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I $\begin{cases} \dot{x} = x - 2xy \\ \dot{y} = \frac{x^2}{2} - y \end{cases}$

a) $\begin{cases} x - 2xy = 0 \\ \frac{x^2}{2} - y = 0 \Rightarrow \frac{x^2}{2} = y \end{cases} \Rightarrow x - 2x \cdot \frac{x^2}{2} = 0 \Rightarrow x - x^3 = 0$

$\Leftrightarrow x(1 - x^2) = 0$

$\Rightarrow x_1 = 0 \Rightarrow y_1 = 0$

$x_2 = 1 \Rightarrow y_2 = \frac{1}{2}$

$x_3 = -1 \Rightarrow y_3 = \frac{1}{2}$

$\Rightarrow$ The three equilibrium points are $(0,0)$, $(1, \frac{1}{2})$ $(-1, \frac{1}{2})$

b) $J(x,y) = \begin{pmatrix} \frac{\partial f_1}{\partial x}(x,y) & \frac{\partial f_1}{\partial y}(x,y) \\ \frac{\partial f_2}{\partial x}(x,y) & \frac{\partial f_2}{\partial y}(x,y) \end{pmatrix} = \begin{pmatrix} 1-2y & -2x \\ x & -1 \end{pmatrix}$ - the Jacobian matrix

Matrix of the linearised system around $(0,0)$ is $J(0,0) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. The eigenvalues are the solutions of the equation: $\det(J(0,0) - \lambda I_2) = 0 \Leftrightarrow$

$\Leftrightarrow \begin{vmatrix} 1-\lambda & 0 \\ 0 & -1-\lambda \end{vmatrix} = 0 \Rightarrow (1-\lambda)(-1-\lambda) = 0 \Rightarrow \lambda_1 = 1$

$\lambda_2 = -1$

The eigenvalues are real, different from 0, therefore the equilibrium $(0,0)$ is hyperbolic $\xrightarrow[\text{method}]{\text{Linearization}}$ $\dot{X} = J(0,0)X$ has a saddle, so the equilibrium point $(0,0)$ of $\dot{x} = f(x)$ is unstable

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Matrix of the linearized system around $(1,\frac{1}{2})$ is $J(1,\frac{1}{2}) = \begin{pmatrix} 0 & -2 \\ 1 & -1 \end{pmatrix}$

$\det(J(1,\frac{1}{2}) - \lambda I_2) = 0 \Leftrightarrow \begin{vmatrix} \lambda & -2 \\ 1 & -1-\lambda \end{vmatrix} = 0 \Leftrightarrow \lambda(1+\lambda)+2=0$

$\Rightarrow \lambda^2+\lambda+2=0 \Rightarrow \Delta = 1-8=-7 \Rightarrow \lambda_1 = \frac{-1+i\sqrt{7}}{2}, \lambda_2 = \frac{-1-i\sqrt{7}}{2}$

Both eigenvalues have the real part different from 0 therefore the equilibrium $(1,\frac{1}{2})$ is hyperbolic

Linearisation $\Longrightarrow$ method Because the eigenvalues are complex conjugates with the real part negative, the linearised system ~~around~~ has an attracting focus, therefore the nonlinear system has a attractor and is stable in $(1,\frac{1}{2})$

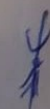
Matrix of the linearized system around $(-1,\frac{1}{2})$ is $J(-1,\frac{1}{2}) = \begin{pmatrix} 0 & 2 \\ -1 & -1 \end{pmatrix} \Rightarrow \det(J(-1,\frac{1}{2}) - \lambda I_2) = \begin{vmatrix} -\lambda & 2 \\ -1 & -1-\lambda \end{vmatrix}$

$\Rightarrow \lambda(1+\lambda)+2=0 \Rightarrow \lambda^2+\lambda+1=0 \Rightarrow$ the same eigenvalues as before therefore the nonlinear system is stable in $(-1;\frac{1}{2})$ and has an attractor.

c) $x=0 \Rightarrow 0=0$
$\dot{y}=-y$

$\Rightarrow$

y	$-\infty$	0	∞
$-y$	$+$	0	$-$

$\Rightarrow$ 

$y=0 \Rightarrow \dot{x}=x$
$0=\frac{x^2}{2} \Leftrightarrow x=0$

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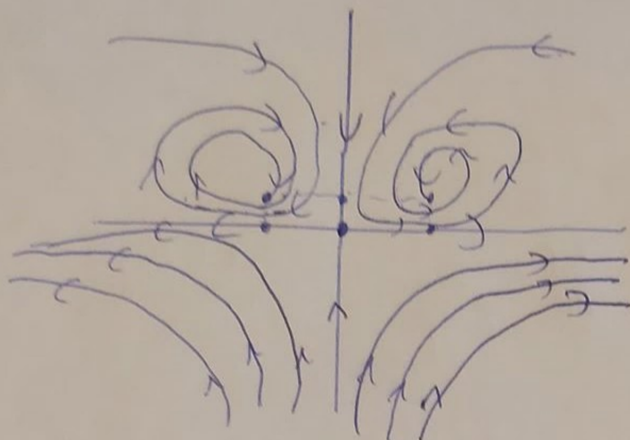

II $\begin{cases} \dot{x} = x - xy \\ \dot{y} = -0,3y + 0,3xy \end{cases}$

a) $\begin{cases} x - xy = 0 \\ -0,3y + 0,3xy = 0 \Rightarrow y = xy \end{cases} \Rightarrow x = y$

$\Rightarrow x = x^2 \Rightarrow x_1 = 0, y_1 = 0$
$x_2 = 1, y_2 = 1$ $\Rightarrow$ The equilibrium points are $(0,0)$ and $(1,1)$

$$J(x,y) = \begin{pmatrix} \frac{\partial f_1}{\partial x}(x,y) & \frac{\partial f_1}{\partial y}(x,y) \\ \frac{\partial f_2}{\partial x}(x,y) & \frac{\partial f_2}{\partial y}(x,y) \end{pmatrix} = \begin{pmatrix} 1-y & -x \\ 0,3y & -0,3+0,3x \end{pmatrix}$$

$J(1,1) = \begin{pmatrix} 0 & -1 \\ 0,3 & 0 \end{pmatrix} \Rightarrow \det(J(1,1) - \lambda I_2) = 0 \Leftrightarrow \begin{vmatrix} -\lambda & -1 \\ 0,3 & -\lambda \end{vmatrix} = 0$

$\Rightarrow \lambda^2 + 0,3 = 0 \Leftrightarrow \lambda_1 = i\sqrt{0,3}$ $\lambda_2 = -i\sqrt{0,3}$

The real parts of the eigenvalues are 0 therefor $(1,1)$ is non-hyperbolic

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b) $\frac{dy}{dx} = \frac{-0,3y + 0,3xy}{x - xy}$

$\Leftrightarrow \frac{dy}{dx} = \frac{y(-0,3+0,3x)}{x(1-y)} \Leftrightarrow \frac{dy}{dx} = \frac{y}{1-y} \cdot \frac{-0,3+0,3x}{x}$

$\Leftrightarrow \frac{1-y}{y}\,dy = \frac{0,3x - 0,3}{x}\,dx \Leftrightarrow \int \frac{1-y}{y}\,dy = 0,3\int \frac{x-1}{x}\,dx$

$\Leftrightarrow \int \frac{1}{y} - 1\,dy = 0,3\int 1 - \frac{1}{x}\,dx \Leftrightarrow \ln y - y + C_1 = 0,3(x - \ln x) + C_2$

$\Rightarrow$ ~~A~~ first integral $H(x,y)$, $H:(0,\infty)\times(0,\infty)\to\mathbb{R}$

is $H(x,y) = y - \ln y + 0,3x - 0,3\ln x$

Now we check if it is a first integral

$\frac{\partial H}{\partial x}(x,y)\cdot f_1(x,y) + \frac{\partial H}{\partial y} f_2(x,y) = 0$

$\Leftrightarrow \left(0,3 - 0,3\frac{1}{x}\right)(x - xy) + \left(1 - \frac{1}{y}\right)(-0,3y + 0,3xy) = 0 \;|:0,3$

$\Leftrightarrow \left(1 - \frac{1}{x}\right)(x - xy) + \left(1 - \frac{1}{y}\right)(xy - y) = 0$

$\Leftrightarrow \cancel{x} - \cancel{1} - \cancel{xy} + y + \cancel{xy} - y - \cancel{x} + \cancel{1} = 0$ „A"

$\Rightarrow H(x,y)$ is a first integral

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c) $x = 0 \Rightarrow 0 = 0$

$\dot{y} = -0,3y$

y	$-\infty$	0	∞
$\dot{y}$	$+$	0	$-$

$y = 0 \Rightarrow \dot{x} = x$

$0 = 0$

x	$-\infty$	0	∞
$\dot{x}$	$-$	0	$+$