8. X, Y binomial distributions $\Rightarrow$

$$X\begin{pmatrix} k \\ C_m^k p^k (1-p)^{m-k} \end{pmatrix}_{k=\overline{0,m}} \quad Y\begin{pmatrix} k \\ C_n^k p^k (1-p)^{n-k} \end{pmatrix}_{k=\overline{0,n}}$$

X, Y independent $(*) \Rightarrow$

$$P(X,Y) = P(X) \cdot P(Y)$$

$$X+Y\begin{pmatrix} x_i + y_j \\ p_{ij} \end{pmatrix}_{i,j \in I \times J}$$

where $I = \overline{0,m}$; $J = \overline{0,n}$; $\cancel{p_{ij} = P(X = x_i, Y = y_j)}$

$$p_{ij} = P(X = x_i, Y = y_j)$$

$$P(X+Y=k) = P((X=0, Y=k) \cup (X=1, Y=k-1) \cup \text{— — — —} \cup (X=k, Y=0))$$

$$\overset{(*)}{\underset{m.e}{=}} P(X=0) \cdot P(Y=k) + P(X=1) \cdot P(Y=k-1) + \text{— — —} + P(X=k) \cdot P(Y=0)$$

$$= \sum_{i=0}^{k} P(X=i) \cdot P(Y=k-i)$$

$$= \sum_{i=0}^{k} C_m^i p^i (1-p)^{m-i} \cdot C_n^{k-i} p^{k-i} (1-p)^{n-k+i}$$

$$= \sum_{i=0}^{k} C_m^i C_n^{k-i} p^k (1-p)^{m+n-k}$$

$$= p^k (1-p)^{m+n-k} \sum_{i=0}^{k} C_m^i C_n^{k-i} \quad (1)$$

The sum $\sum_{i=0}^{k} C_m^i C_n^{k-i}$ can be computed using the hypergeometric distribution.

For the following parameters ~~we have~~:

$m+n$ - total number of objects

m - marked objects

k - trials

we have the following hypergeometric p.d.f.

~~pdf~~ $\begin{pmatrix} i \\ \dfrac{C_m^i C_n^{k-i}}{C_{m+n}^k} \end{pmatrix} \quad i = \overline{0,k}$

Also we know that the sum of ~~each~~ all probabilities in the p.d.f. is 1 $\Rightarrow$

$$1 = \sum_{i=0}^{k} \frac{C_m^i C_n^{k-i}}{C_{m+n}^k} \Leftrightarrow 1 = \frac{1}{C_{m+n}^k} \sum_{i=0}^{k} C_m^i C_n^{k-i}$$

$$\Rightarrow C_{m+n}^k = \sum_{i=0}^{k} C_m^i C_n^{k-i} \quad (2)$$

(1) $\Rightarrow P(X+Y=k) = C_{m+n}^{k} p^{k} (1-p)^{m+n-k}$

(2)

$$\Rightarrow X+Y \begin{pmatrix} k \\ C_{m+n}^{k} p^{k} (1-p)^{m+n-k} \end{pmatrix}_{k=0,\overline{m+n}}$$

p.d.f. of $X+Y$

$\Rightarrow$ $X+Y$ has the binomial distribution with $m+n$ trials, and probability p of succes at each trial.