Combinatorics

Assignment 8

October 24, 2017

Reading list

- The slides
- Section 10.3, except
  - Representing Graphs
  - Adjacency Matrices
- Section 10.4
- Section 10.5
- Section 10.7
- Section 10.8

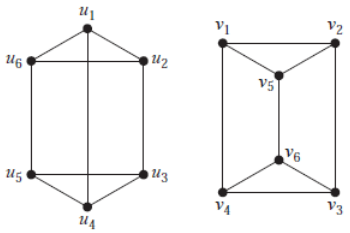
Note:

- Because the list of the exercises is quite long this week, it will not be possible to present all exercises. Therefore the focus will be on the exercises that other students have questions about.
- In particular there are so many topics discussed this week, that it is not possible to cover them all in the set of exercises that can be handed in. Therefore, if you only look at the exercises to be handed in, you will definitely skip some topics that are likely to be asked at the exam.
- If it is too difficult to draw graphs directly in LaTeX, please don't waste your time on that, but simply include a clear picture of a graph drawn by hand.
- Note that your homework will be printed in black and white, so do not rely on colors to distinguish between nodes, but use different shapes.

- You can hand in your solutions as a single PDF via the assignment module in Blackboard. Note that the document should be typeset with LaTeX, Word or a similar program. It should not be a scan or picture of your handwritten notes.
- Make sure that your name, student number and group number are on top of the first page!
- **Note that your submission should be an individual submission because it can influence your final grade for this course. If we detect that your work is not completely your own work, we will ask the exam committee to investigate whether it is plagiarism or not!**

Exercises to be presented during the exercise hours

Exercise 1
Determine whether the given pair of graphs is isomorphic. Exhibit an isomorphism or provide a rigorous argument that none exists.

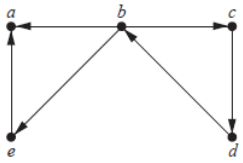


Exercise 2
Show that isomorphism of simple graphs is an equivalence relation.

Exercise 3
Determine Engelbert's Erdös number and give a path in the collaboration graph of mathematicians. Do not only give the name of the mathematicians, but also the title of the articles that prove the collaboration.

Exercise 4
Determine whether each of these graphs is strongly connected and if not, whether it is weakly connected.

a)



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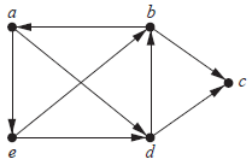

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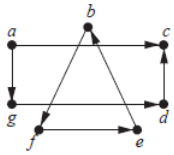
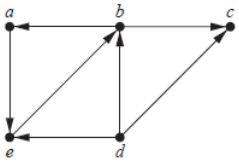


Exercise 5
Find the strongly connected components of each of these graphs.

a)



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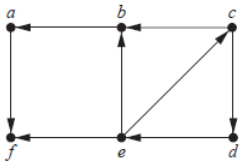

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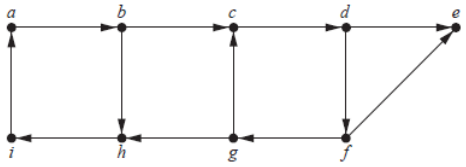


Exercise 6
How many nonisomorphic connected simple graphs are there with n vertices when n is

a) 2?

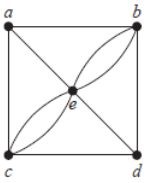
b) 3?

c) 4?

d) 5?

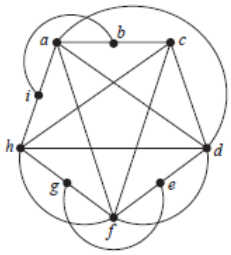
Exercise 7
Suppose that you have a three-gallon jug and a five-gallon jug. You may fill either jug with water, you may empty either jug, and you may transfer water from either jug into the other jug. Use a path in a directed graph to show that you can end up with a jug containing exactly one gallon. [*Hint:* Use an ordered pair (a, b) to indicate how much water is in each jug. Represent these ordered pairs by vertices. Add an edge for each allowable operation with the jugs.]

Exercise 8
Determine whether the given graph has an Euler circuit. Construct such a circuit if one exists. If no Euler circuit exists, determine whether the graph has an Euler path and construct such a path if one exists.



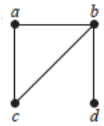
Exercise 9
Suppose that a connected planar graph has eight vertices, each of degree three. Into how many regions is the plane divided by a planar representation of this graph?

Exercise 10
Use Kuratowski's theorem to determine whether the given graph is planar.



Exercise 11
Decide whether it is possible to decrease the chromatic number by removing a single vertex and all edges incident with it.

a)



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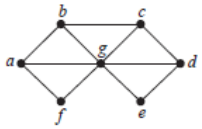

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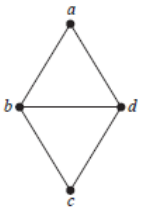
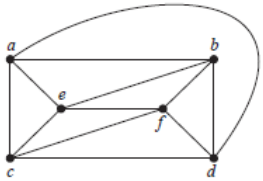


d)



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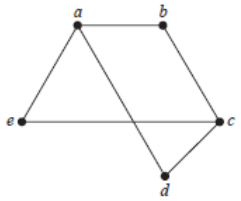
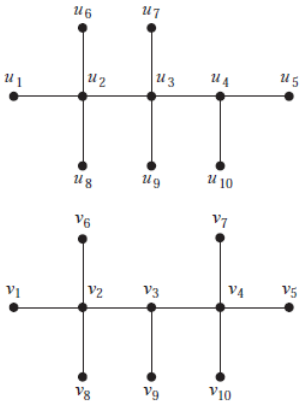


Exercise 12
A zoo wants to set up natural habitats in which to exhibit its animals. Unfortunately, some animals will eat some of the others when given the opportunity. How can a graph model and a coloring be used to determine the number of different habitats needed and the placement of the animals in these habitats?

Exercises to be handed in

> You are expected to explain your answers, even if this is not explicitly stated in the exercises themselves.

Exercise 13
Determine whether the given pair of graphs is isomorphic. Exhibit an isomorphism or provide a rigorous argument that none exists. 2 pt



Exercise 14

Do only d.

Find the number of paths of length n between two different vertices in K_4 if n is

2 pt

a) 2.

b) 3.

c) 4.

d) 5.

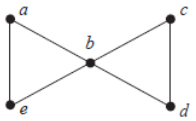
Exercise 15

Do only c and d.

For each of these graphs, find $\kappa(G)$, $\lambda(G)$, and $\min_{v \in V} \deg(v)$, and determine which of the two inequalities in $\kappa(G) \leq \lambda(G) \leq \min_{v \in V} \deg(v)$ are strict.

6 pt

a)



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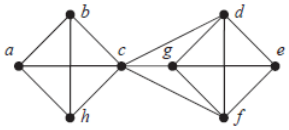

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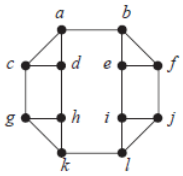

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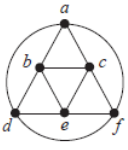
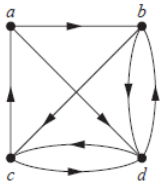


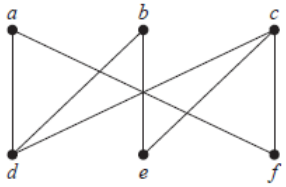
Exercise 16
Determine whether the directed graph shown has an Euler circuit. Construct an Euler circuit if one exists. If no Euler circuit exists, determine whether the directed graph has an Euler path. Construct an Euler path if one exists. [3 pt]



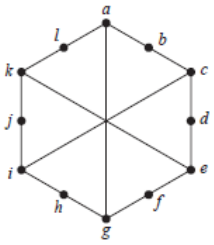
[*Hint:* You may use the results of exercises 10.5.10 and 10.5.11 in the book.]

Exercise 17
Can you find a simple graph with n vertices with $n \geq 4$ that does not have a Hamilton circuit, yet the degree of every vertex in the graph is at least $(n-1)/2$? [2 pt]

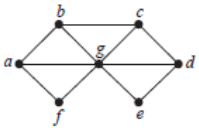
Exercise 18
Determine whether the given graph is planar. If so, draw it so that no edges cross. [2 pt]



Exercise 19
Determine whether the given graph is homeomorphic to $K_{3,3}$. [3 pt]



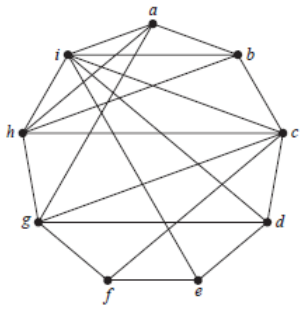
Exercise 20
Find the chromatic number of the given graph. [2 pt]



Exercise 21

Decide whether it is possible to decrease the chromatic number by removing a single vertex and all edges incident with it. [2 pt]

a)



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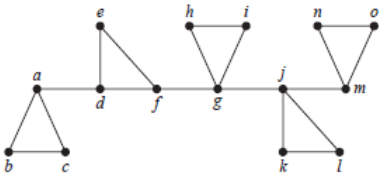
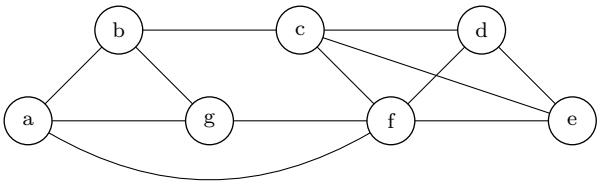


Exercise 22

This exercise combines several aspects of the theory of this week.

a) Draw a simple graph which is nonplanar and has exactly one cut edge. Make sure that your graph is minimal with respect to the number of vertices. Explain why your graph has one cut edge and prove that your graph is nonplanar. [3 pt]

b) Consider the following graph H: [3 pt]



Determine its chromatic number and prove that your value is correct without giving an explicit vertex coloring.

Your final grade is the sum of your scores divided by 3.0.