1 Solve the equation

$$3e^{2x} - 4e^{-2x} = 5.$$

Give the answer correct to 3 decimal places. [3]

$u = e^{2x}$

$3u - 4u^{-1} = 5$

$3u^2 - 5u - 4 = 0$

$u = \dfrac{5 \pm \sqrt{5^2 - 4(3)(-4)}}{2(3)}$

$u = 2.257, -0.591$

$e^{2x} = 2.257$

$2x = \ln(2.257)$

$x = 0.407$

2 **(a)** Sketch the graph of $y = |2x + 3|$. [1]

(b) Solve the inequality $3x + 8 > |2x + 3|$. [3]

$3x+8 = 2x+3$ $\quad$ $3x+8 = -2x-3$

$x = -5$ $\quad$ $5x = -11$

$x = -11/5$

no $\quad$ no $\quad$ yes

-5 $\quad$ $-11/5$

$x > -11/5$

3 Find the coefficient of x^3 in the binomial expansion of $(3 + x)\sqrt{1 + 4x}$. [4]

$(3+x)(1+4x)^{1/2}$

$(3+x)\left(1+(1/2)(4x)+\frac{1/2(-1/2)}{2}(4x)^2+\frac{1/2(-1/2)(-3/2)}{6}(4x)^3\right)$

$(3+x)(1+2x-2x^2+4x^3)$

$12x^3-2x^3$

$10x^3$

4 **(a)** Show that the equation $\sin 2\theta + \cos 2\theta = 2\sin^2\theta$ can be expressed in the form

$$\cos^2\theta + 2\sin\theta\cos\theta - 3\sin^2\theta = 0.$$ [2]

$\sin 2\theta + \cos 2\theta - 2\sin^2\theta = 0$

$2\sin\theta\cos\theta + 1 - 2\sin^2\theta - 2\sin^2\theta = 0$

$2\sin\theta\cos\theta + \sin^2\theta + \cos^2\theta - 4\sin^2\theta = 0$

$\cos^2\theta + 2\sin\theta\cos\theta - 3\sin^2\theta = 0$

(b) Hence solve the equation $\sin 2\theta + \cos 2\theta = 2\sin^2\theta$ for $0° < \theta < 180°$. [4]

$\cos^2\theta + 2\sin\theta\cos\theta - 3\sin^2\theta = 0$

$(\cos\theta + 3\sin\theta)(\cos\theta - \sin\theta) = 0$

$\cos\theta + 3\sin\theta = 0$

$3\sin\theta = -\cos\theta$

$\frac{3\sin\theta}{\cos\theta} = -1$

$\tan\theta = -1/3$

$\theta = 161.6°$

$\cos\theta - \sin\theta = 0$

$\cos\theta = \sin\theta$

$1 = \frac{\sin\theta}{\cos\theta} = \tan\theta$

$\theta = 45°$

1 Solve the inequality $|5x - 3| < 2|3x - 7|$. [4]

$5x-3=2(3x-7)$ $\quad$ $-5x+3=2(3x-7)$

$5x-3=6x-14$ $\quad$ $-5x+3=6x-14$

$-x=-11$ $\quad$ $-11x=-17$

$x=11$ $\quad$ $x=17/11$

yes $\quad$ no $\quad$ yes

$17/11$ $\quad$ 11

$x<17/11$ or $x>11$

2 Solve the equation $\ln(2x^2 - 3) = 2\ln x - \ln 2$, giving your answer in an exact form. [3]

$\ln(2x^2-3) = \ln x^2/2$

$2x^2-3 = x^2/2$

$4x^2-6 = x^2$

$-6 = -3x^2$

$2 = x^2$

$x = \pm\sqrt{2}$

$x > 0$

$x = \sqrt{2}$

4 Solve the equation $2\cos x - \cos\frac{1}{2}x = 1$ for $0 \leqslant x \leqslant 2\pi$. [5]

$u = \frac{1}{2}x$

$2\cos 2u - \cos u = 1$

$2(2\cos^2 u - 1) - \cos u = 1$

$4\cos^2 u - \cos u - 3 = 0$

$z = \cos u$

$4z^2 - z - 3 = 0$

$(4z+3)(z-1) = 0$

$z = -3/4,\ 1 = \cos u$

$u = \frac{1}{2}x = \cos^{-1}(-3/4),\ \cos^{-1}(1)$

$x = 2(2.419),\ 2(3.864),\ 2(0)$

$x = 0,\ 4.84$

1 Solve the equation $\ln(x+5) = 5 + \ln x$. Give your answer correct to 3 decimal places. [4]

$\ln(x+5) - \ln x = 5$

$\ln \frac{x+5}{x} = 5$

$\frac{x+5}{x} = e^5$

$x+5 = xe^5$

$5 = 147.41x$

$x = 0.0339$

2 Find the quotient and remainder when $2x^4 - 27$ is divided by $x^2 + x + 3$. [3]

$$
\begin{array}{r|l}
 & 2x^2 - 2x - 4 \\
\hline
x^2 + x + 3 & 2x^4 + 0x^3 + 0x^2 + 0x - 27 \\
 & -(2x^4 + 2x^3 + 6x^2) \\
\hline
 & -2x^3 - 6x^2 + 0x \\
 & -(-2x^3 - 2x^2 - 6x) \\
\hline
 & -4x^2 + 6x - 27 \\
 & -4x^2 - 4x - 12 \\
\hline
 & 10x - 15
\end{array}
$$

$2x^2 - 2x - 4$ r $10x - 15$

6 **(a)** Express $3\cos x + 2\cos(x - 60°)$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. State the exact value of R and give α correct to 2 decimal places. [4]

$3\cos x + 2(\cos x \cos 60° + \sin x \sin 60°)$

$3\cos x + \cos x + \sqrt{3}\sin x$

$4\cos x + \sqrt{3}\sin x$

$R = \sqrt{4^2 + 3} = \sqrt{19}$

$\alpha = \tan^{-1}\left(\frac{\sqrt{3}}{4}\right) = 23.4°$

$\sqrt{19}\cos(x - 23.4)$

(b) Hence solve the equation

$$3\cos 2\theta + 2\cos(2\theta - 60°) = 2.5$$

for $0° < \theta < 180°$. [4]

$\sqrt{19}\cos(x - 23.4) = 2.5$

$x - 23.4 = \cos^{-1}(2.5/\sqrt{19})$

$x = 39.2°, 164.2°$

10 Let $f(x) = \dfrac{21 - 8x - 2x^2}{(1 + 2x)(3 - x)^2}$.

(a) Express $f(x)$ in partial fractions. [5]

$$\frac{21-8x-2x^2}{(1+2x)(3-x)^2} = \frac{A}{1+2x} + \frac{B}{3-x} + \frac{C}{(3-x)^2}$$

$$21-8x-2x^2 = A(3-x)^2 + B(1+2x)(3-x) + C(1+2x)$$

$$21-8x-2x^2 = A(9-6x+x^2) + B(3+5x-2x^2) + C(1+2x)$$

$x=3$ $\quad 21-8(3)-2(3)^2 = C(1+2\times 3)$

$C=-3$

$x=-1/2$ $\quad 21-8(-1/2)-2(-1/2)^2 = A(3+1/2)^2$

$A=2$

$x=0$ $\quad 21 = 2(9) + 3B + (-3)$

$B=2$

$2/1+2x + 2/3-x - 3/(3-x)^2$

(b) Hence obtain the expansion of f(x) in ascending powers of x, up to and including the term in x^2. [5]

$2(1+2x)^{-1} + 2(3-x)^{-1} - 3(3-x)^{-2}$

$2\left(1 + (-1)(2x) + \frac{(-1)(-2)}{2}(2x)^2\right) + 2\left(3^{-1}\left(1 + (-1)\frac{-x}{3} + \frac{(-1)(-2)}{2}\left(\frac{-x}{2}\right)^2\right)\right.$

$-3\left(3^{-2}\left(1 + \frac{-2x}{3} + \frac{-2}{2}\left(\frac{-x}{3}\right)^2\right)\right.$

$2 - 4x + 8x^2 + \frac{2}{3} + \frac{2}{9}x + \frac{2}{27}x^2 - \frac{1}{3} - \frac{2}{9}x - \frac{x^2}{9}$

$7/3 - 4x + 215/27\,x^2$

3

The variables x and y are related by the equation $y = ab^x$, where a and b are constants. The diagram shows the result of plotting $\ln y$ against x for two pairs of values of x and y. The coordinates of these points are (1, 3.7) and (2.2, 6.46).

Use this information to find the values of a and b. [4]

$\ln y = \ln ab^x$

$\ln y = \ln a + x \ln b$

$\begin{cases} 3.7 = \ln a + \ln b \\ 6.46 = \ln a + 2.2 \ln b \end{cases}$

$-2.76 = -1.2 \ln b$

$b = 9.97$

$a = 4.06$

5 **(a)** Given that

$$\sin\left(x+\tfrac{1}{6}\pi\right)-\sin\left(x-\tfrac{1}{6}\pi\right)=\cos\left(x+\tfrac{1}{3}\pi\right)-\cos\left(x-\tfrac{1}{3}\pi\right),$$

find the exact value of $\tan x$. [4]

$(\sin x\cos\pi/6+\cos x\sin\pi/6)-(\sin x\cos\frac{\pi}{6}-\cos x\sin\pi/6)=$

$(\cos x\cos\pi/3-\sin x\sin\pi/3)-(\cos x\cos\pi/3+\sin x\sin\pi/3)$

$\sqrt{3}/2\sin x+\frac{1}{2}\cos x-\frac{\sqrt{3}}{2}\sin x+\frac{1}{2}\cos x=\frac{\sqrt{3}}{2}\cos x-\frac{1}{2}\sin x-\frac{\sqrt{3}}{2}\cos x-\frac{1}{2}\sin x$

$\cos x=-\sqrt{3}\sin x$

$-1/\sqrt{3}=\tan x$

(b) Hence find the exact roots of the equation

$$\sin\left(x + \tfrac{1}{6}\pi\right) - \sin\left(x - \tfrac{1}{6}\pi\right) = \cos\left(x + \tfrac{1}{3}\pi\right) - \cos\left(x - \tfrac{1}{3}\pi\right)$$

for $0 \leqslant x \leqslant 2\pi$. [2]

$-\frac{1}{\sqrt{3}} = \tan x$

$x = \frac{5\pi}{6}, \frac{11\pi}{6}$

10 Let $f(x) = \dfrac{24x + 13}{(1 - 2x)(2 + x)^2}$.

(a) Express $f(x)$ in partial fractions. [5]

$\frac{24x+13}{(1-2x)(2+x)^2} = \frac{A}{1-2x} + \frac{B}{2+x} + \frac{C}{(2+x)^2}$

$24x+13 = A(2+x)^2 + B(1-2x)(2+x) + C(1-2x)$

$x = -2$

$-48+13 = C(1-2(-2))$

$-35 = 5C$

$C = -7$

$x = 1/2$

$24(1/2)+13 = A(2+1/2)^2$

$25 = 25A/4$

$4 = A$

$x = 1$

$24(1)+13 = 4(2+1)^2 + B(1-2)(2+1) - 7(1-2)$

$37 = 36 - 3B + 7$

$37 = 43 - 3B$

$B = 2$

$4/1-2x + 2/2+x - 7/(2+x)^2$

(b) Hence obtain the expansion of f(x) in ascending powers of x, up to and including the term in x^2. [5]

$4(1-2x)^{-1} + 2(2+x)^{-1} - 7(2+x)^{-2}$

$4(1-(-2x)+(-2x)^2) + 2(2^{-1}(1+(-1)\frac{x}{2}+(\frac{x}{2})^2) - 7(2^{-2}+(-2)(\frac{x}{2})+3(\frac{x}{2})^2)$

$4(1+2x+4x^2) + 1 - \frac{x}{2} + \frac{x^2}{4} - 7/4 - x + \frac{3x^2}{4}$

$13/4 + 37/4\, x + 239/16\, x^2$

(c) State the set of values of x for which the expansion in **(b)** is valid. [1]

$-1/2 < x < 1/2$

1 **(a)** Sketch the graph of $y = |4x - 2|$. [1]

(b) Solve the inequality $1 + 3x < |4x - 2|$. [4]

$1 + 3x = 4x - 2$

$3 = x$

$1 + 3x = -4x + 2$

$7x = 1$

$x = 1/7$

yes | no | yes

$1/7$ 3

$x < 1/7$, $x > 3$

3 The polynomial $2x^3 + ax^2 - 11x + b$ is denoted by $p(x)$. It is given that $p(x)$ is divisible by $(2x - 1)$ and that when $p(x)$ is divided by $(x + 1)$ the remainder is 12.

Find the values of a and b. [5]

$$\begin{cases} 2(1/2)^3 + a(1/2)^2 - 11(1/2) + b = 0 \\ 2(-1)^3 + a(-1)^2 - 11(-1) + b = 12 \end{cases}$$

$$\begin{cases} \frac{1}{4} + \frac{1}{4}a - \frac{11}{2} + b = 0 \\ -2 + a + 11 + b = 12 \end{cases}$$

$$\begin{cases} 4 + a - 22 + 4b = 0 \\ a + b = 3 \end{cases}$$

$a = -3$

$b = 6$

7 **(a)** By expressing 3θ as $2\theta + \theta$, prove the identity $\cos 3\theta \equiv 4\cos^3\theta - 3\cos\theta$. [3]

$\cos(2\theta+\theta) \equiv 4\cos^3\theta - 3\cos\theta$

$\cos 2\theta \cos\theta - \sin 2\theta \sin\theta$

$(2\cos^2\theta - 1)\cos\theta - 2\sin\theta\cos\theta \times \sin\theta$

$2\cos^3\theta - \cos\theta - 2\sin^2\theta\cos\theta$

$2\cos^3\theta - \cos\theta - 2\cos\theta(1-\cos^2\theta)$

$4\cos^3\theta - 3\cos\theta$

(b) Hence solve the equation

$$\cos 3\theta + \cos\theta\cos 2\theta = \cos^2\theta$$

for $0° \leqslant \theta \leqslant 180°$. [5]

$4\cos^3\theta - 3\cos\theta + \cos\theta\cos 2\theta = \cos^2\theta$

$4\cos^3\theta - 3\cos\theta + \cos\theta(2\cos^2\theta - 1) = \cos^2\theta$

$4\cos^3\theta - 3\cos\theta + 2\cos^3\theta - \cos\theta - \cos^2\theta = 0$

$6\cos^3\theta - \cos^2\theta - 4\cos\theta = 0$

$\cos\theta(6\cos^2\theta - \cos\theta - 4) = 0$

$\cos\theta = 0$ $\qquad$ $6\cos^2\theta - \cos\theta - 4 = 0$

$\theta = 90°$

$u = \cos\theta$

$u = \dfrac{1 \pm \sqrt{1^2 - 4(6)(-4)}}{2(6)}$

$\theta = 137.5°$

1 Find the set of values of x satisfying the inequality $|2^{x+1} - 2| < 0.5$, giving your answer to 3 significant figures. [4]

$2^{x+1} - 2 = 1/2$

$2^{x+1} = 5/2$

$\log_2 2^{x+1} = \log_2 5/2$

$x+1 = 1.322$

$x = 0.322$

$2^{x+1} - 2 = -1/2$

$2^{x+1} = 3/2$

$\log_2 (2^{x+1}) = \log_2 (3/2)$

$x+1 = 0.585$

$x = -0.415$

no ~~yes~~ no

-0.415 0.322

$-0.415 < x < 0.322$