Break-Ground:

Slope of a curve

Two young mathematicians discuss the novel idea of the "slope of a curve."

Check out this dialogue between two calculus students (based on a true story):

Devyn: Riley, do you remember "slope?'

Riley: Most definitely. "Rise over run."

Devyn: You know it.

Riley: "Change in y over change in x."

Devny: That's right.

Riley: Brought to you by the letter "m."

Devny: Enough! My important question is: could we define "slope" for a curve that's not a straight line?

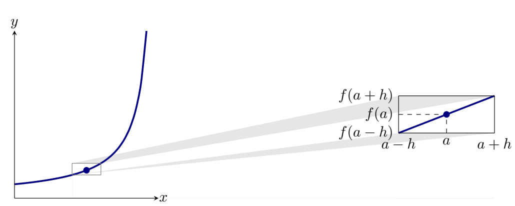
Riley: Well, maybe if we "zoom in" on a curve, it would look like a line, and then we could call it "the slope at that point."

Devyn: Ah! And this "zoom in" idea sounds like a limit!

Riley: This is so awesome. We just made math!

The concept introduced above, of the "slope of a curve at a point," is in fact one of the central concepts of calculus. It will, of course, be completely explained. Let's explore Devyn and Riley's ideas a little more, first.

To find the "slope of a curve at a point," Devyn and Riley spoke of "zooming in" on a curve until it looks like a line. When you zoom in on a *smooth* curve, it will eventually look like a line. This line is called the tangent line.



Learning outcomes: Use limits to find the slope of the tangent line at a point. Understand the definition of the derivative at a point.

Problem 1 *Which of the following approximate the slope of the "zoomed line"?*

Select All Correct Answers:

(a) $\dfrac{(f(a)+h)-f(a)}{(a+h)-a}$

(b) $\dfrac{f(a+h)-f(a)}{(a+h)-a}$ ✓

(c) $\dfrac{(f(a)-h)-f(a)}{(a-h)-a}$

(d) $\dfrac{f(a-h)-f(a)}{(a-h)-a}$ ✓

(e) $\dfrac{f(a)-(f(a)+h)}{a-(a+h)}$

(f) $\dfrac{f(a)-f(a+h)}{a-(a+h)}$ ✓

(g) $\dfrac{f(a)-(f(a)-h)}{a-(a-h)}$

(h) $\dfrac{f(a)-f(a-h)}{a-(a-h)}$ ✓

Problem 2 *Let $f(x) = 3x - 1$. Zoom in on the curve around $a = -2$ so that $h = 0.1$. Use one of the formulations in the problem above to approximate the slope of the curve. The slope of the curve at $a = -2$ is approximately...* $\boxed{3}$

Problem 3 *Repeat the previous problem for $f(x) = x^2 - 1$, $a = 0$, and $h = 0.2$. Choose a formulation that will give you a positive answer for the slope. The (positive) slope of the curve at $a = 0$ is approximately...* $\boxed{0.2}$

Problem 4 *Zoom in on the curve $f(x) = x^2 - 1$ near $x = 0$ again. By looking at the graph, what is your best guess for the actual slope of the curve at zero?*

Multiple Choice:

(a) *impossible to say*

(b) *zero* ✓

(c) *one*

(d) *infinity*