Break-Ground:

Limits and velocity

Two young mathematicians discuss limits and instantaneous velocity.

Check out this dialogue between two calculus students (based on a true story):

Devyn: Hey Riley, I've been thinking about limits.

Riley: That is awesome.

Devyn: I know! You know limits remind me of something... How a GPS or a phone computes velocity!

Riley: Huh. A GPS can calculate our location. Then, to compute velocity from position, it must look at

$$\frac{\text{change in position}}{\text{change in time}}$$

Devyn: And then we study this as the change in time gets closer and closer to zero.

Riley: Just like with limits at zero, we can study something by looking **near** a point, but **not exactly at** a point.

Devyn: O.M.G. Life's a rich tapestry.

Riley: Poet, you know it.

Suppose you take a road trip from Columbus Ohio to Urbana-Champaign Illinois. Moreover, suppose your position is modeled by

$$s(t) = 36t^2 - 4.8t^3 \qquad \text{(miles West of Columbus)}$$

where t is measured in hours and runs from 0 to 5 hours.

Problem 1 *What is the average velocity for the entire trip?*

Hint: *Remember,*

$$\textit{change in distance} = \textit{rate} \cdot \textit{change in time}.$$

Learning outcomes: Consider limits as behavior nearer and nearer to a point.

Hint: *So,*

$$\frac{\Delta distance}{\Delta time} = rate.$$

Hint: *So,*

$$\frac{\Delta distance}{\Delta time} = \frac{300}{5}.$$

The average velocity is $\boxed{60}$ *miles per hour.*

Problem 2 *Use a calculator to estimate the instantaneous velocity at $t = 2$.*

Hint: *Remember,*

$$change\ in\ distance = rate \cdot change\ in\ time.$$

Hint: *So,*

$$\frac{\Delta distance}{\Delta time} = rate.$$

Hint: *Compute*

$$\frac{36(2+\Delta t)^2 - 4.8(2+\Delta t)^3 - \left(36 \cdot 2^2 - 4.8 \cdot 2^3\right)}{\Delta t}$$

for smaller, and smaller values of Δt.

The instantaneous velocity, (rounded to the nearest tenth) is $\boxed{86.4}$ *miles per hour.*

Problem 3 *Considering the work above, when we want to compute instantaneous velocity, we need to compute*

$$\frac{change\ in\ position}{change\ in\ time}$$

when (choose all that apply):

Select All Correct Answers:

(a) *The "change in time" is zero.*

(b) *The "change in time" gets closer and closer to zero.* ✓

(c) *The "change in time" approaches zero.* ✓

(d) *The "change in time" is near zero.*

(e) *The "change in time" goes to zero.* ✓

Computing average velocities for smaller, and smaller, values of Δt as we did above is tedious. Nevertheless, this is exactly how a GPS determines velocity from position! To avoid these tedious calculations, we would really like to have a formula.