Dig-In:

A Sun-line

Here we "play" with mathematics.

A mission gone ary

You are a top-secret agent, on a mission that puts you over the Pacific on a plane flying nearly over the equator. Though you are the world's most respected secret-angent, you had no idea what was about to happen next.

There you were, being your (frankly awesome) spy-self. When "some nonsense" turned into the "some other nonsense" and the next thing you knew, you were tossed out of the plane plunging downward into the dark Pacfic. Never to loose hope, you spread your arms and legs to increase your surface area, and hence atmospheric drag.

What would you need to reduce your frontal area to (drag coefficient?) to ???

Through a clever use of nearly all of your garments, you *are* able to increase your frontal area (and drag coefficient?) to exactly the necessary value to reduce your impact speed to....

kE?

As you splash down into the chilly waters of the Pacific, you see a small desert island nearly 500 m ahead of you...

With your last bit of strength, you manage to swim to the island, and finally know the ressurance of solid ground. As you lay in the warm sand, you realize that you are a pretty awesome secret agent.

The next morning

The next morning you awake and immediatley start to concoct a plan to get your off this island and back to mission.

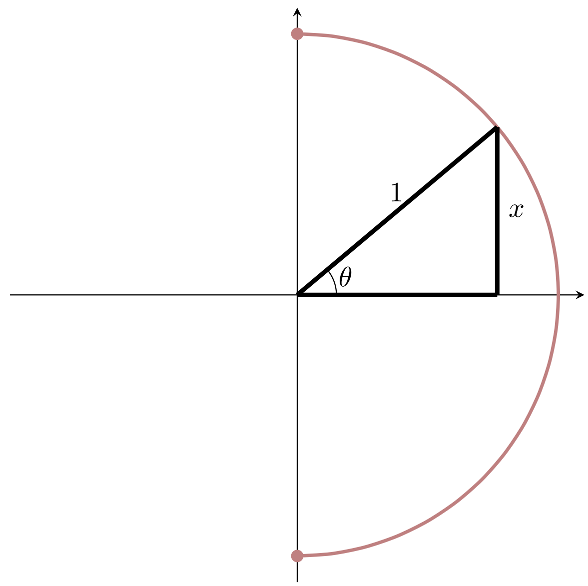
Hidden beneith your skin, is a secret, single-use, transponder beacon. If ever you are in trouble in the world, you can, through a very careful application of pressure, trigger this transponder. It will be active for a mere five seconds. However, if you set off the beacon at exactly 11:51 pm GMT

Learning outcomes:

MAYBE YOU HAVE THE ABILITY TO GET 1 time, and MAKE One call.

TRAPPED ON GALAPAGOSE ISLAND - NEED A CLOCK - USE LINEAR APPROX

Imagine that the world was flat and that everyday, the Sun (infinitely far away) acutally rises up at 6 am in the East, and sets at 6 pm in the west.



In the diagram above, we see a rod of height h casting a shadow. We can see that the slope of the line connecting the point representing the Sun to the tip of the rod is

$$\frac{R\sin\left(\frac{n\pi}{12}\right)-h}{R\cos\left(\frac{n\pi}{12}\right)}$$

If we take the limit as R goes to infinity, then we find,

$$\begin{aligned}
\lim_{R\to\infty}\frac{R\sin\left(\frac{n\pi}{12}\right)-h}{R\cos\left(\frac{n\pi}{12}\right)} &= \lim_{R\to\infty}\frac{R\sin\left(\frac{n\pi}{12}\right)-h}{R\cos\left(\frac{n\pi}{12}\right)}\cdot\frac{1/R}{1/R} \\
&= \lim_{R\to\infty}\frac{\sin\left(\frac{n\pi}{12}\right)-h/R}{\cos\left(\frac{n\pi}{12}\right)} \\
&= \frac{\sin\left(\frac{n\pi}{12}\right)}{\cos\left(\frac{n\pi}{12}\right)} \\
&= \tan\left(\frac{n\pi}{12}\right)
\end{aligned}$$

What does the graph look like?

We'll use all of our curve sketching techniques to try to understand this function on the interval $[-2\pi, 2\pi]$, as gesture of friendship, we'll tell you that $\mathrm{Si}(x)$ is continuous on $[-2\pi, 2\pi]$. The first thing we should do is compute the first derivative of $\mathrm{Si}(x)$.

Example 1. *Compute:*

$$\frac{d}{dx}\,\mathrm{Si}(x)$$

Explanation. *By the First Fundamental Theorem of Calculus,*

$$\begin{aligned}\frac{d}{dx}\,\mathrm{Si}(x) &= \frac{d}{dx}\int_0^x \frac{\sin(t)}{t}\,dt\\ &= \frac{\sin(x)}{x}.\end{aligned}$$

Now we'll compute the second derivative of $\mathrm{Si}(x)$.

Example 2. *Compute:*

$$\frac{d^2}{dx^2}\,\mathrm{Si}(x)$$

Explanation. *By our previous work and the quotient rule we see*

$$\begin{aligned}\frac{d^2}{dx^2}\,\mathrm{Si}(x) &= \frac{d}{dx}\frac{\sin(x)}{x}\\ &= \frac{x\cos(x)-\sin(x)}{x^2}.\end{aligned}$$

Now we should find the y-intercept.

Example 3. *Compute* $\mathrm{Si}(0)$*.*

Explanation. *Here* $\mathrm{Si}(0) = 0$*, as* Si *is an accumulation function, and at* $x = 0$*, no area has been accumulated.*

Now we'll look for critical points, where the derivative is zero or undefined.

Example 4. *Find the critical points of* $\mathrm{Si}(x)$*.*

Explanation. *The critical points are where* $\mathrm{Si}'(x) = 0$ *or it does not exist. Since*

$$\mathrm{Si}'(x) = \frac{\sin(x)}{x}.$$

We see that this derivative does not exist at $x = 0$*, and for* $x \neq 0$*,* $\mathrm{Si}'(x) = 0$ *precisely when* $\sin(x)$ *is zero. Since* $\sin(x)$ *is zero at* $x = -2\pi, -\pi, \pi, 2\pi$*, we see that the critical points are where*

$$x = -2\pi, -\pi, 0, \pi, 2\pi.$$

We'll identify which of these are maximums and minimums.

Example 5. *Find the local extrema of* $\mathrm{Si}(x)$ *on the interval* $[-2\pi, 2\pi]$.

Explanation. *The critical points are at*

$$x = -2\pi, -\pi, 0, \pi, 2\pi.$$

We will use the first derivative test to identifiy which of these are local extrema.

Inflection points are harder. Let's try our hand.

Example 6. *Find the inflection points of* $\mathrm{Si}(x)$.

Explanation. *We start by looking at the second derivative of* $\mathrm{Si}(x)$,

$$\mathrm{Si}''(x) = \frac{x\cos(x) - \sin(x)}{x^2}.$$

The first candidate for an infection point is $x = 0$, *since the second derivative does not exist. To find other inflection points on* $[-2\pi, 2\pi]$, *we need to find when*

$$x\cos(x) - \sin(x) = 0.$$

This is zero when $x = 0$, *anywhere else?*