Sigma notation may seem scary at first, but it really isn't that bad.

Definition 1. *Let f be a function, and $m \leq n$ integers. Then we can write the sum*

$$f(m) + f(m+1) + f(m+2) + \cdots + f(n-1) + f(n) = \sum_{k=m}^{n} f(k)$$

We read this as "The sum of f of k from k equals m to k equals n."

This is pretty abstract. Let's see if we can sort this out.

Question 1 *What are the terms of this sum?*

$$\sum_{k=2}^{5} \sin(k)$$

$$\sin(\boxed{2}) + \sin(3) + \boxed{\sin(4)} + \sin(5)$$

Example 1. $\displaystyle\sum_{k=3}^{k=4} \frac{1}{1+k} = \frac{1}{4} + \frac{1}{5}$

Question 2 $\displaystyle\sum_{k=1}^{4} k = \boxed{10}$

Hint: $\displaystyle\sum_{k=1}^{4} k = 1 + 2 + 3 + 4 = 10$

The variable k in $\displaystyle\sum_{k=m}^{k=n}$ is called the "index of summation", or just "the index". It can be any variable we like, but the letters i, j, k are used traditionally.

Question 3 $\displaystyle\sum_{i=2}^{i=3} i^2 = \boxed{13}$

Hint: $\displaystyle\sum_{i=2}^{i=3} i^2 = 2^2 + 3^2 = 4 + 9 = 13$

Question 4 $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \sum_{j=1}^{j=\boxed{4}} \boxed{1/j}$

Hint: *There are 4 terms, so since we start counting at $j = 1$, we must go up to $j = 4$.*

Hint: $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \sum_{j=1}^{j=4} \frac{1}{j}$

Question 5 $\sum_{i=5}^{i=5} i^3 = \boxed{125}$

Hint: *This is kind of funny, but in this case we just have one term, namely $5^3 = 125$*

Question 6 $\sum_{i=2}^{i=6} k = \boxed{20}$

Hint: $\sum_{i=2}^{i=6} k = 2 + 3 + 4 + 5 + 6 = 20$

$\sum_{j=0}^{j=4} (j + 2) = \boxed{20}$

Hint: $\sum_{i=2}^{i=6} k = 2 + 3 + 4 + 5 + 6 = 20$ *again!*

Feedback (attempt): *Did you notice how these two expressions had all the same terms? Both are just shorthands for the sum $2+3+4+5+6$. This is called "reindexing" a sum.*

Question 7 *Reindexing the sum* $\sum_{j=4}^{j=7} \sin(j-2)$ *to start at* $k = 1$*, we have*

$\sum_{j=4}^{j=7} \sin(j-2) = \sum_{k=1}^{k=\boxed{4}} \boxed{\sin(k+1)}$

Hint: $\sum_{j=4}^{j=7} \sin(j-2) = \sin(2) + \sin(3) + \sin(4) + \sin(5) = \sum_{k=1}^{k=4} \sin(k+1)$

Question 8 *The sum* $\sin(4+\frac{3}{n}) + \sin(4+\frac{6}{n}) + \sin(4+\frac{9}{n}) + \ldots$ *has* n *terms. In sigma notation, this sum can be expressed as* $\sum_{k=1}^{k=n} \boxed{\sin(4+\frac{3k}{n})}$

Hint: *The* k^{th} *term is of the form* $4+\frac{3k}{n}$, *so the sum is* $\sum_{k=1}^{k=n} \sin(4+\frac{3k}{n})$

Question 9 *Fix a number* n. *Then* $\sum_{k=1}^{k=n} 1 = \boxed{n}$

Hint: *By definition,* $\sum_{k=1}^{k=n} 1$ *is the sum of* n *ones, which is just* n

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Question 10 *If* $\sum_{k=1}^{k=n} f(k) = n^2$, *then* $f(j) = \boxed{2j-1}$

Hint: *To find* $f(j)$, *we could think of this as* $\sum_{k=1}^{k=j} f(j) - \sum_{k=1}^{k=j-1} f(j)$

Hint: *So* $f(j) = j^2 - (j-1)^2 = j^2 - (j^2 - 2j + 1) = 2j - 1$

Feedback (attempt): *This is kind of cool. It says that the sum of the first* n *odd number is* n^2. *Test it and see! Can you find a geometric interpretation of this? If you are interested by this, talk to your TA!*

Question 11 *Which of the following equations could possibly make any sense at all? Mark all that apply.*

Multiple Choice:

(a) $\sum_{j=1}^{j=n} f(j) = j^3$

(b) $\sum_{j=1}^{j=n} f(n) = nf(n)$ ✓

(c) $\sum_{j=1}^{j=n} f(j) = n^3$ ✓

Hint:

- $\sum_{j=1}^{j=n} f(j) = j^3$ *cannot make any sense, since one one side j is telling us the index of a term we are summing, and on the other side it is a fixed number. These two meanings of j cannot coexist.*
- $\sum_{j=1}^{j=n} f(n) = nf(n)$ *not only makes sense, it is universally true! It is okay that n appears in all three parts of the expression, since it is just a fixed number*
- $\sum_{j=1}^{j=n} f(j) = n^3$ *is also fine. As a bonus challenge, can you find the function f which makes this true?*