A stylized model for energy, population, the economy and the environment

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Abstract

In all integrated assessment models, demography is fixed exogenously by some scenario elements and therefore cannot vary in response to the effects of global warming. In order to bridge this gap, I construct and present a stylized integrated model that portrays endogenous fertility, energy consumption, economic production and the consequences of climate change. This model is built on three pillars: we use a two-sector central unified growth theory framework, whose production function features non-substituability between traditional factor of production and energy in the spirit of ecological economics. Emissions arising from this energy use result in global warming. Finally, this rise in temperature implies differentiated effects of climate change on economic sectors that have consequences on the whole economy and society. With numerical simulation, I assess the attractive performance of the model on 300 years of history and I deem that it provides prospective estimates of interesting quality. Furthermore this model appears quite promising as some improvements and extensions might be of prime interest in order to assess climate policy.

Keywords Computable General Equilibrium Models - Unified Growth Theory - Endogenous Growth - Two Sector Growth Model - Environment and Growth - Energy and the Macroeconomy - Climate Economics - Population Growth - Ecological Economics
JEL classification C68 - J11 - O40 - O41 - O44 - Q43 - Q54 - Q56 - Q57

Introduction

Most global warming scenarios ([IPCC, 2014a]) are the final result of numerous simulations among which some of the most important are the one coming from Integrated Assessment Models.

Following the definition of [IPCC, 2018], Integrated assessment is a "method of analysis that combines results and models from the physical, biological, economic and social sciences and the interactions among these components in a consistent framework to evaluate the status and the consequences of environmental change and the policy responses to it".

Most Integrated Assessment Models can be divided in two categories: Policy Optimization Models (POMs) such as DICE ([Nordhaus, 2017], [Nordhaus, 2013]) or FUND ([Tol, 2002]) and Policy Evaluation Models (PEMs). On the one hand, POMs aim at carrying out a cost-benefit analysis of climate change mitigation that compares the abatement costs with the potential impacts due to climate change and thus determine an optimal policy-trajectory. Most of the time, POMs are often stylized models that feature a damage function - meaning that these model attempt to take into account the damages resulting from the effect of global warming - and present a top-down approach with aggregated data. On the other hand, PEMs focus on the cost-effectiveness of a particular policy to achieve a certain mitigation target with a bottom-up point of view. They are often complex and for reasons of computation time do not have any impact function to assess the consequences of climate change on human society.

In any case, these models and their outputs heavily rely on there drivers. These are scenario elements that sketch a potential future. The most used in the scientific community are the SSPs scenarios[1][2] ([Riahi et al., 2017]. They are a framework of scenarios that aims at spanning for the potential outcomes of the world economy along two socio-economic axis: the challenge for mitigation, and the challenge for adaptation. In [O'Neill et al., 2014], they are defined as "reference pathways describing plausible alternative trends in the evolution of society and ecosystems over a century timescale, in the absence of climate change or climate policies" To each SSP are associated Shared climate Policy Assumptions. These are first quantified as population scenarios [KC and Lutz, 2017] (IIASA) and then as GDP scenarios [Dellink et al., 2017] (OECD), [Crespo Cuaresma, 2017] (IIASA), [Leimbach et al., 2017] (PIK).

Despite a quite comprehensive framework, simulation based on the SSPs cannot take into account all the effects of climate change on human societies. In particular, they ignore the feedbacks of climate change on the economy, and thus its demographic consequences. Furthermore, they cannot consider the effect of demographic deviation on the economy, since, population as well as GDP are the basis and cornerstone of these scenario: there is a potential huge methodological blindspot in the field of prospective modelling.

So, as to tackle this issue, this study is a quite preliminary work in order to build a stylized integrated model that features a complete climate-economics loop considering the long-run growth pattern of the economy and population its related and needed energy consumption, the subsequent greenhouse gases emissions, and finally, damages from climate change. In a first part, I shall present some stylized facts that I deem are especially important and should be to be taken into account in the modelling. As a complement to these guidelines, I present the different disciplinary fields that guide and structure our reflection and modelling. Then, I present, the model in itself and in a third part I focus on the results that are obtained with some numerical simulation.

[1]For Shared Socio-economic Pathways

[2]`https://tntcat.iiasa.ac.at/SspDb/dsd?Action=htmlpage&page=about`

This stylized model might then be adapted to study potential migrations or the effect of international trade on the environment or the demographic transition in developing countries.

1 Some stylized facts and theoretical grounds

First of all, it is appropriate to present the thread of our reflection, the one that led to the construction of our model (section 2). To do so, we will first look at the Kaya identity, which makes it possible to explain the importance of considering the four axes of economy, demography, energy and climate. Next, we will successfully present the three fields whose path and methods we take to develop and justify this model: unified growth theory, ecological economics and climate economics. Finally, we present a few works that are close to our own that allow us to justify the relevance of our approach and to frame it.

1.1 Kaya identity

Kaya identity ([Kaya and Yokoburi, 1997]) is a decomposition of the total level of emissions of CO_2 in relative terms allowing a more refined interpretation of the dynamics of emissions and notably to be able to attribute which components of the socio-economic system can explain an increase or a decrease in these emissions. Kaya identity takes the following form:

$$CO_{2,t} = \frac{CO_{2,t}}{E_t}\frac{E_t}{Y_t}\frac{Y_t}{Population_t}Population_t$$

Where $CO_{2,t}$ denotes the yearly CO_2 emissions, E_t the consumption of primary energy, Y_t the world GDP, i.e. the total production value and finally $Population_t$, the total population at time t. Each relative value present a different point of view on the socio-economic industrial system:

- $\frac{CO_{2,t}}{E_t}$ is the carbon intensity of the energy mix: it denotes the amount of CO_2 that is emitted for the use of one unit of primary energy. So depending of the structure of the energy mix, the carbon intensity may vary a lot, from a very high intensity - for example, an economy that relies only on really pollutant fossil fuels, like coal - to a low intensity - for example, an economy using electricity produced from dispatchable sources, like hydroelectricity.

- $\frac{E_t}{Y_t}$ is the energy intensity of the production engine, telling us how much energy is needed to produce one unit of value added. This term is at the heart of the debate on the policy approach to energy transition. Indeed, some consider that it is possible to decouple energy consumption and production ([Solow and Wan, 1976], [Stiglitz, 1980]) while others consider that economic growth and energy use are consubstantial ([Georgescu-roegen, 1971], [Ayres, 1981], [Kummel, 2013]), knowing that these authors do not really agree on what they consider to be the production of added value (which may, in a certain respect, explain their divergence).

- $\frac{Y_t}{Population_t}$ is the world GDP per capita.

In figure 1 we have represented the four factors underlined by the Kaya identity. What can be concluded from this first graph is that all the components are globally on the rise and that the level of emissions of CO_2 appears particularly collinear to the total primary energy

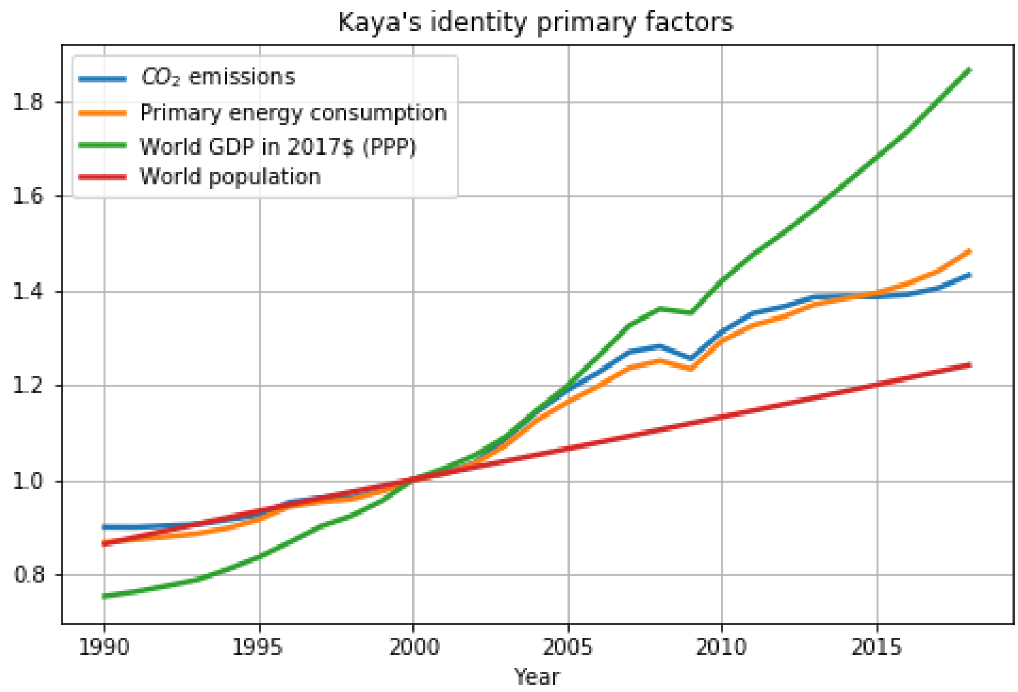

Figure 1: Kaya identity factors 1990-2018 - Sources: British Petroleum ([British Petroleum, 2019a]), World Bank

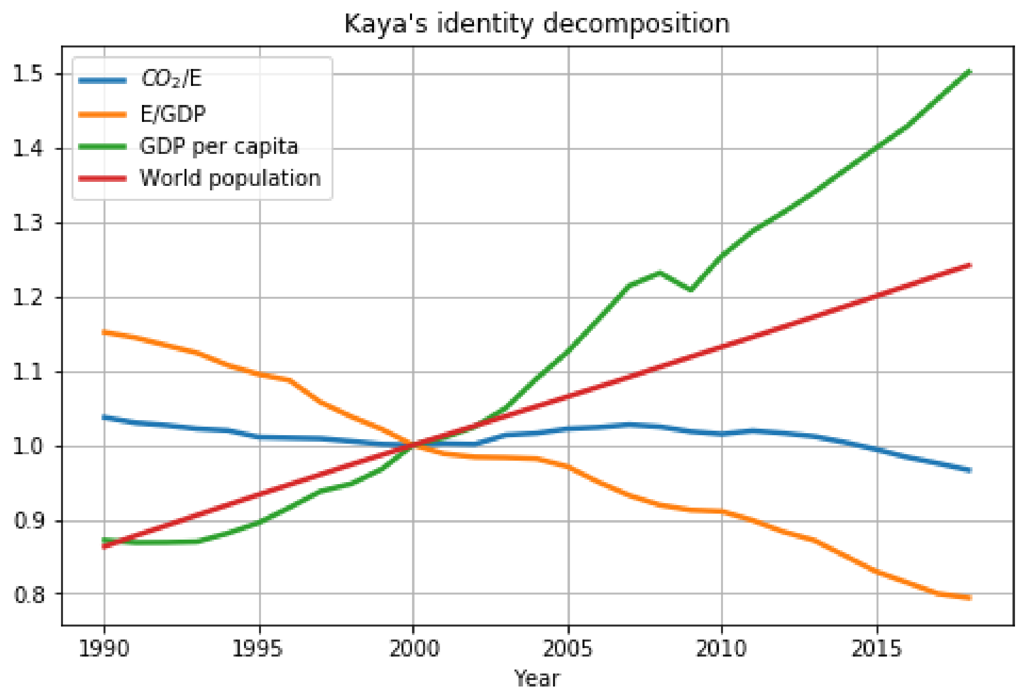

Figure 2: Kaya decomposition 1990-2018 - Sources: British Petroleum ([British Petroleum, 2019a]), World Bank

consumption. We do not learn that much... However, if we look at the decomposition of Kaya's identity, we can conclude to a much more subtle diagnosis. First of all, carbon intensity varies very little over time, which is consistent with the collinearity seen earlier: the productive engine maintains a globally stable energy mix in terms of emissions. Second, on the one hand, the energy intensity of economic production appears to be strictly decreasing: is decoupling possible? In spite of this, we observe, on the other hand, that the GDP per capita increases even faster than the energy intensity decreases, and this is aggravated by a growing population. Thus, finally, we can conclude that the slight decoupling we observe is not sufficient to offset the increase in global production over the period 1990-2015, which explains the rise in global emissions.

Thus, a naive idea would be to study each term independently to determine their future trends and make a final forecast. However, all these terms are intertwined with each other, the value of one of them at an instant t can have an influence on the other at an instant $t+1$: thus, the terms are clearly not independent of each other but moreover it is not obvious which one provokes the other: there is a problem of causality.

What Kaya's identity teaches us can be summed up as follows: the economic, demographic and energy aspects are not negligible when it comes to studying global warming due to greenhouse gas emissions. Moreover, these terms are not independent of each other, so they must be studied in an integrated and endogenous way, and as we want to develop a tool with a prospective meaning, we must take into account the feedback of climate on the socio-economic system.Thus, we propose the following approach: study the endogenous interaction between demography and the economy, which we put into perspective with the need for energy, such as fuel oil, to enable the transition from a Malthusian regime to an industrial regime driven by the accumulation of human capital.

1.2 The long-term interplay between population and economic growth

In his essay ([Malthus, 1826]), Malthus describes an English society in which the search for subsistence prevents any rise in the standard of living: any increase in wealth is absorbed by an increase in population, it is a homeostatic equilibrium. However, only a few years later, following the industrious and the consumer revolution ([de Vries, 1994], [de Vries, 2008]), the economy completely shifted and moved to a "modern growth" regime with the first industrial revolution ([Pomeranz, 2019]). This regime shift can be observed on the figure 3[3] around 1850 in Western countries and around 1950 in South Korea or China.

What is striking is the coordination of this transformation with another no less impressive movement, the demographic transition. In fact, the Malthusian regime is characterised by a high birth rate and a high mortality rate, which do not allow a sustainable increase in population, whereas the modern growth regime is characterised by a low birth rate and low mortality, and therefore a population that is again more or less stable. The transition between these two states, the so-called demographic transition, is characterised by two phases. Firstly, a fall in mortality while the birth rate remains high - leaving a positive natural increase - and secondly a fall in the birth rate with still low mortality. This phase of transition appears to be far too wonderfully synchronised with industrialisation in very many countries to think that this transition can take place independently of industrialisation and vice versa ([Galor, 2012]). This raises the question of the profound interaction between economic development and demography ([Barro and Becker, 1989], [Kremer, 1993]). This consideration leads to an integrated framework where there is no need to resort to a pre-established scenario (contrary, for example, to [Hansen and Prescott, 2002]), unified growth theory ([Galor, 2005]). This modelling

[3]Beware of the log scale !

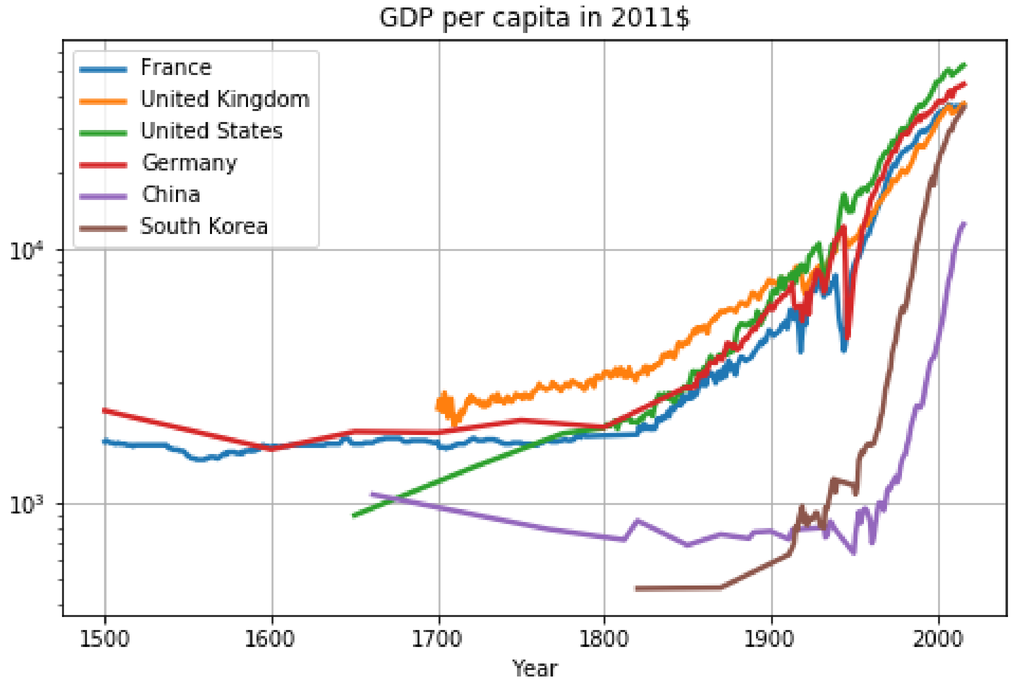

Figure 3: GDP per capita in different countries 1500-2016 - Source: Maddison Project Database 2018 [Bolt et al., 2018]

allows for a natural transition between the two growth regimes by stressing certain mechanisms, the main and most commonly used of which is the quality-quantity trade-off concerning offspring: as the need for education increases, a child "costs more to produce" and there are fewer births with a higher level of education, and therefore higher productivity for the next generation, a phenomenon that encourages economic development and so on... It is interesting to note the special status given to agriculture within the unified growth theory framework (as for example in [Strulik and Weisdorf, 2008]) which is consistent with work in economic history ([Federico, 2004], [Federico, 2015]). Futhermore, since it appears that the impacts of global warming will be particularly noticeable in the field of agriculture ([Burke et al., 2015], [Deschênes and Greenstone, 2012]), it seems judicious to distinguish between the agricultural and non-agricultural (manufacturing) sectors in order to study the economic-demographic interaction in a context of global warming ([Casey et al., 2019], [Burzynnskia et al., 2019]).

1.3 Energy as a natural bound on economic growth

Unified growth theory, is a brilliant work, quite elegant, simple and insightfull, but it clearly does not take into account the issue of energy. However, it appears that, just as population dynamics are profoundly linked to economic dynamics, the availability, extraction and consumption of energy is anything but a secondary element in modern economic growth ([Smil, 2018], [Kander, 2002]). Thus, by observing world energy consumption over the last two hundred years it is possible to notice that quantitatively speaking world primary energy consumption strongly correlates with world GDP (see figure 4). Moreover, qualitatively speaking, different growth regimes are not only associated to some amount of primary energy, but also to a specific energy mix (see figure 5), hence malthusian state relates to primary energy, the first industrial revolution to coal and the second to oil and gas.

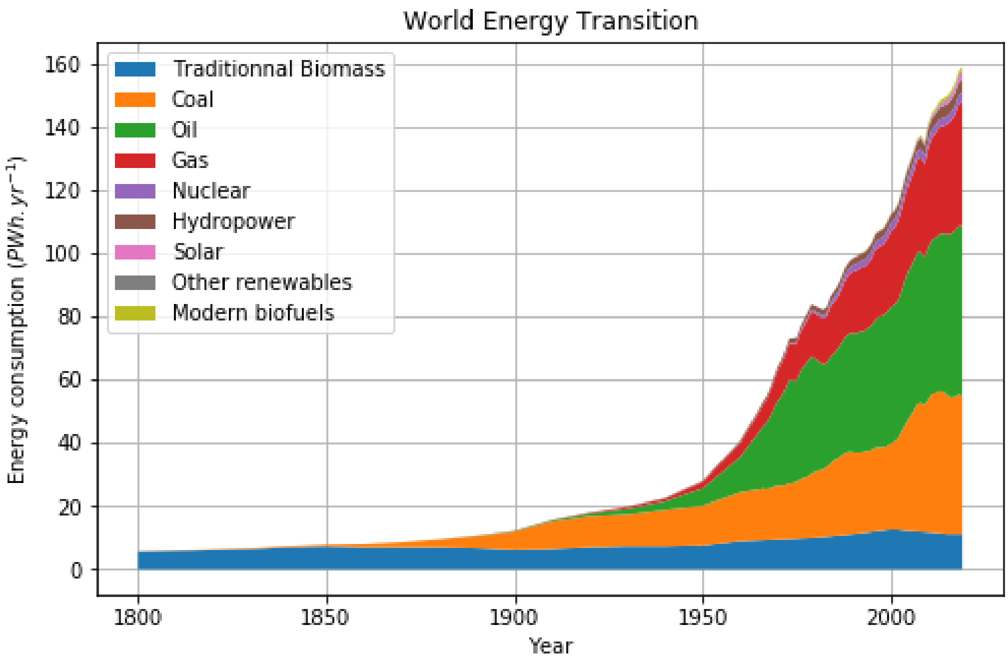

Figure 4: Total world primary energy consumption - Sources: [Smil, 2016], [British Petroleum, 2019a]

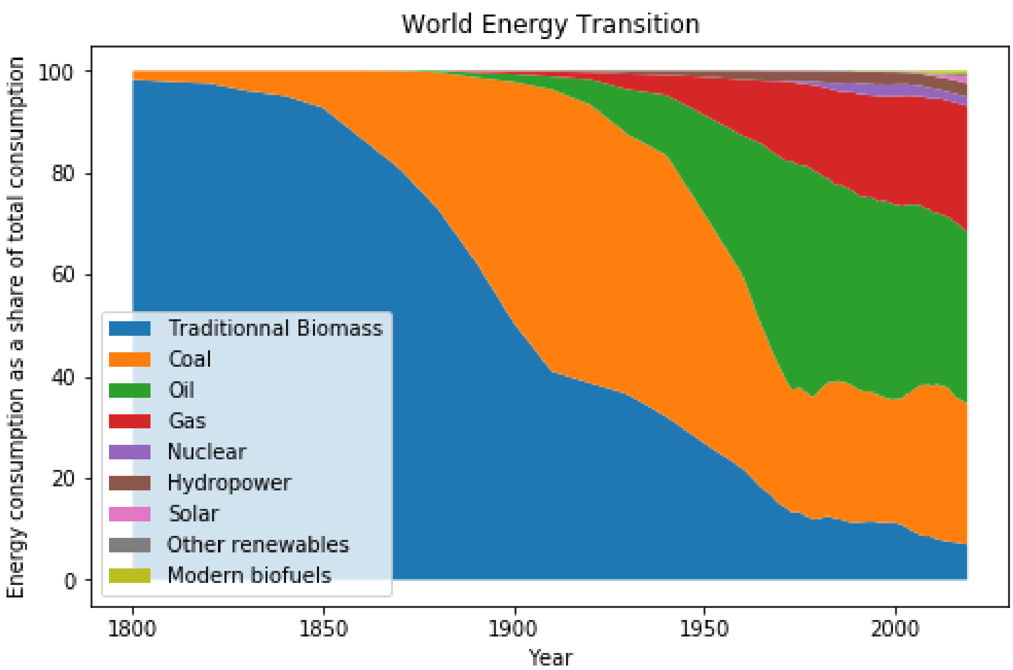

Figure 5: Shares of world primary energy consumption - Sources: [Smil, 2016], [British Petroleum, 2019a]

The question often considered fundamental to the study of growth is the search for the attribution of the primary and fundamental causes of growth. Thus, a model as simple as Solow-Swan model ([Solow, 1956], [Swan, 1956]) makes its contribution by showing the primary importance of capital, labour as well as technical progress. This model, although very simple, describes the mechanism of growth particularly well and does not need to consider energy as a fundamental cause, all the more so as the cost-share of energy is very low, if energy were so limiting, it should have a higher cost-share. However, the discussion on the place of energy in growth is discussed both in terms of the impact of prices ([Kilian, 2008]), and in terms of its elasticity with respect to other factors of production ([Berndt and Wood, 1979], [Griffin, 1981], [Berndt and Wood, 1981]). Faced with this logic, some scholars mobilise another conception of production and growth, centred on the link between mechanical labour and the production of value, which could be described as thermodynamics. From their point of view, since production requires mechanical labour, energy is not only consubstantial to growth, but also has an efficiency limited by its thermodynamic output ([Kummel, 2013], [Kümmel and Lindenberger, 2014]). [Georgescu-roegen, 1971] [Ayres, 1981], [Ayres et al., 2003], [Ayres and Warr, 2009], [Ayres et al., 2013]).

Thus, the question of the importance of energy and the energy transition over the last two centuries is both that of estimating the elasticity of substitution between capital, labour and energy to explain the trajectory of energy consumption over time, and that of estimating the elasticity between different fuels to explain the transition in the use of one energy resource to another.

To consider these two elements, we rely on nested-CES production functions ([Sato, 1967], see section 2.2.2 and 2.2.3) of type KL - E. Concerning the elasticity between capital, labour and energy, it appears to be less than one, so the aggregates KL and E are not substitutes but complements ([Stern, 2011], [Stern and Kander, 2012]). For interfuel, elasticities are above one ([Stern, 2012]).

Thus, it appears that energy, and in particular fossil fuels, if it is not what drives growth and progress (indeed, a resource is not acting as such), the availability of energy resources is necessary for our modern growth regime resulting from the industrial revolutions: energy constitutes a natural physical medium-term limit to growth. As our current regime is particularly dependent on fossil fuels (resulting from the transformation of organic residues under certain conditions of temperature and pressure), the short and medium term availability of fossil energy appears to be a first "carbon limit" of growth to be taken into account.

1.4 Prospective and the influence of climate

The second limit is its dual limit, which is global warming caused by greenhouse gas emissions from the combustion of the aforementioned fossil fuels. Thus, it is clear in the scientific community that global warming will be a major force for transformation of our environment, societies and economies in the 21^th century ([IPCC, 2014a], [IPCC, 2014b], [IPCC, 2014c]). Thus, this is an issue that needs to be taken into account and incorporated into the modelling. For this, we rely on the pioneering work of Nordhaus ([Nordhaus, 2017]) in particular his use of damage functions. In spite of the extremely strong criticism to which they can be subjected ([Pinduyk, 2015], section 4), they remain the only tools that can be used to complete a climate-economy model that would not be completely bottom-up. Thus, since we wish to distinguish between the agricultural and manufacturing sectors, we rely on [Desmet and Rossi-Hansberg, 2015] and [Burke et al., 2015] and we adopt the stylized climate model of [Nordhaus, 2017].

1.5 Current research venues and objectives of the paper

At present, there is no model that, to my knowledge, has all the features that we wish to incorporate into our model. However, there is a lot of work that has many similarities with what we are proposing. Concerning, the interaction between population and energy extraction, [Fröling, 2011] proposes a quality analysis in terms of the study of energy and the interaction between its use and the unified theory of growth. In the same way, [Bovari and Court, 2019] proposes an interpretation in terms of the unified theory of growth of industrial revolutions with particular emphasis on the question of technological progress and the development of knowledge. More focused on the characteristics of energy (with in-depth references to EROI - Energy Returned On Energy Invested and exergy[4]), [Court et al., 2019] also proposes an analysis linking population, economy and energy, but without endogenous demography. On the other hand, the interaction between population and the effect of climate change is also much studied. Indeed, [Desmet and Rossi-Hansberg, 2015] features a spatial economics model to assess potential climatic migration, his model considers CO_2 emissions and displays a stylized climatic model, but do not have endogenous demography. [Burzynnskia et al., 2019] shares the same objective (and add the issue of see level rise) but uses a random utility maximization unified growth theory framework allowing some heterogeneity with the behaviour of agents. However, it does not consider energy extraction and CO_2 emissions and thus resorts to use SSP scenarios as a climatic driver. In addition, [Casey et al., 2019] studies the effect of increasing temperature through a two sector-model but only with a static perspective. Finally, [Gerlagh et al., 2017] and [Gerlagh et al., 2018] study the dynamics of capital in a climate change framework with an overlapping generation model which highlights a very different calculation of the social cost of carbon and obtains very different estimates from state-of-the art model, such as [Golosov et al., 2014] and [Nordhaus, 2017].

So what is expected of the model we are going to present? First of all, it does not pretend to be a perfect model that would make a very exact addition of the models mentioned above. The latter were created to answer a specific research question (migration, cost of social carbon, etc.), so we are inspired by them, but we do not aim to propose a perfect model, which in any case does not exist and will never exist. In the next part, we present a model whose objective is to present in an endogenous way all the stylized facts that we have presented: economy-demography interaction, energy as a boundary for the production of goods, progessive successions of different energy sources, and climate feedback via a decrease in production. This model therefore appears to be a preliminary work aiming at being able, first of all, to assess the extent to which climate feedback causes a deviation from the scenarios used in the major modelling exercises (is it significant, or on the contrary very weak?), but also to carry out climate policy assessment exercises (What is the global effect of international trade on the climate? Is a policy aimed at reducing fertility interesting?) - the endogenous nature of a maximum number of model components would make it possible to capture a maximum number of direct and indirect consequences of the measure tested, in particular general equilibrium effects that could be transmitted from one period to another with unforeseen consequences -, and finally, in the same way, it could be used to carry out contrafactual retrospective analysis (What if this had happened? or this?). The existence of a capital market makes it possible to define a social cost of carbon that could be used as a metric for the evaluation of the policies under consideration.

[4]see [Ayres et al., 2003] for a defintion of exergy

2 A simple model

In this section, I present the model I derived from the concepts and ideas described in the previous section. In a very schematic way, it is an overlapping generation model where each period corresponds to a general equilibrium, which makes it possible to determine the overall production of goods and energy. On the one hand, the production of goods makes it possible to satisfy the needs and preferences of agents and to measure the level of economic development. On the other hand, the production of energy is not only a prerequisite of any kind of production and thus of economic growth, but is also responsible for the emission of greenhouse gases into the atmosphere. These emissions are the trigger that causes global warming, a phenomenon whose effects are beginning to be felt and which promises to be major and structuring in the years to come. Moreover, this balance allows the endogenous determination of the fertility of the agents. This model features two sectors, an agricultural one and a manufacturing one, hence highlighting the subsistence constraint of the agents, and different technology vintages that allow for regime shifts and in particular for the transition from a Malthusian regime to an industrial regime.

2.1 Household

We consider an economy with three overlapping generations with a time step of 30 years that features three stages of life: childhood (0 to 15 years old), parenthood (15 to 45 years old), and old age (45 to 75 years old). There are two types of agents: unskilled agents and skilled agents, that are defined by their relative productivity, and a different rearing time and quantity of food consumed during childhood.

At each period, children consume some of the time of their parent[5] and a fixed amount of food[6]. Thus the relative cost of skilled and unskilled children is γ.

In the second period of life, agents devote their one unit of time to work and child-rearing (they also need to buy food for their children). Hence, they earn an income that they can spend in three different ways: consuming the agricultural good, the manufactured good or saving money for old age (we suppose that the manufactured good is the capital good). However, agents face a subsistence constraint that might bind their consumption and prevent them from consuming manufactured goods or saving some earnings. This might be seen as a necessary condition to grow old. Finally, we suppose that agents all receive an equal share of the total land rent, which is therefore a complement to the income from labor.

During the third period of life, an old agent receive an income from its savings and consume agricultural goods as well as manufactured goods.

We define the utility of a type $i = s, u$ agent by:

$$U_t = u^c_{i,t,t} + \delta u^c_{i,t,t+1} + \nu ln(n^u_{i,t} w^u_{t+1} + n^s_{i,t} w^s_{t+1})$$

Where $u^c_{i,t,T} = \theta ln(c^a_{i,t,T}) + (1-\theta) ln(c^m_{i,t,T})$ and $T = t, t+1$ with the following constraints:

$$\begin{aligned} p^m_t s_t + p^a_t \left(c^a_{i,t,t} + (n^u_{i,t} + \gamma^s n^s_{i,t})\epsilon_a \tilde{c_a}\right) + p^m_t c^m_{i,t,t} &\leq w^i_t \left(1 - \tau(n^u_{i,t} + \gamma^s n^s_{i,t} \tau^s)\right) + \frac{r_{L,t} X}{N_t} \\ p^a_{t+1} c^a_{i,t,t+1} + p^m_{t+1} c^m_{i,t,t+1} &\leq s_t r_{K,t+1} \\ \tilde{c_a} &\leq c^a_{i,t,t} \end{aligned}$$

[5] τ for future unskilled agents and $\gamma\tau$ for future skilled agents

[6] $\epsilon^a \tilde{c^a}$ for future unskilled agents and $\gamma \epsilon^a \tilde{c^a}$ for future skilled agents

In the spirit of [Galor and Mountford, 2008], this utility function features a quality-quantity trade-off when it comes to fertility. Indeed agents derive utility, not only from the number of children that are born, the quantity, but also from their future wage. This presupposes that agents are perfectly capable of anticipating the equilibrium of the following period and therefore the salary of future unskilled and skilled agents. It is on the basis of these perfect anticipations that the distribution in quantity between qualified and unqualified offspring is decided.

With "total income" denoted as $I^i = w_t^i + \frac{r_{L,t}X}{N_t}$ for agent of type $i = s, u$, the income coming from labor and land rents, we obtain the following behaviour when the subsistence constraint is binding (i.e. when $\frac{\theta}{1+\delta+\nu}\frac{I}{p_t^a} \leq \tilde{c_a}$):

$$\begin{cases} c_{i,t,t}^a &= \tilde{c_a} \\ c_{i,t,t}^m &= \frac{1-\theta}{1-\theta+\delta+\nu}\frac{I^i - p_t^a\tilde{c_a}}{p_t^m} \\ c_{i,t,t+1}^a &= \frac{\delta\theta}{1-\theta+\delta+\nu}\frac{(I^i - p_t^a\tilde{c_a})r_{K,t+1}}{p_{t+1}^a p_t^m} \\ c_{i,t,t+1}^m &= \frac{\delta(1-\theta)}{1-\theta+\delta+\nu}\frac{(I^i - p_t^a\tilde{c_a})r_{K,t+1}}{p_{t+1}^m p_t^m} \\ n_{i,t}^u + \gamma^s n_{i,t}^s &= \frac{\nu}{1-\theta+\delta+\nu}\frac{I^i - p_t^a\tilde{c_a}}{w_t^i\tau + p_t^a\epsilon_a\tilde{c_a}} \end{cases}$$

with:

$$s_t^i = \frac{\delta}{1-\theta+\delta+\nu}\frac{I^i - p_t^a\tilde{c_a}}{p_t^m}$$

and:

$$l_t^i = 1 - \tau\frac{\nu}{1-\theta+\delta+\nu}\frac{I^i - p_t^a\tilde{c_a}}{w_t^i\tau + p_t^a\epsilon_a\tilde{c_a}}$$

And the agent features the following behaviour when the subsistence constraint is not binding anymore (i.e. $\frac{\theta}{1+\delta+\nu}\frac{I}{p_t^a} \geq \tilde{c_a}$):

$$\begin{cases} c_{i,t,t}^a &= \frac{\theta}{1+\delta+\nu}\frac{I^i}{p_t^a} \\ c_{i,t,t}^m &= \frac{1-\theta}{1+\delta+\nu}\frac{I^i}{p_t^m} \\ c_{i,t,t+1}^a &= \frac{\delta\theta}{1+\delta+\nu}\frac{I^i r_{K,t+1}}{p_{t+1}^a p_t^m} \\ c_{i,t,t+1}^m &= \frac{\delta(1-\theta)}{1+\delta+\nu}\frac{I^i r_{K,t+1}}{p_{t+1}^m p_t^m} \\ n_{i,t}^u + \gamma^s n_{i,t}^s &= \frac{\nu}{1+\delta+\nu}\frac{I^i}{w_t^i\tau + p_t^a\epsilon_a\tilde{c_a}} \end{cases}$$

with:

$$s_t^i = \frac{\delta}{1+\delta+\nu}\frac{I^i}{p_t^m}$$

and:

$$l_t^i = 1 - \tau\frac{\nu}{1+\delta+\nu}\frac{I^i}{w_t^i\tau + p_t^a\epsilon_a\tilde{c_a}}$$

The functional forms we have chosen allow us to express a "total child aggregate" $M_{t+1} = N_t^u(n_{u,t}^u + \gamma^s n_{u,t}^s) + N_t^s(n_{u,t}^u + \gamma^s n_{u,t}^s) = N_{t+1}^u + \gamma N_{t+1}^s$, which we can use to determine the qualified/unqualified distribution by resolving the general equilibrium of the following period, with the condition: if $N_{t+1}^s \neq 0$ then $\frac{w_{t+1}^s}{w_{t+1}^u} = \gamma$. What is interesting with this aggregate is that, whatever the distribution is between skilled and unskilled at $t + 1$, the consumption of food and the child-rearing time are unaffected.

2.1.1 Total population

At time t, the total population is given by:

$$Population_t = \frac{1}{2}(N^u_{t+1} + N^s_{t+1}) + N^u_t + N^s_t + N^u_{t-1} + N^s_{t-1}$$

2.1.2 Total demand of goods and supply of factor of production

From the solution of the household problem it is possible to derive the total demand of good from the agents. Concerning the agricultural good, the demand comes from the three generations:

$$D^a(r_K, r_L, w^u, w^s) = M_t\epsilon^a\tilde{c^a} + c^a_{u,t,t}N^u_t + c^a_{s,t,t}N^s_t + c^a_{u,t-1,t}N^u_{t-1} + c^a_{s,t-1,t}N^s_{t-1} \quad (1)$$

And for the manufactured good, the demand arises from the consumption of manufactured good but also from the savings, as the manufactured good is the capital good:

$$D^m(r_K, r_L, w^u, w^s) = c^m_{u,t,t}N^u_t + c^m_{s,t,t}N^s_t + c^m_{u,t-1,t}N^u_{t-1} + c^m_{s,t-1,t}N^s_{t-1} + s^u_tN^u_t + s^s_tN^s_t \quad (2)$$

To boot, it is possible to determine the supply in the three factors of primary production: capital, land, and labour. So, the capital supply is given by:

$$K^s_t = s^u_{t-1}N^u_{t-1} + s^u_{t-1}N^u_{t-1}$$

Total supply of land is fixed and is equal to $X^s = X_{land}$. Total supply of unskilled labor is $L^{s,u} = l^uN^u_t$ and total supply of skilled labor is $L^{s,s} = l^sN^s_t$

2.2 Production

In our model, we introduce two sectors that produce consumer and capital goods. However, as explained in the section, it is fundamental to consider energy as an essential input and in particular to be able to distinguish between the different energy sources at work, in order to be able to study the evolution of the energy mix over time.To do this, we consider 4 possible sources of energy: traditional biomass, coal, hydrocarbons (oil and gas) and all renewable energies (wind, solar, hydroelectric and nuclear).

2.2.1 Energy

Some details are to be noted concerning the energies (non-renewable in particular). The cost of extraction increases as the resource diminishes (i.e. the extraction yield decreases), as the latter is increasingly difficult to access. This increase in production cost is offset by technological progress, which lowers extraction costs (i.e. extraction yield increases). We present these two phenomena with capital investment as the only factor of production (except for the biomass).

For each energy source, we consider a representative firm and we suppose that these firms first maximize their profit, and second face competition so that there is also a zero-profit condition.

Traditional biomass We consider traditional biomass as plant or animal material used for energy production (electricity or heat), like wood. This source of energy is therefore easily available and covers a crucial role in accommodating energy demand of over two billion people living in developing countries. Indeed, traditional biomass appears historically as the energy used before the industrial revolutions and requires a very low level of technology. We thus assume that traditional biomass energy comes from two factors of production, land and capital, whose productivity does not vary with technological development.

Furthermore, even if burning plant-derived biomass releases CO_2, we do not take into account these emissions in the model since photosynthesis cycles the CO_2 back into new crops. As, a share of the CO_2 absorbed by the biomass is moved to the soil, the previous cycle might event generate negative emissions.

So we consider the following production function:

$$E^{bio}(K,X) = A^{bio}K^{\alpha^{bio}}X^{1-\alpha^{bio}}$$

Maximizing the profit, we obtain the demand for capital and land:

$$K_t^{bio} = \frac{\alpha p_t^{E^{bio}}}{r_{K,t}}E_t^{bio}$$

$$X_t^{bio} = \frac{(1-\alpha)p_t^{E^{bio}}}{r_{L,t}}E_t^{bio}$$

As well as the price of traditional biomass energy with the no profit condition:

$$p_t^{E^{bio}} = \frac{1}{A^{bio}}\frac{r_{K,t}^{\alpha^{bio}}r_{L,t}^{1-\alpha^{bio}}}{(\alpha^{bio})^{\alpha^{bio}}(1-\alpha^{bio})^{1-\alpha^{bio}}}$$

Coal As presented in section 1.3 coal is the first fossil fuel energy to be extracted and used and appears, with the invention of the steam engine, as a central element of the first industrial revolution ([Smil, 2018] underlines the key role of energy as a driver of the economic engine and [Pomeranz, 2019] considers coal as a technical solution to environmental and ecological constraint).

The main characteristics that we consider for coal are first the need of little technological development so that the exploitation of this resource is profitable (compared for example to renewables, such as solar or wind energy), second a very high stock (reserves as well as resources[7]), third a low energy efficiency (due to a low combustion temperature), and finally a high CO_2 emission intensity.

In this classification system, 'resources' are defined as "concentrations of naturally occurring solid, liquid, or gaseous material in or on the Earth's crust in such form that economic extraction is potentially feasible." The geological dimension is divided into 'identified' and 'undiscovered' resources.

We thus, consider the following production function:

[7] Roughly speaking, resources denotes the total quantity of solid, liquid or gaseous material in the Earth's crust, whereas, reserves is defined as the share of resources that can be recovered by an economic agent. So, if resources can be considered as an objective, physical measure or estimation, reserves is defined "by the current or expected profitability of exploitation, governed by the ratio of future market prices to the long-run cost of production"([Banerjee et al., 2012])

$$E_t^{coal}(K) = f^{coal}(Q_t)\left(\frac{R_t^{coal}}{R_0^{coal}}\right)^{\sigma^{coal}} K \text{ if } f^{coal}(Q_t) > 0$$

with:

$$R_{t+1}^{coal} = R_t^{coal} - E_t^{coal}$$

Maximization of the profit as well as no profit condition allows us to obtain the demand of capital as well as the price of energy:

$$K_t^{coal} = \frac{1}{f^{coal}(Q_t)\left(\frac{R_t^{coal}}{R_0^{coal}}\right)^{\sigma^{coal}}} E_t^{coal} \text{ if } f^{coal}(Q_t) > 0$$

and:

$$p_t^{E^{coal}} = \frac{r_{K,t}}{f^{coal}(Q_t)\left(\frac{R_t^{coal}}{R_0^{coal}}\right)^{\sigma^{coal}}} \text{ if } f^{coal}(Q_t) > 0$$

Hydrocarbon: Oil and Gas Like coal seventy years before, hydrocarbons appear to be the trigger of the second industrial revolution, in the late XIX^{th} century, oil extraction and internal combustion engine brought the possibility of even higher levels of industrial production ([Smil, 2018]). So, in our model hydrocarbons are quite similar to coal, but differs in some way: the level of technological development required for profitability is higher, the stock (in particular reserves) is much smaller compared to the ones of coal[8], the energy efficiency is higher and the CO_2 intensity is lower. Thus, the model we consider has the same form as regarding coal:

$$E_t^{hydro}(K) = f^{hydro}(Q_t)\left(\frac{R_t^{hydro}}{R_0^{hydro}}\right)^{\sigma^{hydro}} K \text{ if } f^{oil}(Q_t) > 0$$

with:

$$R_{t+1}^{hydro} = R_t^{hydro} - E_t^{hydro}$$

So:

$$K_t^{hydro} = \frac{1}{f^{hydro}(Q_t)\left(\frac{R_t^{hydro}}{R_0^{hydro}}\right)^{\sigma^{hydro}}} E_t^{hydro} \text{ if } f^{oil}(Q_t) > 0$$

and:

$$p_t^{E^{hydro}} = \frac{r_{K,t}}{f^{hydro}(Q_t)\left(\frac{R_t^{hydro}}{R_0^{hydro}}\right)^{\sigma^{hydro}}} \text{ if } f^{oil}(Q_t) > 0$$

Renewables Finally, we consider the rest of the different sources of energy that we call (admittedly abusively) renewable energies. The main characteristic of these energies is, first of all, their renewable character, thus, the cumulative volume of energy production appears to be unlimited over time, even if it is limited over a given period of time[9]. However, despite this

[8] According to [British Petroleum, 2019b], the petrol reserve/production ratio is equal to 50 years, 50.9 years for gas, and 132 years for coal

[9] This limitation is quite relative, indeed, the total potential of renewable energy estimated by [Banerjee et al., 2012] is 67510-294625 EJ per year while the total consumption of primary energy in 2005 is 453.4 EJ.

availability, it is more expensive on average than fossil fuels[10]. Thus, we propose the following modelling:

$$E_t^{renew}(K) = min(f^{renew}(Q_t)K, E_{max}^{renew}) \text{ if } f^{renew}(Q_t) > 0$$

So:

$$K_t^{renew} = \frac{1}{f^{renew}(Q_t)} E_t^{renew} \text{ if } f^{renew}(Q_t) > 0$$

and:

$$p_t^{E^{renew}} = \frac{r_{K,t}}{f^{hydro}(Q_t)} \text{ if } f^{renew}(Q_t) > 0$$

Technological progress of energy extraction In order to model the threshold effect concerning the emergence of a new technology, we define the functions $f^i(Q_t)$ by sigmoid functions of the form:

$$f^i(Q_t) = A_{lim}^i tanh(scale^i(Q_t - Q_{min}^i)$$

This functional form is based on two presuppositions, firstly that of emergence and secondly the idea that the extracting yield is bounded and that it converges towards its limit with technological development.

2.2.2 Agricultural good

In our model, agriculture has a very special role. First of all, it represents a very important part of GDP because of the constraint of substance: in a Malthusian regime, agriculture appears as a major productive sector. Moreover, this share of GDP gradually decreases as the economy moves away from the subsistence constraint ([Klein Goldewijk et al., 2011]). Thus, the growth rate of agricultural production is higher than that of the population, but lower than that of GDP ([Federico, 2015]). As explained in the section 1.3, we consider that the production factors classically considered in agriculture, capital, land and labour need a non-substitutable energy complement[11]. According to [Burzynnskia et al., 2019] the agricultural sector presents a very important substitutability between skilled and unskilled work, so we consider that the agricultural sector only takes as input unskilled work. Thus, the model for the production function is a nested-CES function ([Sato, 1967]), which is given by:

$$F^a(K, L^u, E^i, T) = (1 - D^a(T)) \left((A_{LLK}^a LLK)^{\frac{\eta^a - 1}{\eta^a}} + (A_E^a E^a)^{\frac{\eta^a - 1}{\eta^a}} \right)^{\frac{\eta^a}{\eta^a - 1}}$$

where LLK is a land, labor and capital aggregate:

$$LLK(K, L^u, X) = \left({\alpha_{LLK}^a}^{\frac{1}{\eta_{LLK}^a}} K^{\frac{\eta_{LLK}^a - 1}{\eta_{LLK}^a}} + {\beta_{LLK}^a}^{\frac{1}{\eta_{LLK}^a}} X^{\frac{\eta_{LLK}^a - 1}{\eta_{LLK}^a}} + (1 - \alpha_{LLK}^a - \beta_{LLK}^a)^{\frac{1}{\eta_{LLK}^a}} {L^u}^{\frac{\eta_{LLK}^a - 1}{\eta_{LLK}^a}} \right)^{\frac{\eta_{LLK}^a}{\eta_{LLK}^a - 1}}$$

and E is an aggregate of the different energy sources:

$$E^a = \left(\sum_i (\rho^i E^i)^{\frac{\eta_E^a - 1}{\eta_E^a}} \right)^{\frac{\eta_E^a}{\eta_E^a - 1}}$$

[10] Technically, the forms of energy described here often require a heavy investment and then a very low marginal cost. Here, we consider the average cost over the 30-year period in the production function, rather than considering the investment necessary and the resulting marginal cost

[11] Even if the energy intensity is lower than in the modern industrial sector

A^a_{LLK} denotes the TFP of the agricultural sector and is an increasing function of Q_t, the technological/knowledge index. However, the productivity term for energy A^a_E do not vary with technology, on the contrary, to keep up with the increase of the productivity of labor and capital, there is a need of more energy, which is available due to the profitable extraction. ρ^i denotes the energy efficiency factor that converts primary energy into final energy.

It is then possible to define the prices of both aggregates:

$$p^{LLK} = \left(\alpha^a_{LLK} r_K^{1-\eta^a_{LLK}} + \beta^a_{LLK} r_L^{1-\eta^a_{LLK}} + (1-\alpha^a_{LLK}-\beta^a_{LLK}) w^{u\,1-\eta^a_{LLK}}\right)^{\frac{1}{1-\eta^a_{LLK}}}$$

$$p^{E^a} = \left(\sum_i \left(\frac{p^{E^i}}{\rho^i}\right)^{1-\eta^a_E}\right)^{\frac{1}{1-\eta^a_E}}$$

And thus the price of the agricultural good:

$$p^a = \frac{1}{1-D^a(T)}\left(\left(\frac{p^{LLK}}{A^a_{LLK}}\right)^{1-\eta^a} + \left(\frac{p^{E^a}}{A^a_E}\right)^{1-\eta^a}\right)^{\frac{1}{1-\eta^a}}$$

And finally the demand for each production factor is:

$$\begin{cases} K^a &= \alpha^a_{LLK}\left(\frac{r_K}{p^{LLK}}\right)^{-\eta^a_{LLK}} (A^a_{LLK}(1-D^a(T))^{\eta^a-1}\left(\frac{p_{LLK}}{p^a}\right)^{-\eta^a} Y^a \\ X^a &= \beta^a_{LLK}\left(\frac{r_L}{p^{LLK}}\right)^{-\eta^a_{LLk}} (A^a_{LLK}(1-D^a(T))^{\eta^a-1}\left(\frac{p_{LLK}}{p^a}\right)^{-\eta^a} Y^a \\ L^{u,a} &= (1-\alpha^a_{LLK}-\beta^a_{LLK})\left(\frac{w^u}{p^{LLK}}\right)^{-\eta^a_{LLk}} (A^a_{LLK}(1-D^a(T))^{\eta^a-1}\left(\frac{p_{LLK}}{p^a}\right)^{-\eta^a} Y^a \\ E^{a,i} &= \frac{1}{\rho^i}\left(\frac{p^{E^i}}{\rho^i p^{E^a}}\right)^{-\eta^a_E} (A^a_E(1-D^a(T))^{\eta^a-1}\left(\frac{p^{E^a}}{p^a}\right)^{-\eta^a} Y^a \end{cases}$$

2.2.3 Manufactured good

Concerning the manufacturing sector, it displays two different technologies, an old one and a modern one, which represent craftsmanship during the Malthusian phase, as well as the fossil fuel-based industry ([Smil, 2018]) which characterises the industrial revolution economy. In this second case, economic production requires skilled labour (as in [Galor and Mountford, 2008]), which beyond the change in the productive regime introduces a shift in the demographic regime.

Old technology For the old technology, I consider that production only depends on unskilled labor and traditional biomass energy, which means that manufacturing with this technology does not require technical tools, machines, nor heavy investment and that only human labor catalyses the use of biomass energy. Hence, I set the following production function:

$$F^{m,old}(L^u, E^{bio}, T) = (1-D^m(T))\left((A^{m,old}_L L^u)^{\frac{\eta^{m,old}-1}{\eta^{m,old}}} + (A^{m,old}_E \rho^{bio} E^{bio})^{\frac{\eta^{m,old}-1}{\eta^{m,old}}}\right)^{\frac{\eta^{m,old}}{\eta^{m,old}-1}}$$

So the price of the good produced with the old technology is:

$$p^{m,old} = \frac{1}{1-D^m(T)}\left(\left(\frac{w^u}{A^{m,old}_L}\right)^{1-\eta^{m,old}} + \left(\frac{p^{E^{bio}}}{A^{m,old}_E \rho^{bio}}\right)^{1-\eta^{m,old}}\right)^{\frac{1}{1-\eta^{m,old}}}$$

And the demand for both factors of production is:

$$\begin{cases} L^{u,a} &= (A_L^{m,old}(1-D^m(T))^{\eta^{m,old}-1}\left(\frac{w^u}{p^{m,old}}\right)^{-\eta^{m,old}} Y^{m,old} \\ E^{m,old,bio} &= \frac{1}{\rho^{bio}}(A_E^{m,old}(1-D^m(T))^{\eta^{m,old}-1}\left(\frac{p^{E^{bio}}}{\rho^{bio}p^{m,old}}\right)^{-\eta^{m,old}} Y^{m,old} \end{cases}$$

Modern technology On the opposite, the industrial regime is characterised by a need for capital investment, the latter being controlled by a skilled labour force and requiring the use of large volumes of energy. As in the agricultural sector, capital, labour and energy factors are hardly substitutable. Moreover, in the same way, the productivity term $A_{KL}^{m,new}$ is increasing by Q_t and $A_E^{m,new}$ is constant. Moreover, $A_{KL}^{m,new\prime}(Q) > A_{LLK}^{a}{}'(Q)$, which means that in a regime of industrial growth, productivity in the manufacturing sector increases faster than in the agricultural sector. This is consistent with the decline in the importance of agriculture in GDP[12]. The production function takes the form of:

$$F^{m,new}(K,L^u,L^s,E^i,T) = (1-D^m(T))\left((A_{KL}^{m,new}KL)^{\frac{\eta^{m,new}-1}{\eta^{m,new}}} + (A_E^{m,new}E^{m,new})^{\frac{\eta^{m,new}-1}{\eta^{m,new}}}\right)^{\frac{\eta^{m,new}}{\eta^{m,new}-1}}$$

where:

$$KL = \left(\alpha_{KL}^{m,new\,\frac{1}{\eta_{KL}^{m,new}}} K^{\frac{\eta_{KL}^{m,new}-1}{\eta_{KL}^{m,new}}} + (1-\alpha_{KL}^{m,new})^{\frac{1}{\eta_{KL}^{m,new}}} L^{m,new\,\frac{\eta_{KL}^{m,new}-1}{\eta_{KL}^{m,new}}}\right)^{\frac{\eta_{KL}^{m,new}}{\eta_{KL}^{m,new}-1}}$$

$$L^{m,new} = \left(\alpha_L^{m,new\,\frac{1}{\eta_L^{m,new}}}(L^s)^{\frac{\eta_L^{m,new}-1}{\eta_L^{m,new}}} + (1-\alpha_L^{m,new})^{\frac{1}{\eta_L^{m,new}}}(L^u)^{\frac{\eta_L^{m,new}-1}{\eta_L^{m,new}}}\right)^{\frac{\eta_L^{m,new}}{\eta_L^{m,new}-1}}$$

and:

$$E^{m,new} = \left(\sum_i (\rho^i E^{m,new,i})^{\frac{\eta_E^{m,new}-1}{\eta_E^{m,new}}}\right)^{\frac{\eta_E^{m,new}}{\eta_E^{m,new}-1}}$$

The prices of aggregates are:

$$W = \left(\alpha_L^{m,new} w^{s\,1-\eta_L^{m,new}} + (1-\alpha_L^{m,new}) w^{u\,1-\eta_L^{m,new}}\right)^{\frac{1}{1-\eta_L^{m,new}}}$$

$$p^{KL} = \left(\alpha_{KL}^{m,new} r_K^{1-\eta_{KL}^{m,new}} + (1-\alpha_{KL}^{m,new}) W^{1-\eta_{KL}^{m,new}}\right)^{\frac{1}{1-\eta_{KL}^{m,new}}}$$

$$p^{E^{m,new}} = \left(\sum_i \left(\frac{p^{E^i}}{\rho^i}\right)^{1-\eta_E^{m,new}}\right)^{\frac{1}{1-\eta_E^{m,new}}}$$

And thus the price of the agricultural good:

$$p^{m,new} = \frac{1}{1-D^m(T)}\left(\left(\frac{p^{KL}}{A_{KL}^{m,new}}\right)^{1-\eta^{m,new}} + \left(\frac{p^{E^{m,new}}}{A_E^{m,new}}\right)^{1-\eta^{m,new}}\right)^{\frac{1}{1-\eta^{m,new}}}$$

[12] This decline also has to do with the preferences we have assumed for our agents: in a steady state, the share of agriculture in GDP should be close to the θ parameter of preference for the agricultural good.

Finally, the demand for production factors is:

$$\begin{cases} K^{m,new} = & \alpha_{LLK}^{m,new}\left(\frac{r_K}{p^{KL}}\right)^{-\eta_{KL}^{m,new}} (A_{KL}^{m,new}(1-D^m(T))^{\eta^{m,new}-1}\left(\frac{p_{KL}}{p^{m,new}}\right)^{-\eta^{m,new}} Y^{m,new} \\ L^{s,m,new} = & \alpha_{L}^{m,new}\left(\frac{w^s}{W}\right)^{-\eta_{L}^{m,new}} (1-\alpha_{KL}^{m,new})\left(\frac{W}{p^{KL}}\right)^{-\eta_{LLk}^{m,new}} \\ & (A_{KL}^{m,new}(1-D^m(T))^{\eta^{m,new}-1}\left(\frac{p_{KL}}{p^{m,new}}\right)^{-\eta^{m,new}} Y^{m,new} \\ L^{u,m,new} = & (1-\alpha_{L}^{m,new})\left(\frac{w^u}{W}\right)^{-\eta_{L}^{m,new}} (1-\alpha_{KL}^{m,new})\left(\frac{W}{p^{KL}}\right)^{-\eta_{LLk}^{m,new}} \\ & (A_{KL}^{m,new}(1-D^m(T))^{\eta^{m,new}-1}\left(\frac{p_{KL}}{p^{m,new}}\right)^{-\eta^{m,new}} Y^{m,new} \\ E^{a,i} = & \frac{1}{\rho^i}\left(\frac{p^{E^i}}{\rho^i p^{E^{m,new}}}\right)^{-\eta_{E}^{m,new}} (A_{E}^{m,new}(1-D^m(T))^{\eta^{m,new}-1}\left(\frac{p^{E^{m,new}}}{p^{m,new}}\right)^{-\eta^{m,new}} Y^{m,new} \end{cases}$$

2.2.4 Total demand of production factors

Now, that we have derived the demand of production factor for each goods[13], it is possible to define the total demand of each factor of production.

With the old technology There are only three factors of production and the demand is given by:

$$K_d(r_k, r_l, w_u) = K^{bio} + K^{coal} + K^{oil} + K^{renew} + K^a$$
$$X_d(r_k, r_l, w_u) = X^{bio} + X^a$$
$$L_d^u(r_k, r_l, w_u) = L^{u,a} + L^{u,m,old}$$

With the new technology There are four factors of productions and there is the condition $w^s = \gamma w^u$. However, there is still the good number of unknows as the distribution between skilled and unskilled worker is an unknow as well[14]

$$K_d(r_k, r_l, w_u) = K^{bio} + K^{coal} + K^{oil} + K^{renew} + K^a + K^{m,new}$$

$$X_d(r_k, r_l, w_u) = X^{bio} + X^a$$
$$L_d^u(r_k, r_l, w_u) = L^{u,a} + L^{u,m,new}$$
$$L_d^s(r_k, r_l, w_u) = L^{s,m,new}$$

2.3 Human capital accumulation and technological progress

This model presents two different regimes in terms of technical progress: a Malthusian regime and an industrial regime. We resume [Galor and Mountford, 2008] by defining a synthetic indicator of progress Q_t. Let $h_t = \frac{N_t^s}{N_t^s + N_t^u}$ the share of skilled agents in the population at time t. We then set:

$$\frac{Q_{t+1} - Q_t}{Q_t} = g(h_t) \text{ i.e. } Q_{t+1} = (1 + g(h_t))Q_t$$

where g is such that $g(0) > 0$, $g' > 0$ and $g'' < 0$

[13]Agricultural and manufactured goods may be considered as final goods and the different kinds of energy might be considered as intermediate goods.

[14]The condition is: $N_t^u + \gamma N_t^s = constant$

This index can be understood in various ways, it is both a human capital index, since it represents a spillover effect of the appearance of high qualifications. It is also a technology index in that it drives all the technology-dependent parameters that can vary over time. Thus, we can consider that this factor corresponds to a form of useful knowledge ([Strulik et al., 2013]) which allows the emergence of GPTs ([Schaefer et al., 2014]) materialising through the discovery and then the use of a new form of energy.

2.4 Climate dynamics and damages

In the previous sections, I have introduced a closed economy model that allows the unified growth theory and the precepts of ecological economics to be brought together by putting demographics and energy availability at the centre of economic dynamics in a coherent way, in the same way as [Fröling, 2011] or [Bovari and Court, 2019] have done. However, in prospective terms, an effect that was previously negligible appears as a major element of economic evolution: climate change. As explained in the section 1.4, it therefore appears relevant to apply an endogenous climate loop in order to consider the future effects of global warming: whether they concern the economy or, indirectly, demographics.

As our model allows us to know the energy mix over a period of time, it also allows us to deduce the CO_2 emissions resulting from the combustion of fossil fuels, so we deduce the carbon concentration in the different carbon reservoirs and finally derive the radiative forcing and its effect in terms of global warming on the atmosphere. Finally, from the global mean surface temperature anomaly, it is possible to deduce the climate damage in each sector.

2.4.1 Carbon dioxide emissions

First, emissions are relative to the energy mix, which is why it is interesting to present a distinction between the different types of fuel possible. In fact, coal has a much lower emission ratio per unit of useful energy than hydrocarbons such as oil or gas. Thus, emissions are given by:

$$CO_{2,t} = \xi^{coal} E_t^{coal} + \xi^{hydro} E_t^{hydro} + CO_{2,t}^{ext}$$

It is possible to notice that we do not include here the emissions related to the production of energy from biomass, since the latter is cancelled out by the regrowth of biomass that occurs after cutting. However, these emissions are indeed not completely zero and there is also a very important effect of land-use change[15] resulting in a possibly large amount of emissions. All these emissions are included in the term $CO2_t^{ext}$ which is defined and constructed by the climate model calibration strategy.

2.4.2 Carbon cycle

In order to model the carbon cycle and the global warming phenomenon, we use the calibrated simple model featured in [Nordhaus, 2017][16]. The carbon cycle is represented using a 3-box

[15] For example, in the case of deforestation for agricultural purpose, there is not as much regrowth of biomass, so the land-use change induces some emissions that are not taken into account in our model.

[16] More precisely, the climate model featured in the DICE-2016 model is calibrated on the RCP 8.5 prospective scenario and behaves quite pourly on other kind of scenarios, this is something that one needs to bear in mind when dealing with DICE-2016 climate model. Indeed, it would be interesting to feature a more rigorous and complete climate model (in the spirit of [Gasser and Ciais, 2013]), that would, for example, allow us to consider land-use change and non-linearities of the climate system. However, even if this would be an interesting research

model representing the atmospheric CO_2 reservoir, the earth's surface - surface ocean as well as biosphere, and the deep ocean.

$$\begin{cases} m_{t+1}^{atm} = & c_{atm\longrightarrow atm} m_t^{atm} + c_{up\longrightarrow atm} m_t^{up} + CO_{2,t} \\ m_{t+1}^{up} = & c_{atm\longrightarrow up} m_t^{atm} + c_{up\longrightarrow up} m_t^{up} + c_{deep\longrightarrow up} m_t^{deep} \\ m_{t+1}^{deep} = & c_{up\longrightarrow deep} m_t^{up} + c_{deep\longrightarrow deep} m_t^{deep} \end{cases}$$

With:

$$\begin{aligned} c_{up\longrightarrow atm} &= \frac{m_{eq}^{atm}}{m_{eq}^{up}} c_{atm\longrightarrow up} \\ c_{deep\longrightarrow up} &= \frac{m_{eq}^{up}}{m_{eq}^{deep}} c_{up\longrightarrow deep} \\ c_{atm\longrightarrow atm} &= 1 - c_{atm\longrightarrow up} \\ c_{up\longrightarrow up} &= 1 - c_{up\longrightarrow atm} - c_{up\longrightarrow deep} \\ c_{deep\longrightarrow deep} &= 1 - c_{deep\longrightarrow up} \end{aligned}$$

Such that at the equilibrium where $CO_{2,t} = 0$: $m^{atm} = m_{eq}^{atm}$, $m^{up} = m_{eq}^{up}$, $m^{deep} = m_{eq}^{deep}$

2.4.3 Radiative forcing and global warming

With the concentration of CO_2 in the atmosphere defined, it is possible to derive the radiative forcing:

$$RF_{t+1} = \frac{F_{2\times CO_2}}{log(2)} log\left(\frac{m_{t+1}^{atm}}{m_{eq}^{atm}}\right) + RF_{t+1}^{ext}$$

And then the dynamics of global mean surface temperature and mean ocean temperature are given by:

$$T_{t+1}^{atm} = T_t^{atm} + c_1(RF_{t+1} - \frac{F_{2\times CO_2}}{T_{2\times CO_2}^{atm}} T_t^{atm}) - c_1 c_3 (T_t^{atm} - T_t^{ocean})$$

$$T_{t+1}^{ocean} = T_t^{ocean} + c_4(T_t^{atm} - T_t^{ocean})$$

2.4.4 Climate damages

The effects of global warming take the form of a damage function that can be interpreted as either a decrease in productivity or a net loss of production. We propose here damages that are differentiated according to the sector: agriculture appears more vulnerable to climate change and therefore has a higher damage function than the manufacturing sector. Damage function are derived from [Desmet and Rossi-Hansberg, 2015] and have the following form and are displayed on the figure 6:

$$D^a(T) = d_1^a T + d_2^a T^2$$

$$D^m(T) = d_1^m T + d_2^m T^2$$

2.5 General equilibrium and regime shifts

Regardless of the technology used, at each instant t, there is a single vector on the unitary simplex such that excess-demands are less than or equal to zero in all markets. This vector is the supporting price of a general equilibrium.

question, it is not the focus of this study.

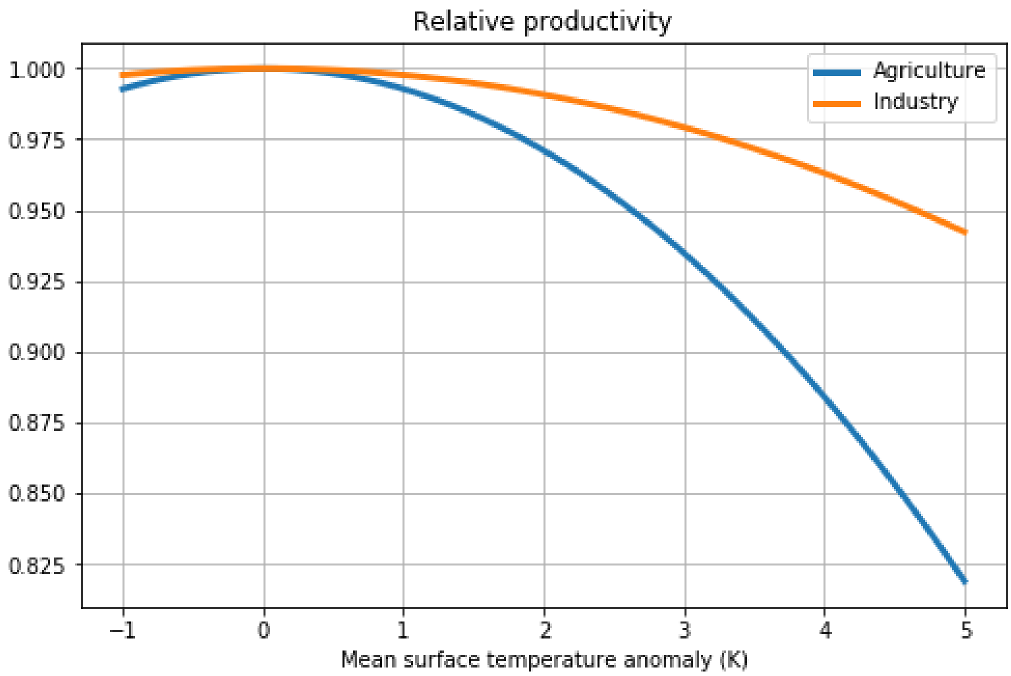

Figure 6: Variation of productivity due to climate change - Source: [Desmet and Rossi-Hansberg, 2015]

As previously written, fertility choices at time t on the part of agents are made with a perfect expectation of the equilibrium that would be obtained at time $t+1$. Since both situations (old and modern technology) admit an equilibrium, it is possible for agents to choose the one that maximizes the hourly wage of an unskilled worker. Thus, the economy shifts from the old technology to the new if:

$$\frac{w^u}{p^m}\,|_{Old\ technology} < \frac{w^u}{p^m}\,|_{Modern\ technology}$$

i.e.

$$\frac{\partial F^{m,old}}{\partial L^u}(L^{u*}, E^{bio*}, T) < \frac{\partial F^{m,new}}{\partial L^u}(K^*, L^{u*}, L^{s*}, E^{i*}, T)$$

In terms of coordination, this tilting criterion also makes sense. Indeed, it means that the regime shift occurs when the marginal productivity of an unskilled worker is higher with the new technology than with the old one. This is consistent with the behaviour of firms in the industrial sector that change technology or are replaced by more productive and therefore more competitive firms.

3 Numerical simulation

3.1 Calibration

In order to perform numerical simulation with the model, the model must be calibrated, i.e. assigning a numerical value to each parameter in order to obtain understandable output data. The objective here is to assess whether the model presented in the previous part by evaluating its ability to approximately reproduce the stylized facts mentioned in the section 1, thus, calibration is not of paramount importance, as our results are too approximate to be used as a quantitative reference. In fact, part of the calibration was done in a very artisanal way, by hand in an iterative way in order to obtain commensurable results with the stylized facts.Therefore, I will not present here the numerical values used, they are however all present in the source code present with this work.

Household The parameters describing the preferences of the agents are chosen from the range of classical parameters of the overlapping generation models and then fine-tuned by hand.

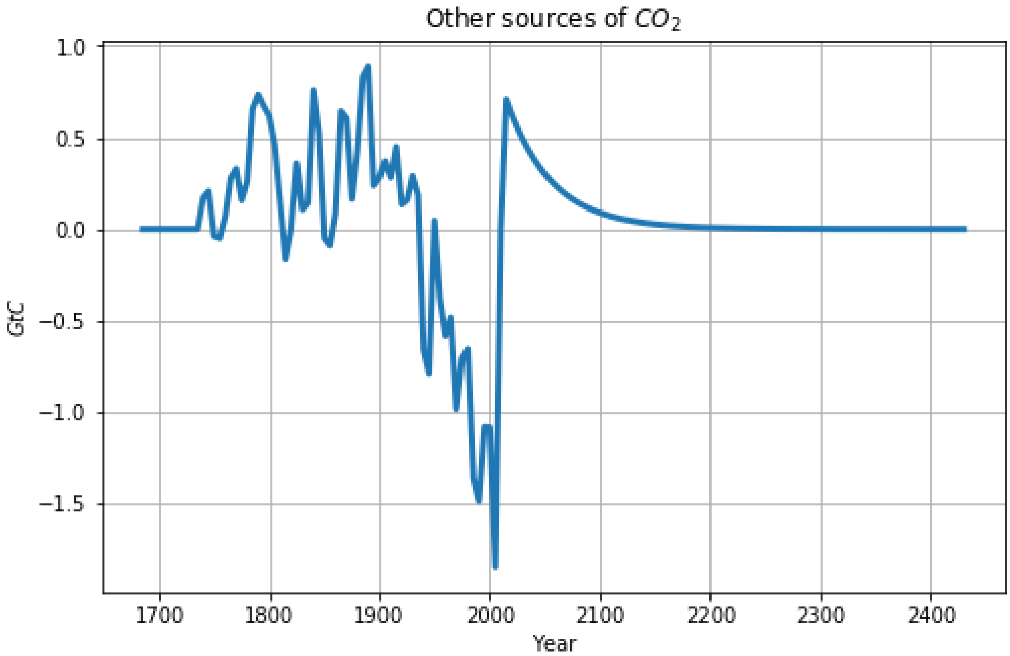

Figure 7: Emissions not taken into account in the model. Emissions from 1700 to 2010 come from the calibration and emissions from 2015 are the one of the DICE-2016 model [Nordhaus, 2017]

Production Regarding production, the elasticity values are taken from the above-mentioned articles. The values of productivity A, as well as the functions defining technical progress have been estimated in order of magnitude and then groped.

Climate system Finally, as far as the climate module is concerned, this is the element for which it is easiest and perhaps most important to carry out a rigorous calibration. From fossil fuel emission data ([Quéré et al., 2018]), atmospheric concentration in CO_2 and global mean surface temperature anomaly ([IPCC, 2014a]), it is possible to deduce precisely the time series of $CO_{2,t}^{ext}$ and RF_t^{ext} by inverting the dynamic system equations representing the climate:

$$\left\{ \begin{array}{rcl} CO_{2,t}^{ext} & = & m_{t+1}^{atm} - (c_{atm \longrightarrow atm} m_t^{atm} + c_{up \longrightarrow atm} m_t^{up}) - E_t^{Fossil} \\ m_{t+1}^{up} & = & c_{atm \longrightarrow up} m_t^{atm} + c_{up \longrightarrow up} m_t^{up} + c_{deep \longrightarrow up} m_t^{deep} \\ m_{t+1}^{deep} & = & c_{up \longrightarrow deep} m_t^{up} + c_{deep \longrightarrow deep} m_t^{deep} \end{array} \right.$$

and:

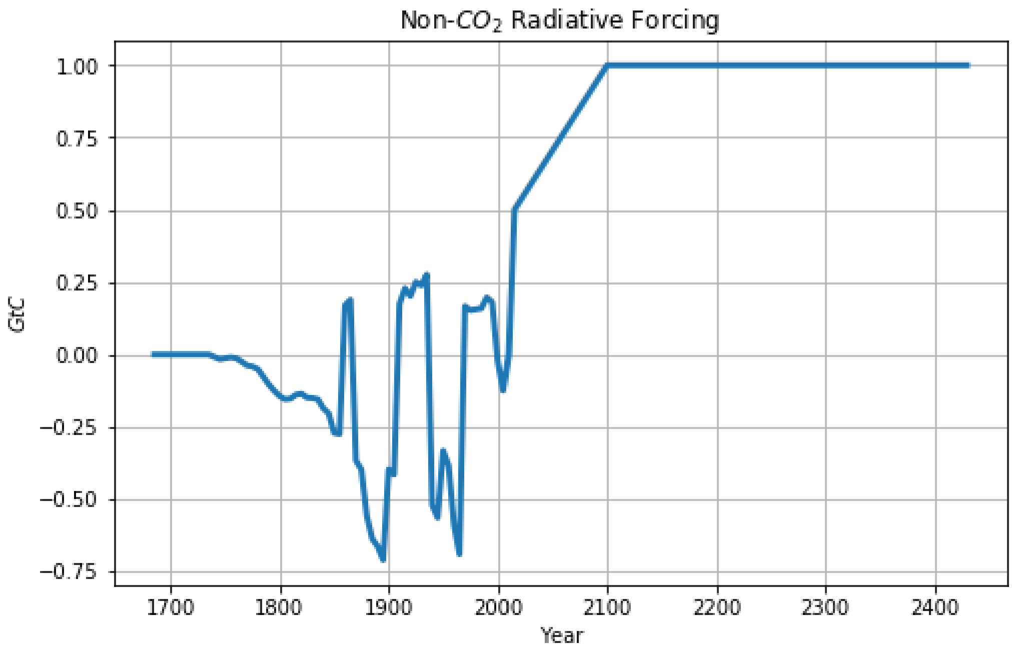

Figure 8: Non-Co_2 radiative forcing. Radiative forcing from 1700 to 2010 comes from the calibration and radiative frocing from 2015 is the one of the DICE-2016 model [Nordhaus, 2017]

$$RF_{t+1} = \frac{F_{2\times CO_2}}{log(2)} log\left(\frac{m^{atm}_{t+1}}{m^{atm}_{eq}}\right) + RF^{ext}_{t+1}$$

And then the dynamics of global mean surface temperature and mean ocean temperature are given by:

$$RF^{ext}_{t+1} = \frac{1}{c_1}(T^{atm}_{t+1} - T^{atm}_t) + \frac{F_{2\times CO_2}}{T^{atm}_{2\times CO_2}} T^{atm}_t + c_3(T^{atm}_t - T^{ocean}_t) - \frac{F_{2\times CO_2}}{log(2)} log\left(\frac{m^{atm}_{t+1}}{m^{atm}_{eq}}\right)$$

$$T^{ocean}_{t+1} = T^{ocean}_t + c_4(T^{atm}_t - T^{ocean}_t)$$

Then we consider the same scenario for $CO^{ext}_{2,t}$ and RF^{ext}_t as in the DICE-2016 code ([Nordhaus, 2017]), both drivers are presented in figure 7 and 8.

3.2 Results

The model has been developed on a Jupyter notebookCode in Python on which all demand and supply functions have been defined. Thus, it is possible to define the different excess demand functions, whose zero is found using the minimize function and the SLSQP algorithm ([Moré et al., 1980]).

Concerning the results, they are, in order of magnitude, of rather good quality and present in all cases a dynamic very close to that of the available historical data. More precisely, let us review the different stylized facts that interest us and study their correspondence with the results of the simulation.

First of all, we must point out the existence of a stable Malthusian state: if the technology remains stuck at its original level, then the dynamic system defined by our model converges towards a limit and this limit can be considered Malthusian insofar as from this point of equilibrium, an increase in per capita wealth is absorbed in the following period by an increase in population. This confers a Malthusian character on our model at the beginning of the simulation. In fact, we start our simulation at this equilibrium state from which, eventually, the population escapes thanks to the accumulation of human capital.

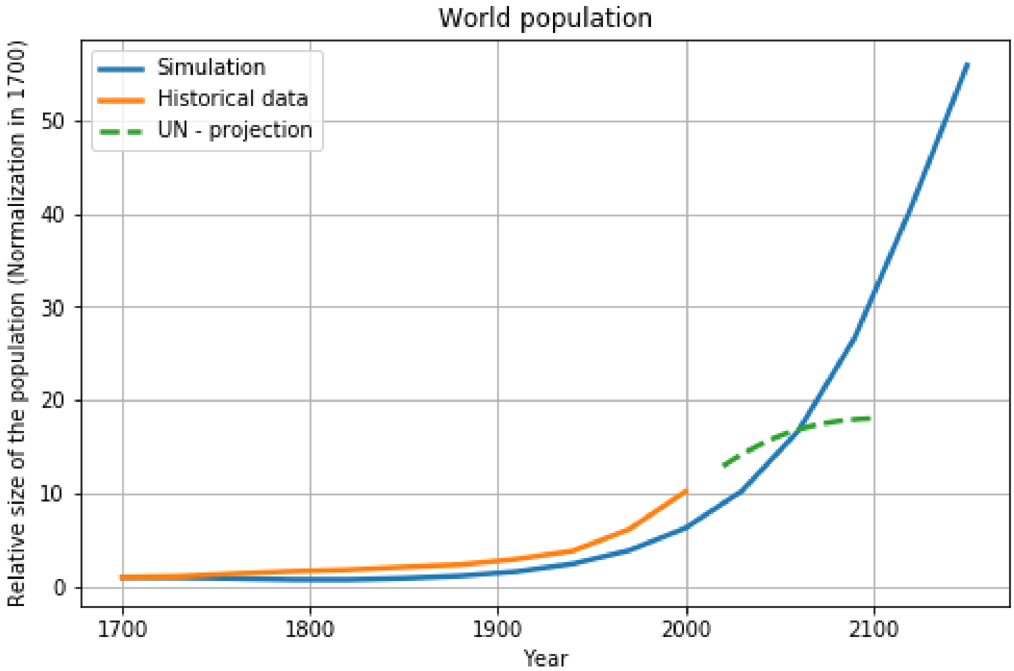

Figure 9: Population simulation, the population in 1700 is normalized to 1 - Sources: Hyde database [Klein Goldewijk et al., 2011], UN World Population Prospects 2019

Figure 10: Population growth rate simulation - Sources: Hyde database [Klein Goldewijk et al., 2011], UN World Population Prospects 2019

Now, regarding the population (9), the trajectory obtained is consistent (albeit with a small difference) with the historical record. However, the forecast is relatively absurd: it is highly unlikely that the population growth keep on such a pace[17]. On the other hand, by observing the simulated and real population growth rate (figure 10), the results appear to be very close to reality. Moreover, a decrease in the growth rate (inversed u-shaped growth rate) can be observed, which rather seems to signify the completion of the demographic transition. Thus the problem here is not a problem of model but a problem of calibration: the transition has to happen more quickly.

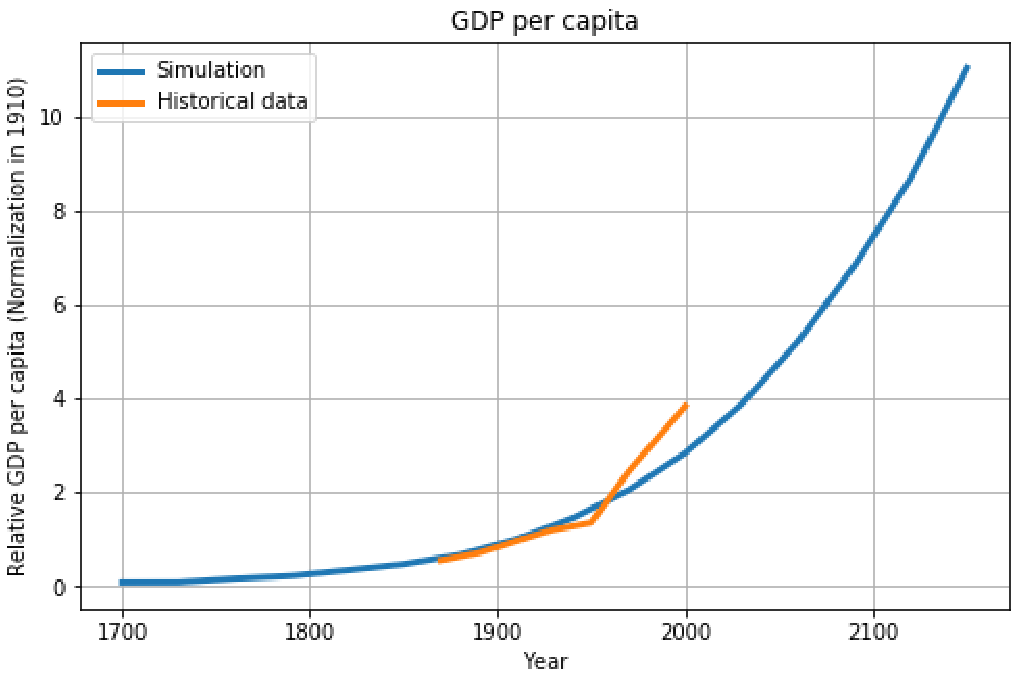

Figure 11: GDP per capita, the GDP per capita in 1910 is normalized to 1 - Source: Maddisson Project Database 2018 [Bolt et al., 2018]

Similarly, GDP per capita (figure 11 fits quite well with the historical data. In fact, it is however rather difficult to comment on this indicator, as there is very little aggregated data at the global level.

However, on the energy side, the results are very encouraging. The overall energy transition (figure 12) appears to be consistent with the real dynamics, both in terms of volume and distribution. Moreover, the consumption of coal on the historical record corresponds very well to that observed in the simulation and the projection corresponds to the discourse carried by [Golosov et al., 2014], namely that the energy of the future, if not oil, is likely to be coal. Concerning hydrocarbons (figure 14), the situation is good as well, in particular in term of dynamics at a ready multiplicative factor.

Finally, climate dynamics (figure 15) features a quite good fit on the historical data. Moreover the orders of magnitude reached in 2100 correspond to RCP8.5, which is certainly the most extreme scenario among all the possibilities considered, but is still conceivable, which means that the overall results of the model are far from being badIt is quite logical that if emissions are

[17]This would give a world population of about 16 billion people, which is certainly far from the 10.8 billion inhabitants forecast by the UN demography service, but in the end remains relatively close in order of magnitude.

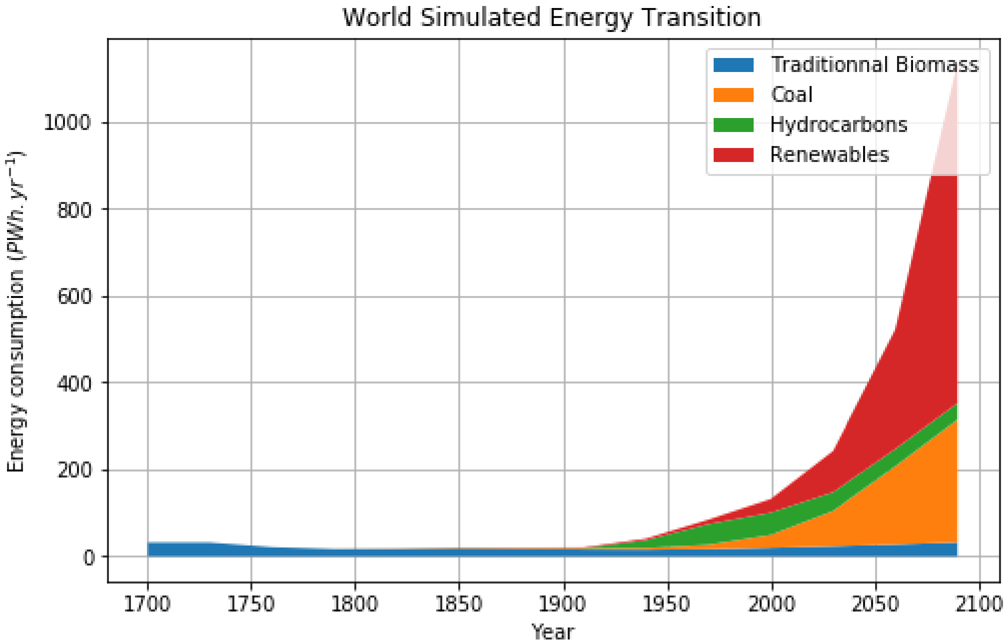

Figure 12: Energy transition simulation

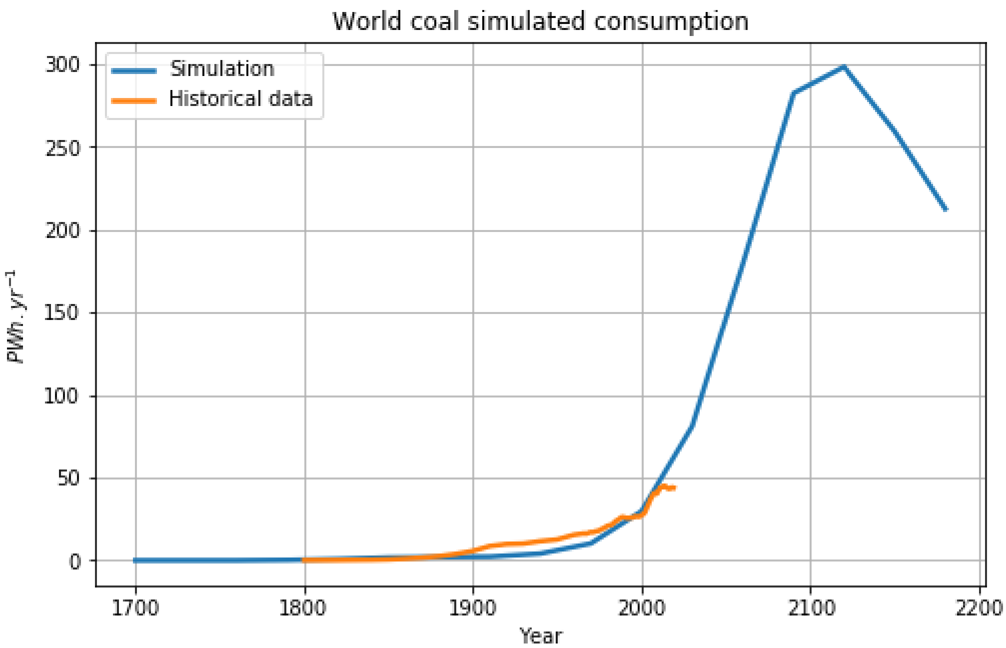

Figure 13: Coal consumption simulation - Source: [Smil, 2016], [British Petroleum, 2019a]

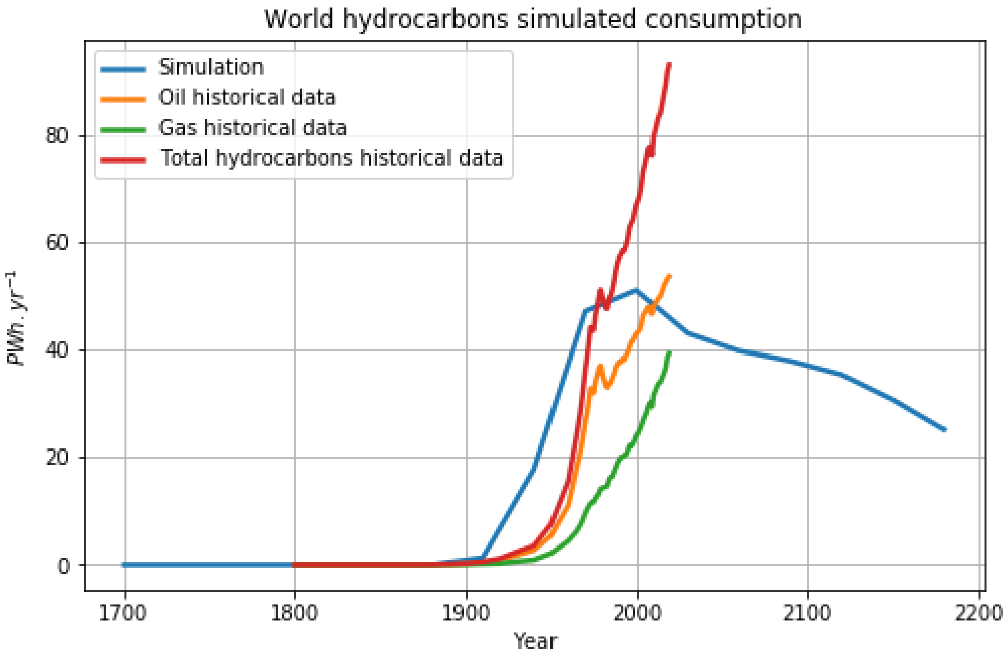

Figure 14: Hydrocarbon consumption simulation - Source: [Smil, 2016], [British Petroleum, 2019a]

globally consistent, then the concentration of CO_2 in the atmosphere as well as the variation in global mean surface temperature.

Overall, the results obtained, without rigorous calibration work, are rather good and indicate that there is a real interest in developing a rigorous and thoughtful calibration procedure because it would then be possible to obtain truly relevant results despite the simplicity and rigidity of the model.

4 Discussions and perspectives

In this section we summarise some of the issues of interest and criticisms that can be made of what has been proposed throughout this study.

4.1 A matter of disaggregation

First of all, as the result of the simulation suggests, the demographic transition appears too late, and in general it is difficult to fit the model's outputs to the world data. This is probably a problem arising from the fact that we consider an aggregation of states that are very heterogeneous with regard to the problems we are dealing with. Talking about world GDP per capita does not make much sense today because there are so many disparities between and within states, but what about this question at the end of the 19^{th} century? By carrying out this aggregation by ease (it is indeed simpler to start with a model with a closed economy before wanting to deal with the case of several economies with trade), the information given by the demographic, income and performance data is diluted and mixed in the worst possible way, which makes it complex to obtain a range of quantitative results of good quality. Thus, it would appear to be an interesting perspective to determine a relevant scale of aggregation,

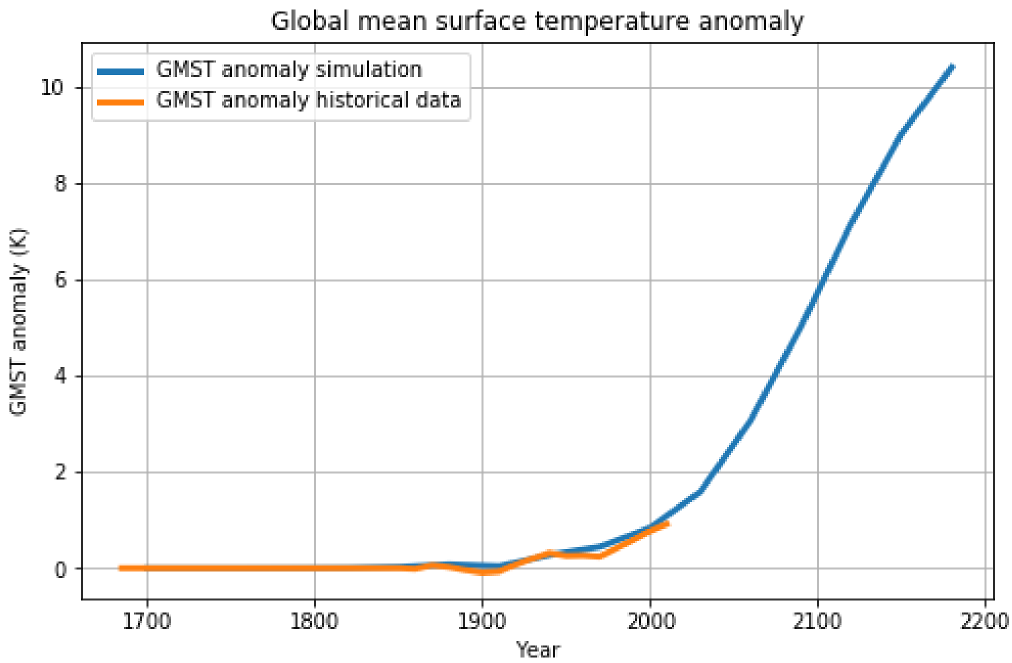

Figure 15: Global mean surface temperature anomaly - Source: [IPCC, 2014a]

particularly by considering areas where the demographic transition is relatively synchronised, such as all OECD countries or China.

4.2 Climate damages

As mentioned above, damage functions are one of the most strongly criticised elements in the field of climate economics ([Pinduyk, 2015]). Indeed, they are a real artifice because it is so difficult to predict future damage and because most estimates of damage functions are made by considering an extreme event (in terms of its duration), such as a particularly long heat wave. However, this way of estimating cannot structurally take into account the impact of a lasting deviation from the original equilibrium, which itself includes extreme events, since what is being estimated for an extreme event of the original balance[18]. If some people consider that in such circumstances it is better to do nothing ([Frisch, 2013]), I still consider it interesting to work with these tools, knowing their many limitations, because as the saying goes: "All models are wrong, but some are useful".

In our work in particular, we have adapted the damage function of [Desmet and Rossi-Hansberg, 2015] which was originally intended to assess damage in a given geographical location. We have therefore chosen to take the worst-case scenario and to define maximum productivity for a global mean surface temperature anomaly of zero, i.e. any warming causes damage. It is then quite difficult to qualitatively assess how well such a damage function is calibrated: 5% loss for 5 degrees of warming may seem too small, but 20% on agriculture may seem extremely important, although, if we don't take adaptation into account, perhaps the damage should be 30% and 60% respectively. Maybe even more, who knows? The only conclusion that can be drawn from this is that we need to multiply the experiments, and study different functions of damages that represent greater climate risk and study all the deviations that this causes.

[18]If this original balance exists.

4.3 Bounds of energy production

The estimates of the renewable energy potential given by [Banerjee et al., 2012] are just extremely important, several orders of magnitude beyond the total current consumption of primary energy (from 100 to 600 times greater). Within this framework, it is then necessary to estimate if these upper bounds are well accessible. Are there no other upper bounds that would be more basic, notably because of the costs of extracting minerals that would prevent the energy from being profitable because the construction of the power plant would not be possible. Seeing the results of the simulation, I couldn't help but think: is it really possible to have such a renewable energy? It seems essential to me (and this is the case for the other energy sources described here) to have a very rigorous approach (which is not the case for the moment) so as not to use any artifice that would deprive the model outputs of all their interest.

Conclusion

In accordance with the expressed objective, this paper presents an extremely stylized model allowing an endogenous interaction of economy, demography, energy use and environment over almost 300 years of history. While it is obvious that not everything is perfect, that many details need to be corrected, the proposed content is very promising. Indeed, the main stylized facts relating to the demographic transition as well as the industrial revolutions are correctly replicated. Many opportunities then open up, starting with the fact that a higher quality calibration can be carried out, allowing not only the replication of stylized facts but also the quantitative reproduction of these facts. On the other hand, it may be interesting from a public policy perspective to try to define a social cost of carbon within the framework of this model, which would make it possible to use this model in a perspective of evaluation and comparison of policies in an integrated manner: what are the consequences not only economic and climatic, but also demographic? And finally, it may also seem very interesting to further disaggregate the model at different levels. Spatially, this would make it easier to reconcile the different dynamics currently at work around the globe, but also to take international trade into account in the equation, at a time when major international trade agreements are strongly criticised by activists and populations alike. On the climate front, it would also be interesting to work on a finer and more robust model that also presents climate non-linearities as the major risks on which a great deal of uncertainty lies, such as the melting of permafrost.

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