Break-Ground:

A secret of the definite integral

Two young mathematicians discuss what calculus is all about.

Check out this dialogue between two calculus students (based on a true story):

Devyn: Ah. So now we have a connection between derivatives and integrals.

Riley: Right, the derivative of the accumulation function is the "inside" function.

Devyn: So how do we use this to compute area?

Sometimes it helps to think about the most basic examples. Consider

$$\int_2^5 4\,dt$$

We know (by geometry) that this computes the area of a 3×4 rectangle which equals 12. On the other hand, if we consider the accumulation function

$$F(x) = \int_2^x 4\,dt,$$

we see that

$$F(5) = \int_2^5 4\,dt.$$

Problem 1 *What is $F(2)$?*

$$F(2) = \boxed{0}$$

Problem 2 *On the other hand, the First Fundamental Theorem of Calculus says that if*

$$F(x) = \int_2^x 4\,dt,$$

then $F'(x) = 4$. Armed with this knowledge, and the fact that $F(2) = 0$, what must $F(x)$ be?

$$F(x) = \boxed{4x - 8}$$

Learning outcomes:

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