Break-Ground:

We can figure it out

Two young mathematicians discuss the derivative of inverse functions.

Check out this dialogue between two calculus students (based on a true story):

Devyn: Riley, I have a calculus question.

Riley: Hit me with it.

Devyn: What's the derivative of $\arctan(x)$?

Riley: Hmmm... we haven't talked about that yet in our class.

Devyn: I know! But maybe we can figure it out.

Riley: Well

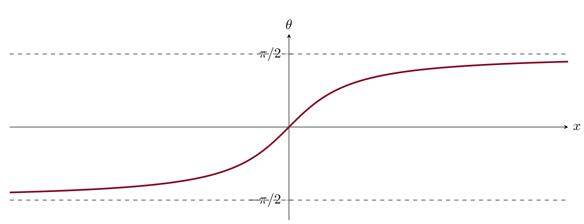
$$\arctan(x) = \tan^{-1}(x)$$

and now we can use the chain rule to take its derivative

$$\begin{aligned}\frac{d}{dx}\tan^{-1}(x) &= -\tan^{-2}(x)\sec^2(x)\\ &= -\frac{\cos^2 x}{\sin^2 x}\cdot\frac{1}{\cos^2 x}\\ &= \frac{-1}{\sin^2 x}\\ &= -\csc^2 x\end{aligned}$$

Devyn: But is this right?

Let's see if we can figure out if Devyn and Riley are correct. Start by looking at a plot of $\theta = \arctan(x)$:



Learning outcomes: Recall the meaning and properties of inverse trigonometric functions. Understand how the derivative of an inverse function relates to the original derivative.

Problem 1 *Let* $f(x) = \arctan(x)$*. Use the plot above to determine the behavior of the derivative of* f *as* x *gets very large. If the limit does not exist, enter "DNE".*

$$\lim_{x\to\infty} f'(x) = \boxed{0}$$

On the other hand,

Problem 2 *Compute the limit of* $-\csc^2(x)$ *as* x *goes to infinity. If the limit does not exist, enter "DNE".*

$$\lim_{x\to\infty} (-\csc^2(x)) = \boxed{DNE}$$

Problem 3 *In light of the problems above, is it possible that*

$$\frac{d}{dx}\arctan(x) = -\csc^2(x)?$$

Multiple Choice:

(a) *yes*

(b) *no* ✓

Problem 4 *When our friends wrote* $\arctan(x) = \tan^{-1}(x)$*, what do they think the "–1" represents? Are they correct?*

Free Response: *Riley thinks that we can use the power rule on the* -1*, which tells us that the students are using* -1 *as an exponent for the tangent function. However, in the case of inverse functions such as* $\arctan(x)$*, the* -1 *is not an exponent.*