Set 1

Problem 1. What is $1 - 2 + 3 - 4 + 5 - 6 + 7 - 8 + \cdots + 19 - 20$?
Answer. -10
Solution. Note that $1 - 2 = 3 - 4 = 5 - 6 = \cdots = 19 - 20 = -1$. Since there are 10 of these such terms, we get that the answer is $10 \cdot -1 = \boxed{-10}$.

Problem 2. What is the area of a triangle with side lengths 5, 12, and 13?
Answer. 30
Solution. Note that $5^2 + 12^2 = 13^2$, so by the Pythagorean Theorem this is just a right triangle with base lengths 12 and 5. The area is thus $\frac{5 \cdot 12}{2} = \boxed{30}$.

Problem 3. How many numbers are between 505 and 700, inclusive?
Answer. 196
Solution. If we subtract 504 from each of the numbers, we get all the numbers from 1 to 196, inclusive. There are 196 of these numbers, so there must have been $\boxed{196}$ numbers in the original set.

Problem 4. What is the greatest common factor of 117 and 156?
Answer. 39
Solution. The greatest common factor of 117 and 156 is the same as the greatest common factor between 117 and $156 - 117 = 39$. Since $117 = 39(3)$, the greatest common factor must be $\boxed{39}$

Set 2

Problem 5. It takes Marbury 1 hour to deliver 6 letters and Madison 3 hours to deliver 60 letters. How many letters can they deliver in an 8 hour work day?
Answer. 208
Solution. Marbury can deliver $6(8) = 48$ letters in 8 hours, and Madison can deliver $\frac{60}{3}(8) = 160$ letters in 8 hours. Summing these up yields $\boxed{208}$ letters in an 8 hour work day.

Problem 6. In quadrilateral $ABCD$, $\angle DAC = 75°$, $\angle ACB = 40°$, $\angle DBC = 75°$, and $\angle BDC = 25°$. Find the measure of angle $\angle DCA$.
Answer. $40°$
Solution. We observe that $\angle DCA = 180° - \angle DBC - \angle ACB - \angle BDC = 40°$.

Problem 7. What are the sum of the factors of 16?
Answer. 31
Solution. Note that the factors of $16 = 2^4$ are 1, 2, 4, 8, and 16. Summing these yields 31.

Problem 8. There are 100 people in math team. If 53 of them do cross country, 27 of them do art club, and 38 of them do neither, how many do both?

Answer. 18
Solution. If 38 do neither, then $100 - 38 = 62$ must do either art club or cross country. Since 53 do cross country, only $62 - 53 = 9$ must do only art club, leaving $27 - 9 = \boxed{18}$ that do both.

Set 3

Problem 9. In a round robin tournament, everyone competes against everyone else. If there are 8 teams, how many matches are there?
Answer. 28
Solution. If every team plays against everyone else, each team plays against 7 other teams. We can sum this over the 8 teams, but since each match has 2 teams playing simultaneously, each match is counted twice in this sum. We thus divide by 2 for a total of $\frac{(8)(7)}{2} = \boxed{28}$ total matches.

Problem 10. If I roll three die, what is the probability the numbers on the three die sum to 16?
Answer. $\frac{1}{36}$
Solution. The only ways to sum to 16 are if I roll $5, 5, 6$, or a $4, 6, 6$. These each have a probability of $\frac{3}{216}$ of occurring, because they can both occur in 3 ways and every roll has probability $\frac{1}{6^3} = \frac{1}{216}$. Summing this up, we have the probability that they sum to 16 is equal to $\frac{6}{216} = \boxed{\frac{1}{36}}$

Problem 11. I have a rectangle with perimeter 36. What is the maximum possible area of the rectangle?
Answer. 81
Solution. Let the side lengths of the rectangle be x and y. Since the perimeter is 36, we see that $x + y$ is 18. Using the AM-GM inequality, we see that $\dfrac{x+y}{2} \geq \sqrt{xy}$. Thus, the maximum possible value of xy, or the area of the rectangle, is $\left(\dfrac{18}{2}\right)^2 = \boxed{81}$

Problem 12. Let $20ABC16$ be a perfect square, with A, B, and C as digits. What is the three-digit number ABC?
Answer. 909
Solution. Note that $\sqrt{2000000} = 1000\sqrt{2}$, or about 1414. Let $20ABC16$ be equal to x^2. We will now use modular arithmetic. $20ABC16$ is congruent to $16 \pmod 1)00$, so it must be congruent to to $16 \equiv 0 \pmod 4)$ and $16 \pmod 2)5$. This leads us to the conclusion that x is either 0 or $2 \mod 4$, and either 4 or $21 \mod 25$. Thus, the last two digits of x must be either $04, 46, 54$, or 96. Note that 1404 is too small, and trying 1446 yields 2090916, giving us our desired answer of $\boxed{909}$.

Time limit: 15 minutes.