Break-Ground:

Same or different?

Two young mathematicians examine one (or two!) functions.

Check out this dialogue between two calculus students (based on a true story):

Devyn: Riley, I have a pressing question.

Riley: Tell me. Tell me everything.

Devyn: Think about the function

$$f(x) = \frac{x^2 - 3x + 2}{x - 2}.$$

Riley: OK.

Devyn: Is this function equal to $g(x) = x - 1$?

Riley: Well if I plot them with my calculator, they look the same.

Devyn: I know!

Riley: And I suppose if I write

$$\begin{aligned} f(x) &= \frac{x^2 - 3x + 2}{x - 2} \\ &= \frac{(x-1)(x-2)}{x-2} \\ &= x - 1 \\ &= g(x). \end{aligned}$$

Devyn: Sure! But what about when $x = 2$? In this case

$$g(2) = 1 \qquad \text{but} \qquad f(2) \text{ is undefined!}$$

Riley: Right, $f(2)$ is undefined because we cannot divide by zero. Hmm. Now I see the problem. Yikes!

Problem 1 *In the context above, are f and g the same function?*

Learning outcomes: Distinguish two functions by considering their domains. Recognize different represenations of the same function.

Multiple Choice:

(a) *yes*

(b) *no* ✓

Problem 2 *Suppose f and g are functions but the domain of f is different from the domain of g. Could it be that f and g are actually the same function?*

Multiple Choice:

(a) *yes*

(b) *no* ✓

Feedback (attempt): *The domain of a function is part of the "data" of the function. A function is not a rule for transforming the input to the output, but rather the relationship between a specified collection of inputs (the domain) and possible outputs (the range).*

Problem 3 *Can the same function be represented by different formulas?*

Multiple Choice:

(a) *yes* ✓

(b) *no*

Problem 4 *Are $f(x) = |x|$ and $g(x) = \sqrt{x^2}$ the same function?*

Multiple Choice:

(a) *These are the same function although they are represented by different formulas.* ✓

(b) *These are different functions because they have different formulas.*

Problem 5 *Let $f(x) = \sin^2(x)$ and $g(u) = \sin^2(u)$. The domain of each of these functions is all real numbers. Which of the following statements are true?*

Multiple Choice:

(a) *There is not enough information to determine if $f = g$.*

(b) *The functions are equal.* ✓

(c) *If $x \neq u$, then $f \neq g$.*

(d) *We have $f \neq g$ since f uses the variable x and g uses the variable u.*