

**Break-Ground:**

## Same or different?

*Two young mathematicians examine one (or two!) functions.*

Check out this dialogue between two calculus students (based on a true story):

**Devyn:** Riley, I have a pressing question.

**Riley:** Tell me. Tell me everything.

**Devyn:** Think about the function

$$f(x) = \frac{x^2 - 3x + 2}{x - 2}.$$

**Riley:** OK.

**Devyn:** Is this function equal to  $g(x) = x - 1$ ?

**Riley:** Well if I plot them with my calculator, they look the same.

**Devyn:** I know!

**Riley:** And I suppose if I write

$$\begin{aligned} f(x) &= \frac{x^2 - 3x + 2}{x - 2} \\ &= \frac{(x - 1)(x - 2)}{x - 2} \\ &= x - 1 \\ &= g(x). \end{aligned}$$

**Devyn:** Sure! But what about when  $x = 2$ ? In this case

$$g(2) = 1 \quad \text{but} \quad f(2) \text{ is undefined!}$$

**Riley:** Right,  $f(2)$  is undefined because we cannot divide by zero. Hmm. Now I see the problem. Yikes!

**Problem 1** *In the context above, are  $f$  and  $g$  the same function?*

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Learning outcomes: Distinguish two functions by considering their domains. Recognize different representations of the same function.

Same or different?

**Multiple Choice:**

- (a) yes
- (b) no ✓

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**Problem 2** Suppose  $f$  and  $g$  are functions but the domain of  $f$  is different from the domain of  $g$ . Could it be that  $f$  and  $g$  are actually the same function?

**Multiple Choice:**

- (a) yes
- (b) no ✓

**Feedback (attempt):** The domain of a function is part of the “data” of the function. A function is not a rule for transforming the input to the output, but rather the relationship between a specified collection of inputs (the domain) and possible outputs (the range).

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**Problem 3** Can the same function be represented by different formulas?

**Multiple Choice:**

- (a) yes ✓
- (b) no

**Problem 4** Are  $f(x) = |x|$  and  $g(x) = \sqrt{x^2}$  the same function?

**Multiple Choice:**

- (a) These are the same function although they are represented by different formulas. ✓
  - (b) These are different functions because they have different formulas.
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Same or different?

**Problem 5** Let  $f(x) = \sin^2(x)$  and  $g(u) = \sin^2(u)$ . The domain of each of these functions is all real numbers. Which of the following statements are true?

**Multiple Choice:**

- (a) There is not enough information to determine if  $f = g$ .
  - (b) The functions are equal. ✓
  - (c) If  $x \neq u$ , then  $f \neq g$ .
  - (d) We have  $f \neq g$  since  $f$  uses the variable  $x$  and  $g$  uses the variable  $u$ .
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