$$q(\alpha) \propto \exp\left[\mathbb{E}\left\{\tfrac{1}{2}\ln(\alpha_k) - \tfrac{1}{2}\alpha_k w_k^2 + (a_0-1)\ln\alpha_k - b_0\alpha_k\right\}\right]$$
$$\exp\left[\left(a_0+\tfrac{1}{2}-1\right)\ln\alpha_k - \left(b_0+\tfrac{1}{2}w_k^2\right)\alpha_k\right]$$

$q(\alpha) \propto \text{Gamma}(a', b')$ $\quad a' = a_0 + \tfrac{1}{2}$

$$b' = b_0 + \tfrac{1}{2}\mathbb{E}_w(w_k^2)$$

$$\Sigma_{k,k} + \mu_{w(k)}^2$$

$$q(w) \propto \exp\left[\mathbb{E}_{\alpha,\gamma}\left\{-\tfrac{\gamma}{2}\sum_{i=1}^{N}(y_i - x_i^T w)^2 - \tfrac{1}{2}\sum_{k=1}^{d}\alpha_k w_k^2\right\}\right]$$
$$\exp -\tfrac{1}{2}\mathbb{E}(\gamma)\sum_{i=1}^{N}\left(y_i^2 - y_i x_i^T w + w^T x_i x_i^T w\right) - \tfrac{1}{2}w^T \text{diag}\, w$$
$$\exp -\tfrac{1}{2}\left[2\mathbb{E}(\gamma)\left(\textstyle\sum y_i x_i^T\right)w + w^T\left[\mathbb{E}(\gamma)\textstyle\sum x_i x_i^T + \mathbb{E}\{\text{diag}(\alpha)\}\right]w\right]$$

$q(w) \propto N(\mu_w, \Sigma_w)$ where

$$\Sigma_w = \mathbb{E}(\gamma)\sum x_i x_i^T + \mathbb{E}\{\text{diag}(\alpha)\}^{-1}$$
$$\mu_w = \Sigma_w\, \mathbb{E}(\gamma)\sum_{i=1}^{N} y_i x_i$$
$$\mathbb{E}(\gamma) = \frac{e'}{f'} \qquad \mathbb{E}(\alpha_k) = \frac{a'_k}{b'_k}$$

(b) Sudo Code

(1) Initalize params $a^0, b^0, e^0, f^0, \mu^0, \Sigma^0$

(2) For $t = 1 \ldots T$

(a) update $q(\gamma) \rightarrow$ $e' = e_0 + N/2$

$$f' = f_0 + \tfrac{1}{2}\sum_{i=1}^{N}(y_i - x_i^T\mu_w)^2 + \sum_{i=1}^{N} x_i^T \Sigma_w x_i$$

(b) update $q(\alpha) \rightarrow$ $a' = a_0 + \tfrac{1}{2}$

$$b' = b_0 + \tfrac{1}{2}\Sigma_{k,k} + \mu_{w(k)}^2$$

(c) update $q(w) \rightarrow$ $\Sigma_w = \frac{e'}{f'}\sum x_i x_i^T + \mathbb{E}\{\text{diag}(\alpha)\}^{-1}$

$$\mu_w = \Sigma_w \frac{e'}{f'}\sum_{i=1}^{N} y_i x_i$$

(3) Compare $\mathcal{L}(a'_t, b'_t, \Sigma'_t, \mu'_t, e'_t, f'_t)$ when change is small it has converged.

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