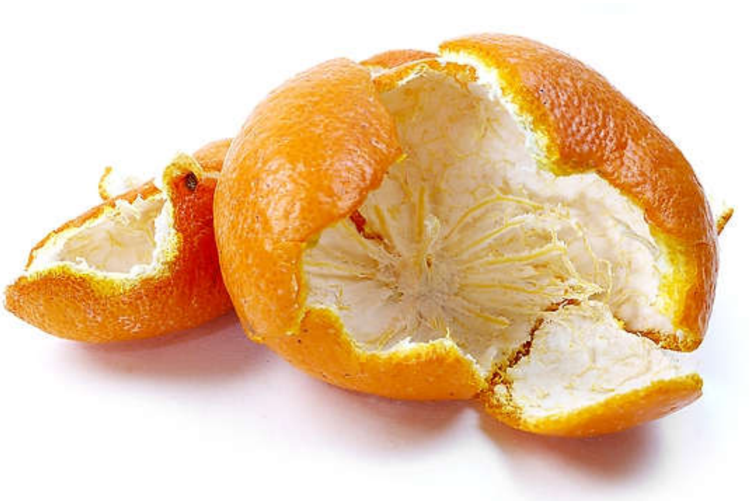
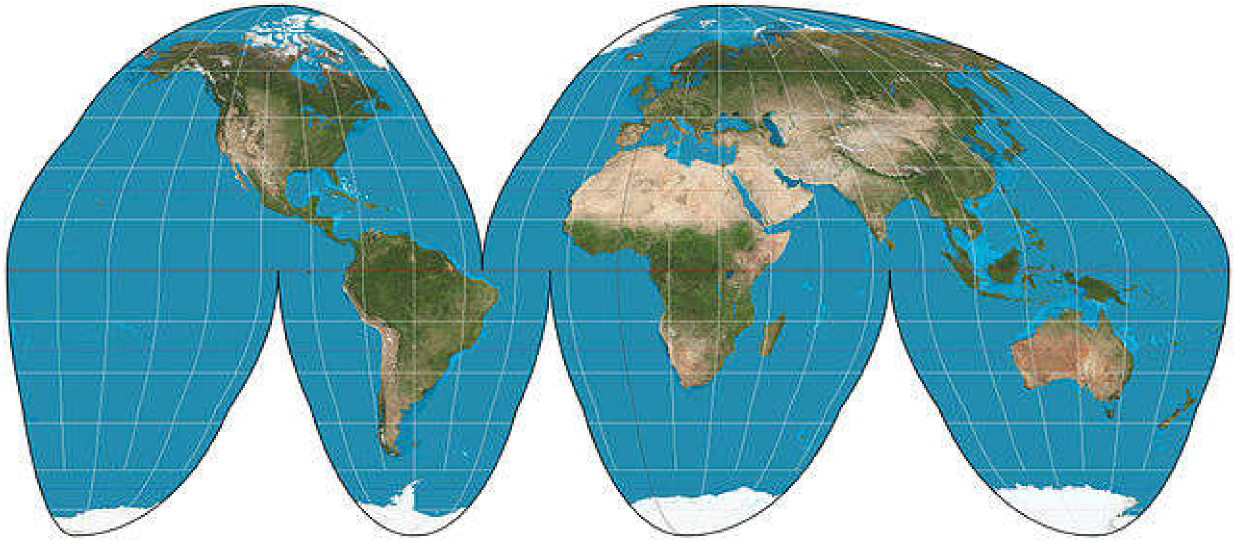
二、Map Projection

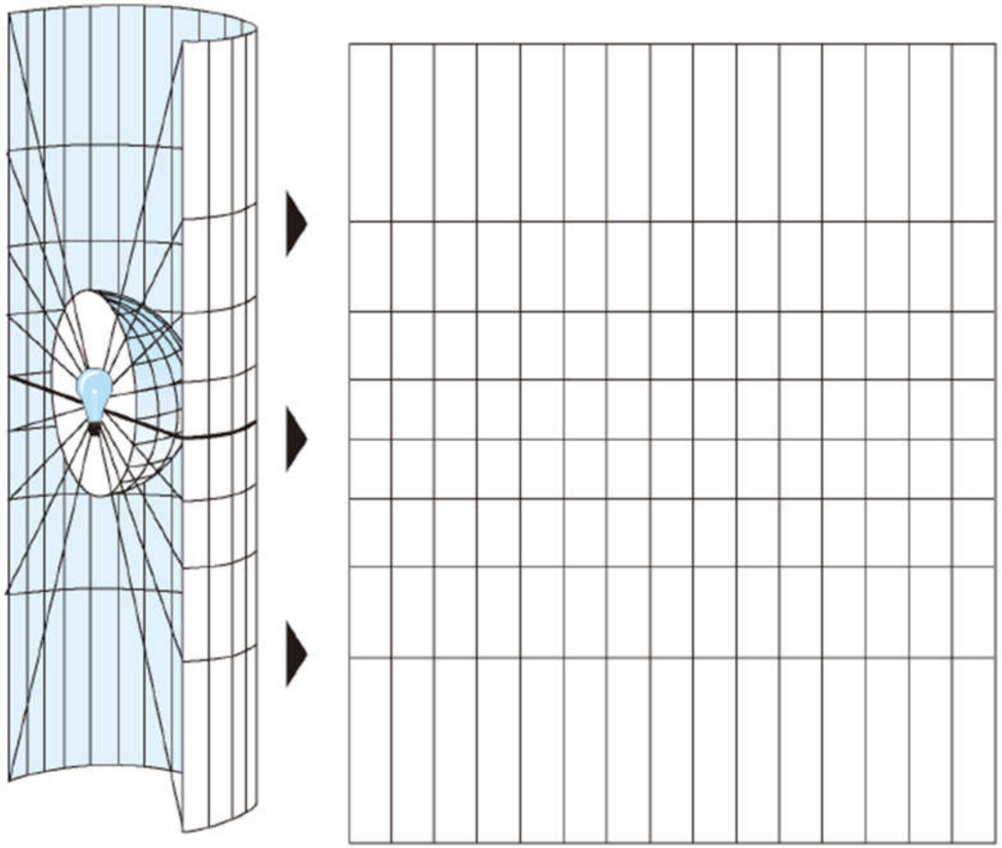
What is projection?

SUSTech | Environment of Science

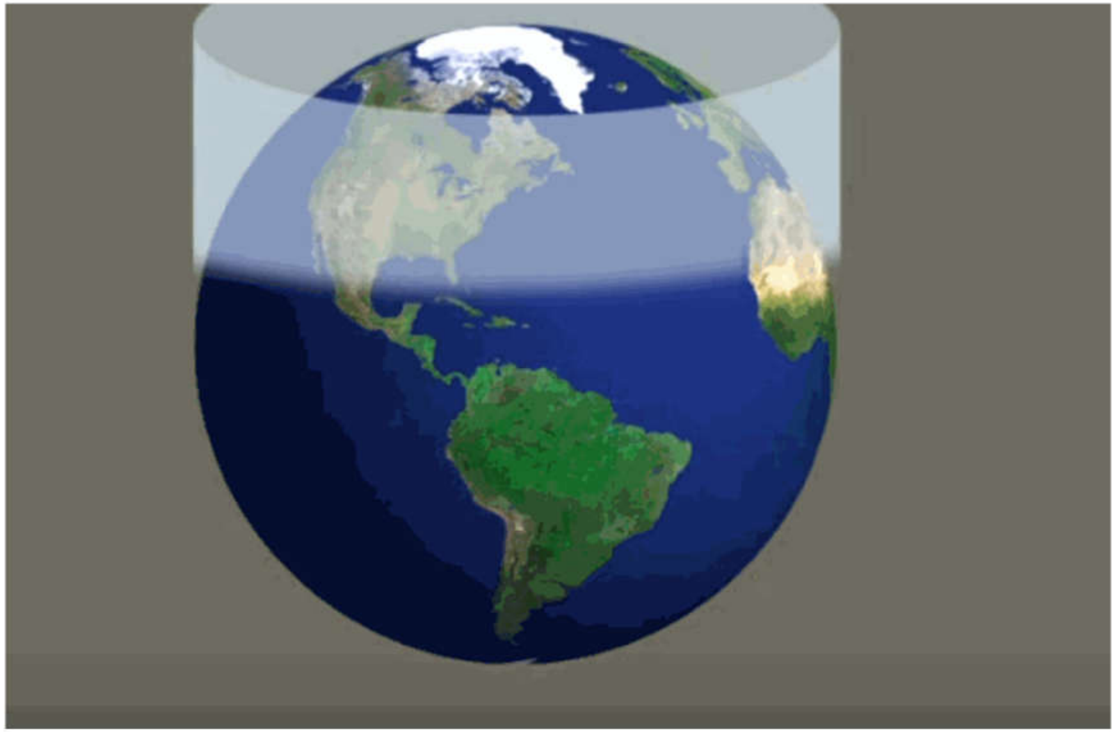
How to understand projection?





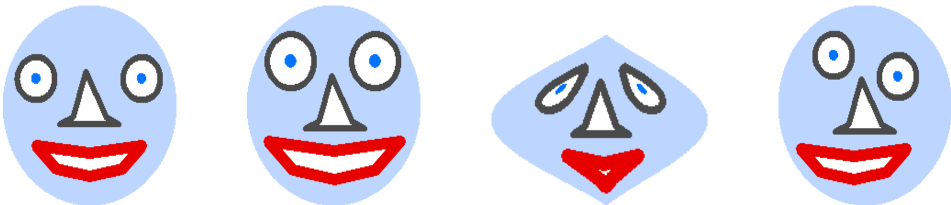


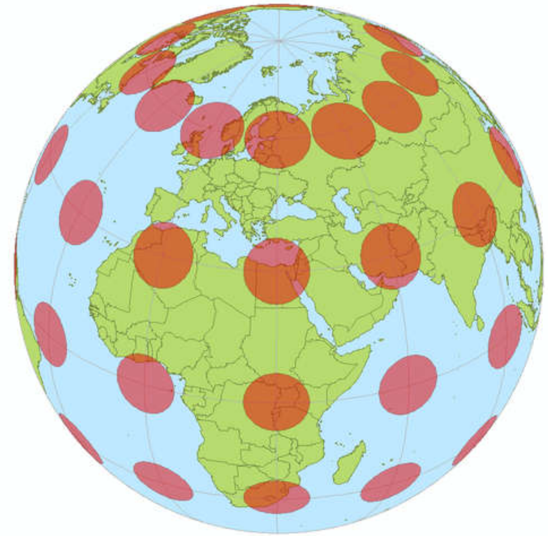
The graticule of a geographic coordinate system is projected onto a cylindrical projection surface.

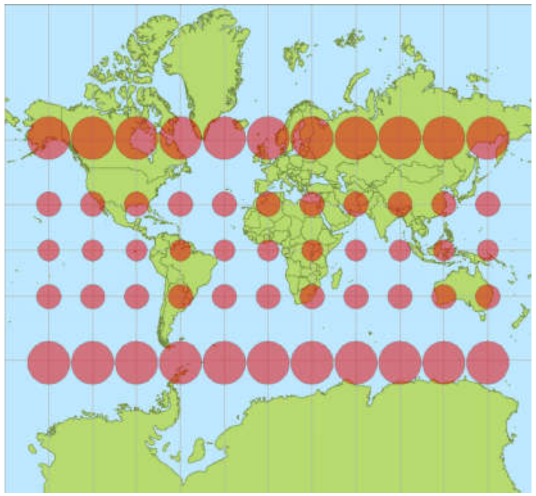


Projection distortion

- 等角投影：
  - 形状相似，矩形房屋投影后仍是矩形
  - 便于判断方向，用于交通图、航海图
- 等面积投影：
  - 面积不变，便于对比大小；
  - 常用于行政图、规划图。

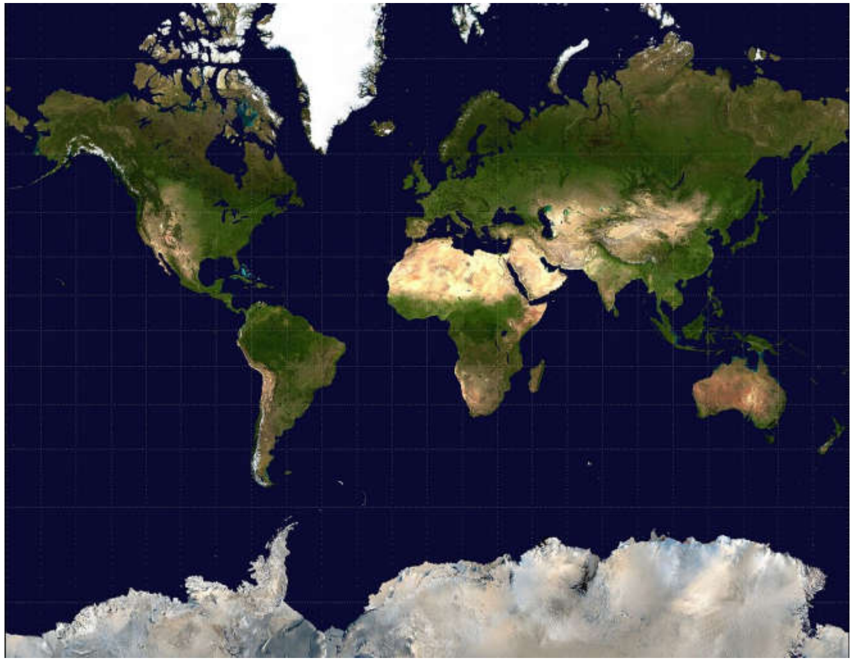




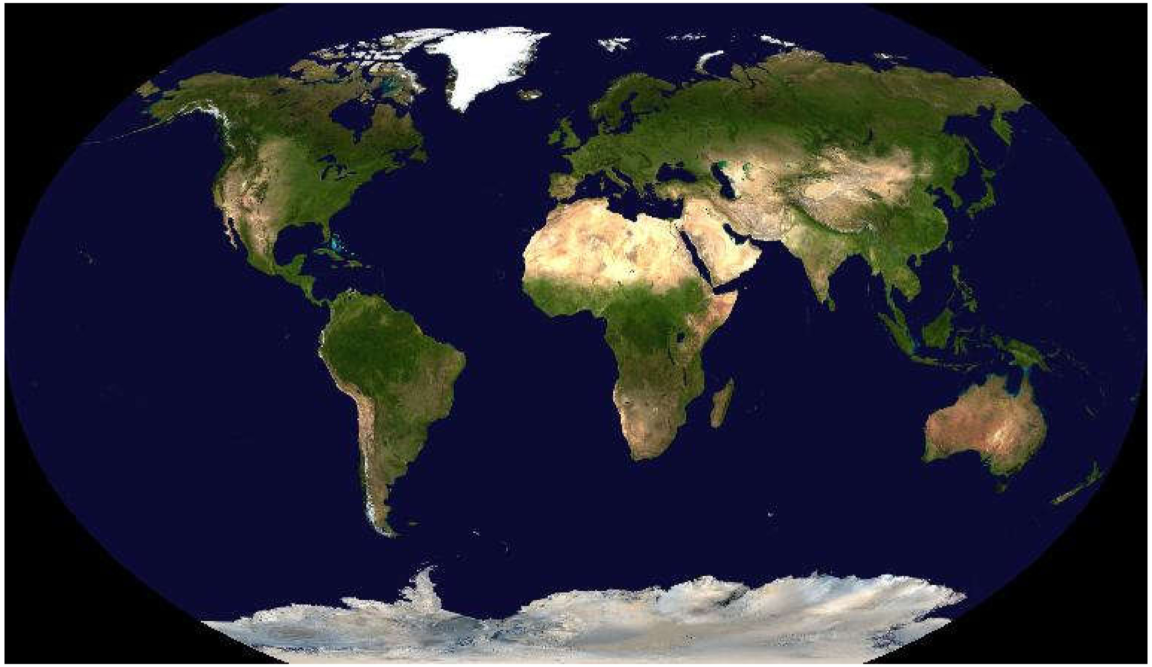


- Conformal projections （等角投影）： preserve local shape.
- Equal area projections （等面积投影）： preserve the area of displayed features.
- Equidistant projections （等距投影）preserve the distances between certain points.
- True-direction (Azimuthal, 等方位投影) projections maintain some of the great circle arcs, giving the directions or azimuths of all points on the map correctly with respect to the center

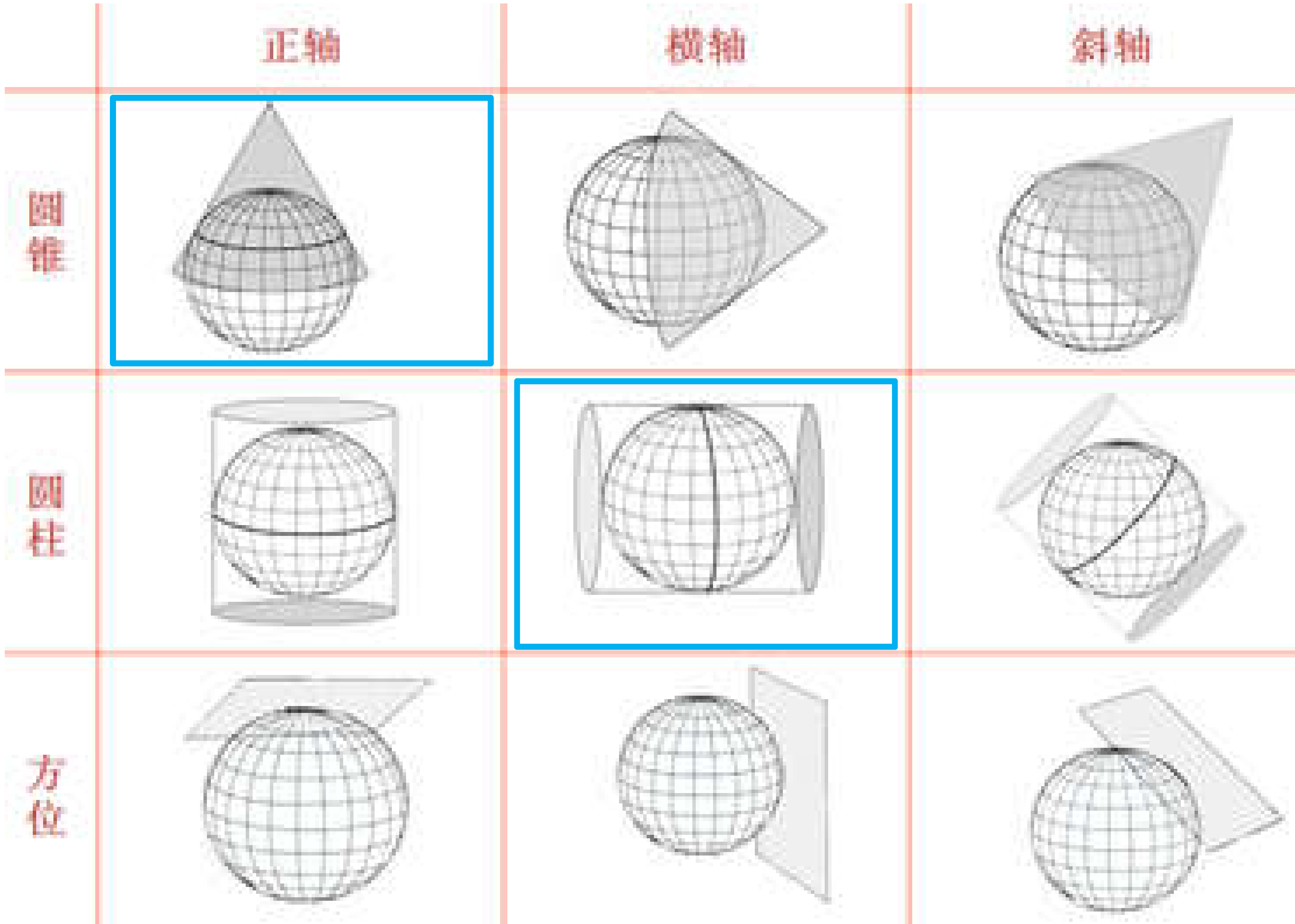
Which map is correct?



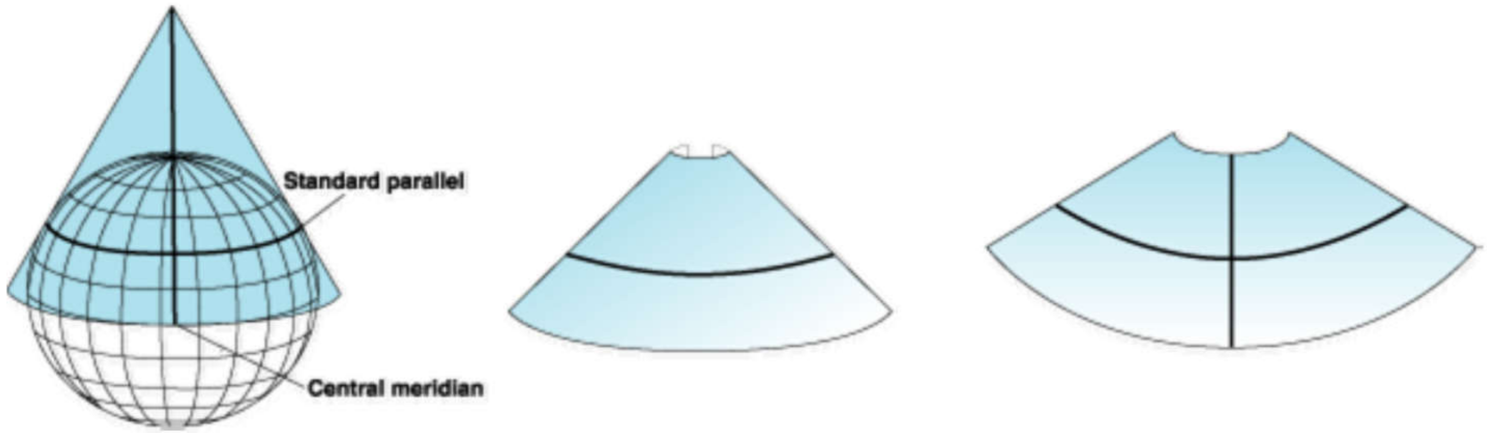
Mercator Projection



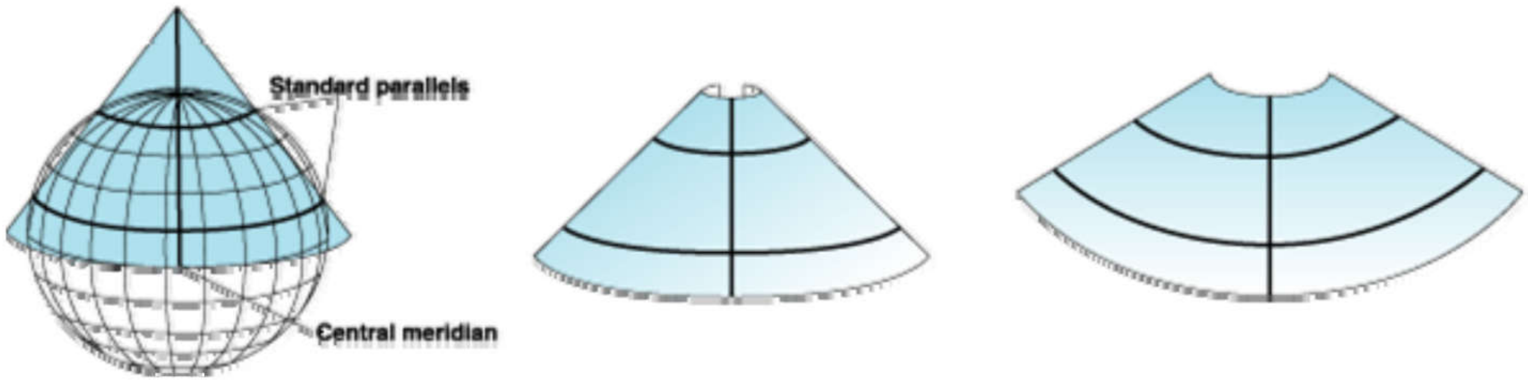
Winkel Tripel Projection


正轴
横轴
斜轴
圆锥
圆柱
方位

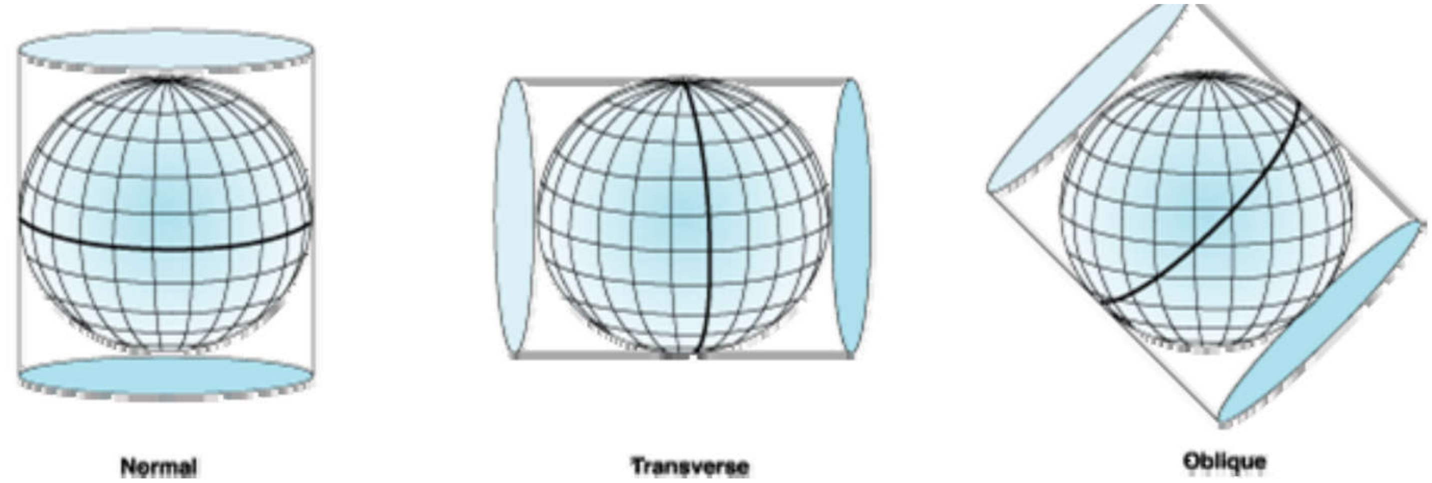
- Conic(tangent)



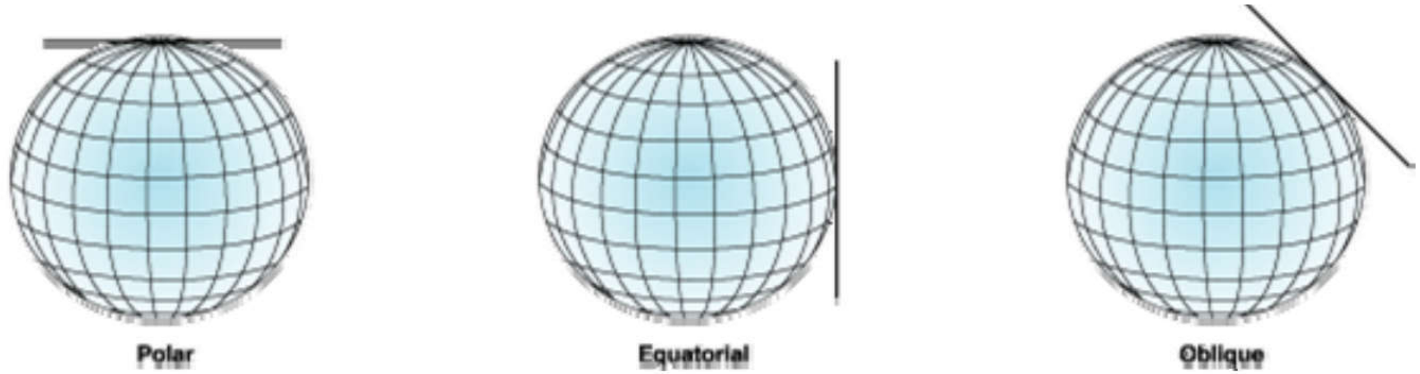
- Conic(secant)



- Cylindrical aspects

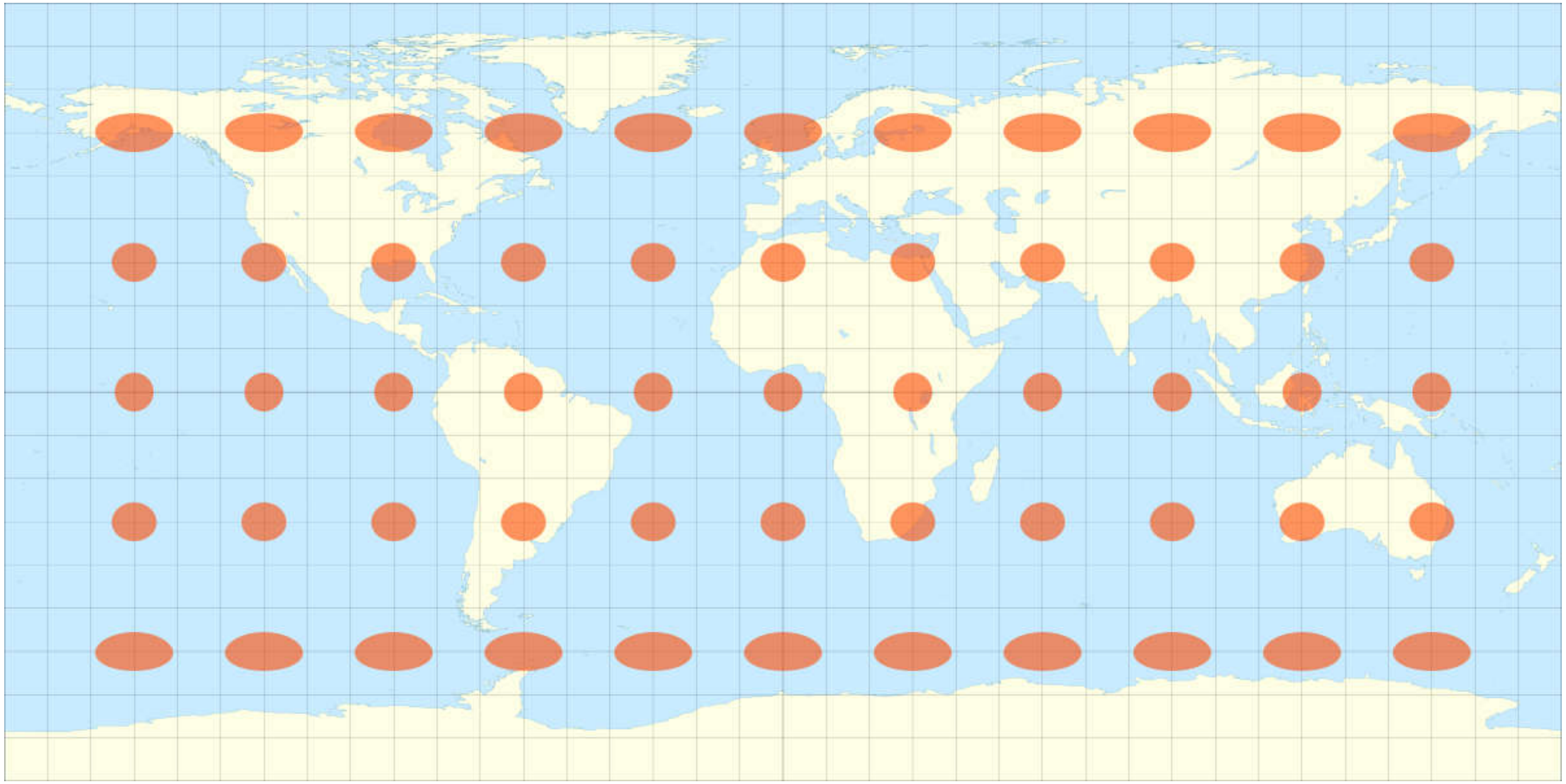


- Planar aspects

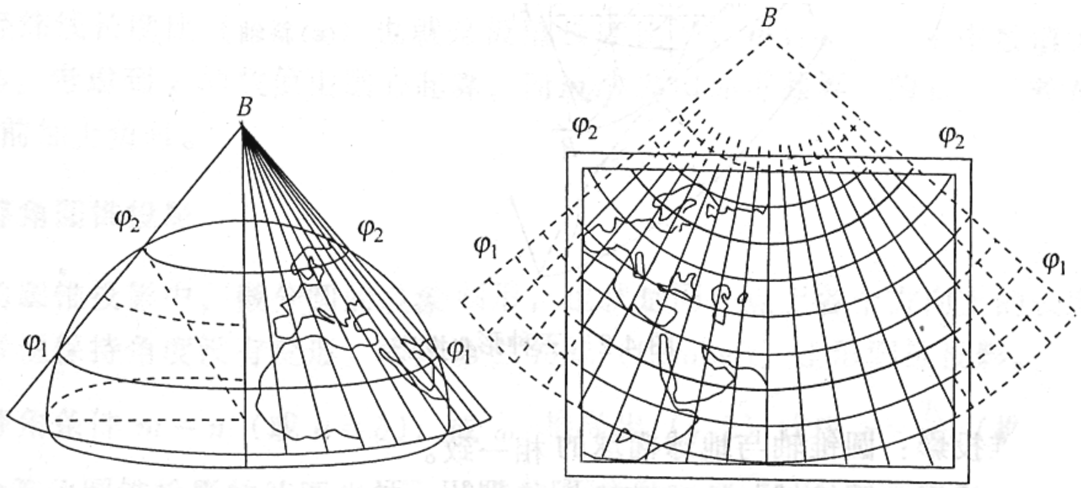


经纬度直投（简易圆柱投影）

X = Longitude Y = Latitude

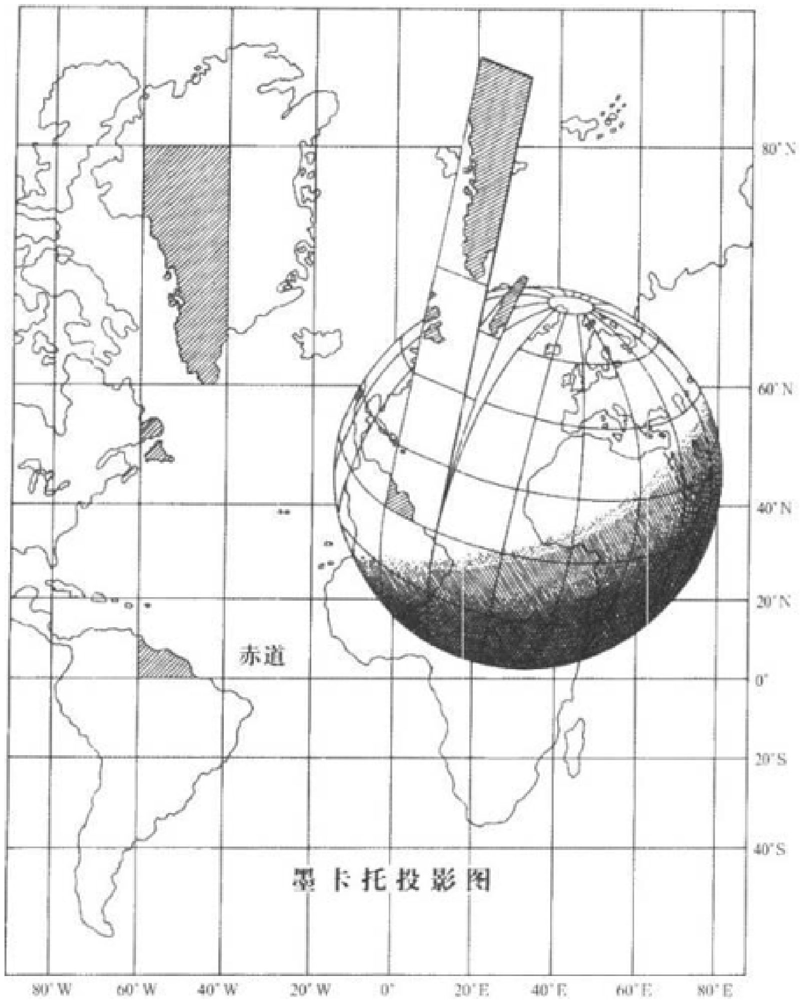


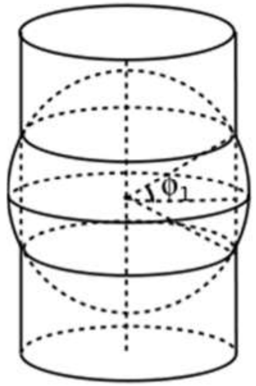
兰勃特(Lambert)或阿尔伯斯(Albers)

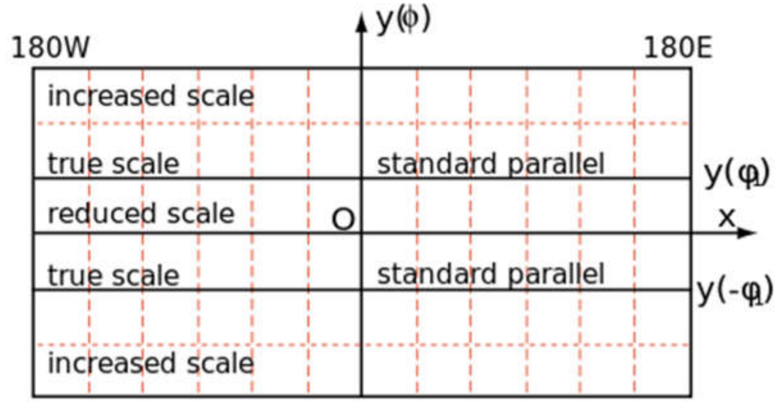


正轴割圆锥投影

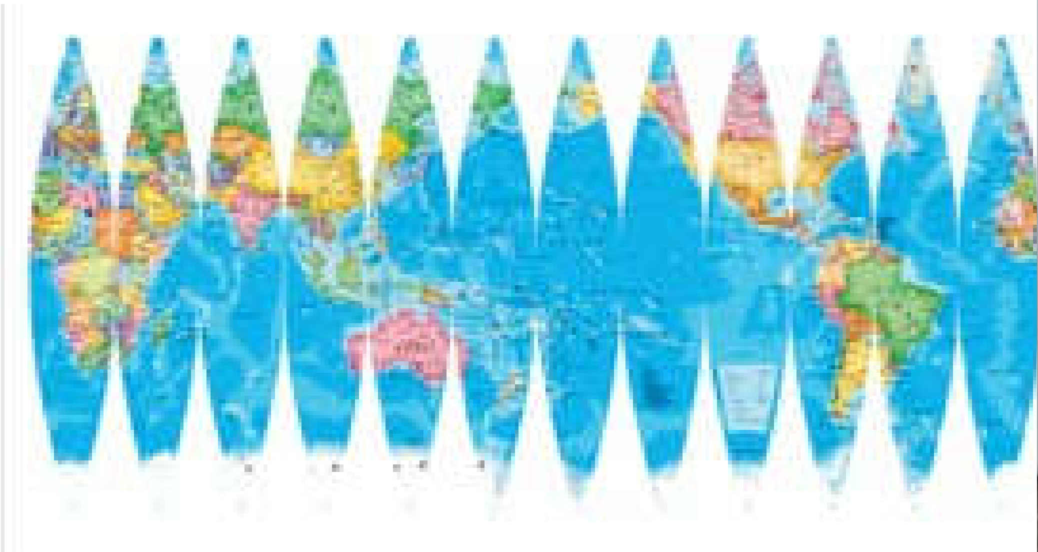
Mercator projection



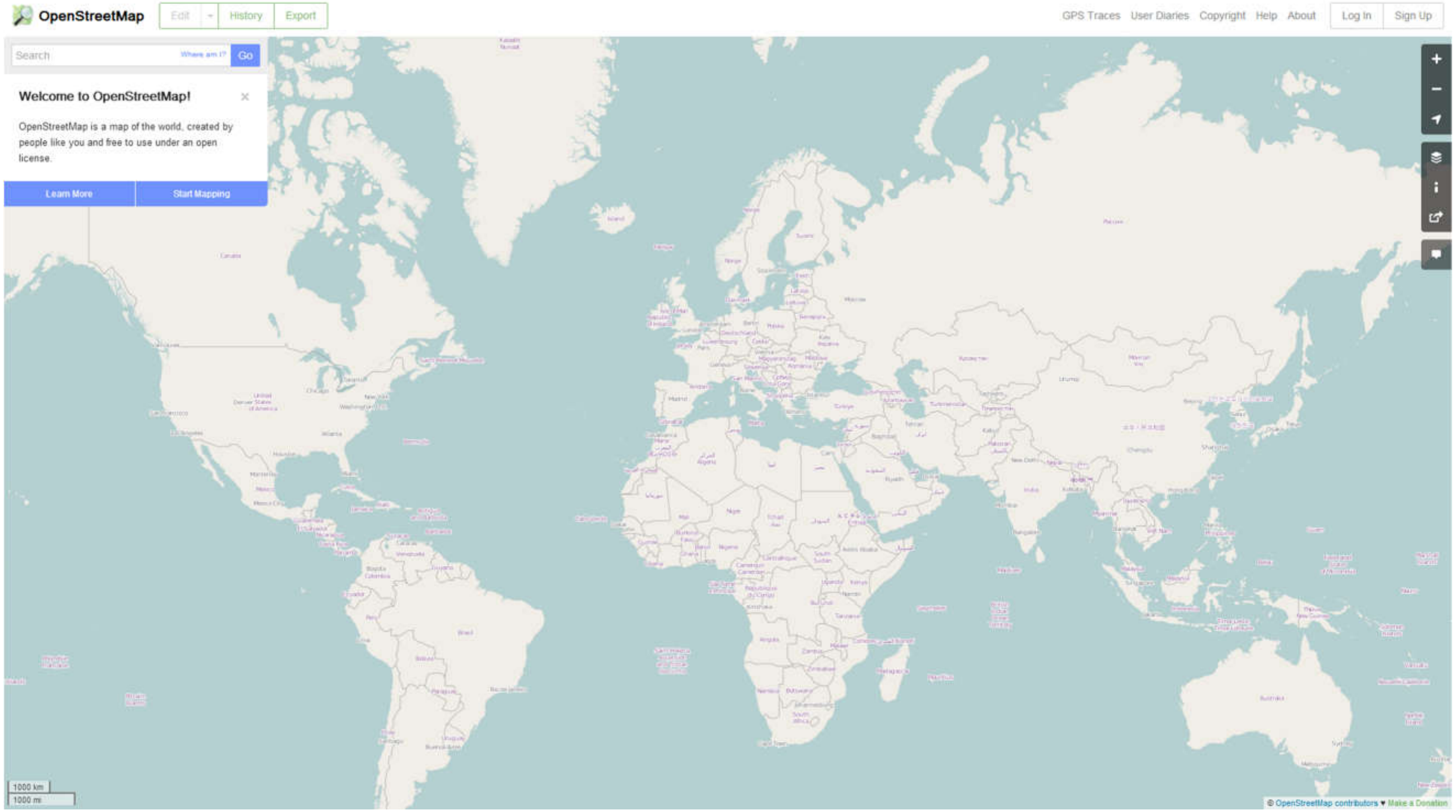




- 分带，像瓜瓣一样



Web Mercator



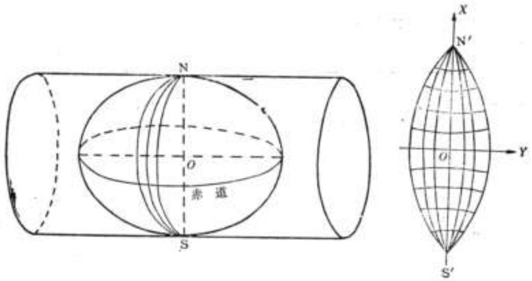
google

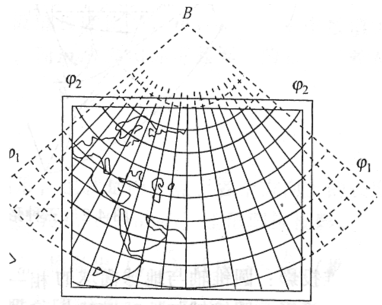
900913 → 3857

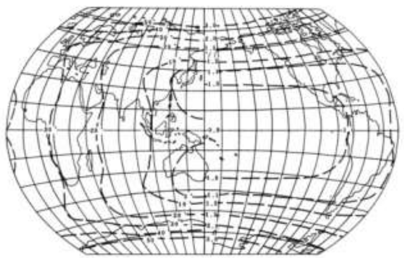
不同投影的适用场景

比例尺 1:1万 | 1:5万 | 1:100万 | 1:1000万

1:1万	1:5万	1:100万	1:1000万
高斯-克吕格 3度分带	高斯-克吕格 6度分带	兰伯特	多圆锥投影

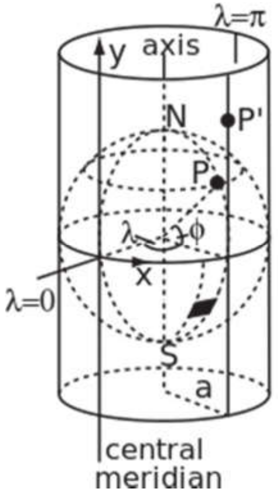


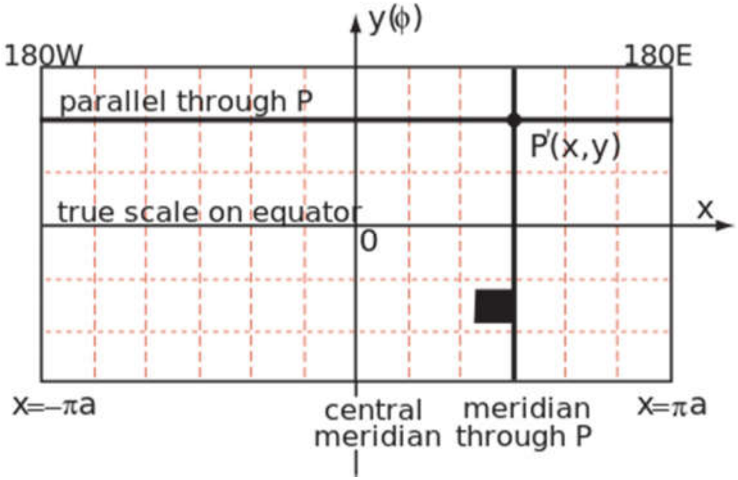


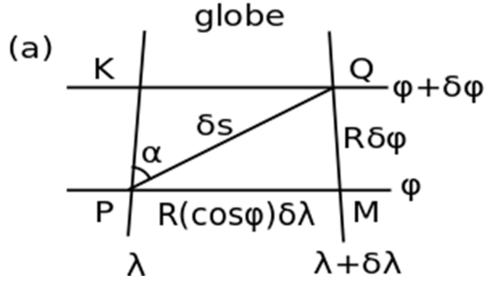


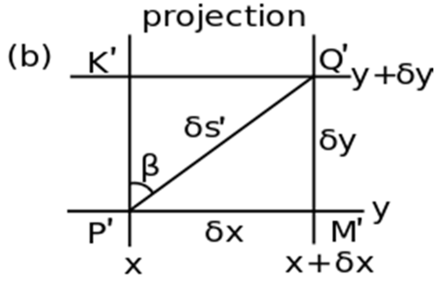
显示范围 城市 | 省域 | 国家 | 世界

Mercator Projection









$$\tan\alpha = \frac{\cos\varphi\delta\lambda}{\delta\varphi}$$

$$\tan\beta = \frac{\delta x}{\delta y} = \frac{R\delta\lambda}{y'\delta\varphi} = \frac{R\tan\alpha}{y'\cos\varphi}$$

$$y = f(\varphi)$$

$$\delta\varphi = \frac{\cos\varphi\delta\lambda}{\tan\alpha}$$

$$y' = \frac{R}{\cos\varphi}$$

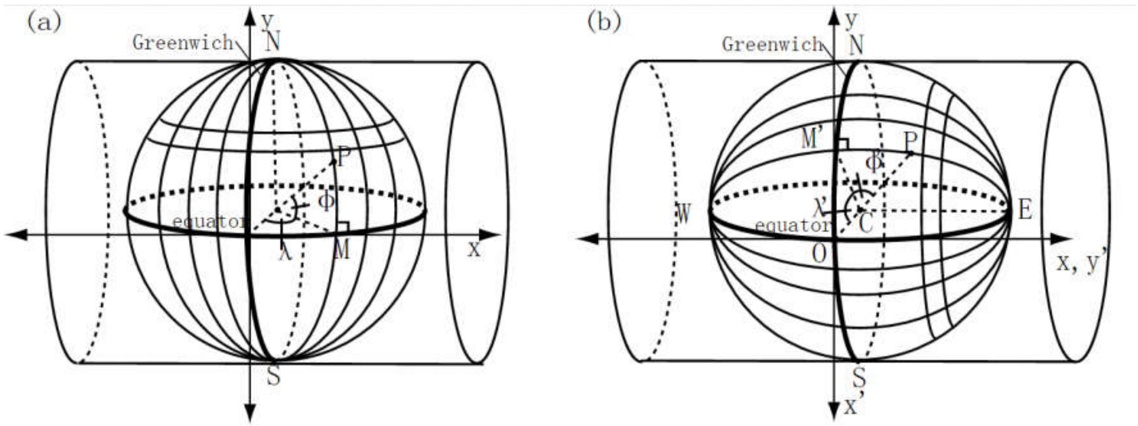
$$x = R\delta\lambda$$

$$u = sin\varphi$$

$$y = \int_0^{\varphi}\frac{R}{cos\varphi}d\varphi = \int_0^{\varphi}\frac{R}{cos\varphi}d\varphi = R\int_0^{\varphi}\frac{cos\varphi}{cos^2\varphi}d\varphi = R\int_0^{u}\frac{1}{1-u^2}du = \frac{R}{2}\int_0^{u}\frac{1}{1+u}+\frac{1}{1-u}du = \frac{R}{2}\ln\left(\frac{1+u}{1-u}\right) = \frac{R}{2}\ln\left(\frac{1+sin\varphi}{1-sin\varphi}\right) = R\ln[\tan(\frac{\pi}{4}+\frac{\varphi}{2})]$$

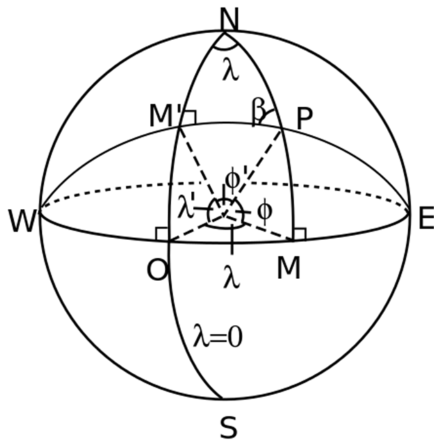
$$x = R(\lambda-\lambda_0), \qquad y = R\ln\left[\tan\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)\right].$$

Transverse Mercator Projection



$$x' = -a\lambda' \qquad y' = \frac{a}{2}\ln\left[\frac{1+\sin\varphi'}{1-\sin\varphi'}\right]$$

Set x = y′ and *y = −x′*

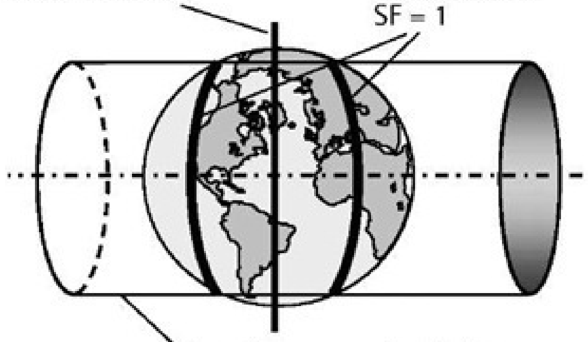


$$\sin\varphi' = \sin\lambda\cos\varphi,$$
$$\tan\lambda' = \sec\lambda\tan\varphi$$

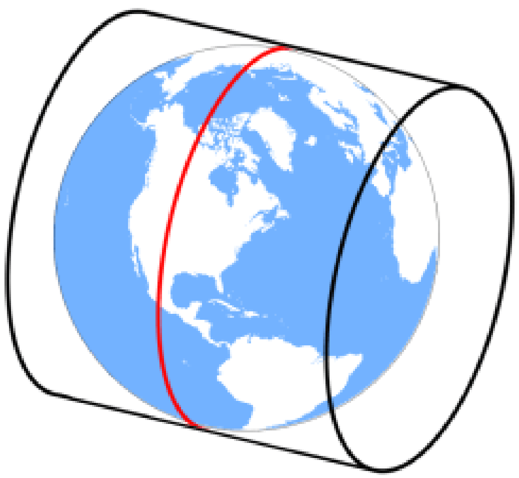
$$x(\lambda,\varphi) = \frac{1}{2}k_0 a\ln\left[\frac{1+\sin\lambda\cos\varphi}{1-\sin\lambda\cos\varphi}\right],$$

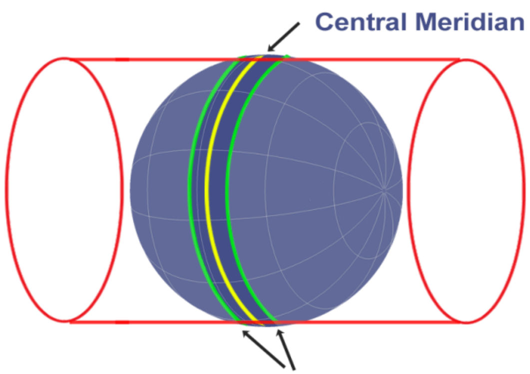
$$y(\lambda,\varphi) = k_0 a\arctan[\sec\lambda\tan\varphi],$$

Universal Transverse Mercator (UTM) projection

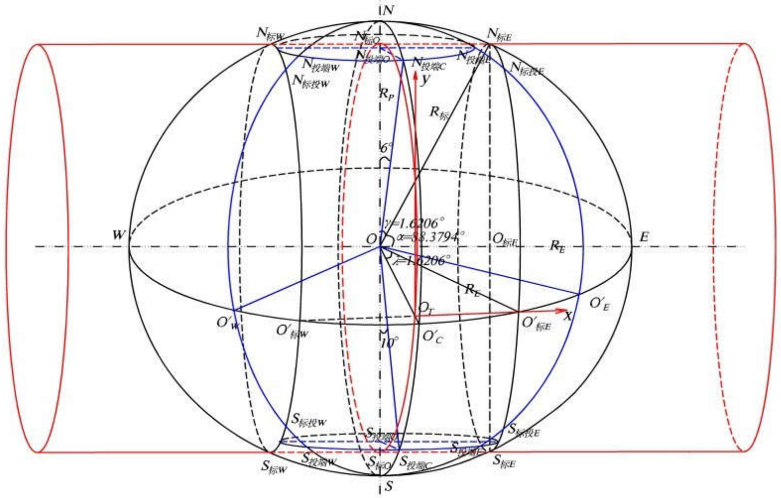


- UTM（Universal Transverse Mercator) Projection
  - ✓ UTM is the transverse secant cylindrical conformal projection
  - ✓ Commonly used by U.S., U.K

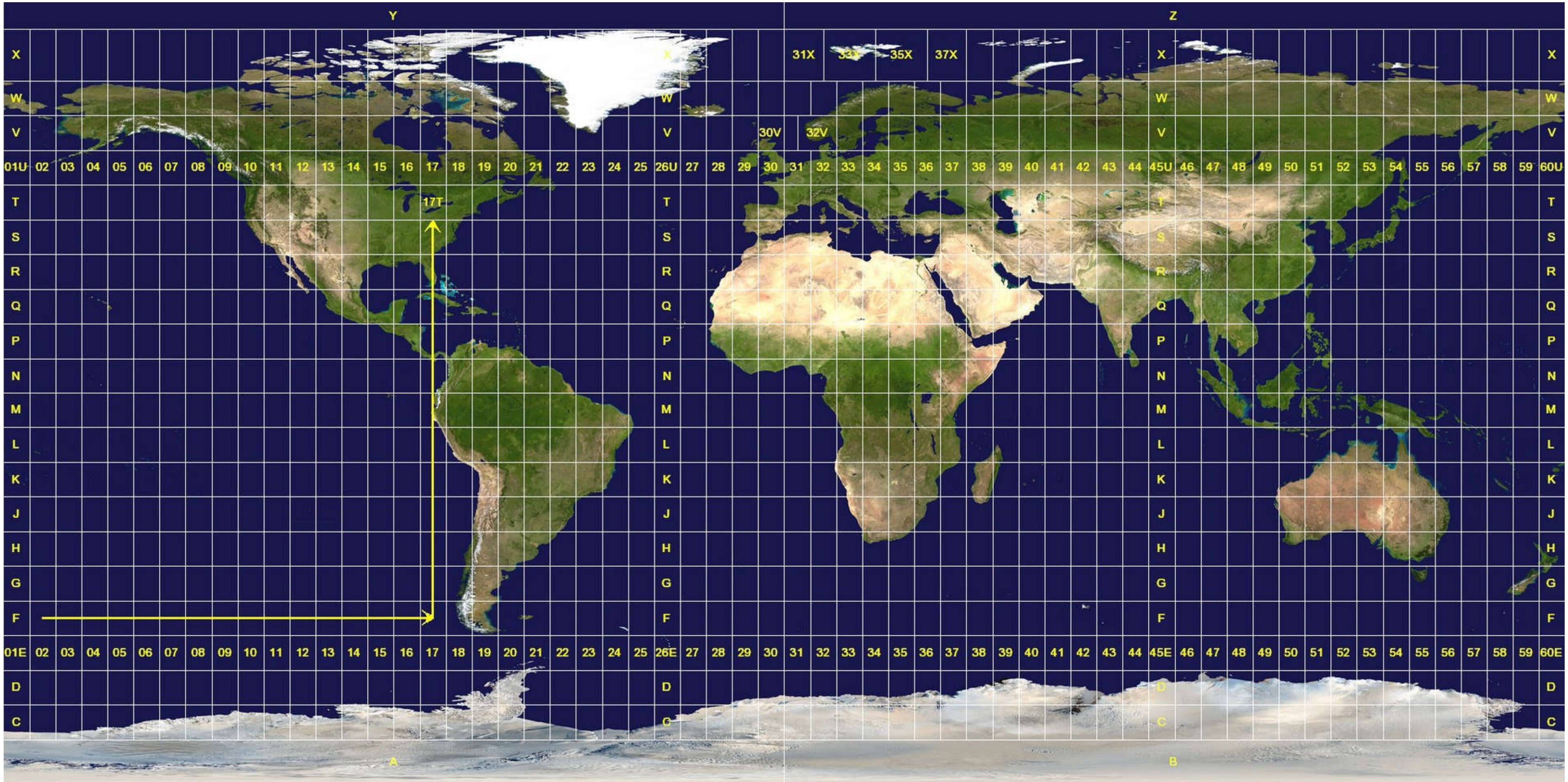




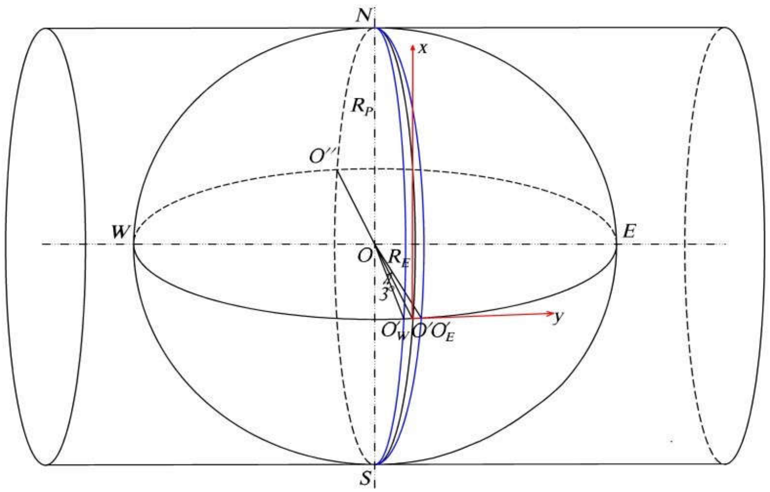
UTM

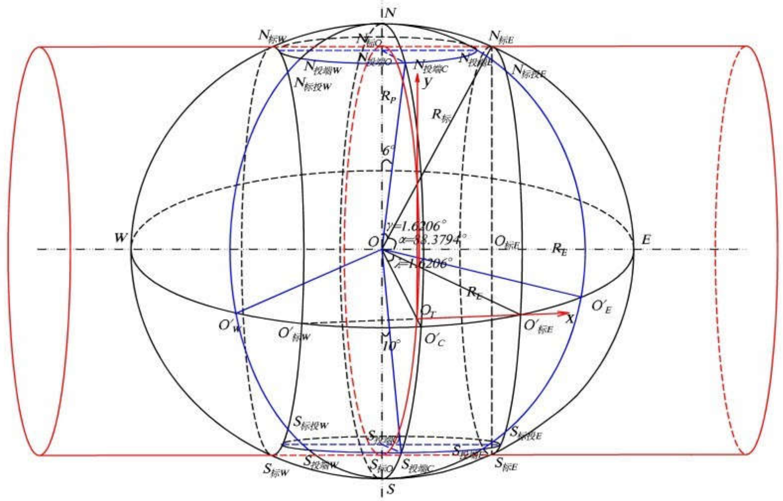


- UTM投影为椭圆柱横正轴割地球椭球体，椭圆柱的中心线位于椭球体赤道面上，且通过椭球体质点。从而将椭球体上的点投影到椭圆柱上。两条割线圆在UTM投影图上长度无变，即2条标准经线圆。两条割线圆之正中间为中央经线圆，中央经线投影后的长度为其投影前的0.999 6倍，比例因子*k*=投影后的长度/投影前的实际长度
- UTM投影替代Gauss-Krüger投影已成大势所趋

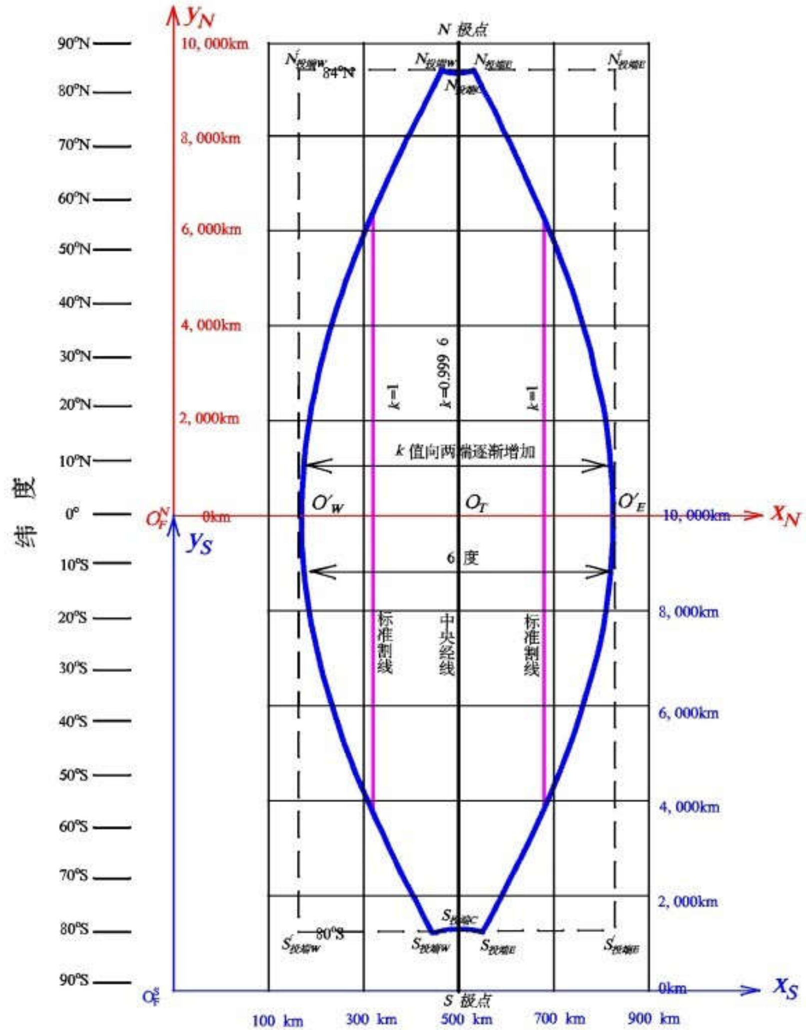

Y
Z
31X
33X
35X
37X
30V
32V
17T
01U
26U
45U
60U
01E
26E
45E
60E
A
B

Gauss-Krüger Projection

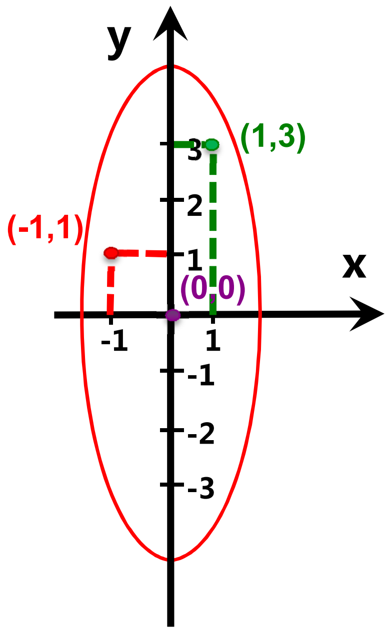


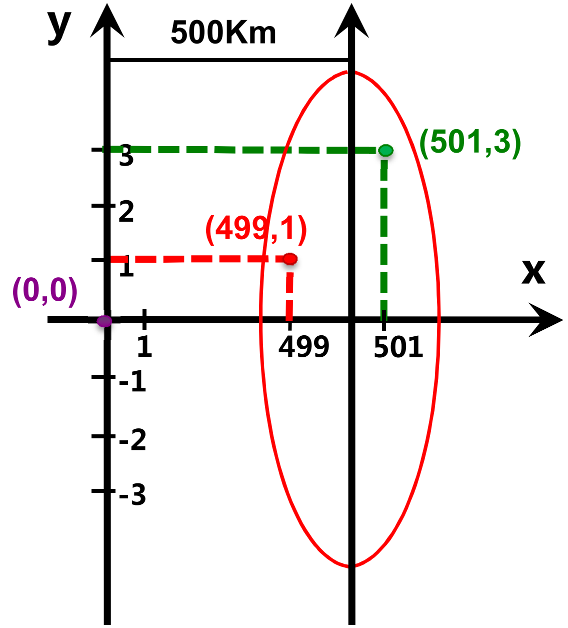


- ✓ 高斯-克吕格投影这个投影是由德国数学家、物理学家、天文学家高斯于19 世纪20 年代拟定，后经德国大地测量学家克吕格于1912 年对投影公式加以补充，故称为高斯-克吕格投影。又称为横轴墨卡托投影、切圆柱投影，是墨卡托投影的变种
- ✓ Gauss-Krüger Projection is the transverse tangent cylindrical conformal projection
- ✓ 为前苏联、中国和德国等国所采用

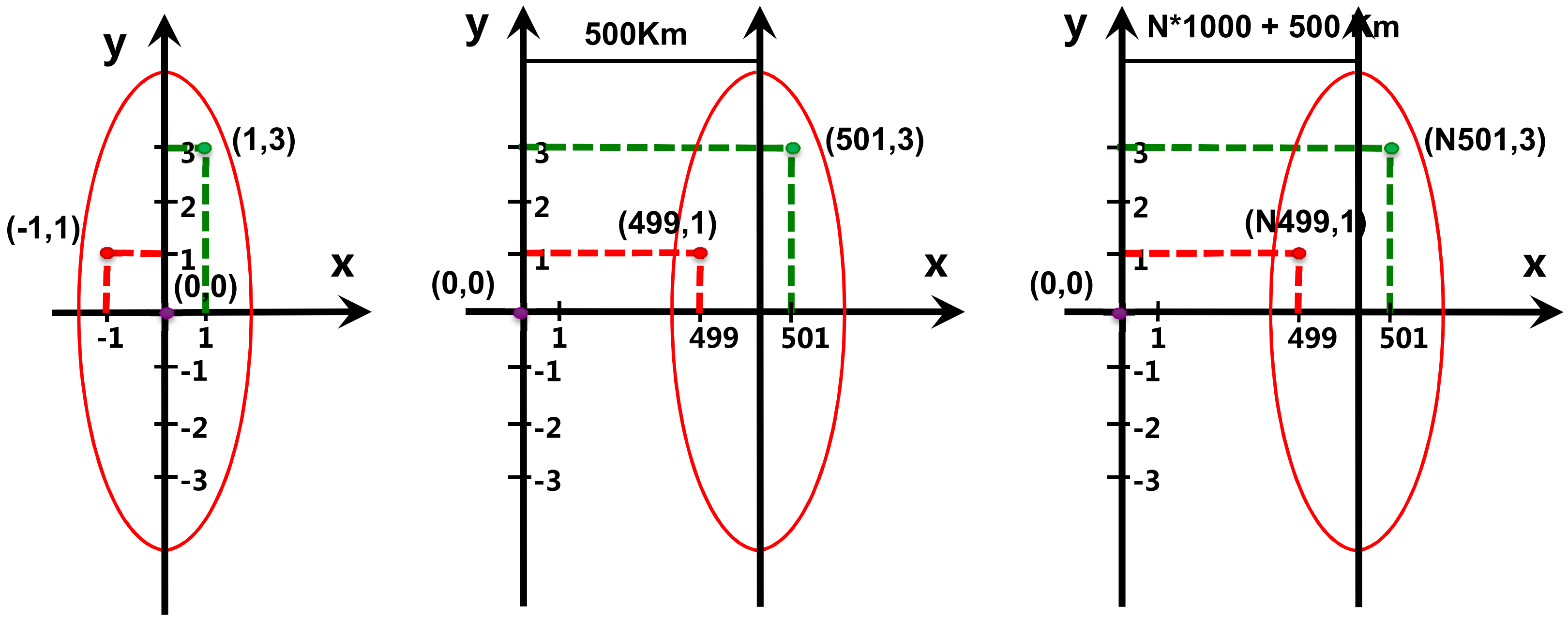


一个UTM投影6度带(据Langley（1998）修改）



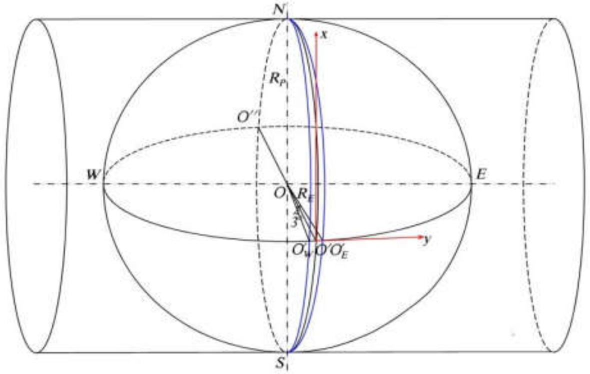


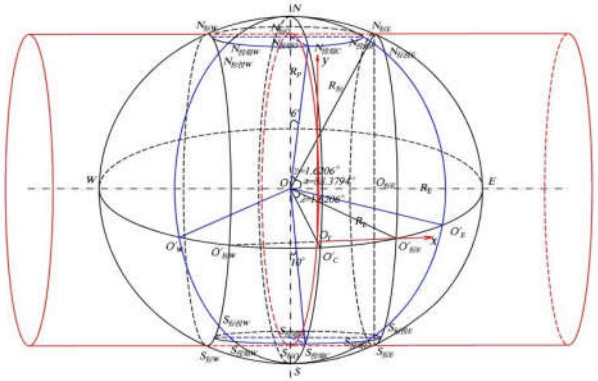
东向偏移(False Easting)　(500 000, 4 231 898)*



东向偏移(False Easting) **(500 000, 4 231 898)*** **(21 500 000, 4 231 898)***

Comparison between UTM and Gauss-Krüger

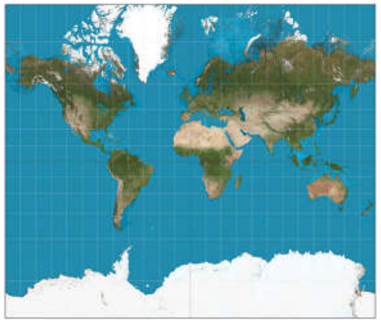


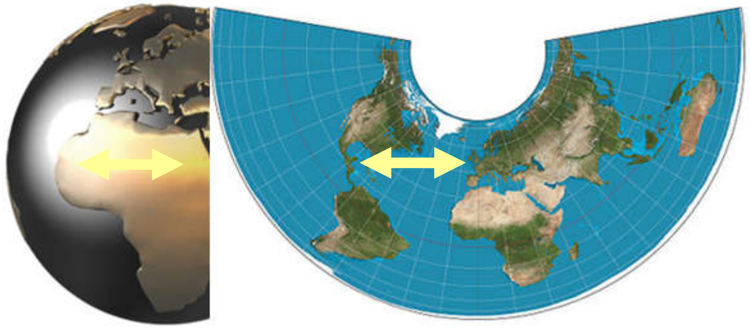


- 每个UTM投影带为经度6°。对一个窄经度（ 6° ）带投影而言，Gauss-Krüger投影以其中央经线保持长度不变，而向中央经线两侧逐渐变形，明显不如UTM的保持中央经线缩短至0.9996，而出现2条长度不变的子午线的整个投影带上的长度变形上合理，改善了该6°带内长度投影变形分布
- UTM投影只适用于80S和84N的纬度范围内
- 国内UTM替换 Gauss-Krüger已经是大势所趋

三、Coordinate Conversions and Transformations

坐标变换（投影转换）





投影转换：源坐标系与目标坐标系基于同一个大地基准（大地坐标系）

虽然公式复杂，但参数明确

因此调用简单，结果精确

$$E = E_0 + k_0 A \left(\eta' + \sum_{j=1}^{3} \alpha_j \cos(2j\xi') \sinh(2j\eta') \right),$$

$$N = N_0 + k_0 A \left(\xi' + \sum_{j=1}^{3} \alpha_j \sin(2j\xi') \cosh(2j\eta') \right),$$

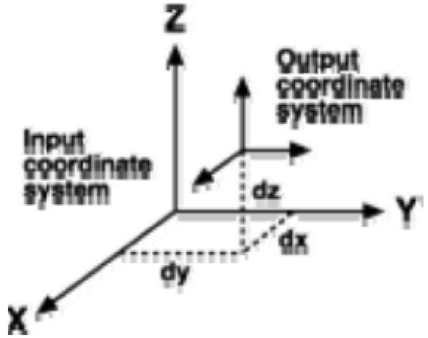
$$k = \frac{k_0 A}{a} \sqrt{ \left\{ 1 + \left(\frac{1-n}{1+n} \tan\varphi \right)^2 \right\} \frac{\sigma^2 + \tau^2}{t^2 + \cos^2(\lambda - \lambda_0)} },$$

$$\gamma = \tan^{-1} \left(\frac{\tau\sqrt{1+t^2} + \sigma t \tan(\lambda - \lambda_0)}{\sigma\sqrt{1+t^2} - \tau t \tan(\lambda - \lambda_0)} \right).$$

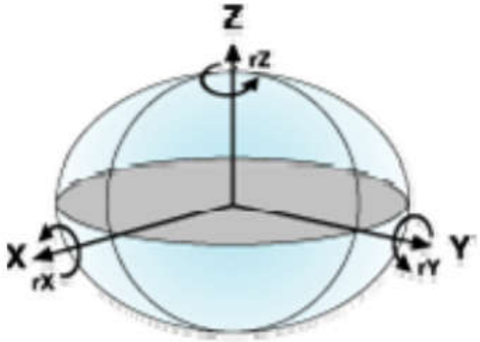
坐标变换（大地基准变化）

- 三参数地心变换（dX，dY，dZ）
- 四参数平面变换（dX，dY，R，M）
- 七参数赫尔默特变换（dX，dY，dZ，Rx，Ry，Rz，M）
  - 位置向量变换(Position Vector)
  - 坐标框架旋转(Coordinate Frame)
- 没有参数
  - 控制点反算参数
  - Georeferencing & Spatial Adjustment

控制点反算参数



$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{new} = \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} + \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{original}$$



$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{new} = \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} + (1+s) \cdot \begin{bmatrix} 1 & r_z & -r_y \\ -r_z & 1 & r_x \\ r_y & -r_x & 1 \end{bmatrix} \cdot \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{original}$$

Special Cases In China

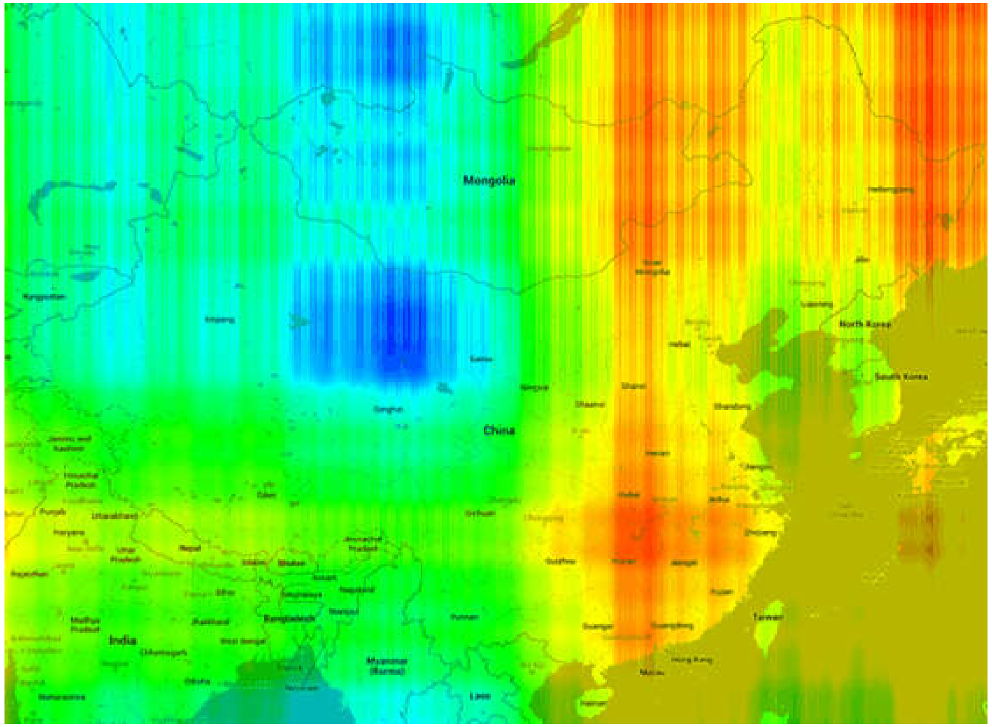
地方独立坐标系

- 为了满足大比例尺地图中高精度的需要
- 通常使用高斯克吕格投影（1度分带）
- 可以理解为基于特定的参心坐标系投影而成

深圳独立坐标系

坐标偏移

- 保密政策
  - 坐标系转换参数
  - 测绘成果不公开
- 先加密再公开
- 地形图非线性保密处理技术
  - 李成名
  - WGS84 --> 国测局坐标系GCJ-02



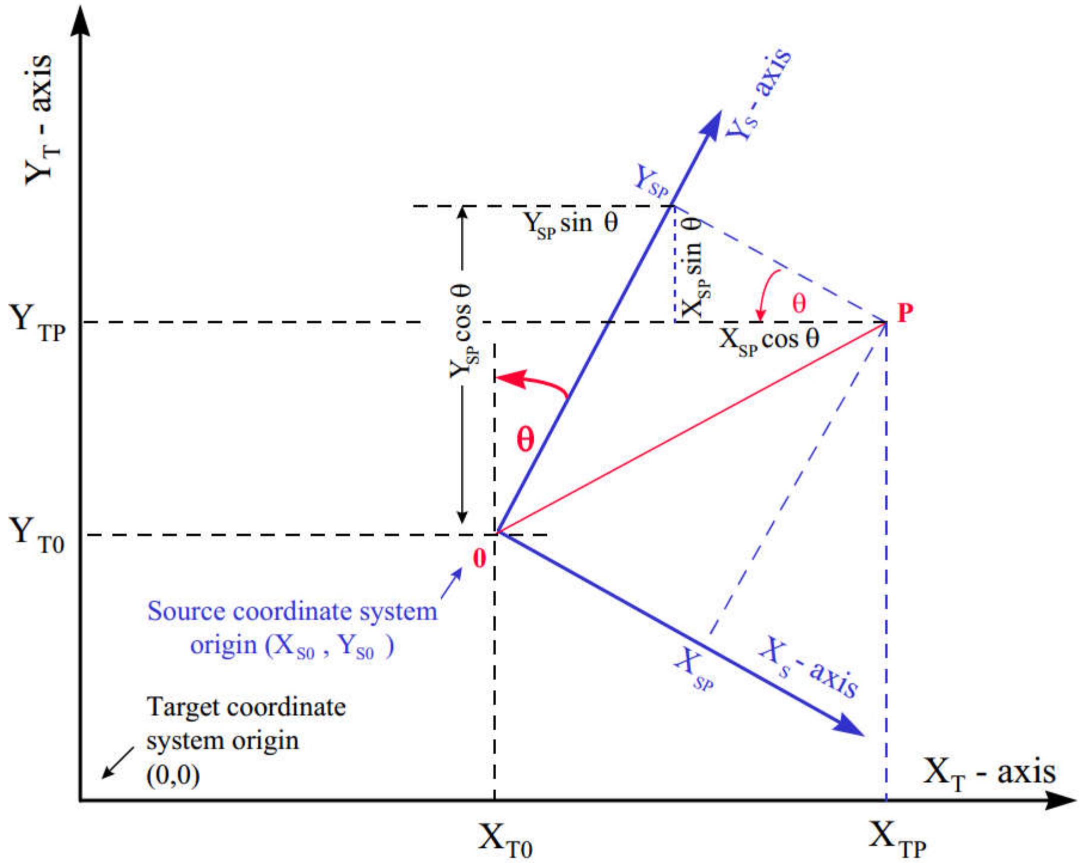
偏移数据的使用

- 早期的导航仪必须安装保密固件
- 地图厂商数据送检
  - 加密处理
- 互联网地图APP中内嵌加密接口
  - 提供加密API
- 百度地图等
  - 大地坐标二次偏移（BD-09）
  - 投影方法保密

Google Map: 22.5979422000,113.9934485100

Baidu Map: 22.6014216192,114.0049045474

四、Practice



$$X_{TP} = X_{T0} + Y_{SP}.\ dS\ .\ \sin\theta\ + X_{SP}.\ dS.\ \cos\theta$$
$$Y_{TP} = Y_{T0} + Y_{SP}.\ dS\ .\ \cos\theta\ - X_{SP}.\ dS.\ \sin\theta$$

$$\begin{pmatrix} X_T \\ Y_T \end{pmatrix} = \begin{pmatrix} X_{T0} \\ Y_{T0} \end{pmatrix} + (1 + dS) * \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} * \begin{pmatrix} X_S \\ Y_S \end{pmatrix}$$

$$x_1 + x_2 + cx_3 + dx_4 = X_1$$

$$x1 + 0x4 + p_{1x}x2 + p_{1y}x3 = t_{1x}$$
$$0x1 + x4 + p_{1y}x2 - p_{1x}x3 = t_{1y}$$
$$x1 + 0x4 + p_{2x}x2 + p2_{1y}x3 = t_{2x}$$
$$0x1 + x4 + p2_{1y}x2 - p_{2x}x3 = t_{2y}$$
$$\begin{pmatrix} 1 & 0 & p_{1x} & p_{1y} \\ 0 & 1 & p_{1y} & -p_{1x} \\ & & & \\ & & & \end{pmatrix} * \begin{matrix} x1 \\ x4 \\ x2 \\ x3 \end{matrix} = \begin{matrix} t_{1x} \\ t_{1y} \end{matrix}$$

X_{T0} , Y_{T0} = the coordinates of the origin point of the source coordinate reference system expressed in the target coordinate reference system;

$1+dS$ = the length of one unit in the source coordinate reference system expressed in units of the target coordinate reference system;

θ = the angle about which the axes of the source coordinate reference system need to be rotated to coincide with the axes of the target coordinate reference system, counter-clockwise being positive. Alternatively, the bearing of the source coordinate reference system Y_s-axis measured relative to target coordinate reference system north.

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