

Unlike the other rounds, just getting the answer right is not enough on the Power Round. Make sure you explain your answer and use words to describe how you arrived at your answer. In the words of middle school math teachers across the nation – no work, no credit!

Feel free to use results from previous problems on this round to prove a later problem (that is, you can use Problem 2 to prove Problem 3, but not vice versa). You do not need to have solved the earlier problem to cite its result.

1 Introduction

Definition. An $n \times n$ *magic square* is an $n \times n$ grid with positive integers in each cell such that the sum of the integers in any row, the sum of the integers in any column, and the sum of the integers in either of the two long diagonals are all equal. This sum is called the *magic sum*.

Problem 1 (3=1+1+1 points). Determine whether each of the following is a magic square. If it is, determine the magic sum. If not, find two sums that are different.

1	4
2	3

(a)

17	17
17	17

(b)

4	2	9
3	7	5
8	6	1

(c)

Definition. A *standard* $n \times n$ magic square is one that uses the integers from 1 through n^2 , inclusive, each once.

Problem 2 (4 points). Fill in the missing entries below so that the result is a standard 4×4 magic square.

16	2	3	13
	7	6	
		15	1

Time limit: 30 minutes.

Problem 3 ($5=1+1+1+2$ points). This problem will guide you through computing the magic sum for a standard 3×3 magic square.

- (a) What are the nine numbers used for the entries of a standard 3×3 magic square?
- (b) What is the sum of these 9 numbers?
- (c) Let the magic sum be S . Recall that this equals the sum of the entries in *any* row. How many rows are there? In terms of S , what is the sum of all these rows?
- (d) Determine the value of S .

This method is called *double counting*, where you count the same thing in two different ways and equate them. In this case, you calculated the sum of all the entries in two different ways.

Problem 4 (3 points). Determine the magic sum for a standard 4×4 magic square.

2 2×2 Magic Squares

Problem 5 ($6=2+2+2$ points). In this problem, you will try to construct a standard 2×2 magic square.

- (a) What would be the magic sum of a standard 2×2 magic square?
- (b) List all the ways you can add two distinct numbers from the set $\{1, 2, 3, 4\}$ to get the magic sum.
- (c) Fill in the entries for a standard 2×2 magic square, or explain why this is impossible.

Problem 6 (4 points). Consider the partially-completed 2×2 *non-standard* magic square.

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Fill in the missing entries, justifying each step and explaining why your solution is the only possible one.

Problem 7 (4 points). Classify all possible 2×2 magic squares with positive integer entries. The one above should be included.

3 Magic Rectangles

Definition. An $m \times n$ *magic rectangle* is a grid with m rows and n columns and positive integers in each cell such that the sum of the integers in each row is the same and the sum of the integers in each column is the same, but these two sums do not necessarily have to be the same. The first is called the *row sum* and the second is called the *column sum*.

Problem 8 (4=2+2 points). Determine whether each of the following is a magic rectangle. If it is, determine the row sum and column sum. If not, find two rows or two columns with different sums.

1	6
5	2
3	4

(a)

4	2	9
3	7	5
8	6	1

(b)

Problem 9 (4 points). Show that any magic rectangle whose row sum equals the column sum must, in fact, be in the shape of a square (so $m = n$ in the definition). Does it have to be a magic square?