8. $X, Y \in N(0,1)$

$\Rightarrow$ pdf. of X is $f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$

pdf. of Y is $f_Y(y) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}y^2}$

$$P((X,Y) \in D_R) = \iint_{D_R} f_{(X,Y)}(x,y)\, dy\, dx$$

$$\underset{\text{indep.}}{\overset{X,Y}{=}} \iint_{D_R} f_X(x) \cdot f_Y(y)\, dy\, dx$$

$$= \iint_{D_R} \frac{1}{2\pi} \cdot e^{-\frac{1}{2}x^2} e^{-\frac{1}{2}y^2}\, dy\, dx$$

$$= \frac{1}{2\pi} \iint_{D_R} e^{-\frac{1}{2}(x^2+y^2)}\, dy\, dx$$

We will use a change of variable:

$(x,y) \to T(r,\theta) = (r\cos\theta, r\sin\theta)$, r - radius, θ - angle

from cartesian to polar.

For this we need the determinant of the Jacobian matrix of T

$$J_T(r,\theta) = \begin{pmatrix} \frac{\partial x}{\partial r}(r,\theta) & \frac{\partial x}{\partial \theta}(r,\theta) \\ \frac{\partial y}{\partial r}(r,\theta) & \frac{\partial y}{\partial \theta}(r,\theta) \end{pmatrix}$$

where ~~$x = r\cos\theta$~~ ~~$x(\theta$~~

$$x(r,\theta) = r\cos\theta$$

$$y(r,\theta) = r\sin\theta$$

$$\Rightarrow J_T(r,\theta) = \begin{pmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{pmatrix}$$

$$\Rightarrow \det J_T(r,\theta) = r\cos^2\theta + r\sin^2\theta$$

$$= r$$

D_R in polar coordinates is $(r,\theta) \in [0,R] \times [0, 2\pi]$

$$\Rightarrow P((x,y) \in D_R) = \frac{1}{2\pi} \int_0^R \int_0^{2\pi} e^{-\frac{1}{2}(r^2\cos^2\theta + r^2\sin^2\theta)} \cdot \det J_T(r,\theta) \cdot d\theta dr$$

$$= \frac{1}{2\pi} \int_0^R \int_0^{2\pi} e^{-\frac{1}{2}r^2} \cdot r \cdot d\theta \, dr$$

$$= \frac{1}{2\pi} \int_0^R e^{-\frac{1}{2}r^2} \cdot r \cdot \theta \Big|_{\theta=0}^{\theta=2\pi} dr$$

$$= \frac{1}{2\pi} \int_0^R 2\pi \cdot e^{-\frac{1}{2}r^2} \cdot r \, dr$$

$$= \int_0^R e^{-\frac{1}{2}r^2} \cdot r \, dr$$

$$= \int_0^R \left(-e^{-\frac{1}{2}r^2}\right)' dr$$

$$= -e^{-\frac{1}{2}r^2}\Big|_0^R$$

$$= -e^{-\frac{1}{2}R^2} + e^0$$

$$= 1 - e^{-\frac{1}{2}R^2}$$

$$P((X,Y) \in D_R) = 0{,}3$$

$$\Leftrightarrow 1 - e^{-\frac{1}{2}R^2} = 0{,}3$$

$$\Leftrightarrow e^{-\frac{1}{2}R^2} = 0{,}7$$

$$\Leftrightarrow e^{\frac{1}{2}R^2} = \frac{10}{7} \Rightarrow \frac{1}{2}R^2 = \ln\frac{10}{7}$$

$$\Rightarrow R^2 = 2\ln\frac{10}{7}$$

$$\Rightarrow R = \sqrt{2\ln\frac{10}{7}}, \text{ because } R \geq 0$$

$$\Rightarrow R = 0{,}8446$$