Dig-In:

The Wallis product

Let

$$I(n) = \int_0^{\pi} \sin^n(x)\,dx.$$

It can be shown that

$$\int_0^{\pi/2} \sin^n(x)\,dx = \frac{n-1}{n} \int_0^{\pi/2} \sin^{n-2}(x)\,dx$$

when $n \geq 2$.

Show that

$$I(2n+2) \leq I(2n+1) \leq I(2n)$$

Show that

$$\frac{I(n)}{I(n-2)} = \frac{n-1}{2n}$$

$$I(0) = \pi$$

$$I(1) = 2$$

$$I(2n) =$$

Learning outcomes: