Homework 2

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4/12/2020

A $100 battery is guaranteed to last 5 years. If it fails to last 5 yeras, the customer received a ful lrefund. It is found that battery life is in fact an exponentional random variable, X, with mean of 30 years. i.e., X has a probability density function,

$$f(x) = \begin{cases} 0, & -\infty < x < 0 \\ \frac{1}{30}e^{-x/30}, & 0 \leq x < \infty \end{cases}$$

a

Compute the expected refund amount for a battery by analytic means.

Let $X_i \sim Exp(\beta = \frac{1}{30})$ and Y be a function of X_i Random Variable.

$$Y_i = f(X_i) = 100 \begin{cases} 1, & 0 \leq x \leq 5 \\ 0, & \text{else} \end{cases}$$

Since $Y_i \sim Bern(p)$, then $E(Y) = 100 \times E(Y) = 100p$

Let $g(x) = \frac{1}{30}e^{-x/30}$ where $g(x)$ is the density function for the aforementioned exponential distribution. Thus $G(x) = 1 - e^{-x/30}$ where $G(x)$ is the Cumulative Distribution Function i.e. integration of $g(x)$.

$p = G(5) = 1 - e^{-1/6} = 0.1535183$

Thus, the expected refund for a given battery is $0.1535183 \times 100 = \$15.35$

b

Assume you don't know the true answer from the above qusetion, and you would use a Monte Carlo method to estimate the expected refund amount. What should be the sumulation size with an error tolerance as 50 cents?

```r
# ratio of acceptable "error" cents to total dollars
init.n <- 1000
rate <- 1/30
cost <- function(x, rate) ifelse(x >= 0 & x <= 5, 100, 0)

init.sample <- rexp(init.n, rate)
init.sample.cost <- cost(init.sample, rate)
init.sample.cost.sigma <- sd(init.sample.cost)

est.n <- floor((2.58 * 1.1 * init.sample.cost.sigma / 0.50)^2)
```

There needs to be 42238 simulations in order to have the estimated value within 50 cents of the true value.

c

> Compute the expected refund amount by using a Monte Carlo method with $n = 10^4$ and $n = 10^6$ samples. What is the error in your method compared to the true answer computed analytically? How much smaller is the error for $n = 10^6$ than $n = 10^4$?

```
refund <- 15.35

est.refund.10000 <- cost(rexp(10^4, rate), rate)
est.refund.10000.mean <- est.refund.10000 %>% mean
est.refund.10000.error <- abs(est.refund.10000.mean - refund)

est.refund.1000000 <- cost(rexp(10^6, rate), rate)
est.refund.1000000.mean <- est.refund.1000000 %>% mean
est.refund.1000000.error <- abs(est.refund.1000000.mean - refund)
```

The expected refund amount with $n = 10^4$ is \$15.79. The error is 0.44.

The expected refund amount with $n = 10^6$ is \$15.3532. The error is 0.0032.

The error for $n = 10^6$ is 137.5 times smaller than the error for $n = 10^4$

d

```
lcl.10000 <- est.refund.10000.mean - 1.96 * sd(est.refund.10000) / sqrt(10^4)
ucl.10000 <- est.refund.10000.mean + 1.96 * sd(est.refund.10000) / sqrt(10^4)

lcl.1000000 <- est.refund.1000000.mean - 1.96 * sd(est.refund.1000000) / sqrt(10^6)
ucl.1000000 <- est.refund.1000000.mean + 1.96 * sd(est.refund.1000000) / sqrt(10^6)
```

95% Confidence Interval for the Average Refund per Battery ($n = 10^4$)

Lower Limit: 15.0752557

Upper Limit: 16.5047443

95% Confidence Interval for the Average Refund per Battery ($n = 10^6$)

Lower Limit: 15.2825421

Upper Limit: 15.4238579

With 95% confidence, each of these intervals contains the true solution. Since the values are randomly generated, there may be realizations of these confidence intervals that do not contain the true value.