Break-Ground:

A limitless dialogue

Two young mathematicians consider a way to compute limits using derivatives.

Check out this dialogue between two calculus students (based on a true story):

Devyn: Yo Riley, guess what I did last night?

Riley: What?

Devyn: I was doing some calculus.

Riley: That. Is. Awesome.

Devyn: I know! Anyway, I noticed something kinda funny. I think you can sometimes take limits by taking the derivative of the numerator and the denominator.

Riley: That's crazy.

Devyn: I know! But check it:

$$\begin{aligned}\lim_{x\to 0}\frac{\sin(x)}{x} &= \lim_{x\to 0}\frac{\frac{d}{dx}\sin(x)}{\frac{d}{dx}x}\\ &= \lim_{x\to 0}\frac{\cos(x)}{1}\\ &= 1.\end{aligned}$$

Riley: Woah. That. Is. Awes...weird. Hmmm, but it seems like cheating. Wait, it doesn't always work, check this out:

$$\lim_{x\to 0}\frac{x^2+1}{x+1} = 1,$$

but

$$\begin{aligned}\lim_{x\to 0}\frac{\frac{d}{dx}\left(x^2+1\right)}{\frac{d}{dx}\left(x+1\right)} &= \lim_{x\to 0}\frac{2x}{1}\\ &= 0.\end{aligned}$$

Learning outcomes: Demonstrate that we can only sometimes find a limit value by finding the limit of quotient of the derivative of the numerator and derivative of the denominator. Explore the hypothesis of L'Hospital's Rule. Make a guess and test this guess.

Problem 1 *Find* ***five*** *examples where this "trick" works, and* ***five*** *examples where it doesn't work.*

Hint: *Start with limits of fractions that you know how to compute. Then take the derivative of the numerator and the denominator, and see if the new limit equals the old limit or not.*

Free Response:

Problem 2 *What is the pattern for when the "trick" works and when it does not work?*

Free Response: