29th TJIMO

Alexandria, Virginia

Round: **Team**

Problem 1. There are 200 animals on a farm. Some are chickens and some are horses. If there are 450 legs in total, how many horses are there? (Chickens have two legs each, and horses have four legs each.)

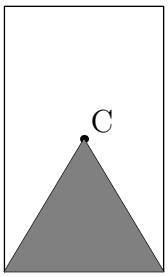
Problem 2. There are n students in a class. The teacher can divide them evenly into groups of 2, 3, 4, 5, and 7. If $1000 < n < 1500$, find n.

Problem 3. An integer is chosen randomly from the first 2017^2 positive integers. What is the probability that the chosen integer is a perfect square?

Problem 4. What is the smallest positive integer that is

- greater than 1
- not prime
- not divisible by 2, 3, or 5?

Problem 5. Point C is the center of the rectangle shown below. If the area of the whole rectangle is 80, find the area of the shaded region.



Problem 6. A four-digit positive integer $\overline{ABCD}$ is called *funny* if each of its digits is between 1 and 9, and both $\overline{ABC}$ and $\overline{BCD}$ are perfect squares. For example, 1441 is *funny* because $144 = 12^2$ and $441 = 21^2$, but 9009 is not *funny* because it has zeros as digits. What is the largest four-digit *funny* integer?

Problem 7. How many positive two-digit integers are $\frac{4}{7}$ of their "reverse"? (For example, 48 is $\frac{4}{7}$ of 84, the number formed by swapping the digits of 48.)

Problem 8. Suppose x, y are positive real numbers such that

$$xy + x^2 = 23$$

$$xy + y^2 = 26.$$

Compute $x + y$.

Time limit: 30 minutes.

Problem 9. Dan the Doubler has an interesting way of doubling numbers. He takes each digit of a number, doubles it, and then puts the answers together. For example, he "doubles" 666 to get 121212 and "doubles" 202 to get 404. For how many three-digit integers from 100 to 999, inclusive, does Dan get the correct answer using his doubling method? (For example, $202 \times 2 = 404$ is correct, but $666 \times 2 = 121212$ is incorrect.)

Problem 10. The product of the lengths of the three altitudes of a certain triangle is 24. If this triangle has an area of 3, then determine the product of the three side lengths of this triangle. (The altitudes of a triangle are the line segments connecting the vertices and the feet of the perpendiculars from those vertices to the opposite sides.)

Time limit: 30 minutes.