

Break-Ground:

Equal or not?

Here we see a dialogue where students discuss combining limits with arithmetic.

Check out this dialogue between two calculus students (based on a true story):

Devyn: Riley, I've been thinking about limits.

Riley: So awesome!

Devyn: Think about

$$\lim_{x \rightarrow a} (f(x) + g(x)).$$

This is the number that $f(x) + g(x)$ gets nearer and nearer to, as x gets nearer and nearer to a .

Riley: You know it!

Devyn: So I think it is the same as

$$\lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} g(x).$$

Riley: Yeah, that does make sense, since when you add two numbers, say

(a number near 6) + (a number near 7)

you get

(a number near 13)

Riley: Right! And I think the same reasoning will work for multiplication! So we should be able to say

$$\lim_{x \rightarrow a} (f(x) \cdot g(x)) = \left(\lim_{x \rightarrow a} f(x) \right) \cdot \left(\lim_{x \rightarrow a} g(x) \right).$$

Devyn: Yes, I think that's right! But what about *division*? Can we use similar reasoning to conclude

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}.$$

Learning outcomes:

Equal or not?

Problem 1 Give an argument (similar to the one above) supporting the idea that

$$\lim_{x \rightarrow a} (f(x) \cdot g(x)) = \left(\lim_{x \rightarrow a} f(x) \right) \cdot \left(\lim_{x \rightarrow a} g(x) \right).$$

Free Response:

For the next problems, suppose L is a number near 1 and that M is a number near 0.

Problem 2 Using the context above,

$$\frac{\text{large}}{\text{small}} = ?$$

Multiple Choice:

- (a) “large” ✓
- (b) “small”
- (c) impossible to say

Problem 3 Using the context above,

$$\frac{\text{small}}{\text{small}} = ?$$

Multiple Choice:

- (a) “large”
- (b) “small”
- (c) impossible to say ✓