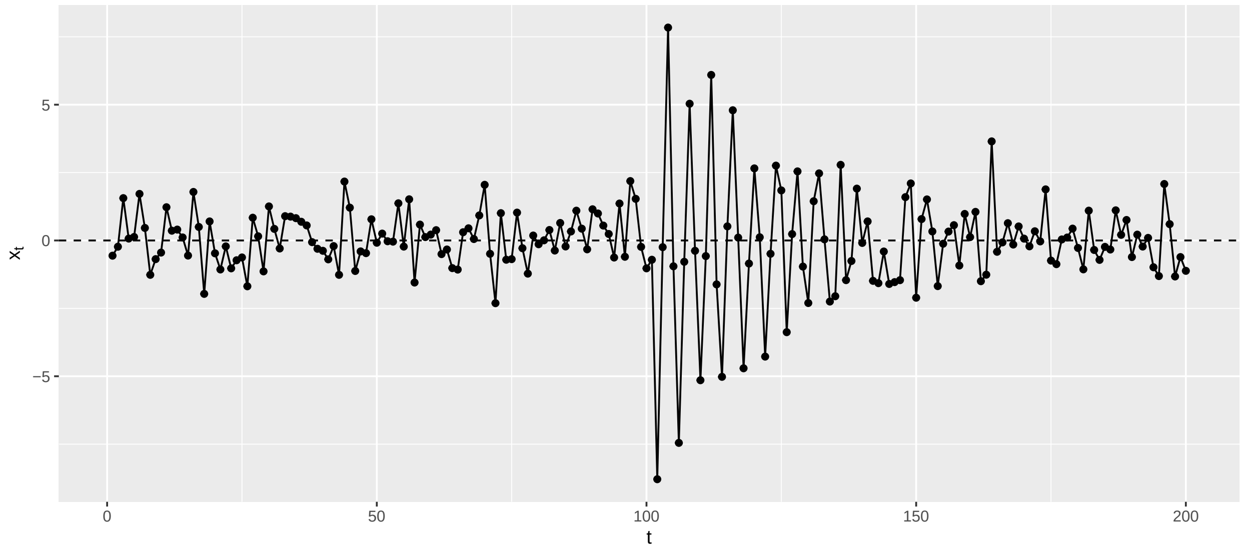
Consider a signal-plus-noise model of the general for $x_t = s_t + w_t$ for t: [1, 100] where $s_t = \begin{cases} 0 & t:[1,100] \\ 10\ exp(-\frac{t-100}{20})cos(\frac{2\pi t}{4}) & t:[101,200] \end{cases}$

and w_t is Gaussian White Noise with $\sigma_w^2 = 1$

a) Simulate (set.seed(123)) and plot n = 200 observations from the model
b) Although the model is not stationary, the ACF can be informative. For the data you generated, plot the ACF then comment

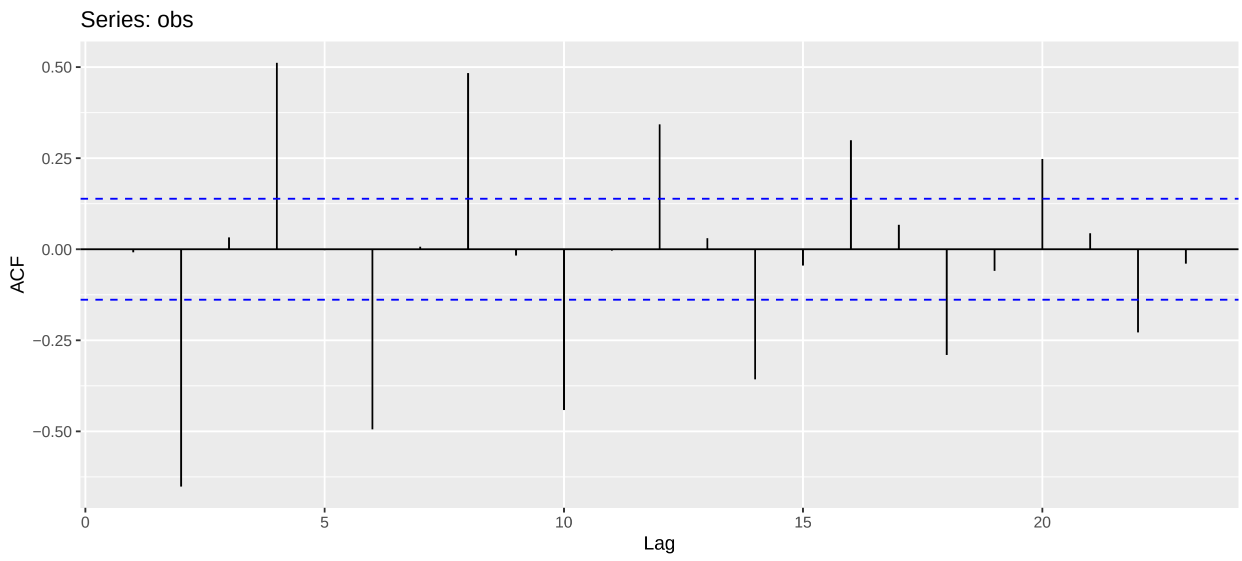
a

```
set.seed(123)

generateSignal <- function(n, cutoff) {
  # assume standard gaussian white noise
  w <- rnorm(n)

  # generate a signal (s)
  s <- function(t, cutoff) {
    ifelse(t <= cutoff, 0, 10 * exp(- (t - 100)/(20)) * cos((2 * pi * t)/(4)))
  }

  # combine white noise and signal
  w + sapply(seq(1, n), function(x) s(x, cutoff))
}

obs <- generateSignal(200, 100)


obs %>%
  enframe %>%
  ggplot(aes(x = name, y = value)) +
    geom_point() +
    geom_hline(yintercept = 0, linetype = "dashed") +
    geom_line() +
    ylab(expression(x[t])) +
    xlab("t")
```



b

```r
autoplot(acf(obs, type = "correlation", plot = FALSE))
```



As the comparison between the current value and the number of lags increases, correlation slightly decreases yet remains significant. This is typically indicative of a trend within the data. The correlation value oscilates between positive and negative values for every other lag indicating that the dependence between the current value and the subsequent lags is *negative*. This can be confirmed by the scatterplot which shows choppiness from t = 100, . . . , 140.

Additionally, even-numbered lags are significantly correlated with the current value of the time series. This may indicate that there is consistent interference with the data based on a multiple of 2. This makes sense since with know $cos(\frac{2\pi t}{4})$ is a component of the signal and thus its frequency being $\frac{1}{4}$.