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Exam on Dynamical Systems (simulation)

1. (2p) Find the flow of $\dot{x} = -2(x-5)$. This dynamical system has a global attractor?

2. (0.5p) Represent in the complex plane the curves

$$\{2e^{it} \; : \; t \in [0, \pi/2]\}, \quad \{2e^{it} \; : \; t \in [0, \pi]\}, \quad \{2e^{it} \; : \; t \in [0, 2\pi)\}.$$

3. Let $A = \begin{pmatrix} 2 & -5 \\ 1 & -2 \end{pmatrix}$.

(a) (3.5p) Find the principal matrix solution of the system $X' = AX$.

(b) (0.5p) Compute e^{At}. (c) (1p) Find $a, b \in \mathbb{R}$ such that $H : \mathbb{R}^2 \to \mathbb{R}$, $H(x, y) = x^2 + ay^2 + bxy$ is a global first integral of $X' = AX$.

4. (0.5p) How many solutions has the following problem?
(a) $x''' + t^2 x = 0$, $x(0) = x'(0) = 1$;
(b) $x''' + t^2 x = 0$, $x(0) = x'(0) = x''(0) = 1$.

5. We consider the map

$$f : \mathbb{R} \to \mathbb{R}, \quad f(x) = \frac{1}{50} x(100 - x).$$

(a) (1p) Find its fixed points and study their stability.

(b) (0.75p) Using the stair-step (cobweb) diagram estimate the basin of attraction of the attractor fixed point. If you do not know how to do it, try another way: in the difference equation $x_{k+1} = \frac{1}{50} x_k(100 - x_k)$, make the change of variable $x_k = 100 y_k$ and use the automatically generated cobweb for the logistic map.

(c) (0.25p) If $(x_k)_{k \geq 0}$ represent the number of fish in some lake at month k and

$$x_{k+1} = \frac{1}{50} x_k(100 - x_k), \quad x_0 = \eta$$

try to predict the fate of the fish in the case $\eta = 80$ and also in the case $\eta = 10$.