29th TJIMO

Alexandria, Virginia

Round: **Practice Team**

Problem 1. Martha is making a banana shake using a recipe she found online. The recipe involves 2 cups of water and 3 bananas and produces 2 liters of banana shake. If Martha wants to make 6 liters of banana shake, how many bananas does she need?

Problem 2. Michael wants to tile his 6 feet by 10 feet floor with 4-inch by 3-inch tiles. How many tiles does he need? (There are 12 inches in a foot).

Problem 3. We know that $15 = 5+5+5$, $16 = 4+4+4+4$, and $17 = 5+4+4+4$. What is the largest positive integer that cannot be expressed as the sum of 4s and 5s?

Problem 4. If m and n are positive integers satisfying $m^2 - n^2 = 28$ and $mn = 48$, then what is $m^2 + n^2$?

Problem 5. William has a jar of jellybeans. If he splits them into groups of 6, he has 1 jellybean left. If he splits them into groups of 11, he has 6 jelly beans left. What is the minimum number of jellybeans William has?

Problem 6. After playing a game together, Justin and Joshua calculate each of their scores and the winner is the individual with the higher score. Justin's score is equivalent to $1 + 3 + 5 + \cdots + 999$ while Joshua's score is equal to $2 + 4 + 6 + \cdots + 1000$. How many more points did the winner have than the loser?

Problem 7. How many different ways are there to scramble the letters in RACER? One way is "ACERR" and another is "RACER".

Problem 8. Jeffery draws triangle A and draws the midpoint of each side. He then connects the three midpoints to form another triangle, B. Jeffery repeats this process for the new smaller triangle to obtain triangle C. What is the ratio of the area of triangle C to the area of triangle A?

Problem 9. Jonathan has two bags of marbles. In the first bag, there are 3 red marbles and 5 blue marbles. In the second bag, there are 4 red marbles and 7 blue marbles. What is the probability that Jonathan picks a blue marble from the first bag and a red marble from the second bag?

Problem 10. Square $ABCD$ has side length 1. Let M be the midpoint of side $\overline{AB}$, and let the line segments $\overline{DB}$ and $\overline{CM}$ intersect at point E inside the square. Determine the area of $\triangle BCE$. Express your answer as a common fraction.

Time limit: 30 minutes.