Dig-in:

The derivative as a function

Here we study the derivative of a function, as a function, in its own right.

The derivative of a function, as a function

We know that to find the derivative of a function at a point $x = a$ we write

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$

However, if we replace the given number a with a varible x, we now have

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

This tells us the instantaneous rate of change at any given point x.

Warning 1. *The notation:*

> $f'(a)$ *means take the derivative of* f *first, then evaluate at* $x = a$.

In other words, given f *a function of* x

$$f'(a) = \left[\frac{d}{dx} f(x)\right]_{x=a}.$$

Given a function f from the real numbers to the real numbers, the derivative f' is also a function from the real numbers to the real numbers. Understanding the relationship between the *functions* f and f' helps us understand any situation (real or imagined) involving changing values.

Question 1 *Let* $f(x) = 3x + 2$. *What is* $f'(-1)$*?*

Multiple Choice:

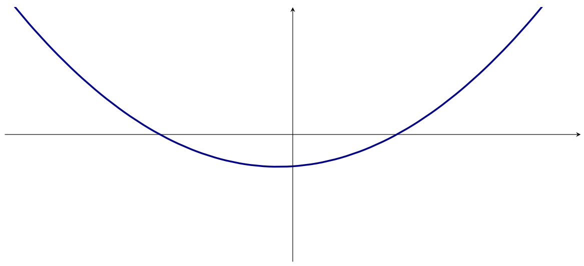
(a) $f'(-1) = 0$ *because* $f'(3)$ *is a number, and a number cooresponds to a horizontal line, which has a slope of zero.*

Learning outcomes: Understand the derivative as a function related to the original definition of a function. Find the derivative function using the limit definition. Relate the derivative function to the derivative at a point. Relate the graph of the function to the graph of its derivative.

(b) *$f'(-1) = 3$ because $y = f(x)$ is a line with slope 3.* ✓

(c) *We cannot solve this problem yet.*

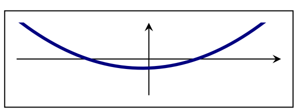
Question 2 *Here we see the graph of f'.*

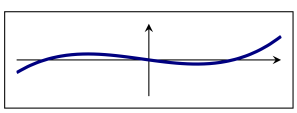


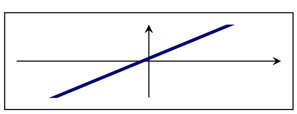
Describe $y = f(x)$ when f' is positive. Describe $y = f(x)$ when f' is negative.
When f' is positive, $y = f(x)$ is (positive/increasing ✓ /negtive/decreasing).
When f' is negative, $y = f(x)$ is (positive/increasing/negtive/decreasing ✓)

Question 3 *Which of the following graphs could be $y = f(x)$?*

Multiple Choice:

(a) 

(b)  ✓

(c) 

The derivative as a function of functions

While writing f' is viewing the derivative of f as a function in its own right, the derivatve itself

$$\frac{d}{dx}$$

is in fact a function that maps functions to functions,

$$\frac{d}{dx}x^2 = 2x$$
$$\frac{d}{dx}f(x) = f'(x).$$

Question 4 *As a function, is*

$$\frac{d}{dx}$$

one-to-one?

Multiple Choice:

(a) *yes*

(b) *no* ✓

Feedback (attempt): *Many different functions share the same derivative since the derivative recordes only the slope of the tangent line and not the value, or height.*