Break-Ground:

Wanted: graphing procedure

Two young mathematicians discuss how to sketch the graphs of functions.

Check out this dialogue between two calculus students (based on a true story):

Devyn: Riley, OK I know how to plot something if I'm given a description.

Riley: Yes, it's kinda fun right?

Devyn: I know! But now I'm not sure how to get the information I need.

Riley: You know, I'd like to make up a procedure based on all these facts, that would tell me what the graph of any function would look like.

Devyn: Me too! Let's get to work!

Problem 1 *Below is a list of features of a graph of a function.*

(a) *Find any vertical asymptotes, these are points $x = a$ where $f(x)$ goes to infinity as x goes to a (from the right, left, or both).*

(b) *Find the critical points (the points where $f'(x) = 0$ or $f'(x)$ is undefined).*

(c) *Identify inflection points and concavity.*

(d) *Determine an interval that shows all relevant behavior.*

(e) *Find the candidates for inflection points, the points where $f''(x) = 0$ or $f''(x)$ is undefined.*

(f) *Compute f' and f''.*

(g) *Find the y-intercept, this is the point $(0, f(0))$. Place this point on your graph.*

(h) *Use either the first or second derivative test to identify local extrema and/or find the intervals where your function is increasing/decreasing.*

(i) *If possible, find the x-intercepts, the points where $f(x) = 0$. Place these points on your graph.*

Learning outcomes: Determine how the graph of a function looks without using a calculator.

(j) *Analyze end behavior: as $x \to \pm\infty$, what happens to the graph of f? Does it have horizontal asymptotes, increase or decrease without bound, or have some other kind of behavior?*

In what order should we take these steps? For example, one must compute f' before computing f''. Also, one must compute f' before finding the critical points. There is more than one correct answer.

Free Response: *Here is one possible answer to this question. Compare it with yours!*

(a) *Compute $f'(x)$ and $f''(x)$.*

(b) *Find the y-intercept, this is the point $(0, f(0))$. Place this point on your graph.*

(c) *Find any vertical asymptotes, these are points $x = a$ where $f(x)$ goes to infinity as x goes to a (from the right, left, or both).*

(d) *If possible, find the x-intercepts, the points where $f(x) = 0$. Place these points on your graph.*

(e) *Analyze end behavior: as $x \to \pm\infty$, what happens to the graph of f? Does it have horizontal asymptotes, increase or decrease without bound, or have some other kind of behavior?.*

(f) *Find the critical points (the points where $f'(x) = 0$ or $f'(x)$ is undefined).*

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(i) *Identify inflection points and concavity.*

(j) *Determine an interval that shows all relevant behavior*