29th TJIMO

Alexandria, Virginia

Round: **Individual**

Problem 1. Compute $2^0 + 1 \cdot 7 + 2(0 \cdot 1 + 8)$.

Problem 2. Aaditya turned 14 years old in the year 2017. Determine in what year Aaditya's age will be double his current age.

Problem 3. A fence encloses a square region with area 2025 ft^2. Compute the perimeter of the fence, in ft.

Problem 4. Larry is working at a job with an odd payscale. On the first day, he receives \$1. On the second day, he receives \$2, and on the third day, he receives 3 dollars. This continues: he receives \$$k$ on the kth day, until he receives \$28 on the 28th day. Calculate the total amount of money in dollars that he receives throughout the 28 days.

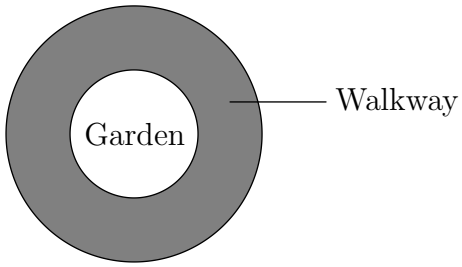
Problem 5. Paula the painter has red paint, yellow paint, green paint, and blue paint. She wants to paint her walls with one color and the floor with a different color. Determine the number of ways she can paint her walls and floor.

Problem 6. The decibel scale is used to measure the intensity of sound, or how loud a sound is. It is designed so that a sound 10 times as loud as another measures 10 decibels higher. For example, if a quiet library is at 40 decibels, a sound 10 times louder would be at 50 decibels. If a vacuum cleaner is 1000 times louder than the quiet library, compute the number of decibels for a vacuum cleaner.

Problem 7. William wants to buy a duck at The General Store. He has one coupon for 10% off and another coupon for \$10 off. If the price of the duck is \$50 and William is only allowed to use one of his two coupons, determine the cheapest price, in dollars, at which William can buy the duck.

Problem 8. Compute the number of ways Alice, Bob, Charlie, David, and Edward can stand in a line if Charlie insists on being the person in the middle.

Problem 9. A circular garden with area 4π has a circular walkway, shaded in the diagram below, around the outside of the garden with width equal to the radius of the garden. Compute the area of the walkway (shaded region) outside the garden.



Problem 10. Today is Saturday, October 28, 2017. Determine the day of the week on which Halloween

Time limit: 60 minutes.

falls *next* year (October 31, 2018).

Problem 11. Determine how many of the following five statements in the box are false.

Exactly one of these statements is false.
Exactly two of these statements are false.
Exactly three of these statements are false.
Exactly four of these statements are false.
Exactly five of these statements are false.

Problem 12. In the city of Mathalopolis, there is a $\frac{1}{3}$ chance that it rains on any given day. Compute the probability it rains in Mathalopolis at some point during two consecutive days.

Problem 13. In acute triangle VMT, all angles are an integer number of degrees, with $m\angle M = 50^\circ$. Compute the minimum possible value of $m\angle T$.

Problem 14. Compute the number of ordered pairs (x, y) of positive even integers such that $x \cdot y = 40$.

Problem 15. Neeyanth has three weeks to read *The Odyssey* for English class. During the first week, he enthusiastically reads half of the book. In the second week, he has to study for a biology test, so he is only able to read another $\frac{1}{6}$ of the book. Having procrastinated, he finishes the remaining 72 pages of the book in the third week. Calculate the number of pages in the book.

Problem 16. For any positive integer n, let $\xi(n)$ be the least common multiple of $1!, 2!, 3!$, and so forth, up through $n!$, where $n! = n(n-1)(n-2)\cdots(2)(1)$. Compute $\frac{\xi(2018)}{\xi(2017)}$.

Problem 17. Point A is at $(3, 0)$, point B is at $(10, 0)$, and point C is at $(0, y)$. If the area of triangle ABC is 42, determine all possible values of y.

Problem 18. There are 100 students at a math competition. Some teams consist of 5 students while others have 6 students. If there are a total of 18 teams, compute the number of teams with 6 students.

Problem 19. Compute the number of positive three-digit integers without leading zeros (so 042 does not count) that have either three even digits or three odd digits.

Problem 20. Harry and William are speedracers. In a race, both of them line up at the same starting point. However, Harry knows that William runs faster than him, so William allows Harry to have a five-second head start. If William runs 20% faster than Harry and the both run at constant speeds, compute the amount of time, in seconds, that Harry runs before William passes him.

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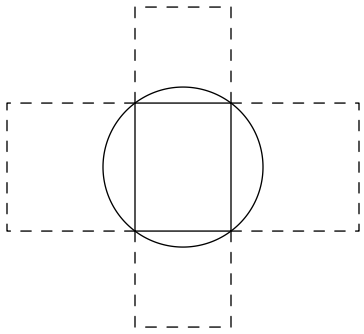
Problem 21. Compute the smallest perfect square greater than 1 that cannot be written as the sum of two (not necessarily distinct) positive prime numbers.

Problem 22. A rectangle is inscribed inside a circle with area 9π. From each side of the rectangle, a square is extruded outward, as shown by the dashed lines in the diagram below. Compute the sum of the areas of the four squares.



Problem 23. Compute the number of positive integers N such that $5 \cdot N$ has two digits while $6 \cdot N$ has three digits.

Problem 24. The General Store sells packs of 5 highlighters for \$3 and packs of 6 highlighters for \$3.50. Compute the maximum number of highlighters that may be purchased with \$41.

Problem 25. One percent of all mathletes are legendary. Some mathletes are oracles, whose job is to predict whether a given mathlete is legendary. Oracles make correct predictions 95% of the time. Kural the mathlete asks an oracle whether he is legendary, and the oracle tells him that he is. Determine the probability that he is actually legendary.

Problem 26. Convex quadrilateral $TJHS$ has side lengths $TJ = JH = 5$, $HS = 1$, and $ST = 7$. If $\angle TJH$ is a right angle, compute the area of quadrilateral $TJHS$.

Problem 27. Compute the positive real number x such that $\sqrt[3]{x + 3\sqrt[3]{x + 3\sqrt[3]{x + \cdots}}} = 3$.

Problem 28. Find all possible areas of rectangles with integer side lengths whose area equals its perimeter.

Problem 29. Compute the last four digits in 101^{2017}.

Problem 30. Quadrilateral $ABCD$ has sides $AB = 1009$, $BC = 2018$, and $CD = 2018$. If $\angle BAD$ is a right angle and $m\angle ADC = \frac{1}{2}m\angle BCD$, compute $m\angle BCD$ in degrees.

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