Combinatorics

Assignment 1

September 5, 2017

Reading list

- The slides
- Section 6.1
- Section 6.2

Note:

- Try to answer the questions without using binomial coefficients. These coefficients will be studied next week.
- You can hand in your solutions as a single PDF via the assignment module in Blackboard. Note that the document should be typeset with LaTeX, Word or a similar program. It should not be a scan or picture of your handwritten notes.
- Make sure that your name, student number and group number are on top of the first page!
- **Note that your submission should be an individual submission because it can influence your final grade for this course. If we detect that your work is not completely your own work, we will ask the exam committee to investigate whether it is plagiarism or not!**

Exercises to be presented during the exercise hours

Exercise 1
How many positive integers between 50 and 100

a) are divisible by 7? Which integers are these?

b) are divisible by 11? Which integers are these?

c) are divisible by both 7 and 11? Which integers are these?

Exercise 2
How many functions are there from the set $\{1, 2, \ldots, n\}$, where n is a positive integer, to the set $\{0, 1\}$

a) that are one-to-one?

b) that assign 0 to both 1 and n?

Exercise 3
In how many ways can a photographer at a wedding arrange six people in a row, including the bride and groom, if

a) the bride must be next to the groom?

b) the bride is not next to the groom?

Exercise 4
A bowl contains 10 red balls and 10 blue balls. A woman selects balls at random without looking at them. We assume that the woman does not replace the balls after drawing them.

a) How many balls must she select to be sure of having at least three balls of the same color?

b) How many balls must she select to be sure of having at least three blue balls?

Exercise 5
How many ordered pairs of integers (a, b) are needed to guarantee that there are two ordered pairs (a_1, b_1) and (a_2, b_2) such that $a_1 \mathbf{mod}\, 5 = a_2 \mathbf{mod}\, 5$ and $b_1 \mathbf{mod}\, 5 = b_2 \mathbf{mod}\, 5$?

Exercise 6
Suppose that there are nine students in a discrete mathematics class at a small college.

a) Show that the class must have at least five male students or at least five female students.

b) Show that the class must have at least three male students or at least seven female students.

Exercise 7
During the lecture a student explained that the number of passwords of length 6 where each symbol is either an uppercase letter or a digit with the extra requirement that a password should at least contain one digit can be computed by the formula: $P_6' = 6 \cdot 36^5 \cdot 10$. This should be a valid formula because of this algorithm:

Task 1 Choose 5 times out of 36 symbols. This can be done in 36^5 ways.

Task 2 Choose 1 time out of 10 digits. This can be done in 10 ways.

Task 3 Choose the position for this obligatory digit. This can be done in 6 ways.

The formula in the book says $P_6 = 36^6 - 26^6$. Computations showed that the formula for P_6' gives a wrong value.

Explain precisely why the method of the student is indeed wrong. [*Hint:* It may be practical to reduce the size of the alphabets and the length of the passwords to see what is going on.]

Exercise 8

You are **not** allowed to use binomial coefficients in this exercise.

a) Dutch licence plates are organized by the institute RDW. They use several series for license plates and several additional requirements. Currently cars get license plates from series 9, which is of the form $XX - NNN - X$, where X can be a letter and N a digit. How many license plates can be used within this series if we take the following requirements into account?

- No vowels are being used.
- The first letter should be G, H, J, K, L, N, P, R, S, T, X or Z.
- The letters C and Q are never used because of their similarity to the digit 0.

Explain your answer.

b) In how many ways can a photographer at a wedding arrange 8 people in a row, including the bride and groom, if the bride is positioned somewhere to the left of the groom and the groom has two neighbors? Explain your answer.

c) There are twelve signs of the zodiac. Use the generalized pigeonhole principle to compute how many people are needed to guarantee that at least sixteen of these people have the same sign?

Exercises to be handed in

You are expected to explain your answers, even if this is not explicitly stated in the exercises themselves.

Exercise 9 6 pt

Do only a, d and g.

How many positive integers between 100 and 999 inclusive

a) are divisible by 7?

b) are odd?

c) have the same three decimal digits?

d) are not divisible by 4?

e) are divisible by 3 or 4?

f) are not divisible by either 3 or 4?

g) divisible by 3 but not by 4?

h) divisible by 3 and 4?

Exercise 10 2 pt

How many functions are there from the set $\{1, 2, \ldots, n\}$, where n is a positive integer, to the set $\{0, 1\}$ that assign 1 to exactly one of the positive integers less than n?

Exercise 11 6 pt

In how many ways can a photographer at a wedding arrange 6 people in a row from a group of 10 people, where the bride and the groom are among these 10 people, if

a) the bride must be in the picture?

b) both the bride and groom must be in the picture?

c) exactly one of the bride and the groom is in the picture?

Exercise 12 3 pt

Show that in a group of five people (where any two people are either friends or enemies), there are not necessarily three mutual friends or three mutual enemies.

Exercise 13

In the slides an example is given of a value n that consists of first 54 7's followed by 25 0's, such that $n \equiv 0 \pmod{19}$. But is there a smaller example of such a value n that consists only of 7's and 0's and $n \equiv 0 \pmod{19}$?

a) Prove that such an example indeed exists, without actually computing such a value n, but by only using the definitions. 1 pt

b) Prove that such an example indeed exists, by actually computing such a value n. 2 pt

Your final grade is the sum of your scores divided by 2.0.