Mathematical Formulation of Constraints for Dienstplanung

Let:

- $x_{e,d,s} \in \{0,1\}$: binary variable, 1 if employee e is assigned to shift s on day d, 0 otherwise
- h_e: total minutes worked by employee e
- duration_s: duration (in minutes) of shift s

1. Mindestbesetzung (Minimum Staffing)

For each day d and shift s:

$$\sum_{e \in \text{Fachkräfte}} x_{e,d,s} \geq R^F_{d,s}$$

$$\sum_{e \in \text{Hilfskräfte}} x_{e,d,s} \geq R^H_{d,s}$$

$$\sum_{e \in \text{Azubis}} x_{e,d,s} \geq R^A_{d,s}$$

2. Fixed Absences

If e is on leave on day d:

$$\sum_{s} x_{e,d,s} = 0$$

3. No Night Shift Before Free Day

If e has a free day on d:

$$x_{e,d-1,N2} = 0$$

4. Monthly Working Time Deviation

$$|h_e - \text{soll}_e| \leq 460.2$$

$$h_e = \sum_{d,s} x_{e,d,s} \cdot \text{duration}_s$$

5. Specific Employee Constraints

- Branz: $x_{\text{Branz,Tue},Z60} = 1$, $x_{\text{Branz,Thu},Z60} = 1$
- $x_{\text{Branz},d,N2} = 0$ for all d
- $x_{e,d,s} = 0$ for $s \neq N2$ and $e \in \{\text{Farniok, Labelle, Wunderlich}\}$
- $x_{e,d,N2} = 0$ for $e \in \{\text{Shoemake, Merriweather, Roberson}\}$
- $x_{e,d,s} = 0$ for $s \neq F2$ on weekdays, $e \in \{\text{Rashid, Hoots}\}$
- $\sum_s x_{\text{Merriweather},d,s} = 0$ for $d = \text{Thu, Fri}$
- $\sum_{e \in \{\text{Branz, Hoots, Vanceet}\}} x_{e,d,F2} \geq 1$ for weekdays d
- $x_{\text{Wunderlich},d,N2} = 0$ if $d \notin \{2-5, 8-15, 20-22\}$

6. No Assignment on Leave Days

$$\sum_s x_{e,d,s} = 0$$

7. One Shift per Day

$$\sum_s x_{e,d,s} \leq 1 \quad \forall e, d$$

8. Arbeitszeitgesetz Constraints

- Max 600 min per day: $\sum_s x_{e,d,s} \cdot \text{duration}_s \leq 600$
- 11 hours (660 min) rest between shifts:

$$\text{if } x_{e,d_1,s_1} = 1 \text{ and } x_{e,d_2,s_2} = 1 \Rightarrow \text{start}_{d_2,s_2} - \text{end}_{d_1,s_1} \geq 660$$

- Weekly average working time not exceeding 2880 min