# Knowledge Representation and Processing

Florian Rabe (for a course given with Michael Kohlhase)

Computer Science, University Erlangen-Nürnberg, Germany

Summer 2020

Administrative Information 2

## Administrative Information

#### Format

#### Zoom

- lectures and exercises via zoom
- participants muted by default for simplicity
- ► interaction strongly encouraged We don't want to lecture we want to have a conversation during which you learn
- let's try out zoom
  - use reactions to say yes no, ask for break etc.
  - feel free to annotate my slides
  - talk in the chat

### Recordings

- maybe prerecorded video lectures or recorded zoom meeting
- ▶ to be decided along the way

# Background

#### Instructors

- ▶ Prof. Dr. Michael Kohlhase Professor of Knowledge Representation and Processing
- ▶ PD Dr. Florian Rabe same research group

#### Course

- ► This course is given for the first time
- ► Always a little bit of an experiment cutting edge vs. unpolished
- ► Could become signature course of our research group same name!

## Prerequisites

### Required

▶ basic knowledge about formal languages, context-free grammars but we'll do a quick revision here

### Helpful

- ► Algorithms and Data Structures mostly as a contrast to this lecture
- ► Basic logic we'll revise it slightly differently here
- ▶ all other courses as examples of how knowledge pervades all of CS

#### General

Curiosity

this course is a bit unusual

Interest in big picture

this course touches on lots of things from all over CS

## **Examination and Grading**

### Suggestion

- grade determined by single exam
- written or oral depends on number of students
- some acknowledgment for practical exercises

to be finalized next week

#### Exam-relevant

- anything mentioned in notes
- anything discussed in lectures

neither is a superset of the other!

## Materials and Exam-Relevance

#### **Textbook**

- does not exist
- normal for research-near specialization courses

#### Notes

- textbook-style but not as comprehensive
- developed along the way

#### Slides

- not comprehensive
- used as visual aid, conversation starters

### Communication

### Open for questions

- open door policy in our offices if the lockdown ever ends
- always room for questions during lectures
- for personal questions, contact me during/after lecture or by email
- forum at https://fsi.cs.fau.de/forum/
  154-Wissensrepraesentation-und-Verarbeitung

#### Materials

official notes and slides as pdf: https://kwarc.info/teaching/WuV/

will be updated from time to time

watch me prepare the materials: https:
//github.com/florian-rabe/Teaching/tree/master/WuV
pull requests and issues welcome

#### **Exercises**

#### **Learning Goals**

- ▶ Get acquainted with state of the art of practice
- ► Try out real tools

#### Homeworks

- one major project as running example
- homeworks building on each other

build one large knowledge-based system details on later slides

# Overview and Essential Concepts

# Representation and Processing

### Common pairs of concepts:

| Representation  | Processing |
|-----------------|------------|
| Static          | Dynamic    |
| Situation       | Change     |
| Be              | Become     |
| Data Structures | Algorithms |
| Set             | Function   |
| State           | Transition |
| Space           | Time       |
|                 |            |

# Data and Knowledge

 $2 \times 2$  key concepts

| Syntax    | Data      |
|-----------|-----------|
| Semantics | Knowledge |

- ▶ Data: any object that can be stored in a computer Example: ((49.5739143, 11.0264941), "2020 - 04 - 21716:
  - 15 : 00*CEST*")
- Syntax: a system of rules that describes which data is well-formed

Example: "a pair of (a pair of two IEEE double precision floating point numbers) and a string encoding of a time stamp"

- ► Semantics: system of rules that determines the meaning of well-formed data
- ► Knowledge: combination of some data with its syntax and semantics

## Knowledge is Elusive

#### Representation of key concepts

- ▶ Data: using primitive objects implemented as bits, bytes, strings, records, arrays, . . .
- Syntax: (context-free) grammars, (context-sensitive) type systems implemeted as inductive data structures
- Semantics: functions for evaluation, interpretation, of well-formed data implemented as recursive algorithms on the syntax
- ► Knowledge: elusive emerges from applying and interacting with the semantics

### Semantics as Translation

- ► Knowledge can be captured by a higher layer of syntax
- ► Then semantics is translation into syntax

| Data syntax                        | Semantics function                    | Knowledge syntax                    |
|------------------------------------|---------------------------------------|-------------------------------------|
| SPARQL query                       | evaluation                            | result set                          |
| SQL query                          | evaluation                            | result table                        |
| program                            | compiler                              | binary code                         |
| program expression                 | interpreter                           | result value                        |
| logical formula                    | interpretation in a model             | mathematical object                 |
| HTML document                      | rendering                             | graphics context                    |
| program expression logical formula | interpreter interpretation in a model | result value<br>mathematical object |

# Heterogeneity of Data and Knowledge

- Capturing knowledge is difficult
- Many different approaches to semantics
  - fundamental formal and methodological differences
  - often captured in different fields, conferences, courses, languages, tools
- Data formats equally heterogeneous
  - ontologies
  - programs
  - logical proofs
  - databases
  - documents

# Challenges of Heterogeneity

### Challenges

- collaboration across communities
- translation across languages
- conversion between data formats
- interoperability across tools

### Sources of problems

- interoperability across formats/tools major source of
  - complexity
    - bugs
- friction in project team due to differing preferences, expertise
- ▶ difficult choice between languages/tools with competing advantages
  - reverting choices difficult, costly
    - maintaining legacy choices increases complexity

# Aspects of Knowledge

- ► Tetrapod model of knowledge active research by our group
- classifies approaches to knowledge into five aspects

| Aspect  | KRLs (examples)   |
|---|---|
| ontologization<br>concretization<br>computation<br>deduction<br>narration | ontology languages (OWL), description logics (ALC) relational databases (SQL, JSON) programming languages (C) logics (HOL) document languages (HTML, LaTeX) |

## Relations between the Aspects

Ontology is distinguished: capture the knowledge that the other four aspects share



# Complementary Advantages of the Aspects

| objects          | characteristic                                |  |   |
|------------------|---|--|---|
|                  | advantage                                     | joint advantage  | application   |
|                  |   | of the other as-   |   |
|                  |   | pects  |   |
| formal proofs    | correctness                                   | ease of use  | verification  |
| programs         | efficiency                                    | well-<br>definedness   | execution   |
| concrete objects | tangibility                                   | abstraction  | storage/retrieval   |
| texts            | flexibility                                   | formal seman-<br>tics  | human understanding   |
|                  | formal proofs<br>programs<br>concrete objects | formal proofs correctness programs efficiency concrete objects tangibility | advantage joint advantage of the other aspects  formal proofs correctness ease of use programs efficiency well-definedness concrete objects tangibility abstraction texts flexibility formal seman- |

| Aspect pair | characteristic advantage   |
|-------------|----------------------------|
| ded./comp.  | rich meta-theory           |
| narr./conc. | simple languages           |
| ded./narr.  | theorems and proofs        |
| comp./conc. | normalization              |
| ded./conc.  | decidable well-definedness |
| comp./narr. | Turing completeness        |

## Structure of the Course

#### Aspect-independent parts

- general methods that are shared among the aspects
- ▶ to be discussed as they come up

### Aspects-specific parts

- one part (about 2 weeks) for each aspect
- ▶ high-level overview of state of the art
- ▶ focus on comparison/evaluation of the aspect-specific results

# Structure of the Exercises

### One major project

- representative for a project that a CS graduate might be put in charge of
  - ▶ challenging heterogeneous data and knowledge
- requires integrating/combining different languages, tools

### unique opportunity in this course because knowledge is everywhere

#### Concrete project

- develop a univis-style system for a university
- lots of heterogeneous knowledge
- course and program descriptions
  - legal texts
    - websites
  - grade tables
  - transcript generation code
- build a completely functional system applying the lessons of the course

Ontological Knowledge 22

# Ontological Knowledge

# Components of an Ontology

8 main declarations

- individual concrete objects that exist in the real world, e.g., "Florian Rabe" or "WuV"
- concept abstract groups of individuals, e.g., "instructor" or "course"
- ► relation binary relations between two individuals, e.g., "teaches"
- properties binary relations between an individuals and a concrete value (a number, a date, etc.), e.g., "has-credits"
- ► concept assertions the statement that a particular individual is an instance of a particular concept
- individual is an instance of a particular concept

  relation assertions the statement that a particular

relation holds about two individuals

- property assertions the statement that a particular individual has a particular value for a particular property
- axioms statements about relations between concepts, e.g., "instructor" □ "person"

# Divisions of an Ontology

#### Abstract vs. concrete

- ► TBox: concepts, relations, properties, axioms

  everything that does not use individuals
- ABox: individuals and assertions

#### Named vs. unnamed

- Signature: individuals, concepts, relations, properties together called entities or resources
- ► Theory: assertions, axioms

# Comparison of Terminology

| Here       | OWL             | Description logics | ER model    | UML                | semantics via logics |
|------------|-----------------|--------------------|-------------|--------------------|----------------------|
| individual | instance        | individual         | entity      | object, instance   | constant             |
| concept    | class           | concept            | entity-type | class              | unary predicate      |
| relation   | object property | role               | role        | association        | binary predicate     |
| property   | data property   | (not common)       | attribute   | field of base type | binary predicate     |

| domain                  | individual     | concept     |
|-------------------------|----------------|-------------|
| type theory, logic      | constant, term | type        |
| set theory              | element        | set         |
| database                | row            | table       |
| philosophy <sup>1</sup> | object         | property    |
| grammar                 | proper noun    | common noun |

<sup>&</sup>lt;sup>1</sup>as in https://plato.stanford.edu/entries/object/

# Ontologies as Sets of Triples

| Assertion          | Triple         |               |              |
|--------------------|----------------|---------------|--------------|
|                    | Subject        | Predicate     | Object       |
| concept assertion  | "Florian Rabe" | is-a          | "instructor" |
| relation assertion | "Florian Rabe" | "teaches"     | "WuV"        |
| property assertion | "WuV"          | "has credits" | 7.5          |

Efficient representation of ontologies using RDF and RDFS standardized special entities.

## Special Entities

RDF and RDFS define special entities for use in ontologies:

- "rdfs:Resource": concept of which all individuals are an instance and thus of which every concept is a subconcept
- "rdf:type": relates an entity to its type:
  - ▶ an individual to its concept (corresponding to is-a above)
    - other entities to their special type (see below)
- rdfs:Class": special class for the type of classes
- "rdf:Property": special class for the type of properties
- "rdfs:subClassOf": a special relation that relates a subconcept to a superconcept
- "rdfs:domain": a special relation that relates a relation to the concepts of its subjects
- "rdfs:range": a special relation that relates a relation/property to the concept/type of its objects

Goal/effect: capture as many parts as possible as RDF triples.

# Declarations as Triples using Special Entities

| Assertion                       | Triple     |                   |                 |
|---------------------------------|------------|-------------------|-----------------|
|                                 | Subject    | Predicate         | Object          |
| individual                      | individual | "rdf:type"        | "rdfs:Resource" |
| concept                         | concept    | "rdf:type"        | "rdf:Class"     |
| relation                        | relation   | "rdf:type"        | "rdf:Property"  |
| property                        | property   | "rdf:type"        | "rdf:Property"  |
| concept assertion               | individual | "rdf:type"        | concept         |
| relation assertion              | individual | relation          | individual      |
| property assertion              | individual | property          | value           |
| for special forms of            | axioms     |                   |                 |
| $c \sqsubseteq d$               | c          | "rdfs:subClassOf" | d               |
| $\operatorname{dom} r \equiv c$ | r          | "rdfs:domain"     | С               |
| $\operatorname{rng} r \equiv c$ | r          | "rdfs:range"      | С               |

# An Example Ontology Language

see syntax of  $\ensuremath{\mathsf{BOL}}$  in the lecture notes

## Semantics as Translation

# Example: Syntax of Arithmetic Language

Syntax: represented as formal grammar

```
Numbers
N ::= 0 \mid 1 \qquad \text{literals}
\mid N + N \qquad \text{sum}
\mid N * N \qquad \text{product}
Formulas
F ::= N \doteq N \qquad \text{equality}
\mid N \leq N \qquad \text{ordering by size}
```

Implementation as inductive data type

# Example: Semantics of Arithmetic Language

Semantics: represented as translation into known language

Problem: Need to choose a known language first Here: unary numbers represented as strings Built-in data (strings and booleans):

 $S ::= \varepsilon$ 

$$S := \varepsilon$$
 empty
| (Unicode) characters
 $B := \text{true}$  truth
| false falsity

Built-in operations to work on the data:

- $\triangleright$  concatenation of strings  $S:=\operatorname{conc}(S,S)$
- replacing all occurrences of c in  $S_1$  with  $S_2$ 
  - S:=replace $(S_1, c, S_2)$ ightharpoonup equality test:  $B::=S_1==S_2$
  - ▶ prefix test: B::=startsWith( $S_1, S_2$ )

# Example: Semantics of Arithmetic Language

### Represented as function from syntax to semantics

- mutually recursive, inductive functions for each non-terminal symbol
- compositional: recursive call on immediate subterms of argument

For numbers n: semantics [n] is a string

- ho  $\llbracket 0 
  rbracket = \varepsilon$
- **▶** [1] = "|"
- ightharpoonup [m+n] = conc([m],[n])
- $\blacktriangleright \ \llbracket m*n \rrbracket = \mathtt{replace}(\llbracket m \rrbracket, "|", \llbracket n \rrbracket)$

For formulas f: semantics  $\llbracket f \rrbracket$  is a boolean

- $| m \le n | = \operatorname{startsWith}([n], [m])$

## Semantics of BOL

| Aspect         | kind of semantic language | semantic language |
|----------------|---------------------------|-------------------|
| deduction      | logic                     | SFOL              |
| concretization | database language         | SQL               |
| computation    | programming language      | Scala             |
| narration      | natural language          | English           |

see details of each translation in the lecture notes

### **General Definition**

A semantics by translation consists of

- syntax: a formal language I
- ▶ semantic language: a formal language *L*different or same aspect as /
- semantic prefix: a theory P in L formalizes fundamentals that are needed to represent I-objects
- interpretation: translates every *I*-theory T to an L-theory P, T

# Common Principles

## Properties shared by all semantics of BOL

not part of formal definition, but best practices

- ▶ I-declaration translated to L-declaration for the same name
- ontologies translated declaration-wise
- ▶ one inductive function for every kind of complex *I*-expression
  - individuals, concepts, relations, properties, formulas
  - maps *l*-expressions to *L*-expressions
- ▶ atomic cases (base cases): I-identifier translated to L-identifier of the same name or something very similar
- complex cases (step cases): compositional

## Compositionality

Case for operator \* in interpretation function compositional iff interpretation of  $*(e_1, \ldots, e_n)$  only depends on on the interpretation of the  $e_i$ 

$$[\![*(e_1,\ldots,e_n)]\!] = [\![*]\!]([\![e_1]\!],\ldots,[\![e_n]\!])$$

for some function  $[\![*]\!]$ 

Example: ;-operator of BOL in translation to FOL

- ▶ translation:  $[\![R_1; R_2]\!] = \exists m : \iota.[\![R_1]\!](x, m) \land [\![R_2]\!](m, y)$
- special case of the above via
  - **\*** \* =:
  - $\triangleright$  n=2
  - $[ ] [ ] [ (p_1, p_2) \mapsto \exists m : \iota.p_1(x, m) \land p_2(m, y) ]$
- ▶ Indeed, we have  $[R_1; R_2] = [; ]([R_1], [R_2])$

## Compositionality (2)

Translation compositional iff

- lacktriangle one translation function for each non-terminal all written  $[\![-]\!]$
- each defined by one induction on syntax

i.e., one case for production mutually recursive

all cases compositional

Substitution theorem: a compositional translation satisfies

$$[\![E(e_1,\ldots,e_n)]\!] = [\![E]\!]([\![e_1]\!],\ldots,[\![e_n]\!])$$

for

- every expression  $E(N_1, ..., N_n)$  with non-terminals  $N_i$
- ▶ some function [E] that only depends on E

## Compositionality (3)

$$[E(e_1,\ldots,e_n)] = [E]([e_1],\ldots,[e_n])$$

for every expression  $E(N_1, ..., N_n)$  with non-terminals  $N_i$ 

#### Now think of

- $\triangleright$  variable  $x_i$  of type  $N_i$  instead of non-terminal  $N_i$
- $\triangleright$   $E(x_1,...,x_n)$  as expression with free variables  $x_i$  of type  $N_i$
- $\triangleright$  expressions e derived from N as expressions of type N
- ▶  $E(e_1,...,e_n)$  as result of substituting  $e_i$  for  $x_i$
- $ightharpoonup [E](x_1,\ldots,x_n)$  as (semantic) expression with free variables  $x_i$

### Then both sides of equations act on $E(x_1, \ldots, x_n)$ :

- left side yields  $[E(e_1, \ldots, e_n)]$  by
  - first substitution  $e_i$  for  $x_i$ 
    - ▶ then semantics ¶—¶ of the whole
- right side yields  $[E]([e_1], \ldots, [e_n])$  by

  - ▶ then substitution  $\llbracket e_i \rrbracket$  for  $x_i$

semantics commutes with substitution

## Non-Compositionality

### Examples

- deduction: cut elimination, translation from natural deduction to Hilbert calculus
- computation: optimizing compiler, e.g., loop unrolling
- concretization: query optimization, e.g., turning a WHERE of a join into a join of WHEREs,
- narration: ambiguous words are translated based on context

#### Typical sources

- subcases in a case of translation function
  - based on inspecting the arguments, e.g., subinduction
  - based on context
- custom-built semantic prefix

Type Systems 41

## Type Systems

## Breakout Question

Is this an improvement over BOL?

#### Declarations

```
D ::= individual ID : C typed atomic individual atomic concept C :=  typed atomic relation C :=  typed atomic relation C :=  typed atomic property C :=  typed atomic property
```

rest as before

## Actually, when is a language an improvement?

### Criteria: orthogonal, often mutually exclusive

- syntax design trade-off
  - expressivity: easy to express knowledge
    - e.g., big grammar, extra production for every user need
  - simplicity: easy to implement/interpret
    - e.g., few, carefully chosen productions
- semantics: specify, implement, document
- intended users
  - skill level
  - prior experience with related languages
  - amount of training needed
- long-term plans: re-answer the above question but now
  - maintainability: syntax was changed, everything to be redone
  - scalability: expressed knowledge content has reached huge sizes

## Church vs. Curry Typing

|                        | intrinsic                           | extrinsic                      |
|------------------------|-------------------------------------|--------------------------------|
| $\lambda$ -calculus by | Church                              | Curry                          |
| type is                | carried by object                   | given by environment           |
| typing is a            | function objects $ ightarrow$ types | relation objects $	imes$ types |
| objects have           | unique type                         | any number of types            |
| types interpreted as   | disjoint sets                       | unary predicates               |
| type given by          | part of declaration                 | additional axiom               |
| example                | individual "WuV":"course"           | individual "Wuv",              |
|                        |                                     | "WuV" is-a "course"            |
| examples               | SFOL, SQL                           | OWL, Scala, English            |
|                        | most logics, functional PLs         | ontology, OO,                  |
|                        |                                     | natural languages              |
|                        | many type theories                  | set theories                   |

## Type Checking

|                        | intrinsic                           | extrinsic                      |
|------------------------|-------------------------------------|--------------------------------|
| type is                | carried by object                   | given by environment           |
| typing is a            | function objects $ ightarrow$ types | relation objects $	imes$ types |
| objects have           | unique type                         | any number of types            |
| type given by          | part of declaration                 | additional axiom               |
| example                | individual "WuV":"course"           | individual "Wuv",              |
|                        |                                     | "WuV" is-a "course"            |
| type inference for $x$ | uniquely infer A from x             | find minimal $A$ with $x : A$  |
| type checking          | inferred=expected                   | prove x : A                    |
| subtyping $A <: B$     | cast from A to B                    | x: A  implies  x: B            |
| typing decidable       | yes unless too expressive           | no unless restricted           |
| typing errors          | static (compile-time)               | dynamic (run-time)             |
| advantages             | easy                                | flexible                       |
|                        | unique type inference               | allows subtyping               |

## Curry Typing in BOL

| language     | objects           | types                          | typing relation                    |
|--------------|-------------------|--------------------------------|------------------------------------|
| Syntax       | individuals       | concepts                       | <i>i</i> is-a <i>c</i>             |
| Semantics in |                   |                                |                                    |
| FOL          | type $\iota$      | predicates $c \subseteq \iota$ | c(i) true                          |
| SQL          | table Individuals | tables containing ids          | id of $i$ in table $c$             |
| Scala        | String            | hash sets of strings           | c.contains(i)                      |
| English      | proper nouns      | common nouns                   | " <i>i</i> is a <i>c</i> " is true |

## Subtyping

### Subtyping works best with Curry Typing

- ightharpoonup explicit subtyping as in  $\mathbb{N} <: \mathbb{Z}$
- ▶ comprehension/refinement as in  $\{x : \mathbb{N} | x \neq 0\}$
- operations like union and intersection on types
- ▶ inheritance between classes, in which case subclass = subtype
- ▶ anonymous record types as in  $\{x : \mathbb{N}, y : \mathbb{Z}\} <: \{x : \mathbb{N}\}$

## A General Definition of a Type System

### A **type system** consists of

- a collection, whose elements are called **objects**,
- a collection, whose elements are called intrinsic types,
- ➤ a function assigning to every object *x* its **intrinsic type** *I*, in which case we write *x* : *I*,
- ► for some intrinsic types *I* 
  - ightharpoonup an intrinsic type  $E_I$
  - ▶ a relation  $\in_I$  between objects with intrinsic types I and  $E_I$ , called the **extrinsic typing** relation for I.

## **Examples**

| System      | intrinsic types    | $E_{I}$           | $\in_I$                                     |
|-------------|--------------------|-------------------|---|
| pure Church | one per type       | none              | none  |
| pure Curry  | objects O, types T | $E_O = T$         | <i>∈o</i> =:                                |
| FOL         | one per type       | none              | none  |
| Scala       | AnyRef , Class     | $E_{Any} = Class$ | $\in_{Any}=	exttt{isInstance}$              |
| BOL         | Ind, Conc          | $E_{Ind} = Conc$  | $\in_{\mathit{Ind}}=\mathtt{is}-\mathtt{a}$ |
| set theory  | Set, Prop          | $E_{Set} = Set$   | $\in_{\mathcal{S}et}=\in$                   |

## **Breakout Question**

What do the following have in common?

- Java class
- ► SQL schema for a table
- logical theory (e.g., Monoid)

## **Breakout Question**

What do the following have in common?

- Java class
- ► SQL schema for a table
- ▶ logical theory (e.g., Monoid)

all are (essentially) abstract data types

## Abstract Data Types: Motivation

Recall subject-centered representation of assertion triples:

```
individual "FlorianRabe"
  is—a "instructor" "male"
  "teach" "WuV" "KRMT"
  "age" 40
  "office" "11.137"
```

Can we use types to force certain assertions to occur together?

- Every instructor should teach a list of courses.
- Every instructor should have an office.

## Abstract Data Types: Motivation

Inspires **subject-centered types**, e.g.,

```
concept instructor
  teach course*
  age: int
  office: string

individual "FlorianRabe": "instructor"
  is—a "male"
```

is—a "male" teach "WuV" "KRMT" age 40 office "11.137"

### Incidental benefits:

- no need to declare relations/properties separately
- reuse relation/property names distinguish via qualified names: instructor .age

## Abstract Data Types: Motivation

```
Natural next step: inheritance
concept person
  age: int
concept male <: person
concept instructor <: person
  teach course*
  office: string
individual "FlorianRabe": "instructor" □ "male"
  "teach" "WuV" "KRMT"
  "age" 40
  "office" "11.137"
```

our language quickly gets a very different flavor

## Abstract Data Types: Examples

### Prevalence of abstract data types:

| aspect         | language | abstract data type                    |
|----------------|----------|---------------------------------------|
| ontologization | UML      | class                                 |
| concretization | SQL      | table schema                          |
| computation    | Scala    | class, interface                      |
| deduction      | various  | theory, specification, module, locale |
| narration      | various  | emergent feature                      |

same idea, but may look very different across languages

## Abstract vs. Concrete Types

### Concrete type: values are

- given by their internal form,
- defined along with the type, typically built from already-known pieces.

examples: products, inductive data types

#### Abstract type: values are

- given by their externally visible properties,
- defined in any environment that understands the type definition.

main example: abstract data types

## Abstract Data Types: Examples

| aspect         | type           | values                            |
|----------------|----------------|-----------------------------------|
| computation    | abstract class | instances of implementing classes |
| concretization | table schema   | table rows                        |
| deduction      | theory         | models                            |

Values depend on the environment in which the type is used:

- class defined in one specification language (e.g., UML), implementations in programing languages Java, Scala, etc. available values may depend on run-time state
- theory defined in logic, models defined in set theories, type theories, programming languages

available values may depend on philosophical position

## Abstract Data Types: Definition

Given some type system, an abstract data type (ADT) is

a flat type

$$\{c_1: T_1[=t_1], \ldots, c_n: T_n[=t_n]\}$$

where

- c<sub>i</sub> are distinct names
- $ightharpoonup T_i$  are types
- $ightharpoonup t_i$  are optional definitions; if given,  $t_i$ :  $T_i$  required
- or a mixin type

$$A_1 * \ldots * A_n$$

for ADTs  $A_i$ .

Languages may or may not make ADTs additional types of the type system

# Abstract Data Types: Class Definitions A class definition in OO:

abstract class a extends  $a_1$  with ... with  $a_m$  {  $c_1$ :  $T_1$ 

$$c_n$$
:  $T_n$ 
}
Corresponding ADT definition:

The usual terminology:

▶ a<sub>i</sub> are super-X or parent-X of a where X is whatever the language calls its ADTs (e.g., X=class)

 $a = a_1 * ... * a_m * \{c_1 : T_1, ..., c_n : T_n\}$ 

## Abstract Data Types: Flattening

### The **flattening** $A^{\flat}$ of an ADT A is

- ightharpoonup if A is flat:  $A^{\flat} = A$
- $(A_1 * ... * A_n)^b$  is union of all  $A_i^b$  where duplicate field names are handled as follows
  - same name, same type, same or omitted definition: merge details may be much more difficult
  - otherwise: ill-formed

## Abstract Data Types: Subtleties

We gloss over several major issues:

- ► How exactly do we merge duplicate field names? Does it always work? implement abstract methods, override, overload
- ▶ Is recursion allowed, i.e., can I define an ADT a = A where a occurs in A?

common in OO-languages: use a in the types of its fields

- What about ADTs with type arguments?
  - e.g., generics in Java, square-brackets in Scala
- Is mutual recursion between fields in a flat type allowed?
  common in OO-languages
- ▶ Is \* commutative? What about dependencies between fields?

no unique answers

incarnations of ADTs subtly different across languages

Context-Sensitive Syntax

U.

## Context-Sensitive Syntax

### **Definition**

### A language system consists of

- context-free syntax
- lacktriangleright distinguished non-terminal symbol  ${\cal V}$

words called vocabularies

ightharpoonup some distinguished non-terminal symbols  ${\cal E}$ 

words called  $\mathcal{E}$ -expressions

▶ unary predicate  $wft(\Theta)$  on vocabularies  $\Theta$ 

well-formed vocabulary  $\Theta$ 

ightharpoonup unary predicates  $\operatorname{wff}_{\Theta}^{\mathcal{E}}(E)$ 

well-formed  $\mathcal{E}$ -expressions E

## Typical Structure

#### Vocabularies

lists of declarations

#### **Declarations**

- named
- at least one for each expression kind
- may contain other expressions
- may contain nested declarations

e.g., type, definition

or fields in an ADT

e.g., fields in an ADT

#### Expressions

- inductive data type
- relative to vocabulary

names occur as base cases

formulas as special case

## Vocabularies and Expressions

| Aspect         | vocabulary Θ      | expression kinds ${\mathcal E}$                  |
|----------------|-------------------|--|
| Ontologization | ontology          | individual, concept, relation, property, formula |
| Concretization | database schema   | cell, row, table, formula                        |
| Computation    | program           | term, type, object, class,                       |
| Logic          | signature, theory | term, type, formula,                             |
| Narration      | dictionary        | phrases, sentences, texts                        |

## **Examples**

See notes made during the lecture for examples

### Concrete Knowledge

## Concrete Knowledge

### Motivation

#### Main ideas

- Ontology abstractly describes concepts and relations
- ► Tool maintains concrete data set
- Focus on efficiently
  - identifying (i.e., assign names)
  - representing
  - processing
  - querying

large sets of concrete data

#### Recall: TBox-ABox distinction

- ► TBox: general parts, abstract, fixed
  - main challenge: correct modeling of domain
- ► ABox: concrete individuals and assertions about them, growing main challenge: aggregate them all

### Concrete Data

#### Concrete is

- Base values: integers, strings, booleans, etc.
- Collections: sets, multisets, lists (always finite)
- Finite aggregations: tuples, records
- User-defined concrete data: enumerations, inductive types
- Advanced objects: finite maps, graphs, etc.

#### Concrete is not

- Free symbols to be interpreted by a model
  - exception: foreign function interfaces

 $\lambda$ -abstraction, quantification

formulas, algorithms

- Variables (free or bound)
- Symbolic expressions
  - Exceptions:
    - expressions of inductive type
    - application of built-in functions
    - queries that return concrete data

## Breakout question

What is the difference between

- ▶ an OWL ontolgoy
- ▶ an SQL database

## Two Approaches

### Based on untyped ontology languages

- Representation based on knowledge graph
- Ontology written in BOL-like language
- Data maintained as set of triples
- ► Typical language/tool design
  - ontology and query language separate
  - triple store and query engine integrated

system = triple store

e.g., OWL, SPARQL e.g., Virtuoso tool

### Based on typed ontology languages

- Representation based on abstract data types
- Ontology written as database schema
- ► Data maintained as tables system = (relational) database
- ► Typical language/tool design
  - ontology and query language integrated
  - table store and query engine integrated

e.g., SQL

e.g., MySQL tool

## **Evolution of Approaches**

#### Our usage is non-standard

- Common
  - ontologies = untyped approach, OWL, triples, SPARQL
  - databases = typed approach, tables, SQL
- Our understanding: two approaches evolved from same idea
  - triple store = untyped database
  - SQL schema = typed ontology

### **Evolution**

- Typed-untyped distinction minor technical difference
- Optimization of respective advantages causes speciation
- ► Today segregation into different
  - jargons
  - languages, tools
  - communities, conferences
  - courses

## Untyped concrete data

### Central data structure = knowledge graph

- nodes = individuals
  - identifier
  - sets of concepts
  - key-value sets of properties
- edges = relation assertions
  - from subject to objects
  - labeled with name of relation

### Processing strengths

- store: as triple set
- edit: Protege-style or graph-based
- visualize: as graph different colors for concepts, relations
- query: match, traverse graph structure
- untyped data simplifies integration, migration

## Typed concrete data

### central data structure = relational database

- tables = abstract data type
- rows = objects of that type
- columns = fields of ADT
- cells = values of fields

## Processing strengths

- store: as JSON, CSV, or similar
- edit: SQL commands or table editors
- visualize: as table view
- query: relational algebra
- typed data simplifies selecting, sorting, aggregating

### **Identifiers**

### Knowledge graph

- concept, relation, property names given in TBox
- individual names attached to nodes

#### **Database**

- table, column names given in schema
- row key = distinguished column options
  - preexistent characteristic column
  - penerated, e.g. UUID, incremental numbering
  - concatenation of characteristic set of columns
- column/row identifiers formed by qualifying with table name

## Breakout question

When using typed concrete data, how to fully realize abstract data types

- nesting: ADTs occurring as field types
- ▶ inheritance between ADTs
- mixins

## ADTs in Typed Concrete Data

### Nesting: field a : A in ADT B

- field types must be base types, a: A not allowed
- allow ID as additional base type
- ▶ use field a : ID in table B
- store value of b in table A

#### Inheritance: B inherits from A

- ▶ add field *parent*<sub>A</sub> to table B
- store values of inherited fields of B in table A

general principle: all objects of type A stored in same table

#### Mixin: A \* B

- essentially join of tables A and B on common fields
- some subtleties depending on ADT flattening

## Open/Closed World

- Question: is the data complete?
  - closed world: yes
  - open world: not necessarily
- Dimensions of openness
  - existence of individual objects
  - assertions about them
- Sources of openness
  - more exists but has not yet been added
  - more could be created later
- Orthogonal to typed/untyped distinction, but in practice
  - knowledge graphs use open world
  - databases use closed world

Open world is natural state, closing adds knowledge

## Closing the World

### Derivable consequences

- ▶ induction: prove universal property by proving for each object
- negation by failure: atomic property false if not provable
- term-generation constraint: only nameable objects exist

#### **Enabled operations**

- universal set: all objects
- complement of concept/type
- defaults: assume default value for property if not otherwise asserted

### Monotonicity problem

- monotone operation: bigger world = more results
- $\triangleright$  examples: union, intersection,  $\exists R.C$ , join, IN conditions
- $\triangleright$  counter-examples: complement,  $\forall R.C$ , NOT IN conditions

technically, non-monotone operations in open world dubious