Problem 1. Martha is making a banana shake using a recipe she found online. The recipe involves 2 cups of water and 3 bananas and produces 2 liters of banana shake. If Martha wants to make 6 liters of banana shake, how many bananas does she need?
Answer. 9
Solution. Because we need 6 liters of shake and the recipe produces 2 liters, we need to use $6/2 = 3$ times as much water and 3 times as many bananas. Thus we need $3 * 3 = \boxed{9}$ bananas.

Problem 2. Michael wants to tile his 6 feet by 10 feet floor with 4-inch by 3-inch tiles. How many tiles does he need? (There are 12 inches in a foot).
Answer. 720
Solution. The total square footage of the floor is 60 square feet. Each tile is $\dfrac{1}{3}$ feet by $\dfrac{1}{4}$ feet, with a total area of $\dfrac{1}{12}$ square feet. Thus we need $\frac{60}{1/12} = \boxed{720}$.

Problem 3. We know that $15 = 5+5+5$, $16 = 4+4+4+4$, and $17 = 5+4+4+4$. What is the largest positive integer that cannot be expressed as the sum of 4s and 5s?
Answer. 11
Solution. We see that 15, 16, and 17 can all be expressed as the sum of 4s and 5s. We can also check that $14 = 5+5+4$, $13 = 5+4+4$, and $12 = 4+4+4$. However, there is no way to express 11 as the sum of 4s and 5s. To prove that 11 is the largest such integer, notice that any positive integer greater than 11 can be formed by repeatedly adding 4 to one of $\{12, 13, 14, 15\}$. Thus, $\boxed{11}$ is the largest such positive integer.

Problem 4. If m and n are positive integers satisfying $m^2 - n^2 = 28$ and $mn = 48$, then what is $m^2 + n^2$?
Answer. 100
Solution. By guessing and checking, we can find $m = 8$ and $n = 6$, so $8^2 + 6^2 = 100$. Alternatively, notice $(m^2 - n^2)^2 + 4(mn)^2 = (m^2 + n^2)^2$, so $28^2 + 4(48)^2 = (m^2 + n^2)^2$. Thus, $(m^2 + n^2)^2 = 10000$, and since m, n are positive integers, $m^2 + n^2 = \sqrt{10000} = \boxed{100}$.

Problem 5. William has a jar of jellybeans. If he splits them into groups of 6, he has 1 jellybean left. If he splits them into groups of 11, he has 6 jelly beans left. What is the minimum number of jellybeans William has?
Answer. 61
Solution. Suppose we give William 5 more jellybeans. Then he would be able to evenly split his jellybeans into groups of 6 or 11. Thus, he must now have a number of jellybeans that is divisible by 6 and 11. The smallest such number is 66. Thus, originally the minimum possible number of jellybeans William could have had is $66 - 5 = \boxed{61}$.

Problem 6. After playing a game together, Justin and Joshua calculate each of their scores and the winner is the individual with the higher score. Justin's score is equivalent to $1 + 3 + 5 + \cdots + 999$ while Joshua's score is equal to $2 + 4 + 6 + \cdots + 1000$. How many more points did the winner have than the loser?

Answer. 500
Solution. First, notice that each player had a total of 500 numbers. Additionally, each of Joshua's numbers was 1 greater than that of Justin's. $(2-1=1,\ 4-3=1,\ 6-5=1$, etc.) That means that Joshua won by $500*1=\boxed{500}$ points.

Problem 7. How many different ways are there to scramble the letters in RACER? One way is "ACERR" and another is "RACER".
Answer. 60
Solution. There are five ways to choose what place to put the A, four ways to choose what place to put the C, and three ways to choose what place to put the E. Then, the remaining letters are R. Thus, our answer is $5\cdot 4\cdot 3=\boxed{60}$.

Problem 8. Jeffery draws triangle A and draws the midpoint of each side. He then connects the three midpoints to form another triangle, B. Jeffery repeats this process for the new smaller triangle to obtain triangle C. What is the ratio of the area of triangle C to the area of triangle A?
Answer. $\frac{1}{16}$
Solution. It is well known that the connection of the midpoints of a triangle forms another triangle with $\frac{1}{4}$ of the area of the original. This implies that the area of B is $\frac{1}{4}$ of the area of A, and the area of C is $\frac{1}{4}$ of the area of B. Thus, the area of C is $\frac{1}{4}\cdot\frac{1}{4}=\boxed{\frac{1}{16}}$ of the area of A.

Problem 9. Jonathan has two bags of marbles. In the first bag, there are 3 red marbles and 5 blue marbles. In the second bag, there are 4 red marbles and 7 blue marbles. What is the probability that Jonathan picks a blue marble from the first bag and a red marble from the second bag?
Answer. $\frac{5}{22}$
Solution. The probability of picking a blue from the first bag is $\frac{5}{8}$, while the probability of picking a red from the second is $\frac{4}{11}$, so the probability of both of these events occurring is $\frac{5}{8}\cdot\frac{4}{11}=\boxed{\frac{5}{22}}$.

Problem 10. Square $ABCD$ has side length 1. Let M be the midpoint of side $\overline{AB}$, and let the line segments $\overline{DB}$ and $\overline{CM}$ intersect at point E inside the square. Determine the area of $\triangle BCE$. Express your answer as a common fraction.
Answer. $\frac{1}{6}$
Solution. Note that triangles $\triangle MBE$ and $\triangle CDE$ are similar by AA similarity (because MB is parallel to CD). Since $\frac{MB}{CD}=\frac{1}{2}$, $\frac{BE}{DE}=\frac{1}{2}$. Notice that since both $\triangle BEC$ and $\triangle DEC$ have the same height with respect to sides BE and DE, so the ratio of their areas is the same as the ratio of BE to DE, which is $\frac{1}{2}$. Since $\triangle DEC$ is twice the area of $\triangle BEC$, we know that $\triangle DBC$ is three times the area of $\triangle BEC$. The area of $\triangle DBC$ is just half the total area, or $\frac{1}{2}$, so the area of $\triangle BCE$ is $\frac{1}{3}\cdot\frac{1}{2}=\boxed{\frac{1}{6}}$.

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