8. Because we have N people we'll have $N!$ possibilities for hats distribution. $\Rightarrow N_t = N!$

Let A_i - the event in which the i'th person gets his hat.

~~Therefore we c~~

The favourable cases are the ones in which no person gets his hat. Therefore we can get this number by subtracting from the total number of cases, the number of cases in which at least one person gets his hat.

This means that we need to find the number of elements of $A_1 \cup A_2 \cup \ldots \cup A_N$.

We can do that with the inclusion - exclusion principle.

$$|A_1 \cup A_2 \cup \ldots \cup A_N| = \sum_{i=1}^{N} |A_i| - \sum_{\substack{\cancel{1 \le i < j \le N} \\ i,j}}^{N} |A_i \cap A_j| + \ldots + (-1)^{N-1} |A_1 \cap A_2 \cap \ldots \cap A_N| \quad (1)$$

$$= \sum_{k=1}^{N} (-1)^{k-1} \sum \cancel{A}$$

$|A_1| = |A_2| = \ldots = |A_N| = (n-1)!$ because if ~~the~~ one person gets his hat, $n-1$ hats remain to be picked up

We can generalise this to k persons

For k persons the ~~probabili~~ number of cases is $(n-k)!$ and we can choose the k persons in C_n^k ways (order does not matter) $\Rightarrow C_n^k(n-k)!$ is the k'th sum in (1)

$$\Rightarrow |A_1 \cup A_2 \cup \dots \cup A_n| = C_N^1(N-1)! - C_N^2(N-2)! + \dots + (-1)^{N-1} C_N^N (N-N)!$$

$$= \sum_{k=1}^{N} (-1)^{k-1} C_N^k (N-k)!$$

$$= \sum_{k=1}^{N} (-1)^{k-1} \frac{N!}{\cancel{(N-k)!}\,k!} \cdot \cancel{(N-k)!}$$

$$= N! \sum_{k=1}^{N} (-1)^{k-1} \cdot \frac{1}{k!}$$

$$\Rightarrow N_f = N! - N! \sum_{k=1}^{N} (-1)^{k-1} \cdot \frac{1}{k!}$$

$$= N!\left(1 - \sum_{k=1}^{N} (-1)^{k-1} \frac{1}{k!}\right)$$

$$= N!\left(1 + \sum_{k=1}^{N} (-1)^{k} \frac{1}{k!}\right)$$

$$= N!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \dots + (-1)^N \frac{1}{N!}\right)$$

$$\Rightarrow P_N = \frac{N_f}{N_t} = \frac{N!\left(1 - \frac{1}{1!} + \frac{1}{2!} + \dots + (-1)^N \frac{1}{N!}\right)}{N!} = 1 - \frac{1}{1!} + \frac{1}{2!} - \dots + (-1)^N \frac{1}{N!}$$

where P_N is the probability that for N people nobody gets his hat

Now we need to find

$$\lim_{n \to \infty} P_n = \lim_{n \to \infty} 1 - \frac{1}{1!} + \frac{1}{2!} - \dots + (-1)^n \frac{1}{n!}$$

$$= \lim_{n \to \infty} \sum_{k=0}^{n} \frac{(-1)^k}{k!}$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k}{k!}$$

$$= e^{-1}$$

$$= \frac{1}{e}$$

$\Rightarrow$ The probability tends to $\frac{1}{e}$