Name: ____

Algorithms and Data Structures
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Quiz 7
given: 2017-05-16

You have 20 minutes.

Problem 1

Points: 5+2+3

Consider the following recursive function:

fun $product(x:\mathbb{Z}):\mathbb{Z} =$
    if $x == 0$
        1
    else
        $x \cdot product(x-1)$

1. Convert it to a dynamic program by completing the blanks below.
2. Give the Θ-class of the space complexity of the resulting dynamic program.
3. Explain in which situations a dynamic program performs better than the recursive variant. Does it matter in this case?

fun $product(x:\mathbb{Z}):\mathbb{Z} =$
    $results :=$ **new** $Array[\mathbb{Z}](x)$
    for i **in** $0,\ldots,n$

        ____

    $results[$____________$]$

Problem 2

Points: 3+2+5

Assume a fixed list D of denominations of coins, sorted decreasingly. We want to find the smallest set of coins whose denominations add up to n.

We use the following greedy-style algorithm:

precondition: D is sorted by $\geq$

fun $greedyCoinChange(D:List[\mathbb{Z}], n:\mathbb{Z}):List[\mathbb{Z}] =$
    $solution := Nil$
    for x **in** D
        while $x + sumList(solution) \leq n$
            $x := prepend(x, solution)$
    $solution$

fun $sumList(l:List[\mathbb{Z}]):\mathbb{Z} =$
    return the sum of all elements in l

1. Give the result for running $greedyCoinChange([50,20,10,5,2,1],14)$.
2. For fixed D, the algorithm's run time is $\Theta(n^2)$. Give a simple improvement that makes the run time $\Theta(n)$.
3. The algorithm is not correct in general. Give an example where $greedyCoinChange(D,n)$ returns a non-optimal result.