Break-Ground:

Wait for the right moment

Two young mathematicians discuss derivatives as functions.

Check out this dialogue between two calculus students (based on a true story):

Devyn: Riley, I might be a calculus genius.

Riley: Yeah? Explain this one to me.

Devyn: Let me first ask you a question. Say you have a function, like $f(x) = x^2$, and you want to know $f'(3)$. Do you plug in the number 3 before or after you find the derivative?

Riley: Hmmmm. Well, my next step is usually

$$f'(3) = \lim_{h\to 0} \frac{f(3+h) - f(3)}{h}.$$

So I guess before.

Devyn: Aha! I think you're wasting time. You see I write

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}.$$

and it means that I can look at the derivative of my function at *any* point. So, I plug in the 3 *after* I've found the derivative.

Riley: That does seem like a pretty genius move. But doesn't working with x, instead of numbers, make all of this more difficult?

Devyn: Not at all. Let's do the problems both ways, at the same time:

$$\underbrace{\begin{aligned} f'(3) &= \lim_{h\to 0} \frac{f(3+h) - f(3)}{h} \\ &= \lim_{h\to 0} \frac{(3+h)^2 - 9}{h} \\ &= \lim_{h\to 0} \frac{9 + 6h + h^2 - 9}{h} \\ &= \lim_{h\to 0} \frac{6h + h^2}{h} \\ &= \lim_{h\to 0} (6+h) \\ &= 6. \end{aligned}}_{\text{plugging in}} \qquad \underbrace{\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h} \\ &= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 - x^2}{h} \\ &= \lim_{h\to 0} \frac{2xh + h^2}{h} \\ &= \lim_{h\to 0} (2x+h) \\ &= 2x, \\ \text{so } f'(3) &= 6. \end{aligned}}_{\text{working with } x}$$

Learning outcomes: Relate the derivative function to the derivative at a point.

Riley: Whoa. So now the derivative is a function. Wait, what's its domain? Its range?

Problem 1 *Suppose you have a function f. Which of the following are true?*

Select All Correct Answers:

(a) *The domain of f' is equal to the domain of f.*

(b) *The range of f' is equal to the range of f.*

(c) *The domain of f' is a subset of the real numbers.* ✓

(d) *The range of f' is a subset of the real numbers.* ✓

(e) *The domain of f' is functions from the real numbers to the real numbers.*

(f) *The range of f' is functions from the real numbers to the real numbers.*

Problem 2 *Find $g'(2)$ for $g(x) = x^2 + 1$ using both methods described above.*

$$g'(2) = \boxed{4}$$