Dig-In:

The Extreme Value Theorem

We examine a fact about continuous functions.

Definition 1.

(a) *A function f has an* ***global maximum*** *at $x = a$, if $f(a) \geq f(x)$ for every x in the domain of the function.*

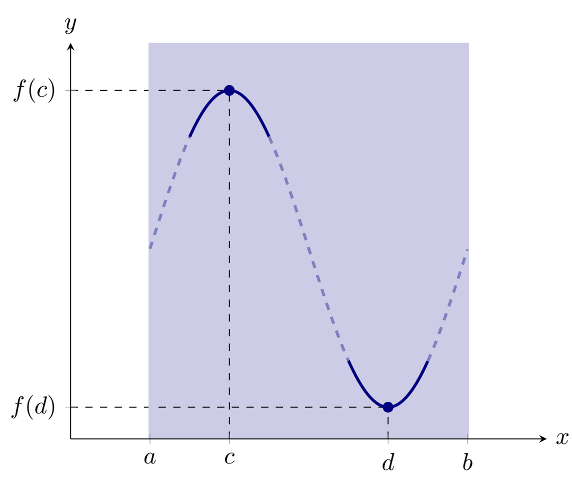
(b) *A function f has an* ***global minimum*** *at $x = a$, if $f(a) \leq f(x)$ for every x in the domain of the function.*

A ***global extremum*** *is either a global maximum or a global minimum.*

If we are working on an finite closed interval, then we have the following theorem.

Theorem 1 (Extreme Value Theorem)**.** *If f is a continuous function for all x in the closed interval $[a, b]$, then there are points c and d in $[a, b]$, such that $(c, f(c))$ is a global maximum and $(d, f(d))$ is a global minimum on $[a, b]$.*

Below, we see a geometric interpretation of this theorem.



Question 1 *Would this theorem hold if we were working on an open interval?*

Multiple Choice:

Learning outcomes: Understand the statement of the Extreme Value Theorem.

(a) *yes*

(b) *no* ✓

Hint: *Consider* $\tan(\theta)$ *for* $-\pi/2 < \theta < \pi/2$. *Does this function achieve its maximum and minimum?*