Problem 1. There are 200 animals on a farm. Some are chickens and some are horses. If there are 450 legs in total, how many horses are there? (Chickens have two legs each, and horses have four legs each.)
Answer. 25
Solution. Let c be the number of chickens, and h be the number of horses. We have the following system of equations:

$$c + h = 200 \tag{1}$$

$$2c + 4h = 450 \tag{2}$$

We can multiply equation (1) by 2 to match the coefficient of c, and subtract the result from equation (2) to get $2h = 50$, or $h = \boxed{25}$.

Problem 2. There are n students in a class. The teacher can divide them evenly into groups of 2, 3, 4, 5, and 7. If $1000 < n < 1500$, find n.
Answer. 1260
Solution. We first want to find the least possible number n that divides 2, 3, 4, 5, and 7, which is just $\operatorname{lcm}(2, 3, 4, 5, 7)$. We can realize that 4 divides 2 evenly, and 3, 4, 5, and 7 have no common factors, so this is equal to $4 \cdot 3 \cdot 5 \cdot 7 = 420$. Notice that any number n that can be the answer must be a multiple of 420 in order to be divisible by 2, 3, 4, 5, and 7. We see that $\boxed{1260}$ is the multiple of 420 in our range.

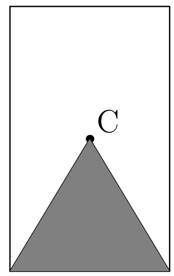
Problem 3. An integer is chosen randomly from the first 2017^2 positive integers. What is the probability that the chosen integer is a perfect square?
Answer. $\frac{1}{2017}$
Solution. Note that between 1 and 2017^2 there are 2017 squares out of a total 2017^2 positive integers. This gives a probability of $\frac{2017}{2017^2} = \boxed{\frac{1}{2017}}$

Problem 4. What is the smallest positive integer that is

- greater than 1
- not prime
- not divisible by 2, 3, or 5?

Answer. 49
Solution. Note that the first integer prime after 2, 3, and 5 is 7. However, because we cannot have our integer be prime, we have to multiply it by another prime. Again, because it cannot be divisible by 2, 3, and 5, we include another factor of 7, so our answer is $7 \cdot 7 = \boxed{49}$.

Problem 5. Point C is the center of the rectangle shown below. If the area of the whole rectangle is 80, find the area of the shaded region.



Answer. 20
Solution. Let l be the length and h be the height. The height of the triangle is $\frac{h}{2}$ because C is the center of the rectangle. L is still the length, so the area of our triangle is $\frac{1}{2} \cdot l \cdot \frac{h}{2} = \frac{l \cdot h}{4} = \boxed{20}$, since $l \cdot h$ is the area of the rectangle, which is 80.

Problem 6. A four-digit positive integer $\overline{ABCD}$ is called *funny* if each of its digits is between 1 and 9, and both $\overline{ABC}$ and $\overline{BCD}$ are perfect squares. For example, 1441 is *funny* because $144 = 12^2$ and $441 = 21^2$, but 9009 is not *funny* because it has zeros as digits. What is the largest four-digit *funny* integer?
Answer. 7841
Solution. We want to prioritize maximizing the thousands, hundreds, and tens digits because those impact the number more than the units digit. We iterate backward from 31^2, the largest 3-digit square, backwards. $31^2 = 961$, but no square has the first 2 digits of 61. This is also true for $30^2 = 900$, and $29^2 = 841$. However, $28^2 = 784$, and as seen by 29^2, there is a square with first 2 digits 84. Thus, our maximized answer is $\boxed{7841}$.

Problem 7. How many positive two-digit integers are $\frac{4}{7}$ of their "reverse"? (For example, 48 is $\frac{4}{7}$ of 84, the number formed by swapping the digits of 48.)
Answer. 4
Solution. Let our number be $\overline{AB}$. We want numbers such that $7 * \overline{AB} = 4 * \overline{BA}$. Expanding our number using $\overline{AB} = 10A + B$, we get $66A = 3B$, or $A = 2B$. Because A and B are digits, they are less than 10, so the total number of solutions is $\boxed{4}$, namely 12 and 21, 24 and 42, 36 and 63, and 48 and 84.

Problem 8. Suppose x, y are positive real numbers such that

$$xy + x^2 = 23$$

$$xy + y^2 = 26.$$

Compute $x + y$.
Answer. 7
Solution. Summing these two equations yields $x^2 + 2xy + y^2 = 49$, so $(x + y)^2 = 49$, and since x and y are positive, we conclude $x + y = \boxed{7}$.

Time limit: 30 minutes.

Problem 9. Dan the Doubler has an interesting way of doubling numbers. He takes each digit of a number, doubles it, and then puts the answers together. For example, he "doubles" 666 to get 121212 and "doubles" 202 to get 404. For how many three-digit integers from 100 to 999, inclusive, does Dan get the correct answer using his doubling method? (For example, $202 \times 2 = 404$ is correct, but $666 \times 2 = 121212$ is incorrect.)
Answer. 25
Solution. Note that for any digit less than 5, the double only consists of 1 digit, so there wouldn't be any overlap. Thus and combination of digits less than 5 works, so there are 9 options for the hundreds digit (any number works for the hundreds digit, as the carryover goes freely to the thousands place), 5 options for the tens digit, and 5 options for the units digit, yielding a total of $\boxed{225}$ total numbers.

Problem 10. The product of the lengths of the three altitudes of a certain triangle is 24. If this triangle has an area of 3, then determine the product of the three side lengths of this triangle. (The altitudes of a triangle are the line segments connecting the vertices and the feet of the perpendiculars from those vertices to the opposite sides.)
Answer. 9
Solution. Let a, b, c be the length of the sides of the triangle, and h_a, h_b, h_c be the lengths of the altitudes of sides a, b, and c respectively. We know $h_a \cdot h_b \cdot h_c = 24$, and $a \cdot h_a = 6$, $b \cdot h_b = 6$, and $c \cdot h_c = 6$. Multiplying the last 3 equations and dividing by the first equation yields $abc = \boxed{9}$