Dig-In:

Polynomial functions

Polynomials are some of our favorite functions.

The functions you are most familiar with are probably polynomial functions.

What are polynomial functions?

Definition 1. *A **polynomial function** in the variable x is a function which can be written in the form*

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

*where the a_i's are all constants (called the **coefficients**) and n is a whole number (called the **degree** when $n \neq 0$). The domain of a polynomial function is $(-\infty, \infty)$.*

Question 1 *Which of the following are polynomial functions?*

Select All Correct Answers:

(a) $f(x) = 0$ ✓

(b) $f(x) = -9$ ✓

(c) $f(x) = 3x + 1$ ✓

(d) $f(x) = x^{1/2} - x + 8$

(e) $f(x) = -4x^{-3} + 5x^{-1} + 7 - 18x^2$

(f) $f(x) = (x+1)(x-1) + e^x - e^x$ ✓

(g) $f(x) = \dfrac{x^2 - 3x + 2}{x - 2}$

(h) $f(x) = x^7 - 32x^6 - \pi x^3 + 45/84$ ✓

Learning outcomes: Know the graphs and properties of "famous" functions.

The phrase above "in the variable x" can actually change.

$$y^2 - 4y + 1$$

is a polynomial in y, and

$$\sin^2(x) + \sin(x) - 3$$

is a polynomial in $\sin(x)$.

What can the graphs look like?

Fun fact:

Theorem 1 (The Fundamental Theorem of Algebra)**.** *Every polynomial of the form*

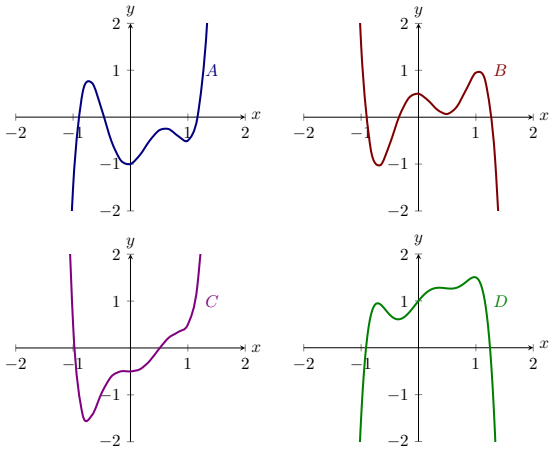
$$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

where the a_i's are real (or even complex!) numbers and $a_n \neq 0$ has exactly n (possibly repeated) complex roots.

Remember, a **root** is where a polynomial is zero. The theorem above is a deep fact of mathematics. The great mathematician Gauss proved the theorem in 1799 for his doctoral thesis.

The upshot as far as we are concerned is that when we plot a polynomial of degree n, its graph will cross the x-axis at most n times.

Example 1. *Here we see the the graphs of four polynomial functions.*



*For each of the curves, determine if the polynomial has **even** or **odd** degree, and if the leading coefficient (the one next to the highest power of x) of the polynomial is **positive** or **negative**.*

Explanation.

- *Curve A is defined by an* (even/ odd ✓) *degree polynomial with a* (positive ✓ / negative) *leading term.*
- *Curve B is defined by an* (even/odd ✓) *degree polynomial with a* (positive/ negative ✓) *leading term.*
- *Curve C is defined by an* (even ✓ / odd) *degree polynomial with a* (positive ✓ / negative) *leading term.*
- *Curve D is defined by an* (even ✓ /odd) *degree polynomial with a* (positive/ negative ✓) *leading term.*