

Possible Guts Problems

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1 Algebra

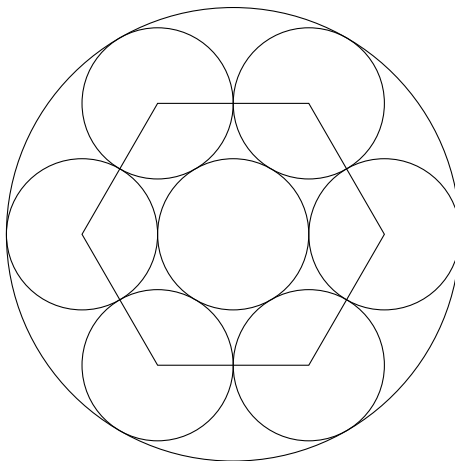
1. What is $1 - 2 + 3 - 4 + 5 - 6 + 7 - 8 + \cdots + 19 - 20$?
2. What is $1 + 2 + 3 + 4 + \cdots + 100$?
3. Gibbons drinks $\frac{1}{3}$ of the water in a bottle, and Ogden drinks $\frac{1}{4}$ of the remaining water. If there are 8mL of water left, how much water was there to begin with?
4. It takes Marbury 1 hour to deliver 6 letters and Madison 3 hours to deliver 60 letters. How many letters can they deliver in an 8 hour work day?
5. If $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots = \frac{\pi^2}{6}$, what is $\frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{6^2} + \cdots$?
6. What is $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{29 \cdot 30}$?
7. For an art project, Katherine is building a model of TJ. If the real auditorium is 30 feet wide but 2 inches on Katherine's model, how long in feet should the model gym be if the real gym is 50 feet long?
8. If $A + B = 4$, $C + D = 8$, and $A + C = 7$, what is $B + D$?
9. Let each letter represent a distinct digit. If $J = 8$, what is the five digit number $TJIMO$?

$$\begin{array}{rccccccccc} & & T & & J & & I & & M & & O \\ + & & & & M & & A & & T & & H \\ \hline & & M & & A & & G & & I & & C \end{array}$$

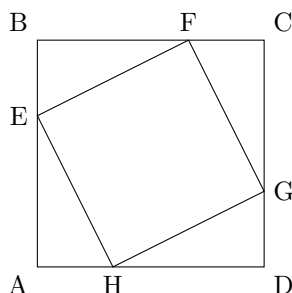
10. If Franklyn can finish two crypto assignments in 10 minutes, Katherine can finish one in 30, and Will Sun can finish one in 15, how long (in hours) will it take for them to finish 6 assignments?
11. Given that the median of 7 integers in the set $\{1, 2, 3, \dots, 15\}$ is 8 and the mode is 9, what is the largest possible mean?
12. If $x + y + z = 7$, $xyz = -48$, and $xy + yz + xz = -14$. If $x \geq y \geq z$, what is (x, y, z) ? Write your answer as an ordered triple.

2 Geometry

1. If an equilateral triangle has an area of $9\sqrt{3}$, what is the perimeter of the triangle?
2. If the radius of the larger circle is 6, and all the smaller circles are the same size, what is the side length of the hexagon?



3. What is the area of a triangle with side lengths 13, 14, and 15?
4. In triangle ABC , let D_1 be on BC such that $2D_1B = D_1C$, D_2 be on BC such that $D_2B = 2D_2C$, E be on AC such that $2AE = CE$, and F be on AB such that $2AF = BF$. Let G be the intersection of D_1E and D_2F . If the area of $\triangle ABC$ is 36, what is the area of $\triangle D_1D_2G$?
5. Let $ABCD$ and $EFGH$ be squares such that E lies on AB , F lies on BC , G lies on CD , and H lies on AD . If the area of $EFGH$ is $\frac{5}{9}$ of the area of $ABCD$. What is $\frac{AE}{BE}$?

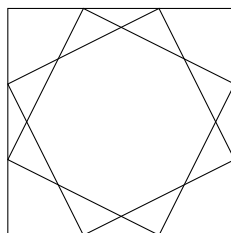


6. In quadrilateral $ABCD$, $\angle DAC = 75^\circ$, $\angle ACB = 40^\circ$, $\angle DBC = 75^\circ$, and $\angle BDC = 25^\circ$. Find the measure of angle $\angle DCA$.
7. Consider point P which lies outside of circle ω . A and B lie on ω , $C \neq A$ is the intersection of AP and ω , and $D \neq B$ is the intersection of BP and ω . If $AC = 7$, $AP = 9$, and $BD = 10$, what is AB ?
8. (Franklyn) Consider triangle ABC . Let D , E , and F be the feet of the altitudes from A , B , and C , respectively. Let H_1 , H_2 , and H_3 be the intersections of AD and BE , CF and BE , and AD and CF , respectively. If the side lengths are $11 + 6\sqrt{3}$, $11 - 6\sqrt{3}$, and 11, what is the area of triangle $H_1H_2H_3$? (You may use the fact that the intersection of the angle bisector of $\angle A$ and (ABC) in $\triangle ABC$ is the center of the circumcircle for cyclic quadrilateral IBI_aC).

3 Combinatorics

1. How many numbers are between 505 and 700, inclusive?
2. Miranda is picking out skirts and crop tops to wear. If she has 5 skirts and 7 crop tops, how many outfits can she make?

3. I am drawing marbles out of a bag. If there are 3 red marbles and 5 blue marbles, what is the probability that the second marble I draw is red?
4. There are 100 people in math team. If 53 of them do cross country, 27 of them do art club, and 38 of them do neither, how many do both?
5. In a round robin tournament, everyone competes against everyone else. How many matches are there?
6. There is a 40% chance of rain on Saturday. If it rains on Saturday, there is a 60% chance of rain on Sunday. Otherwise, there is a 30% chance of rain on Sunday. What is the probability it rains on Sunday?
7. How many ways can I seat Josh, Akshaj, Jeffery, Katherine, Michael, and Wendy at a circular table so that Wendy and Katherine are sitting next to each other?
8. How many ways can I split 7 pieces of candy among 4 people, assuming each person gets at least one?
9. If I roll three die, what is the probability the numbers on the three die sum to 16?
10. How many triangles are in the following figure?



11. What is the probability that if I flip a coin 8 times in a row, I get heads at least 4 times?
12. Allen the alien lives on Lelan. All 52 Lelans start the day by drawing a card without replacement from a deck of 52 cards. If they get an ace, then they get the day off, and if they get a face card, they get the afternoon off. Allen forms a group with 5 other aliens so that if any of them get time off, they secretly swap cards so that Allen gets time off. What is the probability that someone in the group other than Allen will get time off?

4 Number Theory

1. What is the greatest common factor of 117 and 156?
2. What is the least common multiple of 6 and 4?
3. How many factors does 27 have?
4. What are the sum of the factors of 16?
5. Gideon is facing trial, but he only has 10 dollars. If Wainwright offers lawyers for 3 dollars and evidence for 2 dollars each, how many combinations of lawyers and evidence can Gideon buy, assuming that he needs to buy a lawyer?
6. What is the smallest positive integer that gives a remainder of 1 when divided by 3, a remainder of 3 when divided by 5, and 5 when divided by 7?
7. When 2017 is multiplied by a single digit nonzero integer x , all the digits are perfect squares. What is x ?
8. If the sum of 7 consecutive integers is 119, what is the smallest of these 7 integers?

5 Other

1. Katherine, Neeyanth, Wendy, and Ryan are arguing over who is a freshman and who is an upperclassman. Freshmen always lie, while upperclassmen always tell the truth.

Neeyanth says: Ryan is a freshman.

Ryan says: Out of Neeyanth and I, exactly one of us is a freshman.

Wendy says: Katherine and I are either both freshmen or both upperclassmen.

Katherine says: At least one of us is a freshman.

Who is a freshman (list everyone)?

2. Suppose that the following are true:

- Hao always laughs in math class.
- Math class is on Wednesday and Friday.
- If Hao laughs in a class, he will laugh for the rest of the day.

How many of the following are always true?

- Hao only laughs two days a week.
- Hao will laugh on Friday.
- Hao will not laugh on Monday.
- Hao will laugh in Physics, which he has after math.

3. Pick an integer greater than 0. Your score will be equal to

$$\frac{1}{|\text{Log}(\frac{x}{\bar{x}})|}$$

where \bar{x} is the mean of all the answers submitted and x is your answer.

- 4.