Lauren McCarthy - LM2221

HW 3

1 a

$y_i \sim \text{Normal}(x_i^T w, \tau^{-1})$

$w \sim \text{Normal}(0, \text{diag}(\alpha_1, \dots \alpha_d)^{-1})$

$\alpha_k \overset{iid}{\sim} \text{Gamma}(a_0, b_0)$

$\tau \sim \text{Gamma}(e_0, f_0)$

$$\ln p = \sum_{i=1}^{N} \ln p(y_i | w, \tau, x, \alpha) + \ln p(\tau) + \ln p(w|\tau) + \sum_{i=1}^{N} \ln p(\alpha_1, \dots, \alpha_n)$$

$$\sum_{i=1}^{N} \ln p(y_i | w, \tau, x, \alpha) = \sum_{i=1}^{N} \tfrac{1}{2} \ln\left(\tfrac{\tau}{2\pi}\right) - \tfrac{\tau}{2}(y_i - x_i^T w)^2$$

$$\ln w = \tfrac{1}{2} \sum_{k=1}^{d} \ln(\alpha_k) - \tfrac{d}{2} \ln(2\pi) - \tfrac{1}{2} \sum_{k=1}^{d} \alpha_k w_k^2$$

$$\ln(\tau) = e_0 \ln f_0 - \ln \Gamma(e_0) + (e_0 - 1) \ln \tau - f_0 \tau$$

$$\ln(\alpha) = \sum_{k=1}^{d} a_0 \ln b_0 - \ln \Gamma(a_0) + (a_0 - 1) \ln \alpha_k - b_0 \alpha_k$$

$$\underline{\underline{q(\tau)}} \propto \exp\left[\tfrac{N}{2} \ln \tau - \tfrac{\tau}{2} \mathbb{E}_w \left\{ \sum_{i=1}^{N} (y_i - x_i^T w)^2 \right\} + (e_0 - 1) \ln \tau - f_0 \tau \right]$$

$$q(\tau) \propto \text{Gamma}(e', f') \qquad e' = e_0 + \tfrac{N}{2}$$

$$f' = f_0 + \tfrac{1}{2} \underbrace{\mathbb{E}_w \left\{ \sum_{i=1}^{N} (y_i - x_i^T w)^2 \right\}}$$

$$\mathbb{E}_w \left\{ \sum_{i=1}^{N} (y_i - x_i^T w)^2 \right\}$$

$$\mathbb{E}_w \left\{ \sum_{i=1}^{N} (y_i^2 - 2 y_i x_i^T w + x_i^T w w^T x_i) \right\}$$

$$\sum_{i=1}^{N} \boxed{y_i^2 - 2 y_i x_i^T \mu_w} + \sum_{i=1}^{N} x_i^T \underbrace{\mathbb{E}_w(w w^T)}_{\mu_w \mu_w^T} x_i$$

$$\sum_{i=1}^{N} (y_i - x_i^T \mu_w)^2 + \sum_{i=1}^{N} x_i^T \Sigma_w x_i$$

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