Break-Ground:

Derivatives of products are tricky

Two young mathematicians discuss derivatives of products and products of derivatives.

Check out this dialogue between two calculus students (based on a true story):

Devyn: Hey Riley, remember the sum rule for derivatives?

Riley: You know I do.

Devyn: What do you think that the "product rule" will be?

Riley: Let's give this a spin:

$$\frac{d}{dx}\left(f(x)\cdot g(x)\right) = f'(x)\cdot g'(x)?$$

Devyn: Hmmm, let's give this theory an acid test. Let's try

$$f(x) = x^2+1 \qquad \text{and} \qquad g(x) = x^3-3x$$

Now

$$\begin{aligned} f'(x)g'(x) &= (2x)(3x^2-3) \\ &= 6x^3-6x. \end{aligned}$$

Riley: On the other hand,

$$\begin{aligned} f(x)g(x) &= (x^2+1)(x^3-3x) \\ &= x^5-3x^3+x^3-3x \\ &= x^5-2x^3-3x. \end{aligned}$$

Devyn: And so,

$$\frac{d}{dx}\left(f(x)\cdot g(x)\right) = 5x^4-6x^2-3.$$

Riley: Wow. Hmmm. It looks like our guess was incorrect.

Devyn: I've got a feeling that the so-called "product rule" might be a bit tricky.

Learning outcomes: Explain why the product rule is not given by multiplying the derivatives of the products. Apply the sum rule repeatedly to find the derivative of a product. Relate the sum rule, the constant multiple rule, and the product rule.

Problem 1 *Above, our intrepid young mathematicians guess that the "product rule" might be:*

$$\frac{d}{dx}\left(f(x)\cdot g(x)\right) = f'(x)\cdot g'(x)?$$

Does this ***ever*** *hold true?*

Free Response: *Answers will vary. A partial answer is that this will hold when either $f(x)$ or $g(x)$ are zero, or when both are constants.*