1 Solve the equation

$$3e^{2x} - 4e^{-2x} = 5.$$

Give the answer correct to 3 decimal places. [3]

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2 **(a)** Sketch the graph of $y = |2x + 3|$. [1]

(b) Solve the inequality $3x + 8 > |2x + 3|$. [3]

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3 Find the coefficient of x^3 in the binomial expansion of $(3 + x)\sqrt{1 + 4x}$. [4]

4 **(a)** Show that the equation $\sin 2\theta + \cos 2\theta = 2\sin^2\theta$ can be expressed in the form

$$\cos^2\theta + 2\sin\theta\cos\theta - 3\sin^2\theta = 0.$$ [2]

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(b) Hence solve the equation $\sin 2\theta + \cos 2\theta = 2\sin^2\theta$ for $0° < \theta < 180°$. [4]

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1 Solve the inequality $|5x - 3| < 2|3x - 7|$. [4]

2 Solve the equation $\ln(2x^2 - 3) = 2\ln x - \ln 2$, giving your answer in an exact form. [3]

4 Solve the equation $2\cos x - \cos \frac{1}{2}x = 1$ for $0 \leqslant x \leqslant 2\pi$. [5]

1 Solve the equation $\ln(x + 5) = 5 + \ln x$. Give your answer correct to 3 decimal places. [4]

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2 Find the quotient and remainder when $2x^4 - 27$ is divided by $x^2 + x + 3$. [3]

6 **(a)** Express $3\cos x + 2\cos(x - 60°)$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. State the exact value of R and give α correct to 2 decimal places. [4]

(b) Hence solve the equation

$$3\cos 2\theta + 2\cos(2\theta - 60°) = 2.5$$

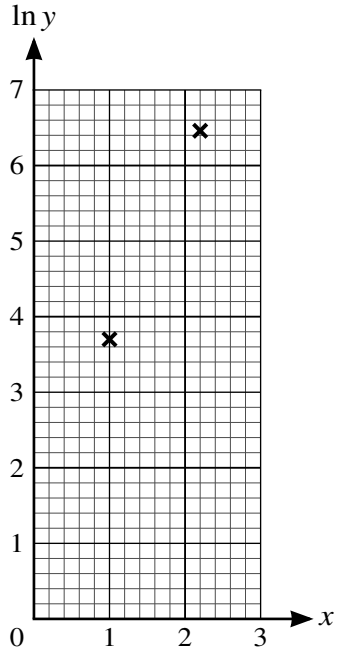
for $0° < \theta < 180°$. [4]

10 Let $\mathrm{f}(x) = \dfrac{21 - 8x - 2x^2}{(1 + 2x)(3 - x)^2}$.

(a) Express $\mathrm{f}(x)$ in partial fractions. [5]

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

3



The variables x and y are related by the equation $y = ab^x$, where a and b are constants. The diagram shows the result of plotting $\ln y$ against x for two pairs of values of x and y. The coordinates of these points are (1, 3.7) and (2.2, 6.46).

Use this information to find the values of a and b. [4]

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5 **(a)** Given that

$$\sin\left(x+\tfrac{1}{6}\pi\right)-\sin\left(x-\tfrac{1}{6}\pi\right)=\cos\left(x+\tfrac{1}{3}\pi\right)-\cos\left(x-\tfrac{1}{3}\pi\right),$$

find the exact value of $\tan x$. [4]

(b) Hence find the exact roots of the equation

$$\sin\left(x + \tfrac{1}{6}\pi\right) - \sin\left(x - \tfrac{1}{6}\pi\right) = \cos\left(x + \tfrac{1}{3}\pi\right) - \cos\left(x - \tfrac{1}{3}\pi\right)$$

for $0 \leqslant x \leqslant 2\pi$. [2]

10 Let $\mathrm{f}(x) = \dfrac{24x + 13}{(1 - 2x)(2 + x)^2}$.

(a) Express $\mathrm{f}(x)$ in partial fractions. [5]

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

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(c) State the set of values of x for which the expansion in **(b)** is valid. [1]

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1 **(a)** Sketch the graph of $y = |4x - 2|$. [1]

(b) Solve the inequality $1 + 3x < |4x - 2|$. [4]

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3 The polynomial $2x^3 + ax^2 - 11x + b$ is denoted by $\mathrm{p}(x)$. It is given that $\mathrm{p}(x)$ is divisible by $(2x - 1)$ and that when $\mathrm{p}(x)$ is divided by $(x + 1)$ the remainder is 12.

Find the values of a and b. [5]

7 **(a)** By expressing 3θ as $2\theta + \theta$, prove the identity $\cos 3\theta \equiv 4\cos^3\theta - 3\cos\theta$. [3]

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(b) Hence solve the equation

$$\cos 3\theta + \cos \theta \cos 2\theta = \cos^2 \theta$$

for $0° \leqslant \theta \leqslant 180°$. [5]

1 Find the set of values of x satisfying the inequality $\left|2^{x+1} - 2\right| < 0.5$, giving your answer to 3 significant figures. [4]