

Dig-In:

The Extreme Value Theorem

We examine a fact about continuous functions.

Definition 1.

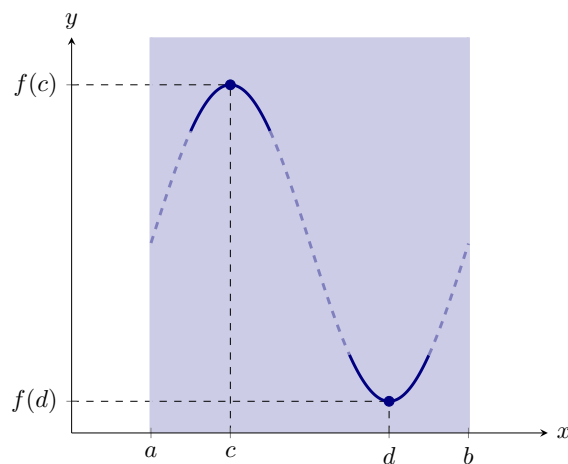
- (a) A function f has an **global maximum** at $x = a$, if $f(a) \geq f(x)$ for every x in the domain of the function.
- (b) A function f has an **global minimum** at $x = a$, if $f(a) \leq f(x)$ for every x in the domain of the function.

A **global extremum** is either a global maximum or a global minimum.

If we are working on an finite closed interval, then we have the following theorem.

Theorem 1 (Extreme Value Theorem). *If f is a continuous function for all x in the closed interval $[a, b]$, then there are points c and d in $[a, b]$, such that $(c, f(c))$ is a global maximum and $(d, f(d))$ is a global minimum on $[a, b]$.*

Below, we see a geometric interpretation of this theorem.



Question 1 Would this theorem hold if we were working on an open interval?

Multiple Choice:

Learning outcomes: Understand the statement of the Extreme Value Theorem.

The Extreme Value Theorem

(a) yes

(b) no ✓

Hint: Consider $\tan(\theta)$ for $-\pi/2 < \theta < \pi/2$. Does this function achieve its maximum and minimum?

