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Exam on Dynamical Systems, June 10, 2020

1. (1p=0.2+0.4+0.4) Find the linear homogeneous difference equation with constant coefficients of minimal order that has as solutions the following sequences.
(a) $3, \quad -6, \quad 12, \quad -24, \quad 48, \quad -96, \dots$.
(b) $3, \quad -3, \quad -3, \quad 3, \quad 3, \quad -3, \dots$ and $0, \quad 1, \quad 0, \quad -1, \quad 0, \quad 1, \dots$.
(c) $5(1/2)^k - 7(1/3)^k$, $k \in \mathbb{Z}$.

2. (1p) We consider the IVP $x^2y'' + xy' + x^2y = 0, \quad y(0) = 1, \quad y'(0) = 0$ (the unknown is denoted by $y(x)$). Writing the solution as a power series $y(x) = \sum_{n=0}^{\infty} a_n x^n$, find the coefficients $a_0, a_1, a_2, a_3, a_4 \in \mathbb{R}$.

3. (2p=0.75+0.25+0.5+0.5) We consider the scalar dynamical system $\dot{x} = x^2 - x^4$. Represent the phase portrait. List the orbits. Specify the properties of $\varphi(t, 0)$ and $\varphi(t, 3)$. If there is an attractor, find its basin of attraction.

4. (a) (1.5p=0.5+0.5+0.5) Specify the type and stability of the linear system $\dot{x} = 2x$, $\dot{y} = -3y$. Find a global first integral $H : \mathbb{R}^2 \to \mathbb{R}$ of this system. Check that H is indeed a global first integral using the definition.

5. (a) (0.5p) Represent in the complex plane the curve
$\{(1 + i)e^{-t-3it} \ : \ t \geq 0\}$.
(b) (2p) Find the solution of the IVP $\dot{x} = -x + 3y, \quad \dot{y} = -3x - y$, $x(0) = y(0) = 1$. Represent in the phase plane the corresponding orbit.
(c) (0.5p) Specify the type and stability of the linear system $\dot{x} = -x + 3y$, $\dot{y} = -3x - y$. If this system has a global first integral, find it.
(d) (0.5p) Find $\lim_{t\to\infty} e^{At}$, where A is the matrix of the system from (c).