Homework 5

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1.1 a

Change the step size h to 0.03 and run the program. What are the output values of x and y? After how many iterations?

[x, y] = [0.12463, 2.97990] after **495** iterations

1.2 b

Change the step size h to 0.01, set $\epsilon = 0.00005$, set the maximum number of iterations to be 5000, and run.

What are the output values of x and y? After how many iterations?

[x, y] = [0.052461, 2.991510] after **3253** iterations

1.3 c

Modify the program so that it solves for x and y so that

$$(x-10)^4(y-1)^2 = 0,\ (y-12)^4(x-1)^2 = 0$$

Do not forget to change the jacobian matrix. Use the initial estimates $[x_0, y_0] = [11, 11]$.

Use step size $h = 0.00002$ [note the number of zeros], set $\epsilon = 0.0004$.

Set the maximum number of iterations to be 80,000 and run.

What are the output values of x and y? After how many iterations?

```
Gvect = [(x - 10)^4 * (y - 1)^2, (y - 12)^4 * (x - 1)^2]';

%%% compute the Jacobian matrix

Jmat = [4 * (x - 10)^3 * (y - 1)^2, (y - 12)^4 * 2 * (x - 1); (x - 10)^4 * 2 * (y
```

[x, y] = [10.111, 11.873] after **69490** iterations

2 2

Suppose we want to find the minimum value of

$$f(x,y) = (x-1)^4 + (y-3)^4 + (x-y)^6$$

Apply the method of steepest descent.

For the initial point, set $x_0 = y_0 = 10$.

Here x_0 and y_0 are the x and y coordinates of the vector, x_0.

At each iteration k,

$$x_{k+1} = x_k - hg_k$$

where g_k is the gradient vector of f evaluated at x_k.

Exit the Loop if we reach the max number of iterations or if

$$\frac{1}{2} g_k^T g_k < \epsilon$$

Use the step size $h = 0.00002$, set $\epsilon = 0.0004$, and max iterations to 80,000.

```
h = 0.00002; eps = 0.0004; N = 80000;
x = 10; y = 10;
xvect = [x, y]'
x_and_y = zeros(N,2);

for i = 1:N
  G = [(x - 1)^4 + (y - 3)^4 + (x - y)^6]';
  J = [4 * (x - 1)^3 + 6 * (x - y)^5; 4 * (y - 3)^3 - 6 * (x - y)^5];
  xvect = xvect - h * J;

  x_and_y(i,:) = xvect;
  x = xvect(1);
  y = xvect(2);

  Fvalue = 0.5*(J'*J);

  if Fvalue < eps
    break
  end
end

disp('The approximate solution found by Gradient Descent is ');
disp('x = '); disp( x); disp('y = '); disp(y);

disp('Output after '); disp(i); disp(' iterations  is [x, y] = ');
disp(xvect);
```

2.1 a

What are the output values of x and y? After how many iterations?

[x, y] = [1.6494, 2.3586] after 60112 iterations

2.2 b

What is the minimum value of $f(x, y)$?

0.47433