29^{th} TJIMO
ALEXANDRIA, VIRGINIA

Round: **Guts**

Set 1

Problem 1. What is $1 + 2 + 3 + 4 + \cdots + 100$?

Problem 2. Miranda is picking out skirts and crop tops to wear. If she has 5 skirts and 7 crop tops, how many outfits can she make?

Problem 3. What is the least common multiple of 6 and 4?

Problem 4. How many multiples of 7 are between 100 and 200?

Time limit: 40 minutes.

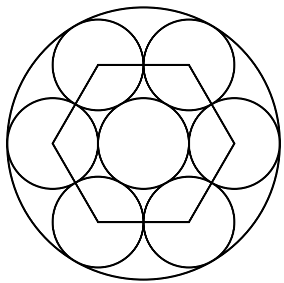
Set 2

Problem 5. Gibbons drinks $\frac{1}{3}$ of the water in a bottle, and Ogden drinks $\frac{1}{4}$ of the remaining water. If there are 8 milliliters of water left, how much water (in milliliters) was there to begin with?

Problem 6. I am drawing marbles out of a bag. If the bag contains 3 red marbles and 5 blue marbles, then what is the probability that the second marble I draw is red?

Problem 7. How many positive integer factors does 27 have?

Problem 8. If the radius of the larger circle is 6, and all the smaller circles are the same size, then what is the side length of the hexagon?



Time limit: 40 minutes.

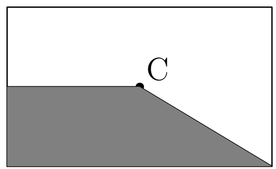
Set 3

Problem 9. If $A + B = 4$, $C + D = 8$, and $A + C = 7$, what is $B + D$?

Problem 10. There is a 40% chance of rain on Saturday. If it rains on Saturday, there is a 60% chance of rain on Sunday. Otherwise, there is a 30% chance of rain on Sunday. What is the probability it rains on Sunday?

Problem 11. The least common multiple of the first ten positive integers is 2520. What is the least common multiple of the first eleven positive integers?

Problem 12. Point C is the center of the rectangle shown below. If the area of the whole rectangle is 128, find the area of the shaded region.



Time limit: 40 minutes.

Set 4

Problem 13. Let the function $f(x)$ be defined as $f(x) = 3x^2 + 2x + 3^{800}$. Find $f(3) - f(2)$.

Problem 14. Suppose a and b are positive integers such that $ab = 1296$ and neither a nor b is divisible by 6. Find $a + b$.

Problem 15. Let a be a solution to the equation $x^2 + 7x - 16 = 0$. Find the product of a and $a + 7$.

Problem 16. How many ways can I seat Josh, Akshaj, Jeffery, Katherine, Michael, and Wendy at a circular table so that Wendy and Katherine are sitting next to each other?

Time limit: 40 minutes.

Set 5

Problem 17. For an art project, Katherine is building a model of TJ. If the real auditorium is 30 feet wide but 2 inches wide in Katherine's model, how long in inches should the model gym be if the real gym is 75 feet long?

Problem 18. A 5-year-old, a 6-year-old, a 7-year-old, and an 8-year-old come to Akshaj's house for trick-or-treating during Halloween. Akshaj has 7 identical pieces of candy to give out. How many ways can Akshaj distribute his 7 pieces of candy among the four children, assuming each child must receive at least one piece of candy?

Problem 19. What is the probability that if I flip a coin 7 times in a row, I get heads at least 4 times?

Problem 20. Gideon is facing trial, but he only has 10 dollars. If Wainwright offers lawyers for 3 dollars and evidence for 2 dollars each, how many combinations of lawyers and evidence can Gideon buy, assuming that he needs to buy a lawyer and he does not need to spend all 10 dollars? (For example, Wainwright can buy one lawyer and no evidence.)

Set 6

Problem 21. Seven not necessarily distinct integers in the set $\{1, 2, 3, \ldots, 15\}$ have a median of 8 and a unique mode of 9. What is the largest possible mean of these seven integers? Express your answer as a common fraction.

Problem 22. What is the smallest positive integer that gives a remainder of 1 when divided by 3, a remainder of 3 when divided by 5, and a remainder of 5 when divided by 7?

Problem 23. Let $ABCD$ and $EFGH$ be squares such that E lies on $\overline{AB}$, F lies on $\overline{BC}$, G lies on $\overline{CD}$, and H lies on $\overline{AD}$. If the area of $EFGH$ is $\frac{5}{9}$ of the area of $ABCD$ and $AE \geq BE$, what is $\frac{AE}{BE}$?

Problem 24. Katherine, Neeyanth, Wendy, and Ryan are arguing over who is a freshman and who is an upperclassman. Freshmen always lie, while upperclassmen always tell the truth.

Neeyanth says: Ryan is a freshman.

Ryan says: Out of Neeyanth and I, exactly one of us is a freshman.

Wendy says: There are not more freshmen than upperclassmen.

Katherine says: At most three of us are freshman.

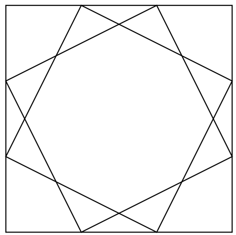
Who are all of the freshmen?

Time limit: 40 minutes.

Set 7

Problem 25. If $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\cdots=\frac{\pi^2}{6}$, what is $\frac{1}{2^2}+\frac{1}{4^2}+\frac{1}{6^2}+\ldots$?

Problem 26. How many triangles are in the following figure?



Problem 27. When 2017 is multiplied by a single-digit nonzero integer x, all the digits are perfect squares. What is x?

Problem 28. Let each letter represent a distinct digit. If $J=8$ and A is even, what is the five digit number $MAGIC$?

$$\begin{array}{cccccc} & \mathrm{T} & \mathrm{J} & \mathrm{I} & \mathrm{M} & \mathrm{O} \\ + & & \mathrm{M} & \mathrm{A} & \mathrm{T} & \mathrm{H} \\ \hline & \mathrm{M} & \mathrm{A} & \mathrm{G} & \mathrm{I} & \mathrm{C} \end{array}$$

Time limit: 40 minutes.

Set 8

Problem 29. What is $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\cdots+\frac{1}{29\cdot 30}$?

Problem 30. Allen, Bobby, Charles, and 49 other people are in a math class together. The teacher randomly gives each of the 52 students one card each from a single deck of 52 cards (everyone receives a different card). Any student who receives either an ace, king, queen, or jack is required to do extra homework. What is the probability that at least one of Allen, Bobby, and Charles has to do extra homework? (Note: There are four aces, four kings, four queens, and four jacks in a standard deck of 52 cards.)

Problem 31. Franklyn can finish two cryptography assignments in 10 minutes, Katherine can finish one in 30 minutes, and Will Sun can finish one in 15 minutes. How long (in hours) will it take for them working together to finish six cryptography assignments?

Problem 32. How many times in a day does the internal angle bisector (the bisector of the acute angle) between the hour hand and the minute hand on a clock pass through the 12 or the 6?