

Break-Ground:

An unnoticed composition

Two young mathematicians discuss the chain rule.

Check out this dialogue between two calculus students (based on a true story):

Devyn: Riley! Something is bothering me.

Riley: What is it?

Devyn: I have broken calculus.

Riley: How?

Devyn: Check this out, we know that:

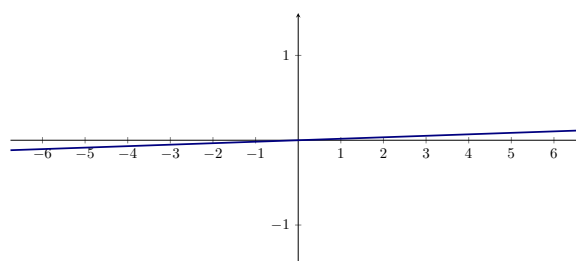
$$\frac{d}{d\theta} \sin(\theta) = \cos(\theta)$$

Riley: Right.

Devyn: But I also know that the derivative **is** the slope of the tangent line at a point.

Riley: Right!

Devyn: But check out this plot of $\sin(\theta)$ that's zoomed in around zero:



Riley: Ok. . .

Devyn: Well, $\cos(0) = 1$, but the slope of this line is totally **not one!** What's going on here?

Learning outcomes: Recognize a composition of functions. Understand rate of change when quantities are dependent upon each other. Apply chain rule to relate quantities expressed with different units.

This problem that Riley and Devyn are having is somewhat subtle.

Question 1 *What mistake is being made?*

Multiple Choice:

- (a) *Riley did not plot $\sin(\theta)$.*
- (b) *Riley did not take the derivative correctly.*
- (c) *Riley was working in degrees, not radians. ✓*
- (d) $\cos(0) \neq 1$
- (e) *Riley computed the slope incorrectly.*

Feedback (attempt): *In calculus, we work with radians. Working in degrees will produce erroneous answers.*
