Improving Predictive Modelling for Mining Accidents

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Abstract

This study improves on a previously implemented predictive analytics model that forecasts severe accidents in mines.[1] The underlying model was created using data from the United States Department of Labor's Mine Safety and Health Administration (MSHA) to predict the likelihood of any severe accidents that may occur in the upcoming quarter. In order to increase accuracy and decrease the high false positive rate, some of the initial parameters were altered, and a unique post-processing model was tailor fit to each individual mine. By constructing a specialized corrective exponential discount factor for each mine, the new model is able to incorporate a company's perceived safety culture within its predictions. The final accuracy of this new model is 79.0%.

Keywords: Mines, Exponential Discount Model, Fix Effect Model, R, Python

1. Introduction

MSHA was formed in 1977 as part of the Federal Mine Safety and Health Act in order to enforce compliance with mandatory health and safety standards in mines. Before this act was passed, the mining industry was one of the most dangerous professions to work in due to the long hours, toxic fumes, and heavy equipment. This in turn left a long trail of deaths, disabilities, and illnesses in its wake, prompting the federal government to intervene in order to protect workers and enforce a higher safety standard.[2]

While this is an extremely valiant goal, the reality of achieving it leaves much to be desired. In the past, MSHA has often been criticized for not being strict enough on regulation, allowing several disastrous accidents to occur over the years. In order to aid in their fundamental mission of ensuring a safe working environment, this research is aimed at helping MSHA prevent systemic risk within the industry. Constructed using the database on violation data, this model acts as a warning system that can notify a company, its workers, and its investors about a mine's current safety status. If mines are preemptively notified that a potential major accident is imminent, they can implement preventive safety measures to minimize any potential fallout.

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[1] Catherine Zhao. A Data-Driven Early Warning System for Mining Accidents. 13 October 2017.

[2] https://www.msha.gov/about/mission

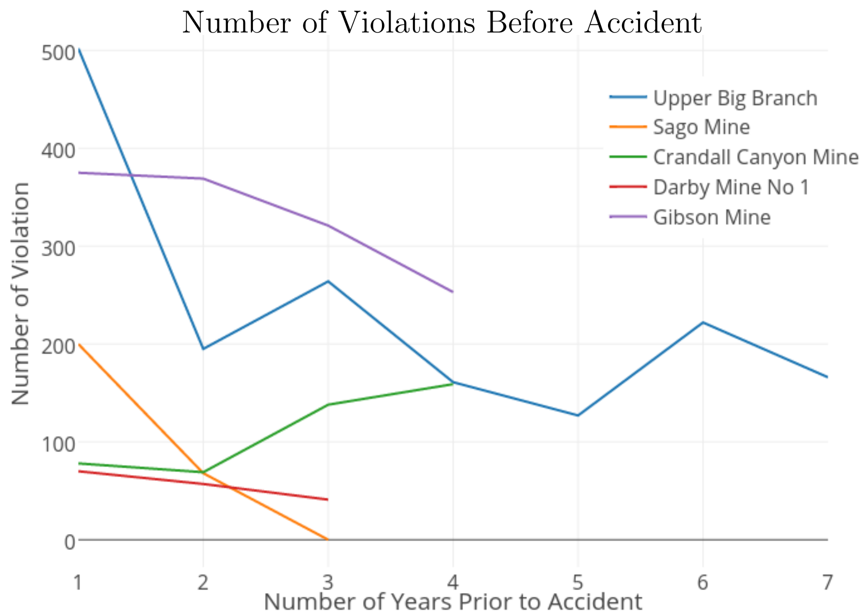

Figure 1: Violations Prior to the Top Five Largest Mine Accidents

According to Figure 1, there is generally a rising trend of violations right before a major accident occurs, suggesting that the compounded effect of the rising number of safety violations likely contributed to the cause of the accident. Throughout this research, a severe accident is defined by any death or permanent disability that occurs in a mine.

2. Examining the Underlying Model

The previous predictive model was based off the Assessed Violation[3], Accidents [4], and Mines [5] datasets from the MSHA database. In order to avoid potentially skewed data from the past, when MSHA safety regulations were less strict, only accident and violation data between 2000 and 2015 was used.

In order to make the data more manageable, daily records were consolidated into quarters, which allowed for a cleaner analysis of overall trends. For example, if a mine has one violation the first month, none the next month, then two violations the following month, these would be consolidated and represented as three violations for the given quarter. This process was applied to both the Assessed Violation and Accidents datasets, leaving the following input columns:

[3]Retrieved 4/12/2017, from https://arlweb.msha.gov/OpenGovernmentData/DataSets/AssessedViolations.zip
[4]Retrieved 4/12/2017, from https://arlweb.msha.gov/OpenGovernmentData/DataSets/Accidents.zip
[5]Retrieved 4/12/2017, from https://arlweb.msha.gov/OpenGovernmentData/DataSets/Mines.zip

Dataset	Column Extracted
Assessed Violation	mine_ID, cal_qtr, cal_yr, violation and proposed_penalty_amt
Accidents	mine_ID, cal_qtr, cal_yr, days_lost, days_restrict, death, and perm_dis
Mines	mine_ID, curr_mine_name, coal_metal_ind, current_mine_type, and no_employees

Table 1: Columns Extracted

Rollover time measurements are also calculated for the last quarter, last year, and last three years. For instance, in order to calculate the number of deaths within the last three years, the number of deaths from the previous 12 quarters are summed together. The variable is then continuously updated when new information is provided in the following quarters. This calculation is done for days lost, days restrict, death, permanent disability, and violation.

```
 [1] "MINE_ID"                       "CURRENT_MINE_NAME"             "COAL_METAL_IND"
 [4] "CURRENT_MINE_TYPE"             "QUARTER"                       "YEAR"
 [7] "ACTIVE"                        "NUM_DAYS_LOST"                 "LAST_QUARTER_DAYS_LOST"
[10] "LAST_YEAR_DAYS_LOST"           "LAST_THREE_YEARS_DAYS_LOST"    "NUM_DAYS_RESTRICT"
[13] "LAST_QUARTER_DAYS_RESTRICT"    "LAST_YEAR_DAYS_RESTRICT"       "LAST_THREE_YEARS_DAYS_RESTRICT"
[16] "NUM_DEATH"                     "LAST_QUARTER_DEATH"            "LAST_YEAR_DEATH"
[19] "LAST_THREE_YEARS_DEATH"        "NUM_DIS"                       "LAST_QUARTER_DIS"
[22] "LAST_YEAR_DIS"                 "LAST_THREE_YEARS_DIS"          "VIOLATION_QUANTITY"
[25] "LAST_QUARTER_VIOLATION"        "LAST_YEAR_VIOLATION"           "LAST_THREE_YEARS_VIOLATION"
[28] "PROPOSED_PENALTY"              "LAST_QUARTER_PENALTY"          "LAST_YEAR_PENALTY"
[31] "LAST_THREE_YEARS_PENALTY"
```

Figure 2: Final Data Frame Columns from Initial Model

Figure 2 shows all of columns that were present in the final consolidated dataset, which was used to generate the initial predictive model. In the end, the data contained 8,877 unique mines that each had 31 attributes associated with it.

This data was then fit to a conditional logistic regression with fixed effects model. The fixed effect variables used were whether the mine produces coal or metal, and what type of mine it is (i.e. facility, surface, or underground). These can be represented mathematically using the normal logistic function:

$$Pr(Y = 1|\mathbf{X}, \mathbf{i}) = \frac{\mathbf{1}}{\mathbf{1} + \mathbf{e}^{-(\alpha_\mathbf{i} + \beta\mathbf{X})}}$$

Here, X represents a single mine along with its attributes at a specific quarter in time, and the parameter i represents the added fixed effects for this given mine's type and category.

When considering the data used, there are very few mines that actually have had severe accidents, which may be why the initial model has such a high false positive rate. The data for severe accidents breaks down as follows:

Severe	Number	Percent(%)
FALSE	364146	99.46
TRUE	1962	0.54

Table 2: Severe Accident Distribution

As for the model's performance, the predictive output results from running the old logistic regression model are as follows:

	Reference	
Prediction	FALSE	TRUE
FALSE	281028	827
TRUE	82395	1135

Table 3: Prediction VS. Actual of Original Model

Accuracy	Sensitivity	Specificity	Precision	F1
0.772	0.578	0.773	0.014	0.027

Table 4: Prediction Output Summary of Original Model

While this old model has a venerable accuracy rate, there is still room for improvement. One possible change is altering the parameters that the model takes. There are some redundancies inherent within the variables that can negatively skew the results. For instance, `LAST_QUARTER_VIOLATION` is the same as `LAST_YEAR_VIOLATION` because inspections only occur annually. Therefore, in order to avoid unnecessary penalization, the `LAST_QUARTER_VIOLATION` variable should be removed in the new model.

Another aspect to consider is that some of the variables may be linked, which requires the creation of interaction variables to properly represent their respective relationships. The most probable relationships exist between days lost and days restrict, death and disability, and violation and penalty. By considering how one variable can affect another, we can reduce the amount of noise within our dataset and model.

Finally, the most pressing problem in the old model is the high number of false positive predictions. From a safety standpoint, it is technically better for the model to overpredict rather than underpredict so that the mines are constantly on alert for any safety issues. However, economically speaking, false positive predictions can be extremely expensive for a company due to the fact that the mine could be expending a significant amount of time and resources unnecessarily. Because this is meant to be an early warning system, it is expected that the model should overpredict in order to alert the possibility of an upcoming accident, but the current false positive rate is much too high. Therefore, the number of false positives should be minimized as much as possible without compromising the integrity of the underlying model.

3. Implementing Model Improvements

3.1. Revisiting Variable Selection

By simply removing the `LAST_QUARTER_VIOLATION` parameter from the regression, the accuracy of the model increased 0.2%, and the specificity, which the likelihood the model will predict no accidents given no severe accidents, also increased by 0.3% Unfortunately, the sensitivity, which represents the likelihood the model will predict an accident given a severe accident, fell by .7%.

Accuracy	Sensitivity	Specificity	Precision	F1
0.774	0.571	0.776	0.014	0.027

Table 5: Prediction Output Summary from Removing `LAST_QUARTER_VIOLATION`

	Reference	
Prediction	FALSE	TRUE
FALSE	282403	841
TRUE	81743	1121

Table 6: Prediction VS. Actual from Removing `LAST_QUARTER_VIOLATION`

From analyzing this new confusion matrix, it is clear that both the number of false positive and true positive predictions fell slightly, which speaks to the added penalization that the `LAST_QUARTER_VIOLATION` variable created. This updated model was used as the basis for all further optimizations, and all subsequent corrections outlined in this paper were built around this reference state.

Although this change may seem minimal, it just goes to show that redundancies are prevalent within the model's parameters. If all such redundancies were removed from the model, that could have a significant combined effect on the results. However, this proved to be difficult given the current model and dataset, especially with regards to the interactive variables.

In order to account for some possible relationships between variables, the following eight interaction variables were added to the new model:

Interactive Variables
Last Quarter Days Lost:Last Quarter Days Restrict
Last Year Days Lost:Last Year Days Restrict
Last 3 Years Days Lost:Last 3 Years Days Restrict
Last Quarter Death:Last Quarter Disability
Last Year Death:Last Year Disability
Last 3 Years Death:Last 3 Years Disability
Last Year Violation:Last Year Penalty
Last 3 Years Violation:Last 3 Years Penalty

Table 7: New Interaction Variables Added to Underlying Model

Since an interaction is formed by the product of two or more predictors, these new interaction variables were created by simply multiplying together their component parts. For example, `Last_Quarter_Days_Lost:Last_Quarter_Days_Restrict` was represented as `Last_Quarter_Days_Lost * Last_Quarter_Days_Restrict` in the conditional logistic regression function.

The output summary of running the model with the added interaction variables is shown below in Table 8. The exact in-sample output results from the constructed model can be found in Appendix B.

Accuracy	Sensitivity	Specificity	Precision	F1
0.772	0.578	0.773	0.014	0.027

Table 8: Prediction Output Summary from Adding Interaction Variables

Adding in the interaction variables produced the exact some results as the original model, and removed any improvements that the previously updated model created. This is likely because with no indication of the importance or effects of these relationships, the model is acting blindly with regards to these interaction variables, and does not know how to consider them properly. In order to determine which subset of variables actually have the largest impact, the glmulti function in R was used.

Unfortunately, this task proved computationally unreasonable since there are now 24 dynamic variables to consider, so the function would need to compute all 2^{24} models. At several minutes of run time per 100 models, an ordinary computer does not have enough computing power to effectively process all of these models. Building this model may be more feasible if this process can be run on a supercomputer or computer cluster.

3.2. Tackling False Positive Predictions

In order to reduce the model's false positive rate, the false positive predictions must first be analyzed. Upon examination, the false positive predictions can be broken down into three categories: Definitely Wrong, Early Warning, and Post Accident.

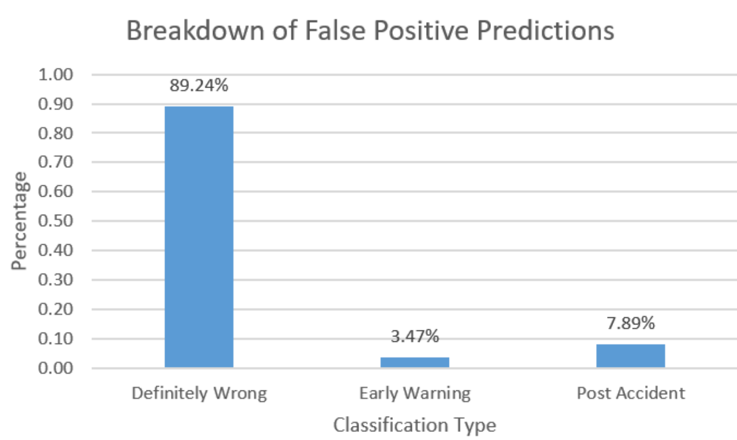

Figure 3: Frequency of the Three Types of False Positive Predictions

As the label states, predictions that are definitely wrong are considered to be definite false positives in the model. This includes scenarios where the model predicts that an accident will occur in the mine, but one never does throughout the entire mine's lifespan. It also includes cases in which the model predicts an accident will happen more than a year before or more than a year after it actually does occur, since it fails to accurately pinpoint the time of the accident. It is difficult to handle these predictions since they are caused by underlying flaws in the initial model. Therefore, in order to fix these issues, the entire core model would need to be redesigned.

Next, predictions that fall under the early warning label are ones that say there will be an accident within one year prior to the accident actually happening. For instance, if the model predicts that an accident will happen in Q1, Q2 and Q3 of 2010, then an accident does occur in Q4 of 2010, the three previous predictions would not be considered wrong, since they correctly indicated that a major accident was imminent. Therefore, they should be considered good predictions and not be classified under the standard definition of a false positive.

Finally, predictions that occur within one year after a severe accident has occurred are labeled post accident, which according to Figure 3, account for around 8% of all false positive predictions. Based on the current model's design, after a severe accident occurs, there is a major spike in the probability of an accident happening in the quarters that directly follow it. For example, if there is a major accident that occurs in Q1 of 2003, the model will continue to predict that an accident is very likely to occur in Q2, Q3, and Q4 of 2003, even though the likelihood of back-to-back accidents in extremely rare. In the event that there are accidents within close proximity to each other, some predictions may be classified as both early warning and post accident.

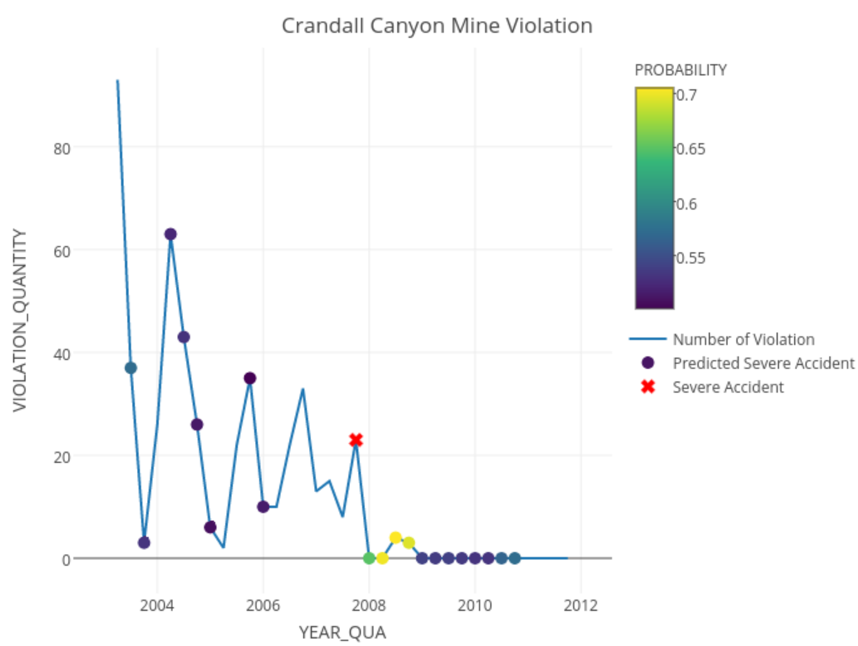

Figure 4: Crandall Canyon Mine Violation Data and Initial Model Predictions

Figure 4 illustrates an example of this post accident phenomenon. The dots overlaying the violation plot represent points where the model predicts there will be an accident, since the calculated probability of an accident occurring is greater that 0.5. After the severe accident at the end of 2007, there is a string of high-probability positive predictions that follow it, even though the number of violations has dropped drastically.

The reason that the model tends to overpredict in these cases is because the heavy weight assigned to a recent known accident in the underlying model is so large that it overshadows the rest of the data. Given that there is a substantial amount of false positives due to this phenomenon, the new model specifically focuses on improving the predictions in this category.

In order to do this, the new model must "reset" after every major accident. Therefore, the new model must track information on when the mine's accidents occur, and how many accidents have occurred up until a certain point in time. This means that when splitting the training and test data, the data must be split in accordance to `MINE_ID`, so that all of the information from one mine stays together in the output of the initial model.

With this new information, the model is able to construct a specialized discount function for each individual mine that considers each mine's individual safety culture. The discount is calculated based off the following exponential function:

$$DISCOUNT = a - \frac{exp(Q\_PAST)}{b}$$

`Q_PAST` tracks the number of accidents since the last accident occurred if it is within 12 quarters. The reason 12 quarters is used is because the average time between accidents across all mines is slightly over 11 quarters, so after three years time, the discount factor should completely disappear. The parameters a and b are then calculated for each individual mine, using the points (1,s) and (12,1) to fit the function, where s is the initial predicted probability of the first quarter after the first accident from the original model.

The old probabilities are then multiplied by this calculated discount factor to produce new lowered probabilities, which is what the new predictions are based off of.

$$NEW\_PRED = PREDICTION * DISCOUNT$$

In order to give some leeway to the mines, the model completely forgives the first accident, meaning that the probability that an accident will occur in the quarter following the first accident is zero. This is the only special case; for all subsequent quarters, the discount follows the function shown above, and exponentially returns to one over the course of the allocated 12 quarters.

	MINE_ID	MINE_NAME	YEAR	Q	NUM_DEATH	NUM_DIS	SEVERE	PROBABILITY	PREDICTION	COUNT	Q_PAST	DISCOUNT	NEW_PROB	NEW_PRED
75	100004	Brierfield Quarry	2009	1	0.0	0.0	0	0.513	1	0	0	1.000000	0.513	1
76	100004	Brierfield Quarry	2009	2	0.0	0.0	0	0.519	1	0	0	1.000000	0.519	1
77	100004	Brierfield Quarry	2009	3	0.0	0.0	0	0.511	1	0	0	1.000000	0.511	1
78	100004	Brierfield Quarry	2009	4	0.0	0.0	0	0.510	1	0	0	1.000000	0.510	1
79	100004	Brierfield Quarry	2010	1	0.0	0.0	0	0.509	1	0	0	1.000000	0.509	1
80	100004	Brierfield Quarry	2010	2	1.0	0.0	1	0.502	1	1	0	1.000000	0.502	1
81	100004	Brierfield Quarry	2010	3	0.0	0.0	0	0.537	1	1	1	0.000000	0.000	0
82	100004	Brierfield Quarry	2010	4	0.0	0.0	0	0.541	1	1	2	0.536637	0.290	0
83	100004	Brierfield Quarry	2011	1	0.0	0.0	0	0.543	1	1	3	0.829543	0.450	0
84	100004	Brierfield Quarry	2011	2	0.0	0.0	0	0.542	1	1	4	0.937297	0.508	1
85	100004	Brierfield Quarry	2011	3	0.0	0.0	0	0.525	1	1	5	0.976938	0.513	1
86	100004	Brierfield Quarry	2011	4	0.0	0.0	0	0.525	1	1	6	0.991521	0.520	1
87	100004	Brierfield Quarry	2012	1	0.0	0.0	0	0.531	1	1	7	0.996886	0.529	1
88	100004	Brierfield Quarry	2012	2	0.0	0.0	0	0.530	1	1	8	0.998859	0.529	1
89	100004	Brierfield Quarry	2012	3	0.0	0.0	0	0.532	1	1	9	0.999585	0.531	1
90	100004	Brierfield Quarry	2012	4	0.0	0.0	0	0.535	1	1	10	0.999852	0.535	1
91	100004	Brierfield Quarry	2013	1	0.0	0.0	0	0.524	1	1	11	0.999951	0.524	1
92	100004	Brierfield Quarry	2013	2	0.0	0.0	0	0.536	1	1	12	0.999987	0.536	1
93	100004	Brierfield Quarry	2013	3	0.0	0.0	0	0.516	1	1	0	1.000000	0.516	1
94	100004	Brierfield Quarry	2013	4	0.0	0.0	0	0.514	1	1	0	1.000000	0.514	1

Figure 5: New Probabilities and Predictions for the Brierfield Quarry Using the Discount Model

Figure 5 is an example of a mine that has only had one accident. The discount is greatest in the few quarters directly following the accident, specifically targeting the post accident predictions. In this case, the discount model was enough to overcome the negative effects of the accident and fix the initially incorrect predictions. For further analysis, a graph of this specific discount function can be found in Appendix C.

In cases where a mine has more than one accident, the initial discount after each accident becomes less forgiving, since the model should not be as generous with mines that have a history of repeated accidents. In order to represent this, the starting discount factor gets shifted later one quarter for each successive accident. Also, if the subsequent accident is within 12 quarters of the previous one, the discount factor will reset itself upon the presence of the second accident.

Figure 6 below is an example of a mine that has two accidents within relatively close proximity to each other. After the second accident occurs, the `Q_PAST` time variable gets reset in order to keep up to date with the new data. When looking at the discount factors, it is clear that the model is much less forgiving after the second accident than it is on the first. For the first accident, the first quarter after has a discount of 0 (completely forgiven), the second quarter after has a discount of 0.791, and the third quarter after has a discount of 0.923. However, for the second accident, the initial discount gets shifted back a quarter, so the first quarter after has a discount of 0.791, the second quarter after has a discount of 0.923, and the third quarter after has a discount of 0.972. In the case of a third accident, the first quarter after would have an initial discount of 0.923, and so on.

	MINE_ID	MINE_NAME	YEAR	Q	NUM_DEATH	NUM_DIS	SEVERE	PROBABILITY	PREDICTION	COUNT	Q_PAST	DISCOUNT	NEW_PROB	NEW_PRED
311	100011	Imerys Sylacauga Operations	2003	1	0.0	0.0	0	0.896	1	0	0	1.000000	0.896	1
312	100011	Imerys Sylacauga Operations	2003	2	0.0	0.0	0	0.884	1	0	0	1.000000	0.884	1
313	100011	Imerys Sylacauga Operations	2003	3	0.0	0.0	0	0.856	1	0	0	1.000000	0.856	1
314	100011	Imerys Sylacauga Operations	2003	4	0.0	1.0	1	0.738	1	1	0	1.000000	0.738	1
315	100011	Imerys Sylacauga Operations	2004	1	0.0	0.0	0	0.791	1	1	1	0.000000	0.000	0
316	100011	Imerys Sylacauga Operations	2004	2	0.0	0.0	0	0.847	1	1	2	0.791184	0.670	1
317	100011	Imerys Sylacauga Operations	2004	3	0.0	1.0	1	0.887	1	2	0	0.923183	0.819	1
318	100011	Imerys Sylacauga Operations	2004	4	0.0	0.0	0	0.896	1	2	1	0.791184	0.709	1
319	100011	Imerys Sylacauga Operations	2005	1	0.0	0.0	0	0.926	1	2	2	0.923183	0.855	1
320	100011	Imerys Sylacauga Operations	2005	2	0.0	0.0	0	0.926	1	2	3	0.971743	0.900	1
321	100011	Imerys Sylacauga Operations	2005	3	0.0	0.0	0	0.914	1	2	4	0.989607	0.905	1
322	100011	Imerys Sylacauga Operations	2005	4	0.0	0.0	0	0.854	1	2	5	0.996179	0.851	1
323	100011	Imerys Sylacauga Operations	2006	1	0.0	0.0	0	0.864	1	2	6	0.998596	0.862	1
324	100011	Imerys Sylacauga Operations	2006	2	0.0	0.0	0	0.845	1	2	7	0.999486	0.845	1
325	100011	Imerys Sylacauga Operations	2006	3	0.0	0.0	0	0.862	1	2	8	0.999813	0.862	1
326	100011	Imerys Sylacauga Operations	2006	4	0.0	0.0	0	0.836	1	2	9	0.999933	0.836	1

Figure 6: Multiple Accidents Occurring in the Imerys Sylacauga Operations Mine

Overall, this discount model was able to reduce the number of post accident false positive predictions by 43.38% from 6,453 to 3,654.

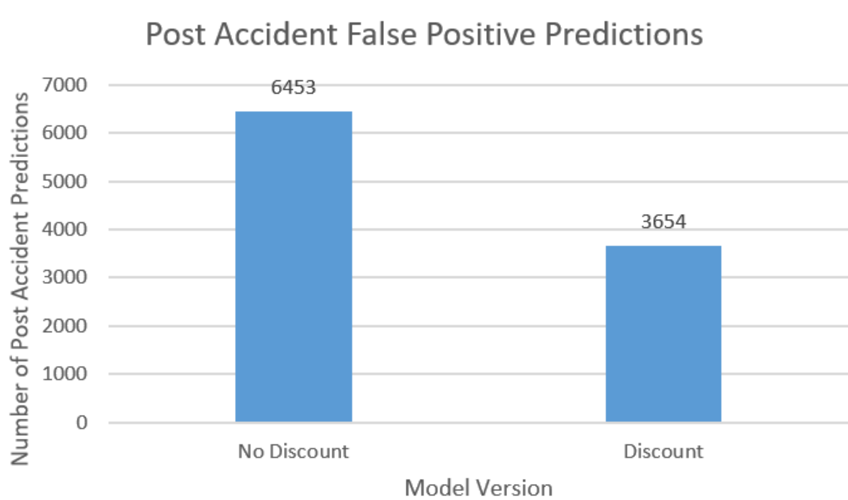

Figure 7: Post Accident False Positive Count

Given this new discount model and the classification of early warning predictions as being correct, the new breakdown of predictive output results are as follows:

	Reference	
Prediction	FALSE	TRUE
FALSE	285613	874
TRUE	75847	1088
Early Warning	2686	N/A

Table 9: Prediction VS. Actual with New Discount Model (See Table 6 for Reference Values)

Accuracy	Sensitivity	Specificity	Precision	F1
0.790	0.812	0.790	0.061	0.027

Table 10: Prediction Output Summary with New Discount Model

Because the early warning predictions are technically considered good predictions, the output summary values were calculated slightly differently. The 2,686 early warning predictions were considered "true positive" predictions when calculating accuracy, sensitivity, and precision, and were not considered "False" predictions when calculating specificity.

Although the new model seemed effective for the few quarters directly following an accident, it did not have enough of an impact to counteract the poor results of the initial model. From Figure 6, it is apparent that the applied discount factor was not enough to counteract the high probabilities generated from the initial model. There continue to be numerous incorrect predictions, both significantly before and after an accident actually occurs, indicating that the original model is too imprecise and needs further reworking.

4. Conclusion

By adjusting parameters and pipelining the results from the previous model into a post-processing discount model, the model's accuracy increased to 79.0%, and the number of post accident false positive predictions fell by over 40%. Overall, adjusting the parameters, applying the discount model, and redefining early warning predictions as correct caused the number of false positive predictions to fall by 7.95%. In order to further reduce these false positive results, more significant changes need to be made to the underlying model.

If interactive variables can be incorporated and computed more efficiently, that may improve the performance of the overall model. Alternatively, the entire model could be re-implemented by fitting the data to a Poisson time series model instead of the conditional logistic regression with fixed effects model that is currently being used. Another aspect to consider in the future is the possibility of distinguishing between different classes of accidents. The current model only predicts for severe accidents, which is defined by the occurrence of a death or permanent disability. However, it would be possible to add some other variable, D, which could define the degree of severity the predicted accident will have (i.e. minor, intermediate, major). This severity index can then be used to classify the model's warnings and predictions, providing additional information on a mine's safety status.

Finally, it is important to note that while all of these models and analyses are extremely useful, they require a strong and comprehensive dataset at its core to build around. Therefore, in order to apply this research to other similar industries and expand into the domain of OSHA's (Occupational Safety and Health Administration) databases, there must be a conscious effort to implement a systematic data collection process across all industries.

Appendix A. Accuracy Model Calculations

	Formula
Accuracy	$\frac{True\_pos+True\_neg}{True+False}$
Sensitivity	$\frac{True\_pos}{True}$
Specificity	$\frac{True\_neg}{False}$
Precision	$\frac{True\_pos}{True\_pos+False\_pos}$
F1	harmonic mean of sensitivity and precision

Table A.11: Formulas Used to Calculate Summary Statistics

Appendix B. Output Results for the Interactive Variable Model

	Coefficient	P-Value
Last Quarter Days Lost	1.48e-05	0.943833
Last Year Days Lost	1.802e-04	0.102927
Last 3 Years Days Lost	2.586e-05	0.502158
Last Quarter Days Restrict	1.225e-03	0.064242
Last Year Days Restrict	6.754e-04	0.072115
Last 3 Years Days Restrict	5.714e-04	3.99e-05
Last Quarter Death	-5.004e-02	0.721088
Last Year Death	1.381e-01	0.087894
Last 3 Years Death	4.772e-02	0.287302
Last Quarter Disability	-2.152e-01	0.212736
Last Year Disability	2.304e-01	0.014398
Last 3 Years Disability	2.840e-01	1.42e-08
Last Year Violation	2.870e-03	8.60e-07
Last 3 Years Violation	4.143e-04	0.061217
Last Year Penalty	1.291e-06	0.000239
Last 3 Years Penalty	4.785e-08	0.767686
Last Quarter Days Lost:Last Quarter Days Restrict	-9.547e-07	0.457944
Last Year Days Lost:Last Year Days Restrict	-1.883e-07	0.413088
Last 3 Years Days Lost:Last 3 Years Days Restrict	-1.037e-07	0.002298
Last Quarter Death:Last Quarter Disability	7.223e-02	0.936458
Last Year Death:Last Year Disability	-3.290e-01	0.151739
Last 3 Years Death:Last 3 Years Disability	-3.339e-02	0.579331
Last Year Violation:Last Year Penalty	-2.529e-09	1.58e-05
Last 3 Years Violation:Last 3 Years Penalty	-6.307e-11	0.408758

Table B.12: In-Sample Model Using Logistic Regression With Fixed Effects Including Interaction Variables

Appendix C. Example of Exponential Discount Function

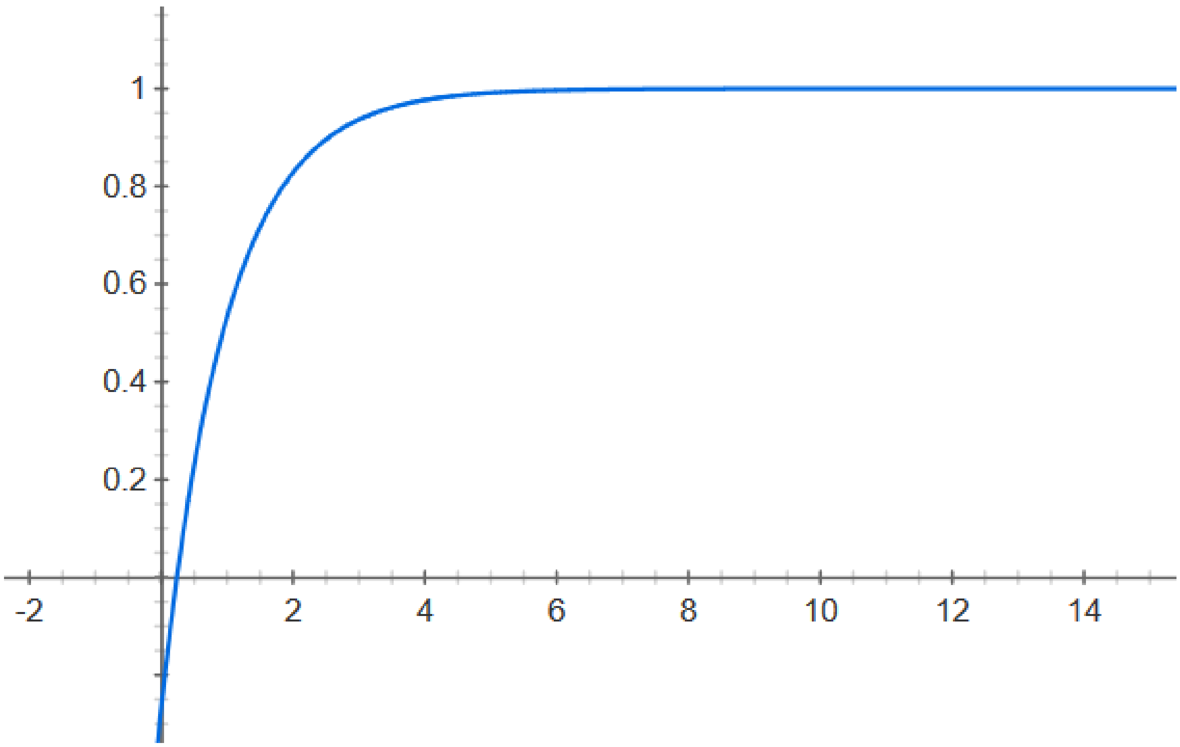

Figure C.8: Exponential Discount Function for the Brierfield Quarry Shown in Figure 5