Mathematical Formulation of Constraints for Dienstplanung

Let:

- $x_{e,d,s} \in \{0,1\}$: binary variable, 1 if employee e is assigned to shift s on day d, 0 otherwise
- h_e: total minutes worked by employee e
- duration_s: duration (in minutes) of shift s

1. Mindestbesetzung (Minimum Staffing)

For each day d and shift s:

$$\sum_{e \in \text{Fachkräfte}} x_{e,d,s} \geq R^F_{d,s}$$

$$\sum_{e \in \text{Hilfskräfte}} x_{e,d,s} \geq R^H_{d,s}$$

$$\sum_{e \in \text{Azubis}} x_{e,d,s} \geq R^A_{d,s}$$

2. Fixed Absences

If e is on leave on day d:

$$\sum_{s} x_{e,d,s} = 0$$

3. No Night Shift Before Free Day

If e has a free day on d:

$$x_{e,d-1,N2} = 0$$

4. Monthly Working Time Deviation

$$|h_e - \text{soll}_e| \leq 460.2$$

$$h_e = \sum_{d,s} x_{e,d,s} \cdot \text{duration}_s$$

5. Specific Employee Constraints

- Branz: $x_{\text{Branz,Tue},Z60} = 1$, $x_{\text{Branz,Thu},Z60} = 1$
- $x_{\text{Branz},d,N2} = 0$ for all d
- $x_{e,d,s} = 0$ for $s \neq N2$ and $e \in \{\text{Farniok, Labelle, Wunderlich}\}$
- $x_{e,d,N2} = 0$ for $e \in \{\text{Shoemake, Merriweather, Roberson}\}$
- $x_{e,d,s} = 0$ for $s \neq F2$ on weekdays, $e \in \{\text{Rashid, Hoots}\}$
- $\sum_s x_{\text{Merriweather},d,s} = 0$ for $d = \text{Thu, Fri}$
- $\sum_{e \in \{\text{Branz, Hoots, Vanceet}\}} x_{e,d,F2} \geq 1$ for weekdays d
- $x_{\text{Wunderlich},d,N2} = 0$ if $d \notin \{2-5, 8-15, 20-22\}$

6. No Assignment on Leave Days

$$\sum_s x_{e,d,s} = 0$$

7. One Shift per Day

$$\sum_s x_{e,d,s} \leq 1 \quad \forall e, d$$

8. Arbeitszeitgesetz Constraints

- Max 600 min per day: $\sum_s x_{e,d,s} \cdot \text{duration}_s \leq 600$
- 11 hours (660 min) rest between shifts:

$$\text{if } x_{e,d_1,s_1} = 1 \text{ and } x_{e,d_2,s_2} = 1 \Rightarrow \text{start}_{d_2,s_2} - \text{end}_{d_1,s_1} \geq 660$$

- Weekly average working time not exceeding 2880 min

9. Max Consecutive Night Shifts

Let $n_{e,d}$ be 1 if employee e is assigned a night shift on day d, 0 otherwise.

To prevent more than 3 consecutive night shifts:

$$\sum_{i=0}^{3} n_{e,d+i} \leq 3 \quad \forall e, d$$

10. Free Days Near Weekend

Let $f_{e,d}$ be 1 if e is free on day d.

If d is a Friday (weekday 5), then encourage $f_{e,d+1} = 1$ or $f_{e,d+2} = 1$:

$$f_{e,d+1} + f_{e,d+2} \geq 1 \quad \text{if } d \mod 7 = 5$$

11. Shift Rotation Forward (Früh → Spät → Nacht)

Let $s_{e,d}$ be the shift index (e.g., Früh=0, Spät=1, Nacht=2).

To enforce forward rotation:

$$s_{e,d+1} \geq s_{e,d} - r_{e,d} \quad \forall e, d$$

Where $r_{e,d}$ is a binary slack variable penalized in the objective.

12. Replacement Days

If a shift on a given day is removed from e, a replacement must exist in a nearby day (e.g., ± 2 days):

$$\sum_{s} x_{e,d-2,s} + x_{e,d-1,s} + x_{e,d+1,s} + x_{e,d+2,s} \geq 1 \quad \text{if } \sum_{s} x_{e,d,s} = 0$$

13. More Free Days After Many Night Shifts

Track night shifts:

$$\sum_{i=0}^{k} n_{e,d-i} > k \Rightarrow f_{e,d+1} = 1$$

For example, if 4 night shifts in a row, enforce a free day:

$$\sum_{i=0}^{3} n_{e,d-i} \geq 4 \Rightarrow \sum_{s} x_{e,d+1,s} = 0$$

14. Not Too Many Consecutive Shifts

Let $w_{e,d} = 1$ if e works any shift on day d.

Limit to 5 consecutive working days:

$$\sum_{i=0}^{5} w_{e,d+i} \leq 5 \quad \forall e, d$$

15. Wishes as Soft Constraints

Let $w_{e,d}$ be 1 if wish day is granted, and $w_{e,d,s}$ be 1 if wished shift is granted.

Encourage satisfying wishes via objective:

$$\text{maximize } \sum_{e,d} w_{e,d} + \sum_{e,d,s} w_{e,d,s}$$

Subject to:

$$x_{e,d,s} \geq w_{e,d,s} \quad \text{(if wished for shift } s\text{)}$$

$$\sum_{s} x_{e,d,s} = 0 \Rightarrow w_{e,d} = 1 \quad \text{(if wished for day off)}$$