29th TJIMO

Alexandria, Virginia

Round: **Individual**

Problem 1. Compute $2^0 + 1 \cdot 7 + 2(0 \cdot 1 + 8)$.

Answer. $\boxed{24}$
Solution. We follow the order of operations to obtain

$$\begin{aligned} 2^0 + 1 \cdot 7 + 2(0 \cdot 1 + 7) &= 1 + 7 + 2(0 + 8) \\ &= 8 + 2(7) \\ &= 8 + 16 \\ &= \boxed{24}. \end{aligned}$$

Problem 2. NAME turned 14 years old in the year 2017. Determine in what year NAME's age will double.

Answer. $\boxed{2031}$
Solution. NAME's age will double in 14 years, or in the year $2017 + 14 = \boxed{2031}$.

Problem 3. A fence encloses a square region with area 2025 ft^2. Compute the perimeter of the fence, in ft.

Answer. $\boxed{180}$ (ft)
Solution. The area of a square with side length s is s^2, so the length of one side enclosed by the fence is $\sqrt{2025 \text{ ft}^2} = 45$ ft. There are four sides, so the perimeter is four times the side length, which is $4 \cdot 45 \text{ ft} = \boxed{180}$ ft.

Problem 4. Today is Saturday, October 28, 2017. Determine the day of the week on which Halloween falls *next* year (October 31, 2018).

Answer. $\boxed{\text{Wednesday}}$
Solution. Since 2018 is not a leap year, October 28, 2018 is 365 days after October 28, 2017. As $365 = 7(52) + 1$ is 1 more than a multiple of 7, October 28, 2018 will be one day after Saturday, or Sunday. October 31 is 3 more days later, which is $\boxed{\text{Wednesday}}$.

Problem 5. Compute the number of positive three-digit integers without leading zeros (so 042 does not count) that have either three even digits or three odd digits.

Answer. $\boxed{225}$
Solution. If the number has three even digits, then the hundreds digit can be either $2, 4, 6$, or 8, for 4 choices, and the tens and ones digit can each be any of $0, 2, 4, 6, 8$, for 5 choices each. This gives $4 \cdot 5 \cdot 5 = 100$ integers for this case.

If the number has three odd digits, then each of the three digits can be any of $1, 3, 5, 7$, or 9, for 5 choices each. This gives $5 \cdot 5 \cdot 5 = 125$ integers for this case.

Time limit: 60 minutes.

In total, we have $100 + 125 = \boxed{225}$ such numbers.

Problem 6. There are 100 students at a math competition. Some teams consist of 5 students while others have 6 students. If there are a total of 18 teams, compute the number of teams with 6 students.

Answer. $\boxed{10}$ (teams)
Solution.

Problem 7. The STORE NAME store sells packs of 5 highlighters for \$3 and packs of 6 highlighters for \$3.50. Compute the maximum number of highlighters that may be purchased with \$41.

Answer. $\boxed{70}$ (highlighters)
Solution. We compare the two deals by comparing the unit price per highlighter for each. The pack of 5 highlighters has a unit price of $\frac{\$3}{5 \text{ highlighters}} = \frac{36}{60}$ dollar per highlighter, while the pack of 6 highlighters has a unit price of $\frac{\$3.50}{6 \text{ highlighters}} = \frac{35}{60}$ dollar per highlighter, so the 6-pack is a better deal. Hence we should start by buying packs of 6 highlighters. However, once we buy 10 of these, we are left with $\$41 - 10(\$3.50) = \$6$. At this point, we can either buy one more 6-pack or two 5-packs. The latter option gives more highlighters, so we can buy a maximum of $10(6) + 2(5) = \boxed{70}$ highlighters.