Dig-In:

Higher order derivatives and graphs

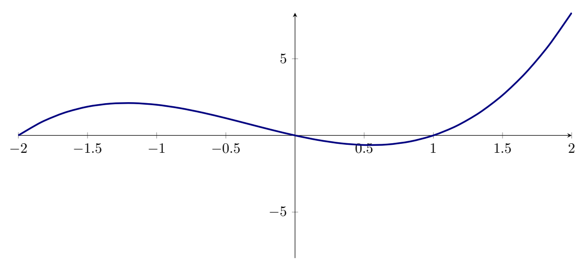
Here we look at graphs of higher order derivatives.

Since the derivative gives us a formula for the slope of a tangent line to a curve, we can gain information about a function purely from the sign of the derivative. In particular, we have the following theorem

Theorem 1. *If f is differentiable on an interval, then*

- $f'(x) > 0$ *on that interval whenever f is increasing as x increases on that interval.*
- $f'(x) < 0$ *on that interval whenever f is decreasing as x increases on that interval.*

Question 1 *Below we have graphed $y = f(x)$:*



Is the first derivative positive or negative on the interval $-1 < x < 1/2$?

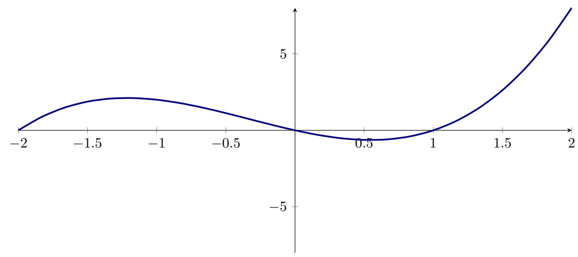
Multiple Choice:

(a) *Positive*

(b) *Negative* ✓

Question 2 *Below we have graphed $y = f'(x)$:*

Learning outcomes: Use the first derivative to determine whether a function is increasing or decreasing. Define higher order derivatives. Compare differing notations for higher order derivatives. Identify the relationships between the function and its first and second derivatives.



Is the graph of $f(x)$ increasing or decreasing as x increases on the interval $-1 < x < 0$?

Multiple Choice:

(a) *Increasing* ✓

(b) *Decreasing*

We call the derivative of the derivative the **second derivative**, the derivative of the derivative of the derivative the **third derivative**, and so on. We have special notation for higher derivatives, check it out:

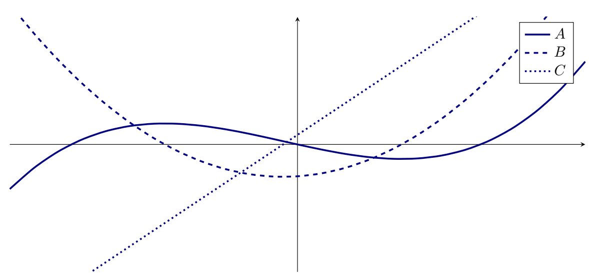
First derivative: $\dfrac{d}{dx}f(x) = f'(x) = f^{(1)}(x)$.

Second derivative: $\dfrac{d^{\,2}}{dx^2}f(x) = f''(x) = f^{(2)}(x)$.

Third derivative: $\dfrac{d^{\,3}}{dx^3}f(x) = f'''(x) = f^{(3)}(x)$.

We use the facts above in our next example.

Example 1. *Here we have unlabeled graphs of f, f', and f'':*



Identify each curve above as a graph of f, f', or f''.

Explanation. *Here we see three curves, A, B, and C. Since A is* (positive/negative/increasing ✓ /decreasing) *when B is positive and* (positive/negative/increasing/decreasing ✓) *when B is negative, we see*

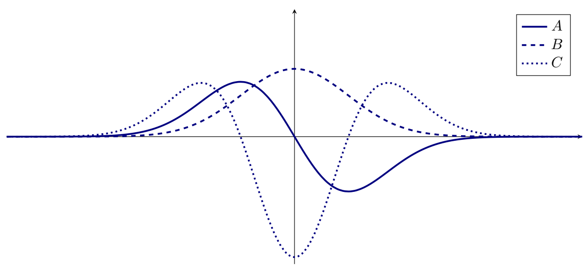
$$A' = B.$$

Since B is increasing when C is (positive ✓ /negative/increasing/decreasing) *and decreasing when C is* (positive/negative ✓ /increasing/decreasing), *we see*

$$B' = C.$$

Hence $f = A$, $f' = B$, and $f'' = C$.

Example 2. *Here we have unlabeled graphs of f, f', and f'':*



Identify each curve above as a graph of f, f', or f''.

Explanation. *Here we see three curves, A, B, and C. Since B is* (positive/negative/increasing ✓ /decreasing) *when A is positive and* (positive/negative/increasing/decreasing ✓) *when A is negative, we see*

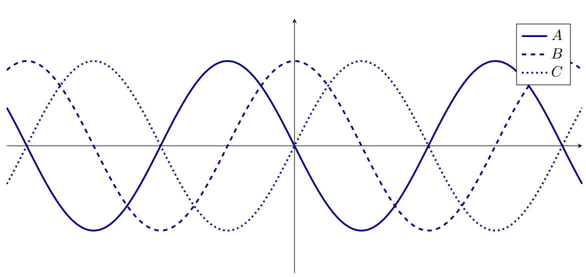
$$B' = A.$$

Since A is increasing when C is (positive ✓ /negative/increasing/decreasing) *and decreasing when C is* (positive/negative ✓ /increasing/decreasing), *we see*

$$A' = C.$$

Hence $f = \underset{\text{given}}{\boxed{B}}$, $f' = \underset{\text{given}}{\boxed{A}}$, *and* $f'' = \underset{\text{given}}{\boxed{C}}$.

Example 3. *Here we have unlabeled graphs of f, f', and f'':*



Identify each curve above as a graph of f, f', or f''.

Explanation. *Here we see three curves, A, B, and C. Since C is* (positive/ negative/increasing ✓ /decreasing) *when B is positive and* (positive/negative/ increasing/decreasing ✓) *when B is negative, we see*

$$C' = B.$$

Since B is increasing when A is (positive ✓ / negative/ increasing/ decreasing) *and decreasing when A is* (positive/negative ✓ /increasing/ decreasing), *we see*

$$B' = A.$$

Hence $f = \underbrace{\boxed{C}}_{\text{given}}$, $f' = \underbrace{\boxed{B}}_{\text{given}}$, *and* $f'' = \underbrace{\boxed{A}}_{\text{given}}$.