(1)(c) objective function $\mathcal{L}$

$$\mathcal{L}(a_1, b_1, e_1, f_1, \mu_1, \Sigma_1) = \mathbb{E}\{\ln p(\tau)\} + \mathbb{E}\{\ln p(w|\alpha_1, \ldots \alpha_d)\} + \sum_{k=1}^{d} \mathbb{E}\{\ln p(\alpha_k)\} + \sum_{i=1}^{N} \mathbb{E}\{\ln p(y_i|x_i, w, \tau)\} - \mathbb{E}\{\ln q(\tau)\} - \mathbb{E}\{\ln q(w)\} - \sum_{k=1}^{d} \mathbb{E}\{\ln q(\alpha_k)\}$$

$$\mathbb{E}\{\ln p(\tau)\} = e_0 \ln f_0 - \ln\Gamma(e_0) + (e_0 - 1)\underbrace{\mathbb{E}\{\ln \tau\}}_{\text{entropy}} - f_0 \mathbb{E}\{\tau\}$$
$$= e_0 \ln f_0 - \ln\Gamma(e_0) + (e_0 - 1)[\Psi(e_1) - \ln(f_1)] - f_0 \frac{e_1}{f_1}$$

$$\mathbb{E}\{\ln p(w|\alpha_1, \ldots \alpha_d)\} = -\frac{d}{2}\ln 2\pi + \frac{1}{2}\sum_{k=1}^{d} \mathbb{E}\{\ln \alpha_k\} - \frac{1}{2}\text{trace}(\text{diag}[\mathbb{E}(\alpha_1), \ldots \mathbb{E}(\alpha_d)]\mathbb{E}(ww^T))$$
$$= -\frac{d}{2}\ln 2\pi + \frac{1}{2}\sum_{k=1}^{d}[\Psi(a_{1k}) - \ln(b_k)] - \frac{1}{2}\text{trace}\left(\frac{a_{11}}{b_{11}}, \ldots \frac{a_{dd}}{b_{dd}}\right)(\Sigma + \mu\mu^T)$$

$$\mathbb{E}\{\ln p(\alpha_k)\} = a_{0,k} \ln b_{0,k} - \ln\Gamma(a_{0,k}) + (a_{0,k} - 1)[\Psi(a_{1,k}) - \ln b_{1,k}] - b_{0,k}\frac{a_{1,k}}{b_{1,k}}$$

$$\mathbb{E}\{\ln p(y_i|x_i, w, \tau)\} = -\frac{1}{2}\ln(2\pi) + \frac{\mathbb{E}\{\ln\tau\}}{2} - \frac{\mathbb{E}\{\tau\}}{2}\mathbb{E}\left\{\sum_{i=1}^{N}(y_i - x_i^T w)^2\right\}$$
$$= -\frac{1}{2}\ln(2\pi) + \frac{1}{2}[\Psi(e_1) - \ln(f_1)] - \frac{1}{2}\frac{e_1}{f_1}\left[\sum_{i=1}^{N}(y_i - x_i^T w)^2 + x_i^T \Sigma x_i\right]$$

$$\mathbb{E}\{\ln q(\tau)\} = e_1 \ln f_1 + \ln\Gamma(e_1) + (1 - e_1)\Psi(e_1)$$

$$\mathbb{E}\{\ln q(w)\} = \frac{N}{2} + \frac{N}{2}\ln(2\pi) + \frac{1}{2}\ln|\Sigma|$$

$$\mathbb{E}\{\ln q(\alpha_k)\} = a_{1,k} \ln b_{1,k} + \ln\Gamma(a_{1,k}) + (1 - a_{1,k})\Psi(a_{1,k})$$

Plugging in these values to the equation above we get the objective.

□