IEEE PIMRC 2019, Tutorial 07

Age of Information: Providing Fresh Data to Real-time Applications

Part I

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September 8, 2019

Tutorial Outline: Part I

(14:00-15:30 by Yin Sun, 15:30-15:45 coffee break)

- Motivation and Introduction:
  - Why is Fresh Data Important?
  - What is the Age of Information (a.k.a. Age or AoI)?
  - Is Age a Good Metric for Freshness?

- Information Freshness Optimization (3 Topics):
  - Sampling for Optimizing Nonlinear Age Functions
  - Sampling for Remote Signal Estimation
  - Age-Optimal Scheduling in Networks

Tutorial Outline: Part II

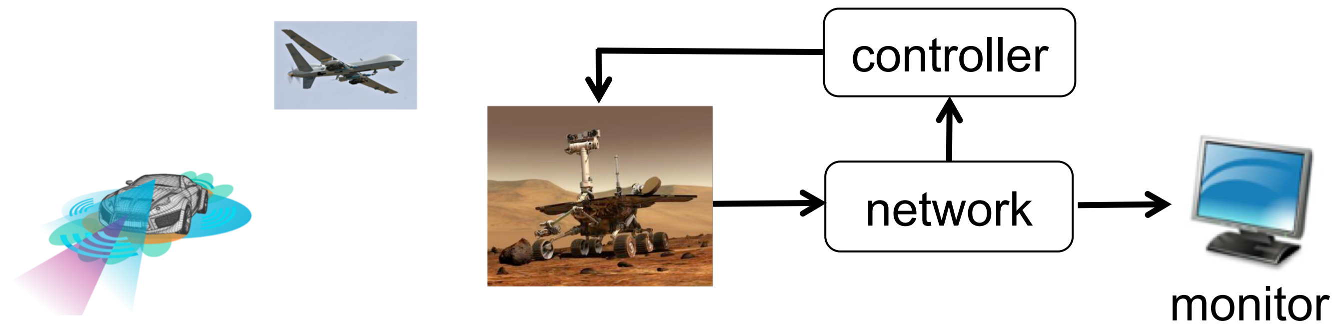
3 Topics: (15:45-17:00 by Elif Uysal)

- Age of Information and Energy
- Age of Information and Channel Coding
- Age of Information in the Internet

Motivation and Introduction

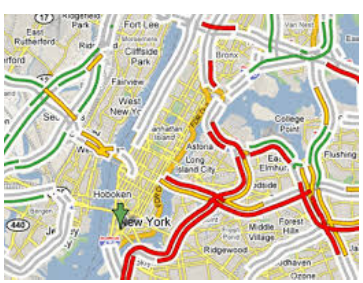
Real-time Services Need Fresh Data

- **Real-time Monitoring and Control**

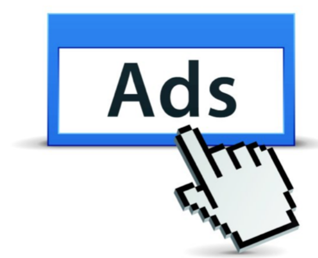


Sensor networks, Internet-of-Things, Cyber-Physical Systems, Robotics, Health, Security, etc.

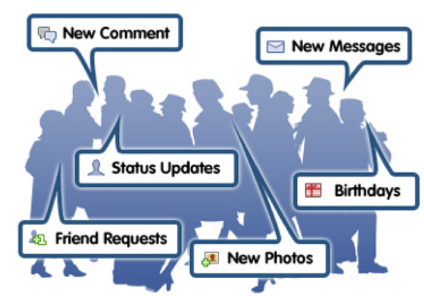
- **Real-time Data Analytics and Learning**



Crowdsourcing

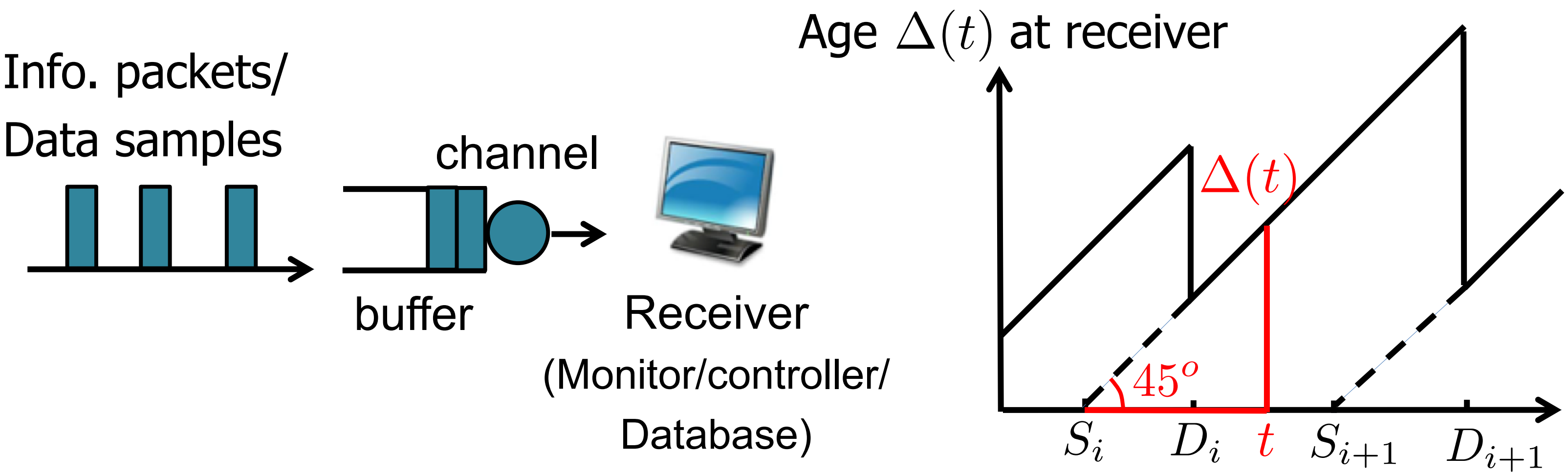


Online Learning (ads bidding)



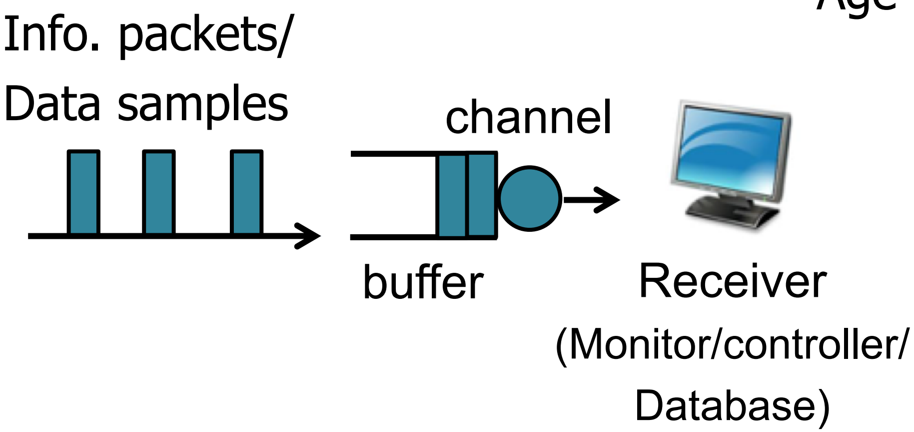
Social networks

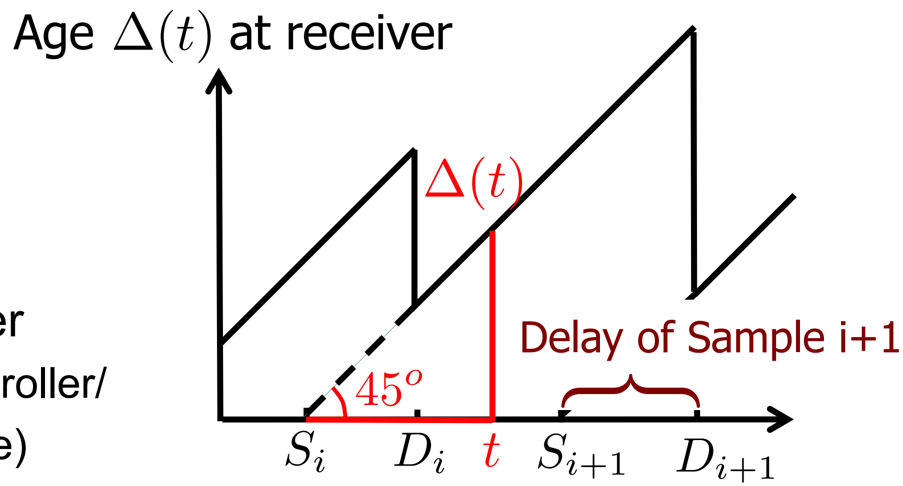
Data Freshness Metric: Age of Information



- In real-time applications, fresh data is more important than stale data
  - E.g., UAV/vehicle/robotic control, wild fire/tornado monitoring …

Age of Information: Definition



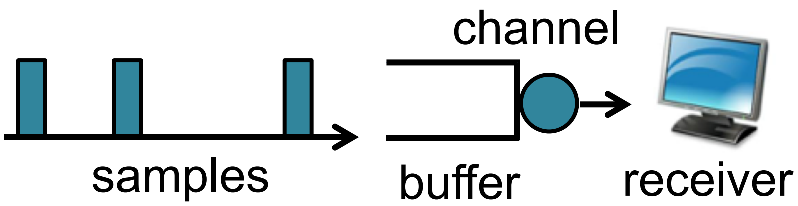


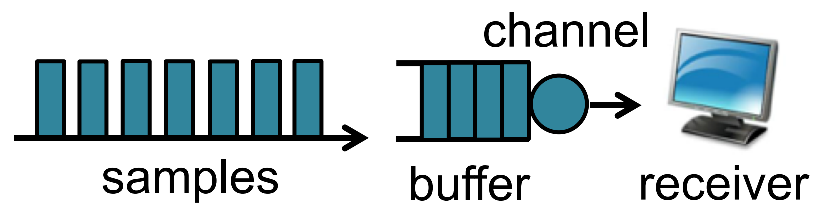
Definition: At time t, the **Age of Information** $\Delta(t)$ is time difference between the current time t and the generation time of the latest received data sample

- If sample i is generated at S_i and delivered at D_i

$$\Delta(t) = \underbrace{t - \max\{S_i : D_i \le t\}}_{\text{Time difference between data generation and usage}}$$

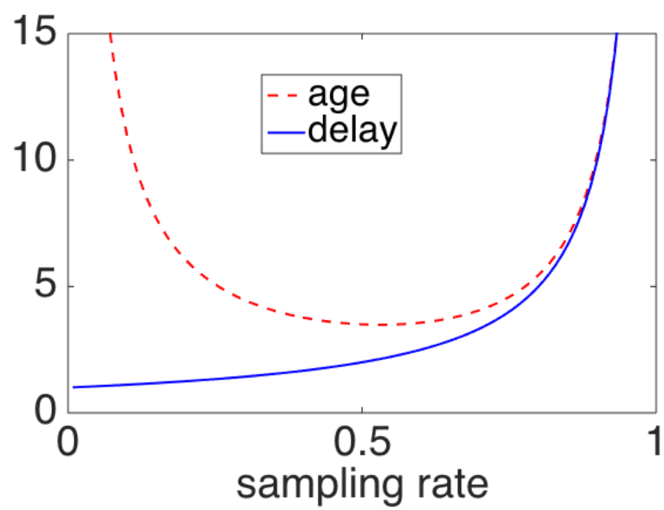
Difference between Age & Delay





In M/M/1 FIFO queues: [Kaul, Yates, Gruteser'12]

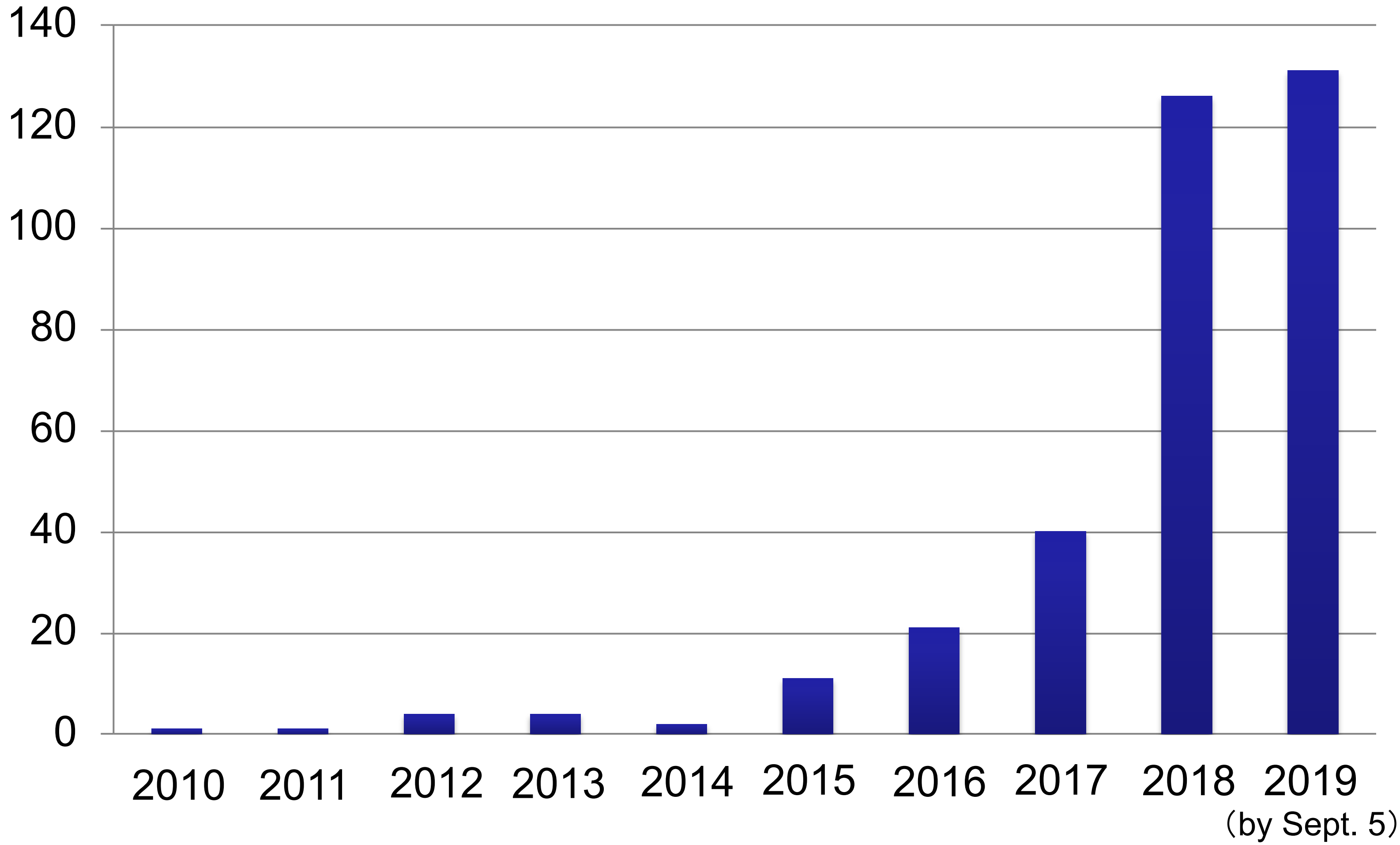
- Age first decreases, then increases with sampling rate
- Delay increases with sampling rate



A Brief History on Age of Information

- Defined in real-time databases [Song-Liu'90,…]
- Queueing analysis [Kaul-Gruteser-Rai-Kenney'11, Yates-Kaul'12, Kam-Kompella-Ephremides'13,14, …]
- Scheduling and age minimization [Altman et al.'10(19), Yates'15, He-Yuan-Ephremides'16, Sun et al.'16, Bedewy-Sun-Shroff'16&17,…]
- Source coding [Zhong-Yates'16, Zhong-Yates-Soljanin'17,…]
- Channel coding [Najm-Yates-Soljanin'17, Yates-Najm-Soljanin-Zhong'17,…]
- Energy harvesting [Bacinoglu-Ceran-Uysal'15, Wu-Yang-Wu'17, Bacinoglu-Uysal'17, Arafa-Ulukus'17,…]
- Freshness of Channel State Information [Costa-Valentin-Ephremides'15, Farazi-Klein-Brown'17,…]
- Caching [Yates-Ciblat-Yener-Wigger'17, Kam et al.'17]
- Game theory [Nguyen et al. 17, Xiao-Sun'18]
- Age of information and Information Theory [Baknina-Ozel-Yang-Ulukus-Yener'18]
- Age of information and learning [Sert-Sonmez-Baghaee-Uysal-Biyikoglu'18, Ceran-Gunduz-Gyorgy'18]
- Age of information and Control [Zhang-Wang'18, Soleymani-Baras-Johansson'18,…]
- Human in the loop [Bastopcu-Ulukus'18, Bin-Liu'19]

Number of papers on AoI



Sources: Google Scholar

AoI paper list: auburn.edu/~yzs0078/AoI.html

Is Age a Good Metric for Freshness?

- **Age:** Time difference between data generation and usage
- **Issue:**
  - Some information changes fast (e.g., car location)
    - Update frequently (sub-second), more bandwidth
  - Other information changes slowly (e.g., temperature)
    - Update infrequently (hourly), less bandwidth

Tony's Question

- Tony asked a question in the Open Problem Session at ITA 2015
  - …There is a correlation structure in the signal. Clearly, the actual age is not a good representation of freshness.
  - What is freshness, can we get an Information Theoretical measure of freshness?
  - There should be a concept of effective age…

Anthony Ephremides

A Possible Answer: Nonlinear Age Functions

- Penalty function $p(\Delta(t))$: p can be any non-decreasing function
  - Dissatisfaction for data staleness, eagerness for refreshing

- Utility function $u(\Delta(t))$: u can be any non-increasing function
  - Utility value of fresh data

- Examples:
  - Temporal Auto-Correlation Function $u(\Delta(t)) = |\mathbb{E}[X_t^* X_{t-\Delta(t)}]|$ [Kosta-Pappas-Ephremides-Angelakis'ISIT2017]
    - For example, auto-correlation function of CSI is a non-increasing age function for small age [Truong-Health'JCN2013]

Nonlinear Age Functions (cont'd)

- More Examples:
  - Estimation Error of Real-time Signal:

Estimate signal value based on causally received samples.

- If sampling times are independent of the signal, estimation error is an age penalty function. [Sun-Polyanskiy-Uysal'ISIT17]
- Wiener process: [Yates-Kaul'ISIT12, Sun-Polyanskiy-Uysal'ISIT17]

$$\mathbb{E}[(W_t - \hat{W}_t)^2] = \mathbb{E}[W^2_{\Delta(t)}] = \Delta(t)$$

- Ornstein-Uhlenbeck process: continuous-time AR (1) [Ornee-Sun'19]

$$\mathbb{E}\left[(X_t - \hat{X}_t)^2\right] = \frac{\sigma^2}{2\theta}\left(1 - e^{-2\theta\Delta(t)}\right) = p(\Delta(t))$$

- **Note:** If sampling times are determined based on causal knowledge of the signal, estimation error is not necessarily a function of age.

Nonlinear Age Functions (cont'd)

- More Examples:
  - Mutual Information and Conditional Entropy:
    - Let $\mathbf{W}_t$ be the set of samples received up to time t
    - Mutual information: $I(X_t; \mathbf{W}_t)$ amount of information contained in $\mathbf{W}_t$ [Sun-Cyr'SPAWC18]
    - Conditional Entropy: $H(X_t|\mathbf{W}_t)$ uncertainty after knowing $\mathbf{W}_t$ [Soleymani-Hirche-Baras'CDC, ECC, WODES2016]
    - If sampling times are independent of the signal and signal is a stationary Markov chain, mutual information is an age utility function, conditional entropy is an age penalty function [Sun-Cyr'JCN2019]
    - Otherwise, they are not necessarily functions of age

Nonlinear Age Functions (cont'd)

- More Examples:
  - Estimation Error in Feedback Control System:

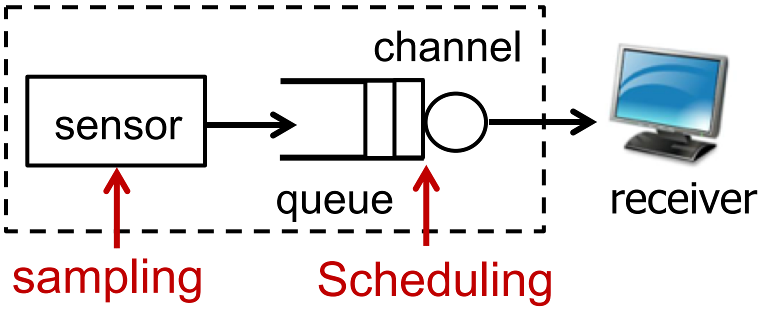
$$X_{t+1} = AX_t + BU_t + N_t$$

    - Samples of the system state X_t are sent to the controller.
    - The controller determines U_t based on causally received samples.
    - The estimation error $\mathbb{E}[(X_t - \hat{X}_t)^2]$ is independent of control.
    - If sampling times are independent of the signal, $\mathbb{E}[(X_t - \hat{X}_t)^2]$ is an age penalty function [Champati et. al'19, Klugel et. al'19]; otherwise, it is not necessarily a function of age.
- Nonlinear age functions date back to 2003. A short survey available in [Sun-Cyr'JCN2019]
- **Summary:** Nonlinear age functions are simple enough and general enough, but not always perfect

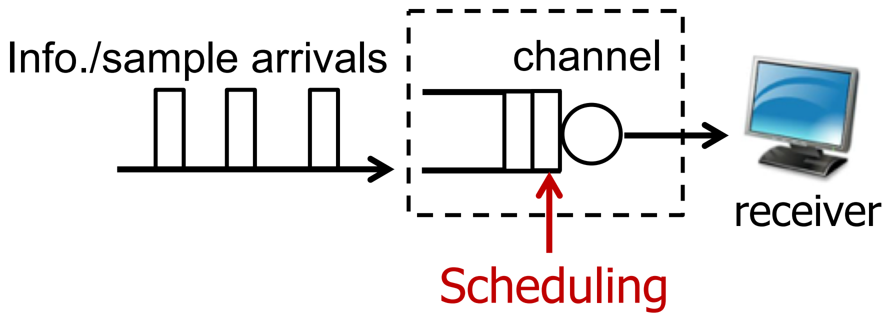
Sun and Cyr, "Sampling for Data Freshness Optimization: Non-linear Age Functions," JCN special issue on Age of Info., 2019.

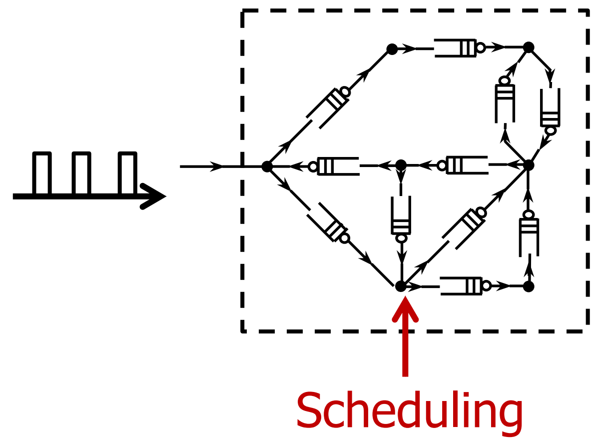
3 Topics on Information Freshness Optimization

- Optimal Sampling and Scheduling



  - From AoI to Sampling theory
- Optimal Scheduling Only





  - Given arrival process
  - optimal scheduling in networks with multiple channels, hops, and/or sources

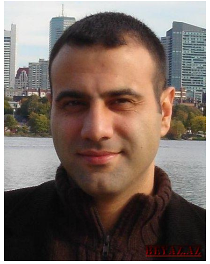
Topic I

Sampling for Optimizing Nonlinear Age Functions

Joint work with

Elif Uysal

Roy D. Yates

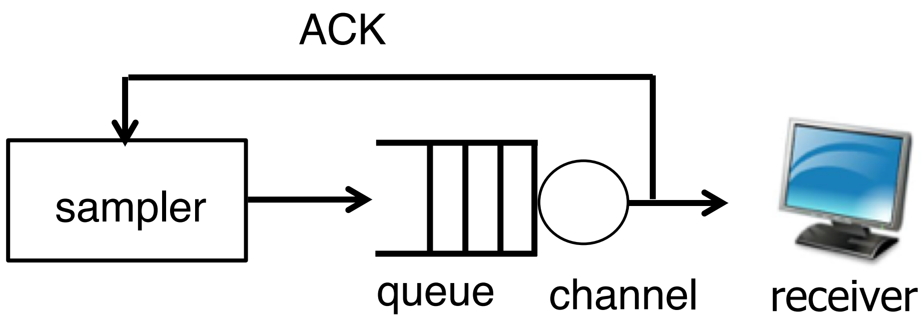


C. Emre Koksal

Ness B. Shroff.

Benjamin Cyr

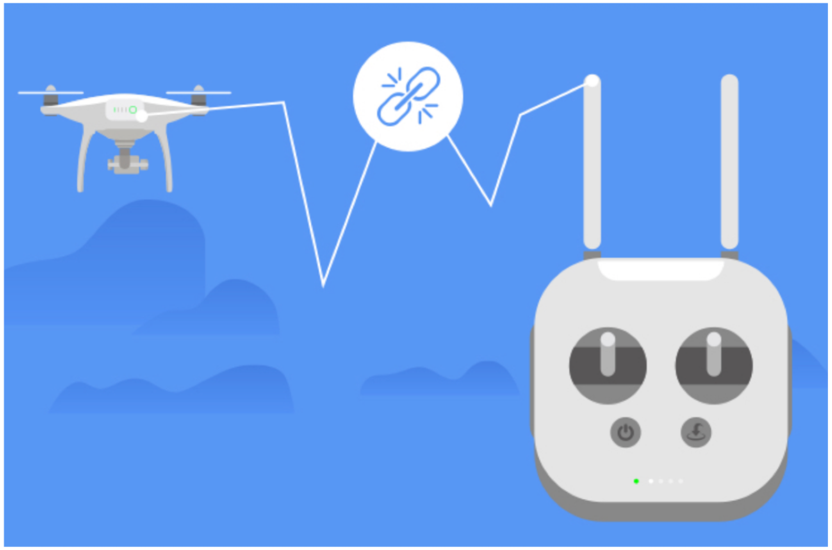
Model:
Fresh Samples through Queues



- Two terminals: **Sampler** sends samples to **Receiver**
- **Channel**: FIFO queue with *i.i.d.* transmission times Y_i
- **Feedback:** ACK
  - Zero delay ➔ sampler knows channel state (idle/busy)

- Sample i is generated at S_i, with channel transmission time Y_i
  - $Y_i \geq 0$ has a **general** distribution
  - Can be discrete/continuous random variable (finite moments)
  - Robustness of CPS under occasionally long commun. delay

Example:
Remote Drone Control Under Interference



Long communication delay
due to interference and jamming

Source: www.dji.com

Source: www.battelle.org

Problem: Continuous-time Sampling

$$\inf_{\pi \in \Pi} \limsup_{T \to \infty} \frac{1}{T} \mathbb{E}\left[\int_0^T \overbrace{p(\Delta(t))}^{\text{age penalty function}} dt\right]$$

Expected time-average age penalty

$$\text{s.t.} \quad \liminf_{n \to \infty} \frac{1}{n} \mathbb{E}\left[\sum_{i=1}^{n} \underbrace{(S_{i+1} - S_i)}_{\text{inter-sample time}}\right] \geq \frac{1}{f_{\max}}$$

Avg. sampling-rate constraint

- $p(\Delta(t))$ is arbitrary non-decreasing function of the age $\Delta(t)$
- Age utility maximization handled by choosing $p(\Delta(t)) = -u(\Delta(t))$
- Sampling policy: $\pi = (S_1, S_2, \ldots)$ sequence of sampling times
- Space of causal policies: Π

  Sampling time S_i decided by the history information of the system

Question: Which sampling policy minimizes the age penalty?

Problem: Continuous-time Sampling

$$\inf_{\pi\in\Pi} \quad \limsup_{T\to\infty} \frac{1}{T}\,\mathbb{E}\left[\int_0^T \overbrace{p(\Delta(t))}^{\text{age penalty function}} dt\right]$$

Expected time-average age penalty

$$\text{s.t.} \quad \liminf_{n\to\infty} \frac{1}{n}\mathbb{E}\left[\sum_{i=1}^{n} \underbrace{(S_{i+1} - S_i)}_{\text{inter-sample time}}\right] \geq \frac{1}{f_{\max}}$$

Avg. sampling-rate constraint

- Constrained continuous-time MDP with a continuous state space
  - Usually difficult to solve

Unconstrained Continuous-time Sampling

Theorem 1. For unconstrained problem, optimal sampling times are given by

$$S_{i+1}(\beta) = \inf\{\underbrace{t \geq D_i(\beta)}_{\text{After previous sample is delivered}} : \underbrace{\mathbb{E}[p(\Delta(t+Y_{i+1}))] \geq \beta}_{\text{Expected age penalty exceeds threshold}}\}$$

Deterministic threshold policy

where $D_i(\beta) = S_i(\beta) + Y_i$ and β is the root of

$$\beta = \frac{\mathbb{E}\left[\int_{D_i(\beta)}^{D_{i+1}(\beta)} p(\Delta(t))dt\right]}{\mathbb{E}[D_{i+1}(\beta) - D_i(\beta)]} = \bar{p}_{\text{opt}}$$

Bisection search (no "curse of dimensionality")

Threshold = optimal objective value

General penalty function, general service time distribution, with finite moments

$$\bar{p}_{\text{opt}} = \inf_{\pi \in \Pi} \limsup_{T \to \infty} \frac{1}{T} \mathbb{E}\left[\int_0^T p(\Delta(t))dt\right]$$

Structure more insightful than [Sun-Uysal-Yates-Koksal-Shroff, TIT17]

Sun and Cyr, "Sampling for Data Freshness Optimization: Non-linear Age Functions," JCN special issue on Age of Info., 2019.

Bisection Search for β

- **Goal:** Solve $\beta = \frac{\mathbb{E}\left[\int_{D_i(\beta)}^{D_{i+1}(\beta)} p(\Delta(t))dt\right]}{\mathbb{E}[D_{i+1}(\beta) - D_i(\beta)]}$

- **Method:** Bisection search

Algorithm 1 Bisection method for solving (17)

given l, u, tolerance $\epsilon > 0$.
repeat
  $\beta := (l+u)/2$.
  $o := \beta - \frac{\mathbb{E}[v(D_{i+1}(\beta) - S_i(\beta)) - v(Y_i)]}{\mathbb{E}[D_{i+1}(\beta) - D_i(\beta)]}$.
  if $o \geq 0$, $u := \beta$; **else**, $l := \beta$.
until $u - l \leq \epsilon$.
return β.

- where $v(t) = \int_0^t p(\Delta(s))ds$

$$\int_{D_i(\beta)}^{D_{i+1}(\beta)} p(\Delta_t)dt = v(D_{i+1}(\beta) - S_i(\beta)) - v(Y_i)$$

Constrained Continuous-time Sampling

Theorem 2. The optimal sampling times are given by **Theorem 1** if

$$\mathbb{E}[S_{i+1}(\beta) - S_i(\beta)] > \frac{1}{f_{\max}}$$

Otherwise, $S_{i+1}(\beta) = \begin{cases} T_{i,\min}(\beta) \text{ with probability } \lambda, \\ T_{i,\max}(\beta) \text{ with probability } 1-\lambda, \end{cases}$

Randomized threshold policy

where $T_{i,\min}(\beta) = \inf\{t \geq D_i(\beta) : \mathbb{E}[p(\Delta(t+Y_{i+1}))] \geq \beta\},$

$T_{i,\max}(\beta) = \inf\{t \geq D_i(\beta) : \mathbb{E}[p(\Delta(t+Y_{i+1}))] > \beta\},$

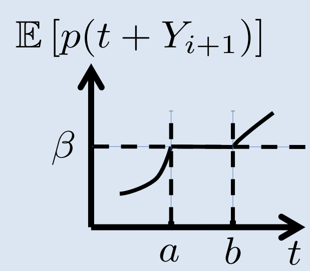
β is determined by solving

Bisection search. No curse of dimensionality

$$\mathbb{E}[T_{i,\min}(\beta) - S_i(\beta)] \leq \frac{1}{f_{\max}} \leq \mathbb{E}[T_{i,\max}(\beta) - S_i(\beta)],$$

Then, λ is given by

$$\lambda = \frac{\mathbb{E}[T_{i,\max}(\beta) - S_i(\beta)] - \frac{1}{f_{\max}}}{\mathbb{E}[T_{i,\max}(\beta) - T_{i,\min}(\beta)]}.$$



Sun and Cyr, "Sampling for Data Freshness Optimization: Non-linear Age Functions," JCN special issue on Age of Info., 2019.

Constrained Continuous-time Sampling

Corollary. Suppose $p(\cdot)$ is strictly increasing. Then, the optimal sampling times are given by **Theorem 1** if

$$\mathbb{E}[S_{i+1}(\beta) - S_i(\beta)] > \frac{1}{f_{\max}}$$

Otherwise, $S_{i+1}(\beta) = \inf\{t \geq D_i(\beta) : \mathbb{E}[p(\Delta(t + Y_{i+1}))] \geq \beta\}$

two thresholds are the same

where β is determined by solving

$$\mathbb{E}[S_{i+1}(\beta) - S_i(\beta)] = \frac{1}{f_{\max}}$$

Bisection search. No curse of dimensionality

Sun and Cyr, "Sampling for Data Freshness Optimization: Non-linear Age Functions," JCN special issue on Age of Info., 2019.

Unconstrained Discrete-time Sampling

Theorem 3. The optimal sampling times are given by

$$S_{i+1}(\beta)=\inf\{\underbrace{t\in\mathbb{N}}_{\text{discrete-time}}:\underbrace{t\geq D_i(\beta)}_{\text{After previous sample is delivered}},\underbrace{\mathbb{E}[p(\Delta(t+Y_{i+1}))]\geq\beta}_{\text{Expected age penalty exceeds threshold}}\}$$

Deterministic threshold policy

where $D_i(\beta)=S_i(\beta)+Y_i$ and β is the root of

$$\beta=\frac{\mathbb{E}\left[\sum_{t=D_i(\beta)}^{D_{i+1}(\beta)-1}p(\Delta(t))\right]}{\mathbb{E}[D_{i+1}(\beta)-D_i(\beta)]}=\bar{p}_{\text{opt}}$$

Threshold = optimal objective value

Bisection search

No curse of dimensionality

$$\bar{p}_{\text{opt}}=\inf_{\pi\in\Pi}\ \limsup_{T\to\infty}\frac{1}{T}\ \mathbb{E}\left[\sum_{t=1}^{T}p(\Delta(t))\right]$$

t are integers.

Scalable with clock rate

Sun and Cyr, "Sampling for Data Freshness Optimization: Non-linear Age Functions," JCN special issue on Age of Info., 2019.

Constrained Discrete-time Sampling

Theorem 4. The optimal sampling times are given by **Theorem 3** if

$$\mathbb{E}[S_{i+1}(\beta) - S_i(\beta)] > \frac{1}{f_{\max}}$$

Otherwise,
$$S_{i+1}(\beta) = \begin{cases} T_{i,\min}(\beta) \text{ with probability } \lambda, \\ T_{i,\max}(\beta) \text{ with probability } 1-\lambda, \end{cases}$$

Randomized threshold policy

where
$$T_{i,\min}(\beta) = \min\{t \in \mathbb{N} : t \geq D_i(\beta), \mathbb{E}\left[p(\Delta(t+Y_{i+1}))\right] \geq \beta\},$$
$$T_{i,\max}(\beta) = \min\{t \in \mathbb{N} : t \geq D_i(\beta), \mathbb{E}\left[p(\Delta(t+Y_{i+1}))\right] > \beta\},$$

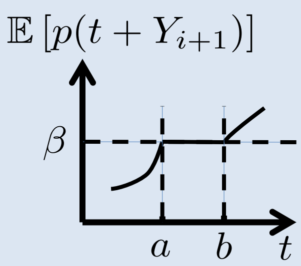
β is determined by solving

Bisection search. No curse of dimensionality

$$\mathbb{E}[T_{i,\min}(\beta) - S_i(\beta)] \leq \frac{1}{f_{\max}} \leq \mathbb{E}[T_{i,\max}(\beta) - S_i(\beta)],$$

Then, λ is given by

$$\lambda = \frac{\mathbb{E}[T_{i,\max}(\beta) - S_i(\beta)] - \frac{1}{f_{\max}}}{\mathbb{E}[T_{i,\max}(\beta) - T_{i,\min}(\beta)]}.$$



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Is the optimal sampling policy deterministic or randomized?

Continuous or discrete time	Constrained or not	Age penalty function	Deterministic policy or not
Continuous time	unconstrained	Non-decreasing	Deterministic
Continuous time	constrained	Non-decreasing	Randomized
Continuous time	constrained	Strict increasing	Deterministic
discrete time	unconstrained	Non-decreasing	Deterministic
discrete time	constrained	Non-decreasing	Randomized

- Why randomized?
  - If one policy cannot meet the sampling rate constraint with equality, a randomized mixture of two policies can.

Three Equivalent Threshold Policies

- **Threshold policy on age penalty:**

$$S_{i+1}(\beta) = \inf\{t \geq D_i(\beta) : \mathbb{E}[p(\Delta(t + Y_{i+1}))] \geq \beta\}$$

  - Define the threshold function

$$w(\beta) = \inf\{\Delta \geq 0 : \mathbb{E}[p(\Delta + Y_{i+1})] \geq \beta\}$$

- **Threshold policy on age:**

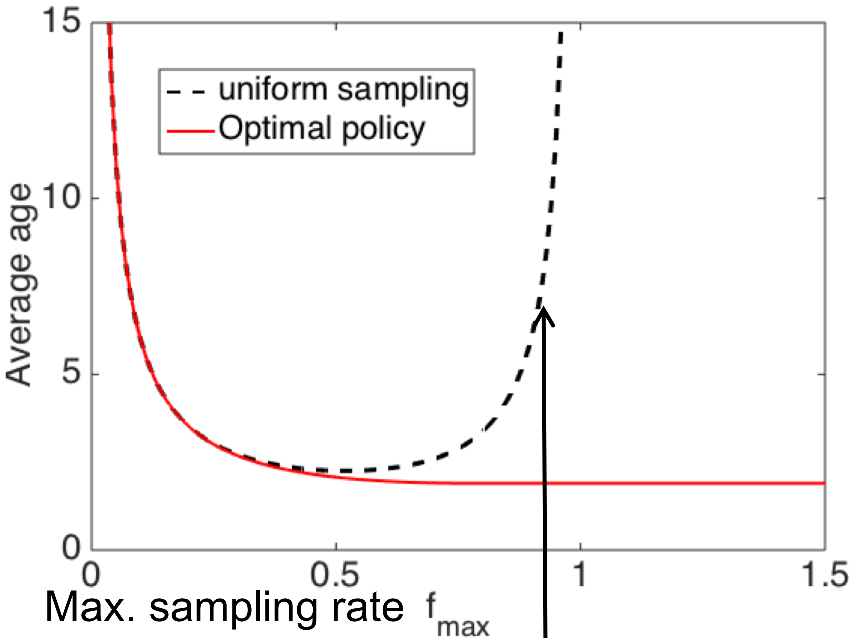
$$S_{i+1}(\beta) = \inf\{t \geq D_i(\beta) : \Delta(t) \geq w(\beta)\}$$

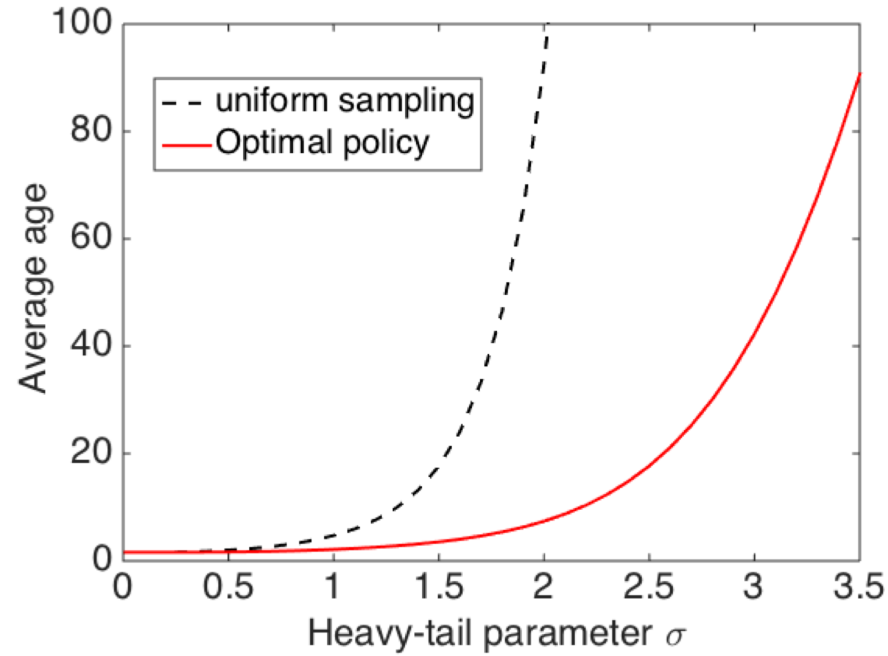
- **Water-filling solution for waiting time:**

$$Z_i(\beta) = \max\{w(\beta) - Y_i, 0\}$$

  - where $Z_i(\beta) = S_{i+1}(\beta) - D_i(\beta)$ is the waiting time after previous sample is delivered
  - All of them are insightful, appeared in different studies

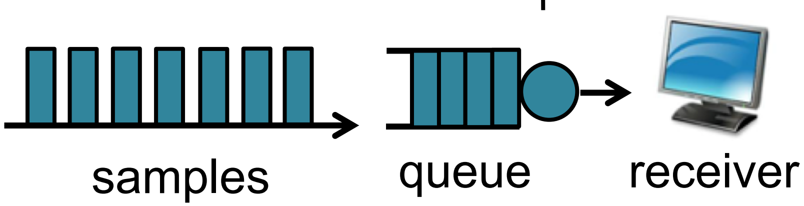
Uniform Sampling can be FAR from Optimal





i.i.d. exponential trans. times $\mu = 1$

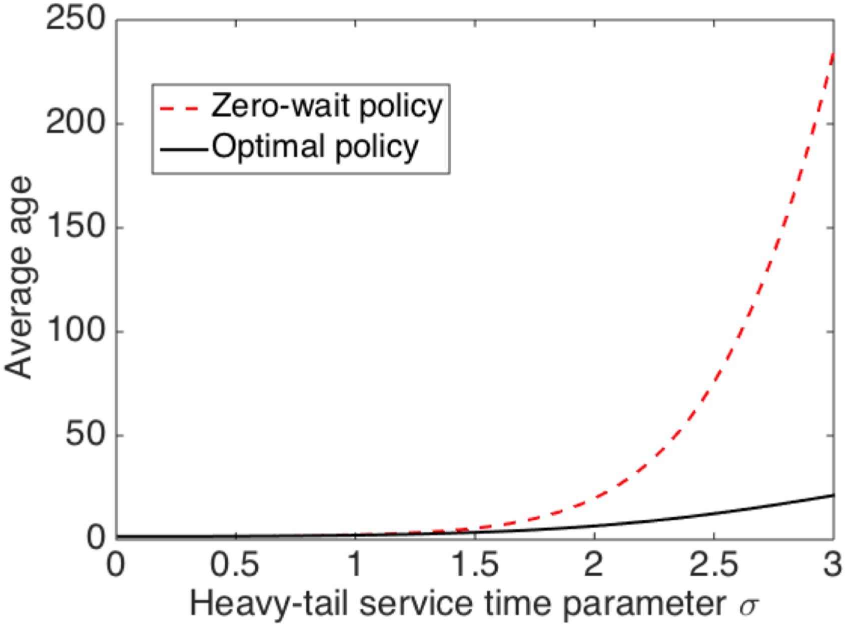
i.i.d. log-normal trans. times $\mu = 1$



Become stale while waiting

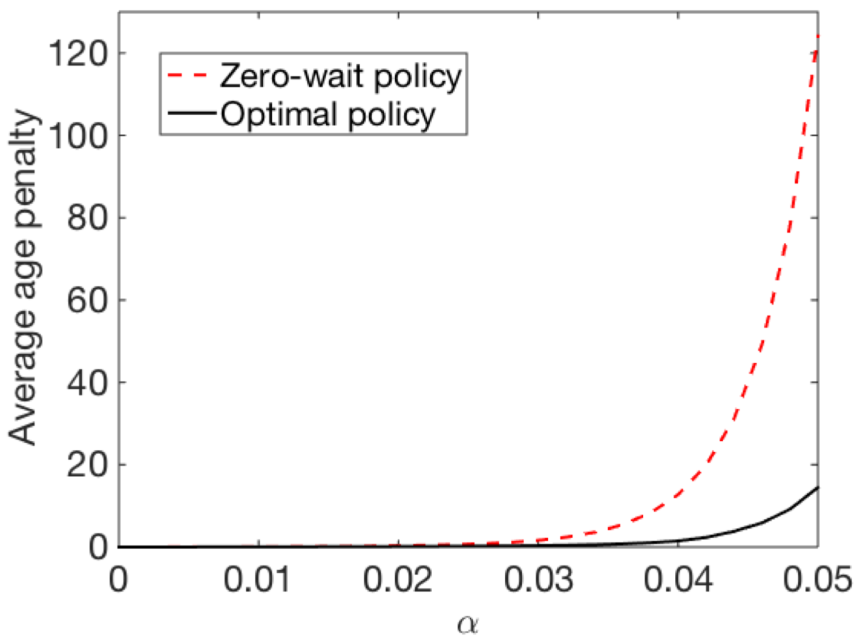
Uniform sampling is far from optimal if
(1) Sampling rate is high
(2) Transmission time is highly random

Zero-Wait can be FAR from Optimal



i.i.d. log-normal trans. times $\mu = 1$

$$S_{i+1} = S_i + Y_i$$



i.i.d. log-normal trans. times $\mu = 1$

$$g(\Delta) = e^{\alpha\Delta} - 1$$

Zero-wait is far from optimal if

(1) Transmission time is highly random

(2) Age penalty grows quickly with age (impatient user, real-time control)

Summary

- Developed optimal sampling policies for optimizing time-average nonlinear age functions

- The optimal policy is a deterministic or randomized threshold policy and the optimal threshold is characterized exactly
  - Both continuous-time & discrete-time sampling
  - Both with & without sampling rate constraint
  - General service time distributions
  - General monotonic age functions
    - Represents the need of real-time applications

- Uniform sampling and Zero-Wait sampling can be far from optimal

Topic II

Sampling for Remote Signal Estimation

Joint work with

Elif Uysal

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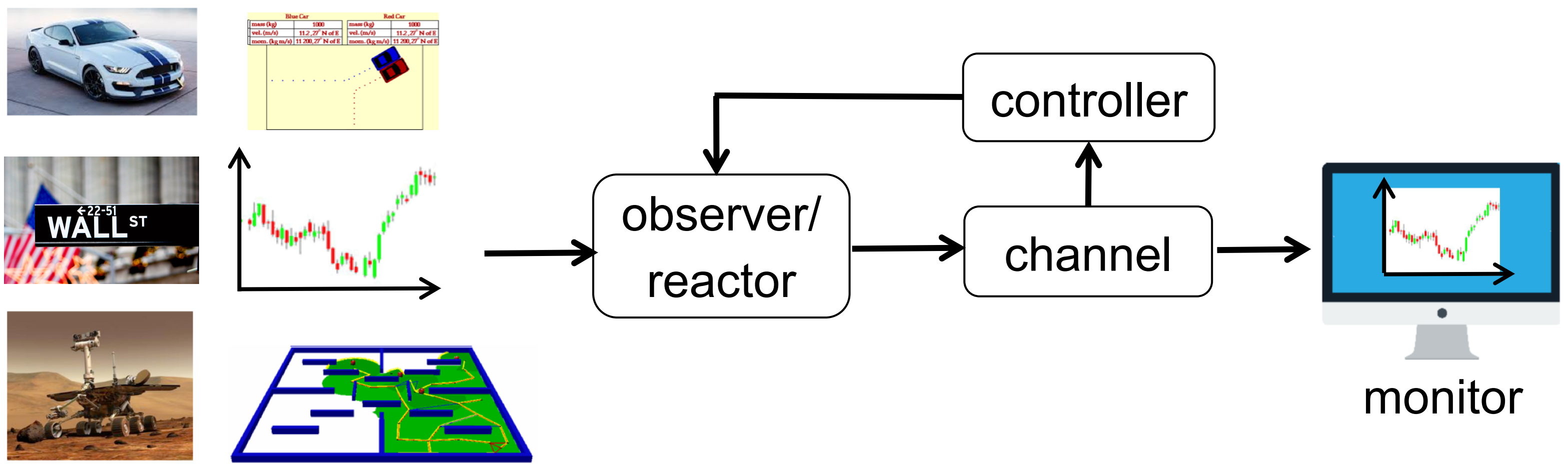
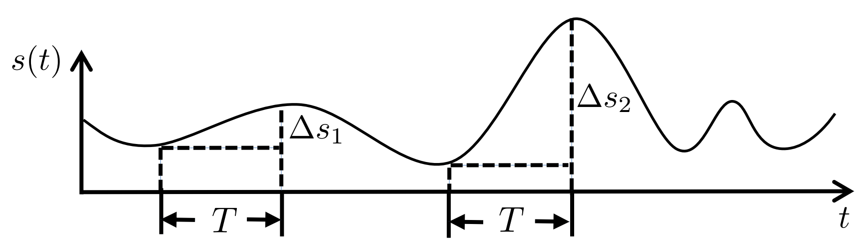
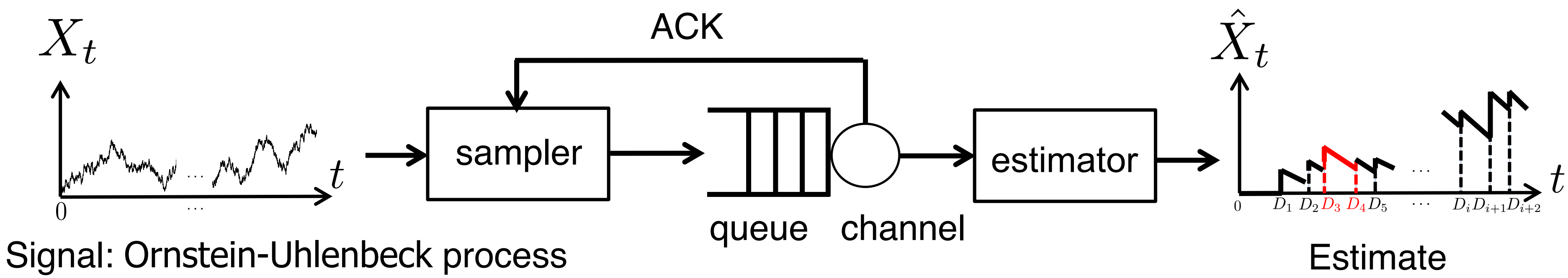
Motivation: Real-time Monitoring & Control



- Real-time information usually in the form of a signal
- Age-of-information: time-difference between source and destination
  - time-difference $\neq$ signal-difference



Model: Timely **Signal** Updates



- **Sampler**: Take samples of an Ornstein-Uhlenbeck (OU) process X_t
  - continuous-time analogue of Autoregressive(1) model
  - Defined by $dX_t = \theta(\mu - X_t)dt + \sigma dW_t$
    - $\theta > 0, \sigma > 0$ and W_t is Wiener process
- **MMSE Estimator**: Use causally received samples to estimate the real-time value of X_t
- **Channel**: FIFO queue with *i.i.d.* service times
  - ACK: Zero delay

Goal: Find optimal sampling policy that minimizes **estimation error** between signal X_t and estimate $\hat{X}_t$

Problem: Optimal Sampling

$$\min_{\pi\in\Pi} \limsup_{T\to\infty} \frac{1}{T}\mathbb{E}\left[\int_0^T \overbrace{(X_t-\hat{X}_t)^2 dt}^{\text{Estimation error}}\right]$$

Time-avg. estimation error (MSE)

$$\text{s.t.} \liminf_{n\to\infty} \frac{1}{n}\mathbb{E}\left[\sum_{i=1}^{n} \underbrace{(S_{i+1}-S_i)}_{\text{inter-sample time}}\right] \geq \frac{1}{f_{\max}}$$

Avg. sampling-rate constraint

- Sampling policy: $(S_1, S_2, ...)$
  - Causal policy space Π: Sampling time S_i decided by the history of signal and channel processes (in math, stopping time)
  - Continuous-time MDP with continuous state space and a constraint
    - Quite challenging to solve

Questions: Which sampling policy is optimal?

What is the relationship with the age of information?

Question 1: Which sampling policy is optimal?

Theorem. For unconstrained problem, a **threshold-based sampler** is optimal:

$$S_{i+1}(\beta) = \inf \Big\{ \underbrace{t \geq D_i(\beta)}_{\text{After previous sample is delivered}} : \underbrace{\big|X_t - \hat{X}_t\big| \geq v(\beta)}_{\text{Signal difference exceeds threshold}} \Big\},$$

Threshold function

After previous sample is delivered   Signal difference exceeds threshold

The optimal β is determined by solving:

$$\beta = \frac{\mathbb{E}\left[\int_{D_i(\beta)}^{D_{i+1}(\beta)} (X_t - \hat{X}_t)^2 dt\right]}{\mathbb{E}[D_{i+1}(\beta) - D_i(\beta)]} = \text{mse}_{\text{opt}}$$

Bisection search
(no "curse of dimensionality")

β = optimal objective value

$$G(x) = \frac{e^{x^2}}{x}\int_0^x e^{-t^2} dt = \frac{e^{x^2}}{x}\frac{\sqrt{\pi}}{2}\operatorname{erf}(x)$$

$$v(\beta) = \frac{\sigma}{\sqrt{\theta}} G^{-1}\left(\frac{\text{mse}_\infty - \text{mse}_{Y_i}}{\text{mse}_\infty - \beta}\right)$$

$$\text{mse}_{Y_i} = \frac{\sigma^2}{2\theta}\mathbb{E}[1 - e^{-2\theta Y_i}] \qquad \text{mse}_\infty = \frac{\sigma^2}{2\theta}$$

$$\text{mse}_{Y_i} \leq \text{mse}_{\text{opt}} \leq \beta \leq \text{mse}_\infty$$

$$\text{mse}_{\text{opt}} = \min_{\pi\in\Pi}\ \limsup_{T\to\infty} \frac{1}{T}\mathbb{E}\left[\int_0^T (X_t - \hat{X}_t)^2 dt\right]$$

Ornee and Sun, "Sampling for Remote Estimation through Queues: Age of Information and Beyond," IEEE WiOpt, 2019, Best Paper Award.

Evaluating Expectations

By Dynkin's formula and optional stopping theorem, we get

$$
\begin{aligned}
&\mathbb{E}[D_{i+1}(\beta) - D_i(\beta)] \\
=&\mathbb{E}[\max\{R_1(v(\beta)) - R_1(O_{Y_i})\}, 0] + \mathbb{E}[Y_i], \\
&\mathbb{E}\left[\int_{D_i(\beta)}^{D_{i+1}(\beta)} (X_t - \hat{X}_t)^2 dt\right] \\
=&\mathbb{E}[\max\{R_2(v(\beta)) - R_2(O_{Y_i})\}, 0] \\
&+ \text{mse}_\infty[\mathbb{E}(Y_i) - \gamma] + \mathbb{E}\left[\max\{v^2(\beta), O_{Y_i}^2\}\right]\gamma,
\end{aligned}
$$

where
$$
\gamma = \frac{1}{2\theta}\mathbb{E}[1 - e^{-2\theta Y_i}],
$$

$$
R_1(v) = \frac{v^2}{\sigma^2}\,{}_2F_2\left(1, 1; \frac{3}{2}, 2; \frac{\theta}{\sigma^2}v^2\right),
$$

$$
R_2(v) = -\frac{v^2}{2\theta} + \frac{v^2}{2\theta}\,{}_2F_2\left(1, 1; \frac{3}{2}, 2; \frac{\theta}{\sigma^2}v^2\right),
$$

Wiener Process vs. Ornstein-Uhlenbeck Process

Theorem: (Wiener Process)

The MSE-optimal sampling policy is

$$S_{i+1}(\beta) = \inf\left\{t \geq D_i(\beta) : \underbrace{\left|X_t - \hat{X}_t\right|}_{\text{instantaneous estimation error}} \geq v(\beta)\right\}$$

optimal β is determined by solving

$$\beta = \frac{\mathbb{E}\left[\int_{D_i(\beta)}^{D_{i+1}(\beta)} (X_t - \hat{X}_t)^2 dt\right]}{\mathbb{E}[D_{i+1}(\beta) - D_i(\beta)]} = \text{mse}_{\text{opt}}$$

β = optimal objective value

threshold function

$$v(\beta) = \sqrt{3(\beta - \mathbb{E}[Y_i])}$$

Theorem: (Ornstein-Uhlenbeck Process)

The MSE-optimal sampling policy is

$$S_{i+1}(\beta) = \inf\left\{t \geq D_i(\beta) : \underbrace{\left|X_t - \hat{X}_t\right|}_{\text{instantaneous estimation error}} \geq v(\beta)\right\}$$

optimal β is determined by solving

$$\beta = \frac{\mathbb{E}\left[\int_{D_i(\beta)}^{D_{i+1}(\beta)} (X_t - \hat{X}_t)^2 dt\right]}{\mathbb{E}[D_{i+1}(\beta) - D_i(\beta)]} = \text{mse}_{\text{opt}}$$

β = optimal objective value

threshold function

$$v(\beta) = \frac{\sigma}{\sqrt{\theta}} G^{-1}\left(\frac{\text{mse}_\infty - \text{mse}_{Y_i}}{\text{mse}_\infty - \beta}\right)$$

The same solution structure, different threshold functions

Ornee and Sun, "Sampling for Remote Estimation through Queues: Age of Information and Beyond," IEEE WiOpt, 2019, Best Paper Award.

Yin Sun, Yury Polyanskiy, and Elif Uysal, "Sampling of the Wiener Process for Remote Estimation over a Channel with Random Delay", *IEEE Transactions on Information Theory*, 2019.

Question 2: Relation with Age of Information?

- Recall that sampling time S_i decided by the history of signal and channel processes

- If the sampling times are independent of signal $\{X_t, t \geq 0\}$, then

$$\mathbb{E}\left[(X_t - \hat{X}_t)^2\right] = \frac{\sigma^2}{2\theta}\left(1 - e^{-2\theta\Delta(t)}\right) = p(\Delta(t))$$

(MSE = Age penalty function)

Our problem reduces to an age penalty minimizaition problem.

$$\inf_{\pi \in \Pi} \quad \limsup_{T \to \infty} \frac{1}{T} \, \mathbb{E}\left[\int_0^T p(\Delta(t))dt\right]$$

$$\text{s.t.} \quad \liminf_{n \to \infty} \frac{1}{n} \mathbb{E}\left[\sum_{i=1}^{n} (S_{i+1} - S_i)\right] \geq \frac{1}{f_{\max}}$$

Age-Optimal vs. MSE-Optimal Sampling (Signal-ignorant vs Signal-aware Sampling)

Theorem: (Age-Optimal Sampling)

The age-optimal sampling policy is

$$S_{i+1}(\beta) = \inf \Big\{ t \geq D_i(\beta) : \underbrace{\mathbb{E}[(X_{t+Y_{i+1}} - \hat{X}_{t+Y_{i+1}})^2]}_{\text{expected estimation error}} \geq \beta \Big\}$$

optimal β is determined by solving

$$\beta = \frac{\mathbb{E}\left[\int_{D_i(\beta)}^{D_{i+1}(\beta)} (X_t - \hat{X}_t)^2 dt\right]}{\mathbb{E}[D_{i+1}(\beta) - D_i(\beta)]} = \text{mse}_{\text{age-opt}}$$

β = optimal objective value

Theorem: (MSE-Optimal Sampling)

The MSE-optimal sampling policy is

$$S_{i+1}(\beta) = \inf \Big\{ t \geq D_i(\beta) : \underbrace{|X_t - \hat{X}_t|}_{\text{instantaneous estimation error}} \geq v(\beta) \Big\},$$

Threshold function

optimal β is determined by solving

$$\beta = \frac{\mathbb{E}\left[\int_{D_i(\beta)}^{D_{i+1}(\beta)} (X_t - \hat{X}_t)^2 dt\right]}{\mathbb{E}[D_{i+1}(\beta) - D_i(\beta)]} = \text{mse}_{\text{opt}}$$

β = optimal objective value

Solutions to constrained problems available in the papers.

Sun and Cyr, "Sampling for Data Freshness Optimization: Non-linear Age Functions," JCN special issue on Age of Info., 2019.

Ornee and Sun, "Sampling for Remote Estimation through Queues: Age of Information and Beyond," IEEE WiOpt, 2019, Best Paper Award.

Summary

- Optimal sampler follows a threshold policy:
  - For signal-aware sampling, the threshold is on instantaneous estimation error, and it is a function of optimal objective value.
  - For signal-ignorant sampling, the threshold is on expected estimation error, and it is equal to the optimal objective value.
  - It is conjectured that this structure holds for a number of strong Markov signals.
  - Tool: Free-boundary method for solving optimal stopping problems of diffusion processes.

- More general than the Queueing model
  - Results should hold for other channel models (e.g., erasure channel)

Topic III

Age-Optimal Scheduling in Networks

Joint work with

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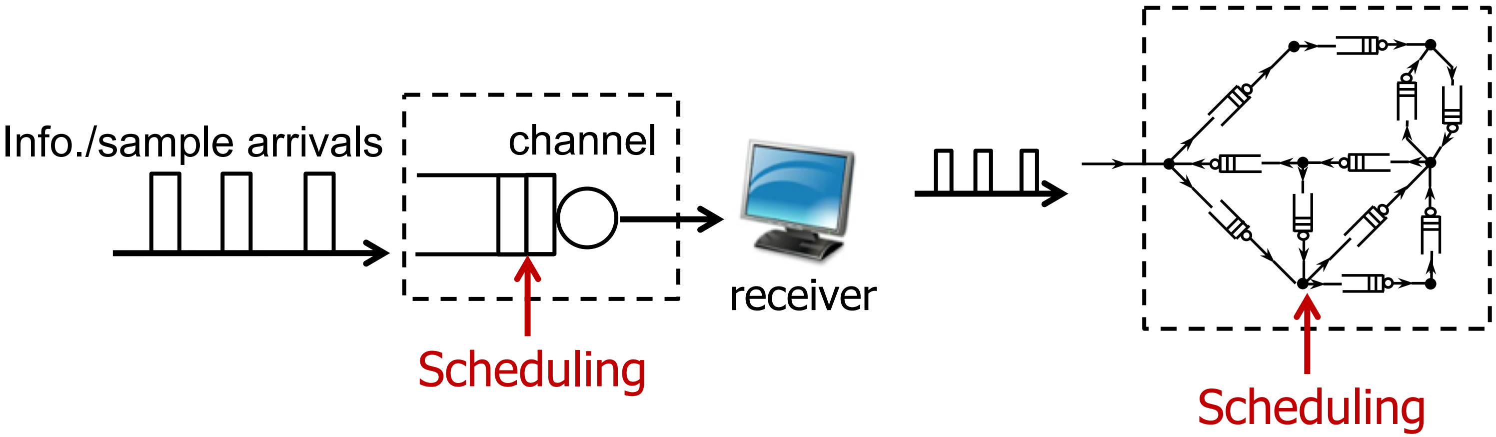
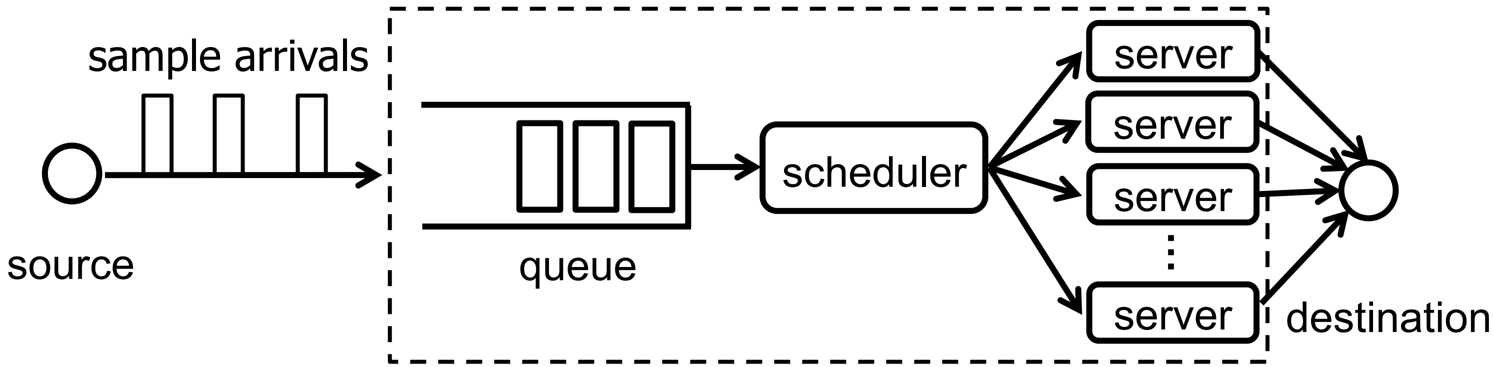
Sastry Kompella

Motivation:



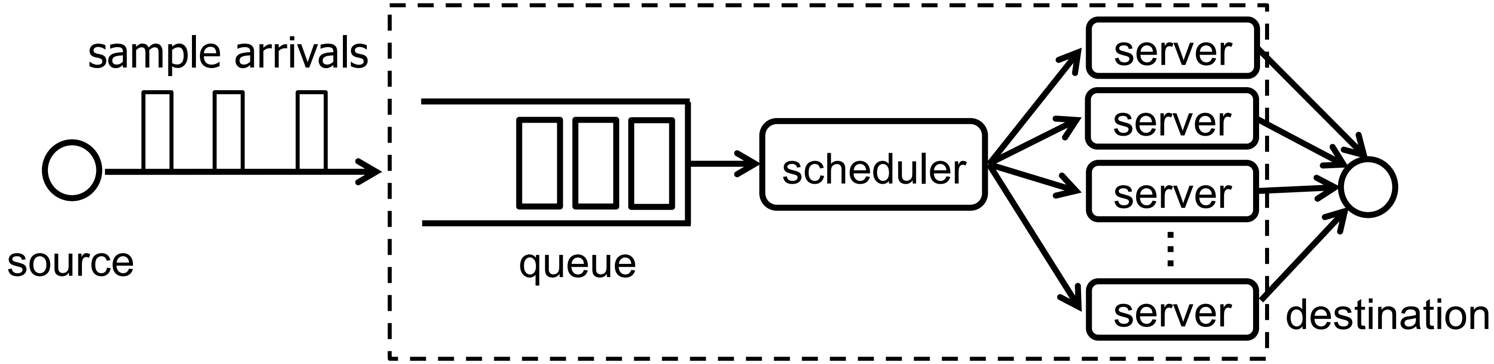
- In news, email, and social updates, the arrival process of data samples cannot be controlled
- Can we improve data freshness by designing the service order of the samples?

Single-Source, Multi-Channel Networks



- Single source, **arbitrarily given arrival** process:
  - Sample i is generated at S_i, arrives at A_i. Hence, $S_i \leq A_i$
  - Arbitrarily given sample generation & arrival times
    - Out-of-order arrivals are possible, e.g., $A_i < A_j, S_i > S_j$
- Multiple **channels,** *i.i.d.* transmission times
- Scheduling Policy:
  - Given sample generation and arrival times, **scheduler** decides which samples to send over time

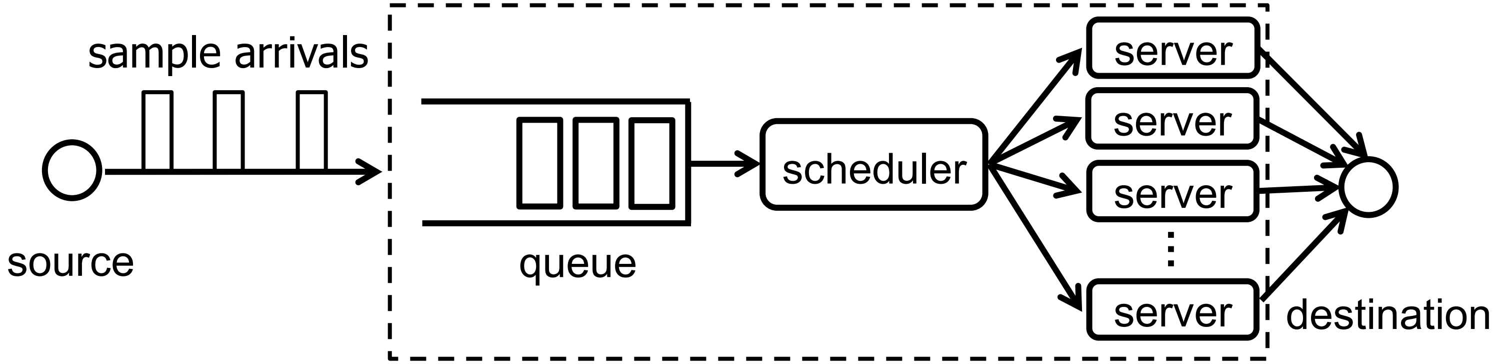
Scheduling Policy Space



- Preemptive Policy:
  - One sample can interrupt the transmission of another sample
- Causal Policy:
  - Scheduler makes decisions based on history and current information
  - Non-causal policies are not practical

Goal: Find the optimal policy that minimizes the age among all casual policies?

Last Generated First Served (LGFS) Policy



Last Generated, First Served (LGFS) policy:

- The last generated sample is sent first, with ties broken arbitrarily

Last Come, First Served (LCFS) policy:

- The last arrived sample is sent first, with ties broken arbitrarily.

- If packets arrive in the same order of their generation times, i.e., $(A_i - A_j)(S_i - S_j) \geq 0$, LGFS becomes LCFS

Metrics: Age Penalty Functional

- ***Age penalty functional*** $p(\{\Delta(t), t \geq 0\})$
  - Any non-decreasing functional p of age process $\{\Delta(t), t \in [0, \infty)\}$, i.e.,

$$\text{If } \Delta_1(t) \leq \Delta_2(t), t \geq 0, \text{ then } p(\{\Delta_1(t), t \geq 0\}) \leq p(\{\Delta_2(t), t \geq 0\})$$

- Examples of age penalty functionals:
  - Avg. age: [Kaul, Yates, Gruteser'12, etc.]

$$p_1(\{\Delta(t), t \geq 0\}) = \frac{1}{T} \int_0^T \Delta(t) dt$$

  - Avg. age penalty function: [Sun, Uysal, Yates, Koksal, Shroff'16, etc.]

$$p_2(\{\Delta(t), t \geq 0\}) = \frac{1}{T} \int_0^T h(\Delta(t)) dt$$

  - Age tail distribution:

Indicator function

$$p_3(\{\Delta(t), t \geq 0\}) = \frac{1}{T} \int_0^T 1_{\{\Delta(t) \geq b\}} dt$$

Very general age metrics

Preemptive LGFS Minimizes Age Process!

Theorem: If the transmission times are i.i.d. exponential, then for all non-decreasing functional p and all sample generation/arrival times $\mathcal{I} = \{n, (S_i, A_i)_{i=1}^n\}$, it holds that

$$\mathbb{E}[p(\{\Delta_{\text{prmp-LGFS}}(t), t \geq 0\})|\mathcal{I}] = \min_{\pi \in \Pi} \mathbb{E}[p(\{\Delta_\pi(t), t \geq 0\})|\mathcal{I}], \ \forall\, \mathcal{I}$$

where Π is the set of all causal policies.

Corollary: If, in addition, packets arrive in the same order of their generation times, then preemptive LCFS is age-optimal, i.e.,

$$\mathbb{E}[p(\{\Delta_{\text{prmp-LCFS}}(t), t \geq 0\})|\mathcal{I}] = \min_{\pi \in \Pi} \mathbb{E}[p(\{\Delta_\pi(t), t \geq 0\})|\mathcal{I}], \ \forall\, \mathcal{I}$$

Preemptive LGFS and preemptive LCFS are age-optimal in a quite strong sense, if service times are i.i.d. exponential.

Bedewy, Sun, Shroff, "Optimizing Data Freshness, Throughput, and Delay in Multi-Server Information-Update Systems", *IEEE ISIT*, 2016.

Bedewy, Sun, Shroff, "Minimizing the Age of Information through Queues", *IEEE Trans. on Inf. Theory*, 2019.

How about Non-Exponential Distributions?

- **New-Better-than-Used (NBU) distribution**:

$$\Pr[X > t] \geq \Pr[X > t + \tau | X > \tau], \ \forall\ t, \tau \geq 0.$$

P{New packet takes more than t sec to complete} $\geq$ P{Old packet takes more than another t sec to complete}

- E.g., constant time, exponential, geometric, gamma distributions

Theorem: If the transmission times are i.i.d. NBU, then for all non-decreasing functional p and all sample generation/arrival times $\mathcal{I} = \{n, (S_i, A_i)_{i=1}^n\}$, it holds that

$$\min_{\pi \in \Pi_{np}} \limsup_{T \to \infty} \frac{1}{T} \mathbb{E}\left[\int_0^T \Delta_\pi(t) dt \middle| \mathcal{I} \right] \leq \limsup_{T \to \infty} \frac{1}{T} \mathbb{E}\left[\int_0^T \Delta_{\text{non-prmp-LGFS}}(t) dt \middle| \mathcal{I} \right]$$

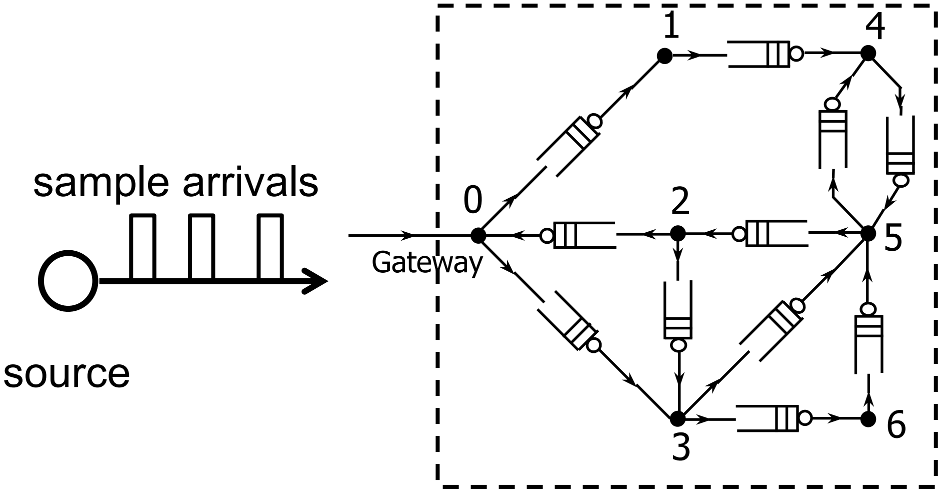
$$\leq \min_{\pi \in \Pi_{np}} \limsup_{T \to \infty} \frac{1}{T} \mathbb{E}\left[\int_0^T \Delta_\pi(t) dt \middle| \mathcal{I} \right] + \mathbb{E}[X],$$

where Π_{np} is the set of non-preemptive causal policies.

Mean service time of a packet

Non-preemptive LGFS is within a small additive gap from age-optimum.

Single-Source, Multi-Hop Networks



Age vector of all nodes:

$$\mathbf{\Delta}(t) = (\Delta_1(t), \ldots, \Delta_N(t))$$

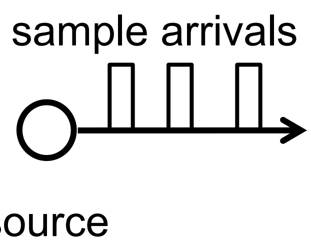
Theorem:

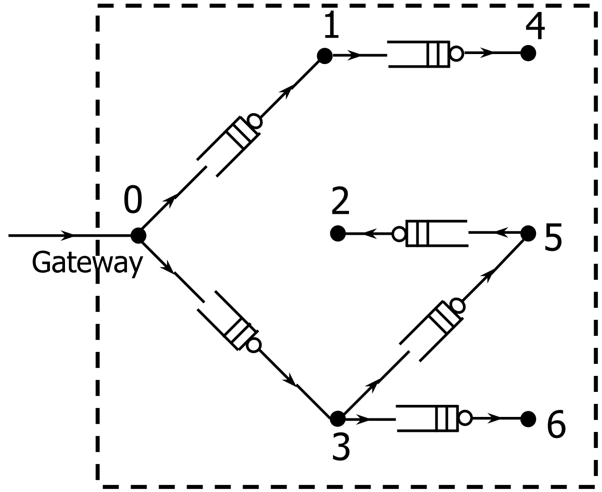
- For independent exponential service times, arbitrary topology
  - Preemptive LGFS minimizes all non-decreasing functional of the age processes at all nodes:

$$\mathbb{E}[p(\{\mathbf{\Delta}_{\text{prmp-LGFS}}(t), t \geq 0\})|\mathcal{I}] = \min_{\pi \in \Pi} \mathbb{E}[p(\{\mathbf{\Delta}_{\pi}(t), t \geq 0\})|\mathcal{I}], \ \forall\ \mathcal{I}$$

Bedewy, Sun, Shroff, "Age-Optimal Information Updates in Multihop Networks", *IEEE ISIT*, 2017.
Bedewy, Sun, Shroff, "Age of Information in Multihop Networks", *IEEE/ACM Transactions on Networking*, 2019.

Single-Source, Multi-Hop Networks





tree topology

Age vector of all nodes:

$$\mathbf{\Delta}(t) = (\Delta_1(t), \ldots, \Delta_N(t))$$

Theorem:

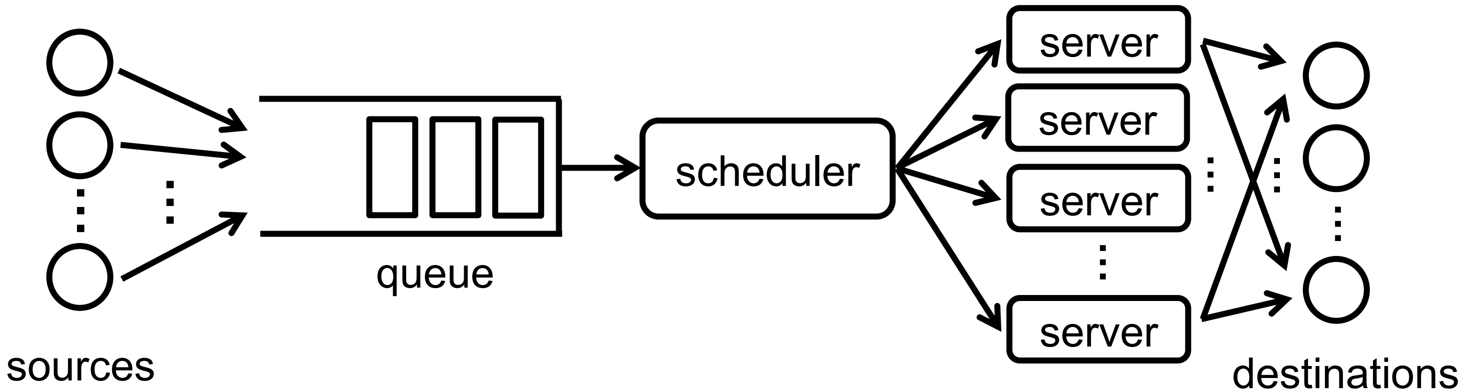
- For independent exponential service times, arbitrary topology
  - Preemptive LGFS minimizes all non-decreasing functional of the age processes at all nodes:

$$\mathbb{E}[p(\{\mathbf{\Delta}_{\text{prmp-LGFS}}(t), t \geq 0\})|\mathcal{I}] = \min_{\pi \in \Pi} \mathbb{E}[p(\{\mathbf{\Delta}_{\pi}(t), t \geq 0\})|\mathcal{I}], \ \forall \ \mathcal{I}$$

- For independent NBU service times, tree topology
  - Non-preemptive LGFS is near age-optimal.

Bedewy, Sun, Shroff, "Age-Optimal Information Updates in Multihop Networks", *IEEE ISIT*, 2017.
Bedewy, Sun, Shroff, "Age of Information in Multihop Networks", *IEEE/ACM Transactions on Networking*, 2019.

Multi-Flow, Multi-Channel Networks



- Multiple flows, each flow with a source and a destination.
- Let $\boldsymbol{\Delta}(t) = (\Delta_1(t), \ldots, \Delta_N(t))$ be the age vector of N flows.
- **Maximum Age First, Last Generated First Served (MAF-LGFS) policy**: The last generated packet from the flow with the maximum age is sent first, with ties broken arbitrarily.

Theorem: Under certain conditions (synchronized arrivals, one server, Exp distribution), preemptive MAF-LGFS minimizes any *symmetric* and *non-decreasing* penalty function of the age vector over time.

- Near age-optimal results for NBU distributions are in

Yin Sun, Elif Uysal-Biyikoglu, and Sastry Kompella, Age-Optimal Updates of Multiple Information Flows, *IEEE INFOCOM Age of Information Workshop (AoI Workshop)*, 2018.

Summary

- For i.i.d. Exponential service times, preemptive LGFS is age-optimal.
- For i.i.d. New-Better-than-Used (NBU) service times, non-preemptive LGFS is near age-optimal.
  - These results hold for
    - Minimizing the age of information process
    - Minimizing non-decreasing functionals of age process
    - Arbitrarily given packet generation times and arrival times (limitations exist for multi-flow case)
    - Several queueing network models
  - In-order arrivals ➔ LGFS becomes LCFS
- Tools:
  - Sample-path method
  - Stochastic ordering
- Some Ads:
  - 3rd Age of Information Workshop

The 3rd Age of Information Workshop

- April 27th 2020 at Beijing, with IEEE INFOCOM 2020
- General Chairs:
  - Ness B. Shroff (The Ohio State University)
  - Zhisheng Niu (Tsinghua University)
- TPC Chairs:
  - Sennur Ulukus (University of Maryland)
  - Sheng Zhou (Tsinghua University)

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