Break-Ground:

A changing circle

Two young mathematicians discuss a circle that is changing.

Check out this dialogue between two calculus students (based on a true story):

Devyn: Riley, I've been thinking about calculus.

Riley: YOLO.

Devyn: Consider a circle of some radius r.

Riley: Ha! What else would we ever call the "radius?"

Devyn: Exactly. Now the formula for the perimeter of a circle is?

Riley: $P = 2 \cdot \pi \cdot r$ baby.

Devyn: And its area?

Riley: You know it's $A = \pi \cdot r^2$.

Devyn: Right, but here's what's bugging me: If I know r', what is P'? What's A'?

Riley: Oooh. Ouch. Hmmm. I wanna say it's

$$P' = 2 \cdot \pi \cdot r' \qquad \text{and} \qquad A' = \pi(r')^2$$

but I'm not sure that is right.

Devyn: Yeah... me too. But I'm not sure that's right either. Are we forgetting something?

Problem 1 *Do you think our young mathematicians above are correct?*

Multiple Choice:

(a) *Yes.* $P' = 2 \cdot \pi \cdot r'$ *and* $A' = \pi(r')^2$.

(b) *No. While* $P' = 2 \cdot \pi \cdot r'$, $A' \neq \pi(r')^2$. ✓

(c) *No. While* $A' = \pi(r')^2$, $P' \neq 2 \cdot \pi \cdot r'$.

Learning outcomes: Solve related rates word problems. Calculate derivatives of expressions with multiple variables implicitly.

(d) *No.* $P' \neq 2 \cdot \pi \cdot r'$ *and* $A' \neq \pi(r')^2$.

(e) *There is no way to tell.*

Problem 2 *Set* $r(t) = 3 \cdot t$. *What is* $r'(t)$ *when* $r = 15$? $r'(t) = \boxed{3}$

Problem 3 *Set* $r = 3 \cdot t$. *Now* $P(t) = 2 \cdot \pi \cdot 3 \cdot t$. *What is* $P'(t)$ *when* $r = 15$?
$P'(t) = \boxed{2 \cdot \pi \cdot 3}$

Problem 4 *Describe what* P' *means in this context.*

Free Response: *It describes how fast the perimeter is changing at a particular instant in time.*

Problem 5 *Set* $r = 3 \cdot t$. *Now* $A(t) = \pi \cdot (3 \cdot t)^2$. *What is* $A'(t)$ *when* $r = 15$?
$A'(t) = \boxed{2 \cdot \pi \cdot 15 \cdot 3}$

Problem 6 *Describe what* A' *means in this context. Does it make sense that* A' *is positive?*

Free Response: *It describes how fast the area is changing. It does make sense that* A' *is positive, since as the radius gets larger, the area should get larger, too.*

Problem 7 *What, if anything, did our two young mathematicians forget about above?*

Free Response: *They forgot the chain rule.*
