9. The total number of cases in which we can place n letters in n envelopes is $n!$.

For the favourable cases we can fix the k'th letter in the k'th envelope. Therefore there are left $n-1$ letters to be placed in $n-1$ envelopes. $\Rightarrow n_f = (n-1)!$

$$\Rightarrow P(X_k = 1) = \frac{n_f}{n_t} = \frac{(n-1)!}{n!} = \frac{1}{n}$$

$$\Rightarrow X_k \sim \begin{pmatrix} 0 & 1 \\ \frac{n-1}{n} & \frac{1}{n} \end{pmatrix}$$

a) $$E(X_k) = 0 \cdot \frac{n-1}{n} + 1 \cdot \frac{1}{n} = \frac{1}{n}$$

$$X_k^2 \sim \begin{pmatrix} 0 & 1 \\ \frac{n-1}{n} & \frac{1}{n} \end{pmatrix} \Rightarrow E(X_k^2) = \frac{1}{n}$$

$$V(X_k) = E(X_k^2) - [E(X_k)]^2$$

$$= \frac{1}{n} - \frac{1}{n^2} = \frac{n-1}{n^2}$$

b) Because S_n is the number of correct mailings we can see it as the sum of all X_k, $k = \overline{1,n}$

$$\Rightarrow E(Z_n) = E\left(\sum_{i=1}^{n} X_i\right) = \sum_{i=1}^{n} E(X_i)$$

$$= \sum_{i=1}^{n} \frac{1}{n} = 1$$

$$V(Z_n) = V\left(\sum_{i=1}^{n} X_i\right) = \sum_{i=1}^{n} V(X_i) = \sum_{i=1}^{n} \frac{n-1}{n^2}$$

$= n \cdot \frac{n-1}{n^2} = \frac{n-1}{n}$, because placing letters in envelopes are independent one from another.

c) The expected number of correct mailings is the expected value of Z_n. Therefore only 1 correct mailing is expected.