Homework #3

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Consider the European call option for an asset price modeled by a geometric Brownian motion with initial asset price \$100, interest rate 3%, volatility 30%, and expiry date 1 year from now

a

According to the formula in the notes, text, or sample computer program on d2l, compute the **exact/true** fair option price for strike prices \$75, \$100, \$130

```
S_0 = 100
r = 0.03
sigma = 0.3
t = 1
K <- c(75, 100, 130)

ExactEuroCall <- S_0 * pnorm(
    (log(S_0/K) + (r + sigma^2/2)*t) /
      (sigma*sqrt(t))
  ) - K * exp(-r * t) * pnorm(
    (log(S_0/K) + (r-sigma^2/2)*t) / (sigma*sqrt(t))
  )

ExactEuroCall
```

```
## [1] 29.106374 13.283308  4.207674
```

b

Using simple Monte Carlo simulation with $n = 10^5$ samples, compute the approximate value of the European call option for the parameter values above. You may use the sample program on d2l. What is the relative error for each strike price? For which strike price is the relative error largest?

```
n <- 10^5 #number of sample paths

#generate n pseudo-random normal numbers (n samples of Brownian motion at maturity)
x <- sqrt(t) * rnorm(n)

#generate n sample stock prices at maturity
S_T <- S_0 * exp((r - sigma^2 / 2) * t + sigma * x)
EuroCallPrice <- c()
EuroPutPrice <- c()
for (k in K) {
  EuroCallPayoff <- pmax(S_T - k, 0) * exp(-r * t)
  EuroPutPayoff <- pmax(k - S_T, 0) * exp(-r * t)
  EuroCallPrice <- c(EuroCallPrice, mean(EuroCallPayoff))
```

```
  EuroPutPrice <- c(EuroPutPrice, mean(EuroPutPayoff))
}

ExactEuroPut <- K * exp(-r*t) * pnorm(
    (log(K / S_0) - (r - sigma^2/2) * t) / (sigma*sqrt(t))
  ) - S_0 *
  pnorm(
    (log(K/S_0) - (r+sigma^2/2)*t) / (sigma*sqrt(t))
  )


error_EuroPut<-abs(EuroPutPrice-ExactEuroPut)/ExactEuroPut
error_EuroCall<-abs(EuroCallPrice-ExactEuroCall)/ExactEuroCall

data <- data.frame(
  "Type" = c("Call", "Call", "Call", "Put", "Put", "Put"),
  "Strike Price" = c(K, K),
  "Exact" = c(ExactEuroCall, ExactEuroPut),
  "Estimate" = c(EuroCallPrice, EuroPutPrice),
  "Estimate Error" = c(error_EuroCall, error_EuroPut)
)

data %>%
  kable(
    caption = "European Option Estimates vs Exact Prices using Black-sholes Formula"
) %>%
  kable_styling(bootstrap_options = "striped", latex_options = "hold_position") %>%
  row_spec(which(data$Estimate.Error >= max(data$Estimate.Error)), color = "black", bold = T)
```

Table 1: European Option Estimates vs Exact Prices using Black-sholes Formula

Type	Strike.Price	Exact	Estimate	Estimate.Error
Call	75	29.106374	29.190503	0.0028904
Call	100	13.283308	13.327388	0.0033184
Call	130	4.207674	4.224599	0.0040224
Put	**75**	**1.889789**	**1.867839**	**0.0116151**
Put	100	10.327862	10.265863	0.0060031
Put	130	30.365593	30.276439	0.0029360

The Strike Price and the row pertaining to the largest relative error is bolded.