Dig-In:

Rational functions

Rational functions are functions defined by fractions of polynomials.

What are rational functions?

Definition 1. *A* ***rational function*** *in the variable x is a function the form*

$$f(x) = \frac{p(x)}{q(x)}$$

where p and q are polynomial functions. The domain of a rational function is all real numbers except for where the denominator is equal to zero.

Question 1 *Which of the following are rational functions?*

Select All Correct Answers:

(a) $f(x) = 0$ ✓

(b) $f(x) = \dfrac{3x+1}{x^2-4x+5}$ ✓

(c) $f(x) = e^x$

(d) $f(x) = \dfrac{\sin(x)}{\cos(x)}$

(e) $f(x) = -4x^{-3} + 5x^{-1} + 7 - 18x^2$ ✓

(f) $f(x) = x^{1/2} - x + 8$

(g) $f(x) = \dfrac{\sqrt{x}}{x^3 - x}$

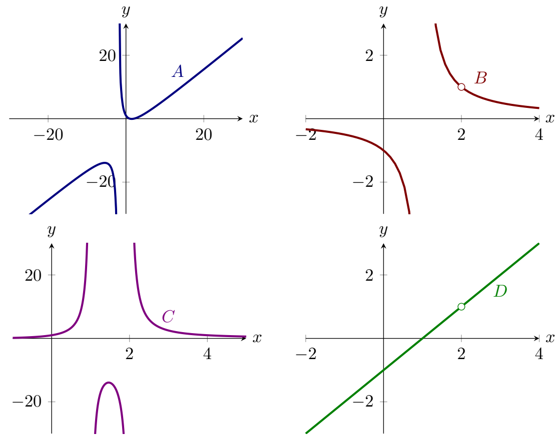
Feedback (attempt): *All polynomials can be thought of as rational functions.*

Learning outcomes: Know the graphs and properties of "famous" functions. Understand the definition of a rational function.

What can the graphs look like?

There is a somewhat wide variation in the graphs of rational functions.

Example 1. *Here we see the the graphs of four rational functions.*



Match the curves A, B, C, and D with the functions

$$\frac{x^2-3x+2}{x-2}, \qquad \frac{x^2-3x+2}{x+2},$$
$$\frac{x-2}{x^2-3x+2}, \qquad \frac{x+2}{x^2-3x+2}.$$

Explanation. *Consider* $\frac{x^2-3x+2}{x-2}$. *This function is undefined only at* $x = 2$. *Of the curves that we see above,* $\underbrace{\boxed{D}}_{\text{given}}$ *is undefined exactly at* $x = 2$.

Now consider $\frac{x^2-3x+2}{x+2}$. *This function is undefined only at* $x = -2$. *The only function above that undefined exactly at* $x = -2$ *is curve* $\underbrace{\boxed{A}}_{\text{given}}$.

Now consider $\frac{x-2}{x^2-3x+2}$. *This function is undefined at the roots of*

$$x^2 - 3x + 2 = (x-2)(x-1).$$

Hence it is undefined at $x = 2$ *and* $x = 1$. *It looks like both curves* B *and* C *would work. Distinguishing between these two curves is easy enough if we* *evaluate at* $x = -2$*. Check it out.*

$$\begin{aligned}\left[\frac{x-2}{x^2-3x+2}\right]_{x=-2} &= \frac{-2-2}{(-2)^2-3(-2)+2}\\ &= \frac{-4}{4+6+2}\\ &= \frac{-4}{12}.\end{aligned}$$

Since this is negative, we see that $\dfrac{x-2}{x^2-3x+2}$ *corresponds to curve* $\underbrace{\boxed{B}}_{\text{given}}$.

Finally, it must be the case that curve $\underbrace{\boxed{C}}_{\text{given}}$ *corresponds to* $\dfrac{x+2}{x^2-3x+2}$*. We should note that if this function is evaluated at* $x = -2$*, the output is zero, and this corroborates our work above.*