Break-Ground:

Modeling the spread of infectious diseases

Two young mathematicians discuss differential equations.

Check out this dialogue between two calculus students (based on a true story):

Devyn: Riley, check out this book, it has some cool applications of calculus:

> The following differential equation can be used to model the spread of an infectious disease:
>
> $$\mathrm{infect}'(t) = k \cdot \mathrm{infect}(t) \cdot (P - \mathrm{infect}(t))$$
>
> where k is a constant, $\mathrm{infect}(t)$ is the number of people infected by the disease on day t, and P is the size of the population vulnerable to the disease.

Riley: Whoa. That's like a formula for a derivative. Wow. Much calculus.

Devyn: I wonder how you solve equations like this?

Riley: I wonder if we can sometimes just use facts about the derivative to give us an approximation for the function that models the spread of infection?

Problem 1 *Suppose your calculus class has had a freak outbreak of the math-philia. Some facts: We have around* 200 *students in our class, we are now on the* 23*rd day of the outbreak, and currently* 100 *students are infected. Using the differential equation*

$$\mathrm{infect}'(t) = k \cdot \mathrm{infect}(t) \cdot (P - \mathrm{infect}(t))$$

we can model the spread of math-philia by setting $k = 0.001$. *What is* $\mathrm{infect}'(23)$*?*

$$\mathrm{infect}'(23) = \boxed{10}$$

Problem 2 *Do your best to explain why the equation*

$$\mathrm{infect}'(t) = k \cdot \mathrm{infect}(t) \cdot (P - \mathrm{infect}(t))$$

is reasonable.

Learning outcomes: Define a differential equation.

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Hint: *Don't worry, just do your best.*

Free Response: *Answers will vary.*