Break-Ground:

Integrals are puzzles!

Two young mathematicians discuss how tricky integrals are puzzles.

Check out this dialogue between two calculus students (based on a true story):

Devyn: Yo Riley, is it just me, or are integrals kind of fun?

Riley: I always feel accomplished when I finish one.

Devyn: I know! Also, even though antiderivatives are difficult, we can always check our work by taking the derivative.

Riley: So awesome!

Devyn: But something is bothering me. When we are doing substitution, we have to find f and g such that

$$\int f(g(x)) \cdot g'(x)\,dx = \int f(g)\,dg.$$

How do we choose f and g?

Riley: Well, never ever pick $g(x) = x$, this doesn't change anything!

Devyn: And never ever pick $g(x)$ to be the entire integrand, this doesn't help either.

Riley: Somehow we must "see" one function "nested" inside of another.

Devyn: I'm not sure there's an easy path to doing, this, I think it's gonna take practice.

In the problems that follow, we will be using the substitution formula

$$\int f(g(x)) \cdot g'(x)\,dx = \int f(g)\,dg$$

While you may use a slightly different method to compute your integrals, the skills developed by answering the problems below will help you in your quest to conquer calculus.

Learning outcomes: Practice the mechanical process of substitution.

Problem 1 *Consider*

$$\int \sin^5(3x)\cos(3x)\,dx = \int f(g(x)) \cdot g'(x)\,dx$$

if $g(x) = 3x$, *and*

$$\int f(g(x)) \cdot g'(x)\,dx = \int f(g)\,dg.$$

what is $f(g)$*?*

$$f(g) = \boxed{\frac{\sin^5(g)\cos(g)}{3}}$$

Problem 2 *Consider*

$$\int \sin^5(3x)\cos(3x)\,dx = \int f(g(x)) \cdot g'(x)\,dx$$

if $f(g) = \dfrac{g^5}{3}$, *and*

$$\int f(g(x)) \cdot g'(x)\,dx = \int f(g)\,dg.$$

what is $g(x)$*?*

$$g(x) = \boxed{\sin(3x)}$$

Problem 3 *In your own words, explain why Devyn and Riley claim we should never pick* $g(x) = x$ *or* $g(x)$ *to be the entire integrand.*

Free Response: *The goal of substitution is to make the integral easier to do. Your choices for* f *and* g *should make things easier, not harder!*

Unless the derivative of $g(x)$ *is* 1, *choosing* $g(x)$ *to be the entire integrand means that you don't have any part of the integrand left to be the derivative of* g. *Choosing* $g(x) = x$ *means that* $g'(x) = 1$, *meaning that you haven't simplified the integral at all.*