Dynamic Analysis and Optimization of Steph Curry's 3-point shot

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Dynamic Analysis and Optimization of Stephen Curry's 3-point shot

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***Abstract*—In this study, we propose a method for analyzing and optimizing the 3-point shooting technique of Stephen Curry, one of the most prominent 3-point shooters in the NBA. While experts claim that Curry has the most efficient shooting form in the NBA, we hypothesize that his form can be optimized further to improve his shooting consistency. Our method applies concepts of kinematics, inverse kinematics, Lagrangian non-linear dynamics, and motion planning to model Curry's 3-point shot. A weighted optimization is then computed to determine an alternate shooting motion that minimizes the energy expended by the body without significantly deviating from his original motion. Results show that by reducing his jump height and upper arm motion, he can reduce his energy expenditure by 23%.**

I. Introduction

THE sport of basketball is evolving, where teams in the National Basketball Association (NBA) now attempt a record high 26.5 [1] 3-point shots a game. As a result, high value is placed on players that excel in this skill.

3-point shooting is a precise motion, requiring near-perfect control of the ball release and jump height to sink the ball into the hoop. Muscle fatigue from extended play reduces a player's control of this ball release, consequently reducing shot accuracy. Fatigue can be reduced by improving shooting efficiency.

The methods described in this paper are applied to the shooting form of Stephen Curry, one of the most prominent 3-point shooters in the NBA. While his physique is below average among NBA players, he often plays the most minutes in a game. Sports analysts attribute this performance to his efficient shooting technique, however there are no studies to back this claim.

This paper proposes a method for analysing the 3-point shooting form of Stephen Curry, and suggests a revised shooting form that reduces the energy of the motion without a significant deviation from his original form. Applying principles of robotics, a dynamic model was developed to analyze his motion. Simulation and optimization are then applied to the model to generate a more efficient technique.

II. Literature Review

Research on anthropomorphic manipulators that mimic throwing motions have applications that range from industry to biomechanics and sports science.

A study [2] found that there are infinitely feasible configurations to model an accurate throw by a 2-Degrees of Freedom (2-DOF) robot. Given specific parameters, a region of possible release points and the minimum joint velocities required to hit a given target were determined.

Other studies [3] established criteria for determining the best release angle for any distance from the hoop when shooting a basketball. Based on release coordinates, an algorithm was derived to find the ideal release angle that achieved minimum take-off velocity.

Data on segments of the human body [4] were used to derive parameters such as length, mass, and center of mass of individual body segments as a ratio of total body height and weight.

Several studies proposed optimization methods for biomechanical systems. While one study [5] compared several techniques to optimize the pitching motion for an underactuated robot arm, another [6] determined the ideal walking motion of a bipedal robot by numerically minimizing an energy cost function.

Most of the prior research modelled 2-DOF manipulators with the base of the arm fixed. To simulate a more human-like motion, this study introduces a prismatic joint which can move the base of the throwing arm. Additionally, many papers only considered improving accuracy through motion planning under the assumption that this motion can be repeated perfectly. However, in the context of basketball, consistency in shooting motion deteriorates proportionally with muscle fatigue.

III. Static Model

To reduce the complexity of the shooting motion, Curry's body was modelled by a 3-DOF manipulator (Figure 1) with one prismatic joint and two revolute joints (P-R-R). The prismatic joint simulated the jumping and the revolute joints simulated the shoulder and elbow joints. The wrist joint was simplified to a single point end-effector. The link parameters were assigned as per the exact length, mass, and center of mass of Curry's body characteristics. Using this mathematical model, a comparison was made between his current form and the optimized form.

A. Physical Attributes

Parameters for the 3-DOF manipulator were determined from official NBA statistics [7], which provided Curry's height with shoes (1.91 m), body weight (82.1 kg), and maximum standing vertical jump (0.749 m). Table I shows parameters for the dynamic model that were derived using human body segment data [4] and the NBA statistics.

TABLE I. STEPHEN CURRY PHYSICAL DATA

Physical Quantity	**Body Part**					
	Half Trunk + Leg [Link 1]		***Upper Arm [Link 2]***		***Forearm + Hand [Link 3]***	
Length (m)	1.45	L_1	0.32	L_2	0.40	L_3
Mass (kg)	34.11	M_1	2.63	M_2	2.22	M_3
Center of Mass (m)	0.65	Lc_1	0.16	Lc_2	0.25	Lc_3
Moment of Intertia (kg m^2)	N/A	I_1	0.09	I_2	0.12	I_3

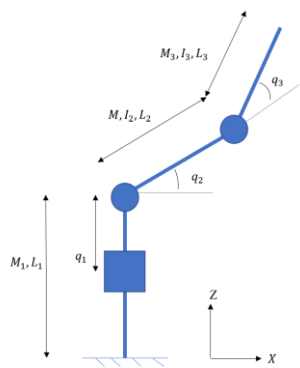

Figure 1. 2-D representation of P-R-R robot.

B. *Projectile Motion and Kinematics*

To derive the kinematics, the robot base is placed at the edge of the 3-point line. The positions of the hoop (X_B,Z_B) and the 3-point line are defined by official NBA standards. Given these, (1) solves for the release velocity, v_0, with respect to a release angle, β.

$$v_0 = \sqrt{\frac{0.5g(X_B - X_E)^2}{\sin\beta\cos\beta\,(X_B - X_E) - \cos^2\beta\,(Z_B - Z_E)}} \tag{1}$$

Equations (2) and (3) give the forward kinematics for the release point using the Denavit-Hartenberg convention.

$$X_E = L_2\cos(q_2) + L_3\cos(q_2 + q_3) \tag{2}$$

$$Z_E = q_1 + L_2\sin(q_2) + L_3\sin(q_2 + q_3) \tag{3}$$

Using (2) and (3), joint angles q_2 and q_3 can be computed for any height q_1 and end effector position (X_E,Z_E). The inverse kinematics are given by (4) and (5).

$$q_2 = Atan(X_E, Z_E - q_1) - Atan(a_2 + L_3\cos(q_3), L_3\sin(q_3)) \tag{4}$$

$$q_3 = Atan\left(\frac{X_E^2 + (Z_E - q_1)^2 - L_2^2 - L_3^2}{2L_2L_3}, \sqrt{1 - \left(\frac{X_E^2 + (Z_E - q_1)^2 - L_2^2 - L_3^2}{2L_2L_3}\right)^2}\right) \tag{5}$$

IV. DYNAMIC MODEL

By computing the velocities of the end-effector, the joint velocities can be obtained by using the Jacobian matrix shown in *Equation (6).*

$$J = \begin{bmatrix} 0 & -L_2\sin(q_2) - L_3\sin(q_2+q_3) & -L_3\sin(q_2+q_3) \\ 0 & 0 & 0 \\ 1 & L_2\cos(q_2) + L_3\cos(q_2+q_3) & L_3\cos(q_2+q_3) \end{bmatrix} \tag{6}$$

In the optimization phase, the velocity of the end-effector for each theoretically computed release point was calculated by applying projectile motion and using the same Jacobian technique. These velocities were used to determine the trajectory of the optimal shooting motion, which is discussed in Section VI.

From (1) through (6) and data from Table I, the joint force required to shoot the ball is evaluated as shown in (7).

$$D(\boldsymbol{q})\ddot{\boldsymbol{q}} + C(\boldsymbol{q}, \dot{\boldsymbol{q}}) + g(\boldsymbol{q}) = \tau \tag{7}$$

V. SIMULATION

On average, Curry takes about 400 ms to complete his shooting motion. Sampling a 30-fps video at 34 Hz, 13 individual frames were isolated. From these frames, data points were extracted that depict his shooting motion when accurately interpolated. A linear regression trendline (accurate to 95%) was fit to the data.

Importing this data to MATLAB (The MathWorks Inc., Natick, MA), the robot was simulated to model Curry's shooting action. From the changes in joint variables, the instantaneous energy at each discretized time instant was computed using (7).

VI. OPTIMIZATION

The cost function, Q, used for optimization is shown in (8). This equation was defined as a ratio of the average change in joint trajectories to the total energy conserved at each joint, i, where joint trajectory and energy conserved are given by (9) and (10) respectively. The motion for each joint was derived as a third order polynomial.

Table II lists the constraints of the cost function determined by the range of motion and limits on forces/torques at each joint.

TABLE II. CONSTRAINTS USED IN OPTIMIZATION

Variable	Lower Bound	Upper bound
$\boldsymbol{\tau_1}$	-600 Nm	600 Nm
$\boldsymbol{\tau_2}$	-25 Nm	45 Nm
$\boldsymbol{\tau_3}$	-10 Nm	60 Nm
$\boldsymbol{q_1}$	1.67 m	2.42 m
$\boldsymbol{q_2}$	40°	90°
$\boldsymbol{q_3}$	10°	100°

Figure 2. From left to right: frames from Curry's actual shooting motion captured at timestamps 60 ms, 120 ms, 210 ms, 300 ms and 330 ms.

Using a constrained search algorithm, energy values for combinations of jump height and release positions were determined. The optimal shooting motion was determined by setting a weight, κ, between the percent energy reduced and the average deviation from Curry's original motion. The weighting is introduced for practical reasons – most professional players are only willing to adopt small changes in their form. The motion with the smallest ratio, Q, was selected to be optimal.

$$\min Q(q_1, X_E, Z_E) = \kappa \frac{\Gamma}{\epsilon' - \epsilon_{TOTAL}} \tag{8}$$

$$\Gamma = \sum_{i=1}^{3} \frac{1}{t_f} \int_0^t |(q_i - q_i')| dt \tag{9}$$

$$\epsilon_{TOTAL} = \int_0^t F_1 \, dq_1 + \sum_{i=2}^{3} \int_0^t \tau_i \, dq_i \tag{10}$$

VII. Results

TABLE III. Results from Optimization Algorithm

Shooting Motion	Jump (q_1)	q_2	q_3	Total Energy	% Change in Motion
True	1.83 m	28.9°	104°	530 J	N/A
Optimal	1.72 m	22.8°	99.8°	405 J	4.21%

The results obtained from optimization are shown in Table III. The best shooting action reduced the overall energy spent by 23.4%. Computing average percent change of the adjustment vector, Γ, the suggested shooting mechanic requires a 4.21% change in his current joint trajectories.

Fig. 3 maps the space of all release points that require less energy than Curry's shot. Simplifications of the wrist joint produced joint combinations that did not distinguish between different methods of release. Consequently, two fields of release points were produced; the right-most region (*red*) corresponds to underarm shots and the left-most region (*green*) corresponds to overhead shots.

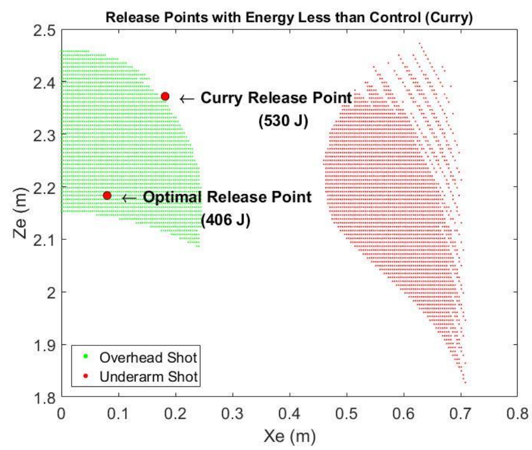

Figure 3. Space of release points that require a lower energy value than Curry.

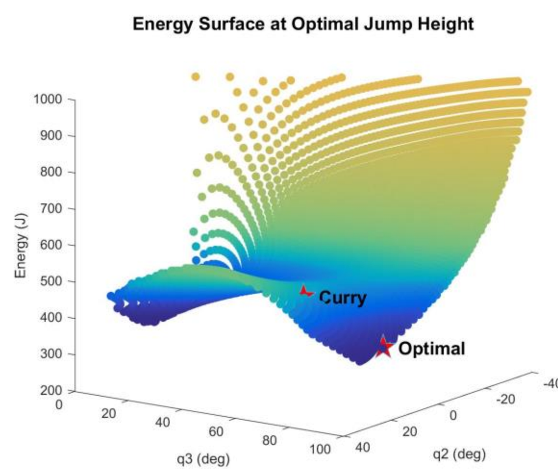

Figure 4. Surface of energy values for all shots at jump height of 1.72 m. (Cool colors represent lower energy while warm colors represent higher energy)

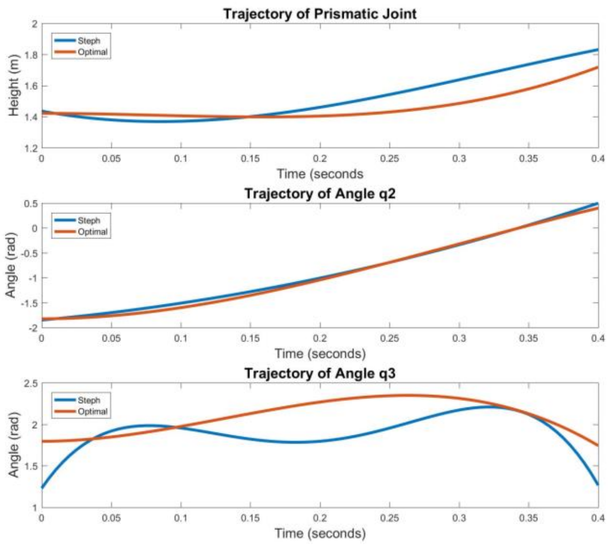

Figure 5 – Comparison of joint variables between Curry and optimal shooting motion.

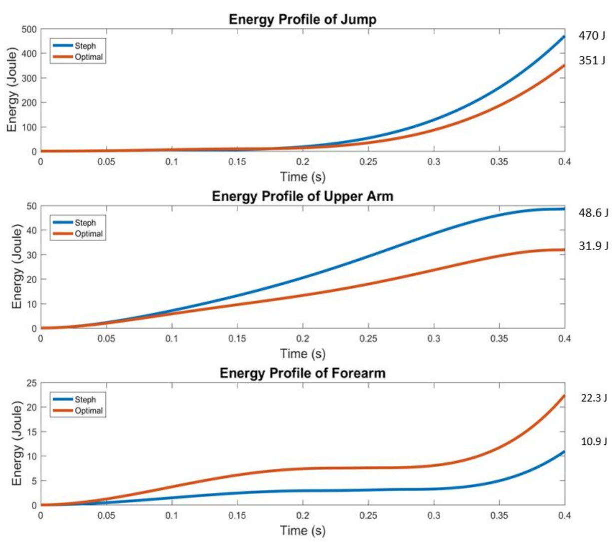

Figure 6 – Energy values of Curry vs optimal shooting motion.

Fig. 4 shows the energy surface plot with respect to the optimal jump height. For reference, the energy expended by Curry is also displayed on the chart. Note that the optimal value does not correspond to the minimum energy because of the trade-off between energy conserved and adjustment in motion.

Fig. 5 and Fig. 6 compare the optimal shooting motion to Curry's original shooting motion with respect to energy and trajectory for each joint. A reduction in jump height by 5.7% decreases the total energy consumption by 119 J. This observation is expected because the larger muscles (leg) used in the motion should expend more energy than the smaller ones (arms).

Furthermore, the optimal shooting motion does not require significant alterations to the upper arm joint trajectory and suggests a smoother catch and release motion for the forearm. While the smooth motion requires more energy from the forearm movement, the effect is outweighed by the decrease in energy of the other two joints. These adjustments mark the best trade-off between energy saved and motion changed.

VIII. Conclusion

A study was performed to model and analyze Curry's 3-point shot and an optimal shooting action was suggested using a weighted optimization algorithm.

Due to simplifications made in the model, the results are only rough estimations of the shooting motion. Realistically, the human body is an extremely complex system with physiological anomalies that cannot be accurately modelled in 2-D by a P-R-R robot.

By constraining the problem, two important factors have been neglected from the optimization. The first factor was generalizing the jump by a prismatic joint, which allowed the robot to continue accelerating upward even after lift-off. In reality, when a player leaves the ground, the ground reaction force that propels him upward disappears. The second factor was the simplification of the wrist joint. Okuboa [8] proved that the wrist adds velocities in the X and Z directions and modulates the upper arm's angular velocity. The dynamic model was tuned to minimize these effects.

To address these complexities, the next steps in this study are to improve the 2-D model by accurately modelling the jump with two revolute joints instead of the prismatic joint. Furthermore, the wrist can be modelled more precisely by using data collected from techniques like motion capture.

This study shows the potential for applying principles of robotics to sports science. In the future, coaches and trainers can adopt similar analysis techniques to improve training methods for athletes at all levels.

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