Dig-In:

Position, velocity, and acceleration

Here we discuss how position, velocity, and acceleration relate to higher derivatives.

Studying functions and their derivatives might seem somewhat abstract. However, consider this passage from a physics book:

> Assuming acceleration a is constant, we may write velocity and position as
>
> $$v(t) = v_0 + at,$$
> $$x(t) = x_0 + v_0 t + (1/2)at^2,$$
>
> where a is the (constant) acceleration, v_0 is the position at time zero, and x_0 is the position at time zero.

These equations model the position and velocity of any object with constant acceleration. In particular these equations can be used to model the motion a falling object, since the acceleration due to gravity is constant.

Calculus allows us to see the connection between these equations. First note that the derivative of the formula for position with respect to time, is the formula for velocity with respect to time.

$$x'(t) = v_0 + at = v(t).$$

Moreover, the derivative of formula for velocity with respect to time, is simply a, the acceleration.

Question 1 *Suppose that s represents the position of a ball tossed at time $t = 0$. Recalling that the acceleration for $t > 0$ is only due to gravity, and knowing that the acceleration due to gravity is -9.8 m/s^2, what is $s''(t)$?*

$$s''(t) = \underbrace{\boxed{-9.8}}_{\text{given}}$$

Learning outcomes: Interpret the second derivative of a position function as acceleration. Calculate higher order derivatives.

Example 1. *You recently took a road trip from Columbus Ohio to Urbana-Champaign Illinois. The distance traveled from Columbus Ohio is roughly modeled by:*

$$s(t) = 36t^2 - 4.8t^3 \qquad \textit{(miles West of Columbus)}$$

where t is measured in hours, and is between 0 and 6. Find a formula for your acceleration.

Explanation. *Here we simply need to find the second derivative:*

$$s'(t) = \underbrace{\boxed{72t - 14.4t^2}}_{\text{given}} \qquad \textit{and} \qquad s''(t) = \underbrace{\boxed{72 - 28.8t}}_{\text{given}}$$

Hence our acceleration is $72 - 28t$ miles/hour^2.