Quiz #1

1. $\int \frac{1+x}{1+x^2} \, dx \longrightarrow \int \frac{1}{x^2+1} \, dx + \int \frac{x}{x^2+1} \, dx$

$u = x^2+1$
$du = 2x \, dx$
$\frac{1}{2} du = x \, dx$

$$= \tan^{-1}(x) + \frac{1}{2}\int \frac{1}{u} \, du$$

$$= \tan^{-1}(x) + \frac{1}{2}\ln|u| + C$$

$$\boxed{= \tan^{-1}(x) + \frac{1}{2}\ln|x^2+1| + C}$$

2. $\int \frac{6t}{t^4 + t^2 + \frac{5}{4}} \, dt \longrightarrow 6\int \frac{t}{t^4 + t^2 + \frac{1}{4} + 1} \, dt$

$$= 6\int \frac{t}{(t^2 + \frac{1}{2})^2 + 1} \, dt$$

$u = t^2 + \frac{1}{2}$
$du = 2t \, dt$
$\frac{1}{2} du = t \, dt$

$$= 6 \cdot \frac{1}{2}\int \frac{1}{u^2+1} \, du$$

$$= 3\tan^{-1}(u) + C$$

$$\boxed{= 3\tan^{-1}\left(t^2 + \frac{1}{2}\right) + C}$$

★ Memorize either of the following for future use

$$\int \frac{1}{x^2+1} \, dx = \tan^{-1}(x) + C$$

$$\int \frac{1}{x^2+a^2} \, dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$