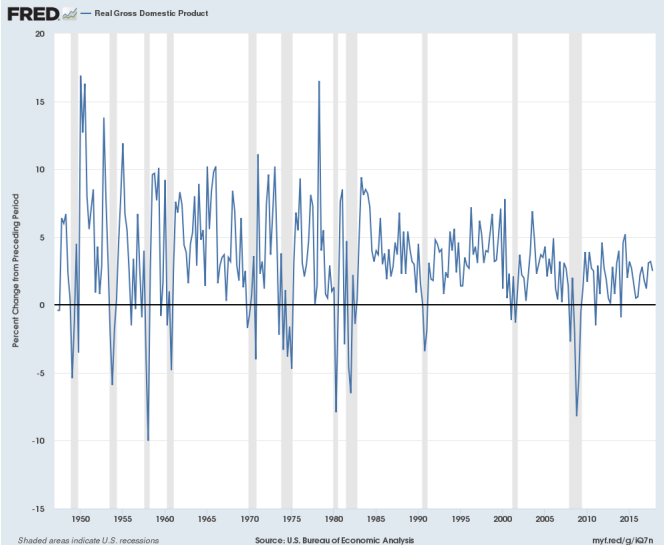
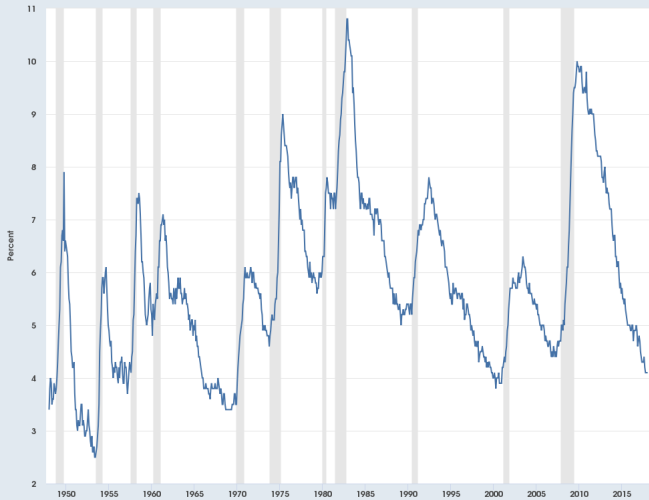


Real Business Cycle model

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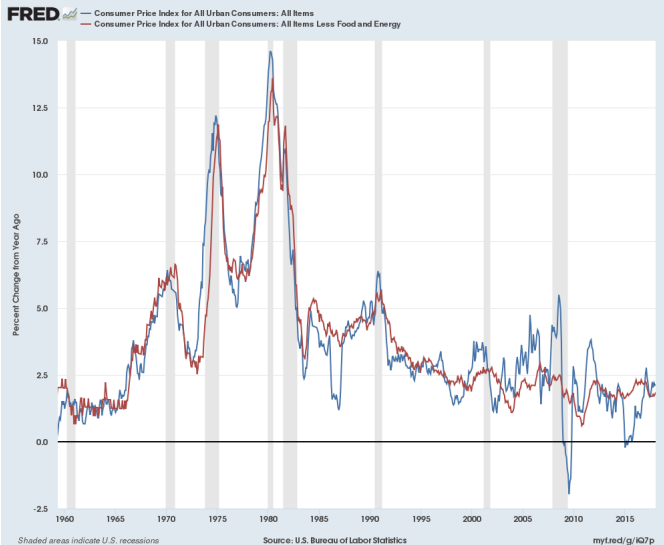


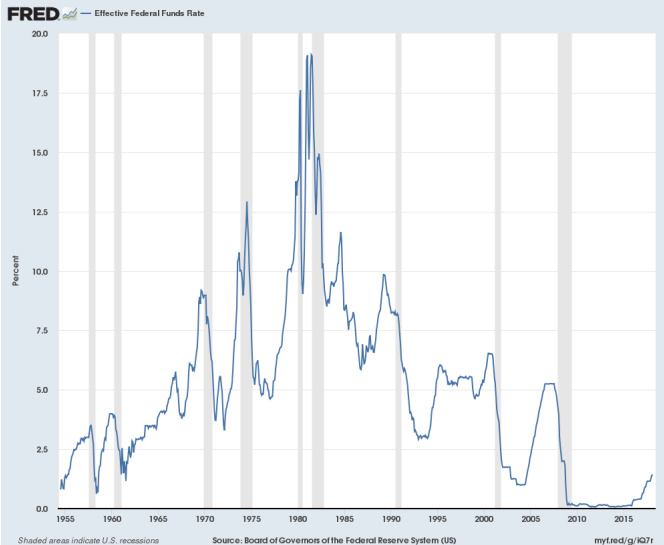


Shaded areas indicate U.S. recessions

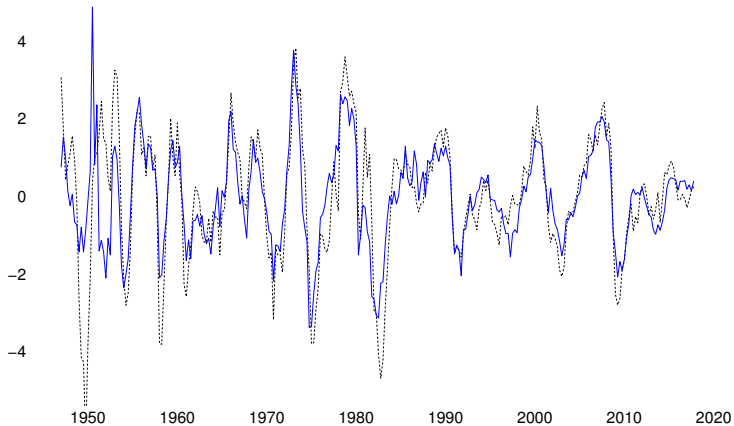
Source: U.S. Bureau of Labor Statistics

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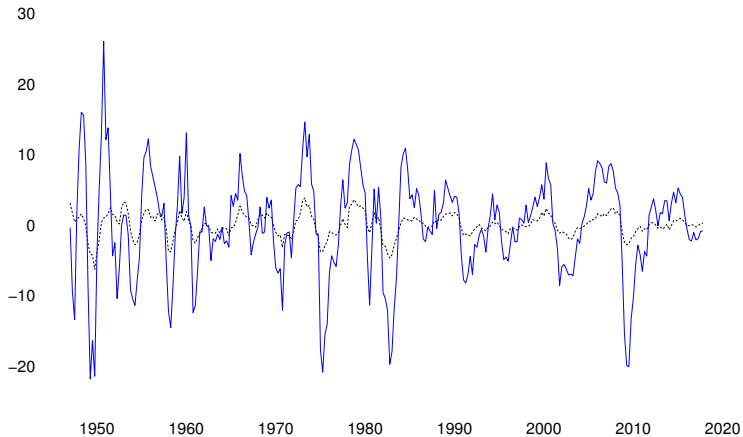




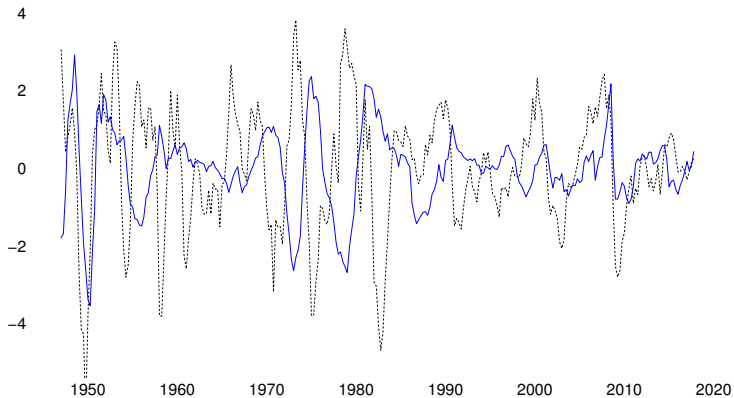
GDP v. Consumption
% deviations from trend



GDP v. Investments
% deviations from trend



GDP v. Price index
% deviations from trend



	<i>Cyclical</i>	<i>Lead/Lag</i>	<i>Variability Relative to GDP</i>
Consumption	Procyclical	Coincident	Smaller
Investment	Procyclical	Coincident	Larger
Price Level	Countercyclical	Coincident	Smaller
Money Supply	Procyclical	Leading	Smaller
Employment	Procyclical	Lagging	Smaller
Real Wage	Procyclical	?	?
Average Labor Productivity	Procyclical	Coincident	Smaller

Source: Williamson (1995), 'Macroeconomics'

Business cycles

Economies fluctuate over time: need to explain

1. Volatility
2. Comovements
3. Persistence (autocorrelation)
4. Effect of expectations on current decisions

Two theoretical approaches

1. Market clearing
2. Non-market clearing

Two competing models

1. Real Business Cycle model (RBC)
2. New-Keynesian model (NK)

Similarities

- ▶ Dynamic general equilibrium
- ▶ Stochastic shocks
- ▶ Forward looking expectations

Main difference concerns information and prices

- ▶ RBC: Complete and flexible
- ▶ NK: Incomplete and sticky

Real Business Cycle model

1. Take Swan-Solow growth model
2. Insert (1) into dynamic optimisation framework
 - ▶ No more constant savings rate
3. Add shocks to total factor productivity (A)
 - ▶ Include uncertainty about shocks
4. Add leisure to account for changes in hours of work

Why real?

Equilibrium is about

- ▶ Household preferences
- ▶ Technology used by firms
- ▶ Government policy decisions

These are **real** factors

Recap: **Swan-Solow model**

$$Y_t = A_t K_t^\alpha N_t^{1-\alpha} \quad (1)$$

$$K_t = (1 + \delta)K_{t-1} + I_t \quad (2)$$

$$I_t = sY_t \quad (3)$$

$$N_t = (1 + n)N_{t-1} \quad (4)$$

$$A_t = (1 + m)A_{t-1} \quad (5)$$

$$Y_t \equiv C_t + I_t \quad (6)$$

$$0 < \alpha < 1, \delta > 0, 0 < s < 1$$

Model's fundamental mechanism: shocks to Total Factor Productivity (TFP)

- ▶ Recall positive correlation between GDP and productivity

Major result: Fluctuations as an equilibrium outcome

- ▶ Work harder when productivity is high: wages increase as labour becomes more productive
- ▶ Save more when productivity is high: interest rates increase as capital becomes more productive

Fluctuations in economy are not that bad

Real Business Cycle model assumes

1. Perfectly functioning competitive markets
2. Rational expectations

Outcomes generated by decentralized decisions of firms and households: can be replicated as solution to a social planner problem who want so maximise

$$\mathbb{E}_t \left[\sum_{i=0}^{\infty} \beta^i (U(C_{t+i}) - V(N_{t+i})) \right] \quad (7)$$

C_t is consumption

N_t hours worked

β is the household's rate of time preference

$$U(C_t) - V(N_t) = \frac{C_t^{1-\eta}}{1-\eta} - \nu N_t \quad (8)$$

Economic constraints

$$Y_t = C_t + I_t = A_t K_{t-1}^\alpha N_t^{1-\alpha} \quad (9)$$

$$K_t = I_t + (1 - \delta)K_{t-1} \quad (10)$$

Technology process A_t is usually a log-linear AR(1) process

- For simplicity assume A_t does not trend over time: economy has average growth rate of zero.

$$\log A_t = (1 - \rho) \log A^* + \rho \log A_{t-1} + \epsilon_t \quad (11)$$

A^* indicates the steady-state for technology.

Criticism

There is some critique on the RBC model specifically concerning

1. Assumption of perfect markets and rational expectations
2. Role of monetary and fiscal policy
3. Role of technology shocks

Perfect markets and rational expectations

- ▶ Can economy be characterized as a perfectly competitive market equilibrium solution describing the behaviour of a set of completely optimising rational agents?
- ▶ Markets are not always competitive; people not always rational in economic decisions

RBC model a benchmark: compare with more complicated models

- ▶ Are market imperfections such as sticky price crucial to understanding macroeconomic fluctuations?
- ▶ Can account for imperfect competition

Monetary & fiscal policy

- ▶ RBC exhibits complete monetary neutrality
- ▶ Including government spending; model would exhibit Ricardian equivalence

Most modern RBC models include mechanisms allowing for monetary/fiscal policy to have Keynesian effects

- ▶ e.g. most DGSE models include sticky prices/wages; leads to real effects for monetary policy

Technology shocks source of economics fluctuations: All variables apart from A_t are deterministic

1. Unclear what these shocks are
2. Credibility of economic fluctuations being optimal response to technology shocks

Technology shocks probably more important than one might think

- ▶ Role of TFP in growth theory: Random TFP fluctuations seem not so strange
- ▶ RBC can generate recessions without technology declines
 - ▶ Output elasticity wrt technology >1

Solving the model

1. Formulate the Lagrangian
2. Find the first order conditions (FOCs)
3. Log-linearise the FOCs
4. Find the steady-state

Constraints

$$\begin{aligned} Y_t &= C_t + I_t = A_t K_{t-1}^\alpha N_t^{1-\alpha} \\ K_t &= I_t + (1 - \delta) K_{t-1} \end{aligned} \tag{12}$$

Can combine in single equation

$$A_t K_{t-1}^\alpha N_t^{1-\alpha} = C_t + K_t - (1 - \delta) K_{t-1} \tag{13}$$

Formulate as **Langrangian** problem

$$\mathcal{L} = \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i [U(C_{t+i}) - V(N_{t+i})] + \quad (14)$$
$$\mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \lambda_{t+i} [A_t K_{t+i-1}^{\alpha} N_t^{1-\alpha} + (1 - \delta) K_{t+i-1} - C_{t+i} - K_{t+i}]$$

The Langrangian involves picking a series of values for consumption and labour, subject to satisfying a series of constraints.

Infinite sum

Equations sums to infinity

- ▶ Infinite number of FOCs for current and expected values of C_t, K_t, N_t .
- ▶ Simplify by looking at when exactly the time t and $t + n$ variables appear

Time t variables appear as

$$\begin{aligned} & U(C_t) - V(N_t) \\ & + \lambda_t (A_t K_{t-1}^\alpha N_t^{1-\alpha} - C_t - K_t + (1 - \delta) K_{t-1}) \\ & + \beta \mathbb{E}_t [\lambda_{t+1} (A_{t+1} K_t^\alpha N_{t+1}^{1-\alpha} + (1 - \delta) K_t)] \end{aligned} \tag{15}$$

t variables only appear once

- ▶ FOCs consist of differentiating the model and setting the derivatives equal to zero

$t + n$ appear exactly as the t variables: only in expectation form and multiplied by discount β^n

- ▶ FOCs are identical to the t variables

Example: Capital

$$\begin{aligned} & U(C_t) - V(N_t) \\ & + \lambda_t (A_t K_{t-1}^\alpha N_t^{1-\alpha} - C_t - K_t + (1 - \delta) K_{t-1}) \\ & + \beta \mathbb{E}_t [\lambda_{t+1} (A_{t+1} K_t^\alpha N_{t+1}^{1-\alpha} + (1 - \delta) K_t)] \end{aligned} \tag{16}$$

$$\frac{\partial \mathcal{L}}{\partial K_t} = -\lambda_t + \beta \mathbb{E}_t \left[\lambda_{t+1} \left(\alpha \frac{Y_{t+1}}{K_t} + 1 - \delta \right) \right] \tag{17}$$

First Order Conditions

$$\frac{\partial \mathcal{L}}{\partial C_t} : U'(C_t) - \lambda_t = 0 \quad (18)$$

$$\frac{\partial \mathcal{L}}{\partial K_t} : -\lambda_t + \beta E_t \left[\lambda_{t+1} \left(\alpha \frac{Y_{t+1}}{K_t} + 1 - \delta \right) \right] = 0 \quad (19)$$

$$\frac{\partial \mathcal{L}}{\partial N_t} : -V'(N_t) + (1 - \alpha) \lambda_t \frac{Y_t}{N_t} = 0 \quad (20)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_t} : A_t K_{t-1}^\alpha N_t^{1-\alpha} - C_t - K_t + (1 - \delta) K_{t-1} = 0 \quad (21)$$

Keynes-Ramsey condition

Define the marginal value of an additional unit of capital next year as

$$R_{t+1} = \alpha \frac{Y_{t+1}}{K_t} + 1 - \delta \quad (22)$$

Write FOC capital as

$$\lambda_t = \beta \mathbb{E}_t(\lambda_{t+1} R_{t+1}) \quad (23)$$

Combine with FOC consumption

$$U'(C_t) = \beta \mathbb{E}_t[U'(C_{t+1}) R_{t+1}] \quad (24)$$

Marginal utility of consumption must equal marginal utility of capital

$$U'(C_t) = \beta \mathbb{E}_t[U'(C_{t+1})R_{t+1}]$$

Decrease in consumption by Δ today at a utility loss of

$$U'(C_t)\Delta$$

Invest to get $R_{t+1}\Delta$ tomorrow which will be worth

$$\beta \mathbb{E}_t[U'(C_{t+1})R_{t+1}\Delta]$$

in terms of today's utility

- ▶ Along an optimal path, the household must be indifferent

Consumption and Separable Consumption-Leisure

$$U(C_t) - V(N_t) = \frac{C_t^{1-\eta}}{1-\eta} - \nu N_t \quad (25)$$

Formulation of the Constant Relative Risk Aversion (CRRA) utility from consumption and separate disutility from labour turns out to be necessary for the model to have a stable growth path solution.

We have

$$U'(C_t) = C_t^{-\eta} = \lambda \quad (26)$$

$$V'(N_t) = -\nu \quad (27)$$

Substituting $U'(C_t)$ into Keynes-Ramsey condition

$$C_t^{-\eta} = \beta \mathbb{E}_t(C_{t+1}^{-\eta} R_{t+1}) \quad (28)$$

FOC N_t for optimal hours worked becomes

$$-\nu + (1 - \alpha) C_t^{-\eta} \frac{Y_t}{N_t} = 0 \quad (29)$$
$$\frac{Y_t}{N_t} = \frac{\nu}{1 - \alpha} C_t^{\eta}$$

RBC model can be defined by six equations

- ▶ Three identities describing resource constraints
- ▶ Two FOCs describing optimal behaviour
- ▶ One definition

$$Y_t = C_t + I_t \quad (30)$$

$$Y_t = A_t K_{t-1}^\alpha N_t^{1-\alpha} \quad (31)$$

$$K_t = I_t + (1 - \delta)K_{t-1} \quad (32)$$

$$C_t^{-\eta} = \beta E_t(C_{t+1}^{-\eta} R_{t+1}) \quad (33)$$

$$\frac{Y_t}{N_t} = \frac{\nu}{1 - \alpha} C_t^\eta \quad (34)$$

$$R_t = \alpha \frac{Y_t}{K_{t-1}} + 1 - \delta \quad (35)$$

RBC model is a nonlinear system of stochastic difference equations

- ▶ Extremely difficult to obtain solutions
- ▶ A trick: linearise the system in vicinity of steady state

System has 7 variables

$$(Y_{t+i}, K_{t+i}, I_{t+i}, C_{t+i}, N_{t+i}, A_{t+i})_{i=0}^{\infty}$$

For any variable x_t in steady state we get

$$x_t = x_{t+1} = \bar{x} \tag{36}$$

Natural way to linearise model is using logs or Δ logs

Taking log differences ($\Delta \log s$)

$$Y_t = 2X_t \Leftrightarrow y = x$$

$$Y_t = 2X_tZ_t \Leftrightarrow y = x + z$$

$$Y_t = 2X_tZ_t^{-3} \Leftrightarrow y = x - 3z$$

$$Y_{t+1} = X_{t+1} + Z_{t+1} \Leftrightarrow y = x \frac{X_t}{Y_t} + z \frac{Z_t}{Y_t}$$

$$Y_{t+1} = X_{t+1} + a \Leftrightarrow y = x \frac{X_t}{Y_t}$$

Taylor series

Non-linear function $F(x_t, y_t)$ can be approximated around any point x_t^*, y_t^* using

$$\begin{aligned} F(x_t, y_t) = & F(x_t^*, y_t^*) \\ & + F_x(x_t^*, y_t^*)(x_t - x_t^*) \\ & + F_y(x_t^*, y_t^*)(y_t - y_t^*) \\ & + F_{xx}(x_t^*, y_t^*)(x_t - x_t^*)^2 \\ & + F_{xy}(x_t^*, y_t^*)(x_t - x_t^*)(y_t - y_t^*) \\ & + F_{yy}(x_t^*, y_t^*)(y_t - y_t^*) + \dots \end{aligned} \tag{37}$$

If gap between (x_t, y_t) and (x_t^*, y_t^*) is small, then terms in second and higher order powers and cross-terms will all be very small and can be ignored leaving something like

$$F(x_t, y_t) \approx \alpha + \beta_1 x_t + \beta_2 y_t \quad (38)$$

If we linearise around point that is far away from (x_t, y_t) , then the approximation will not be accurate.

Steady-state path

DSGE models use particular version of this technique: Log-linearise variables around steady-state path

- ▶ Around this path all real variables grow at same rate

Stochastic economy will on average fluctuate around values given by steady state path

- ▶ Can get therefore an accurate approximation
- ▶ Provides set of linear equations in log-deviations of variables from steady-state values

$$x_t = \log X_t - \log X^*$$

Recall: log-differences are approximately percentage deviations

$$\ln X - \ln Y \approx \frac{X - Y}{Y}$$

Approach provides

- ▶ System of variables expressed in percentage deviations from steady-state path
- ▶ System that can be thought of as business-cycle component of model
- ▶ Coefficients are elasticities (also easy with IRF)
- ▶ Easy to implement

Log-linearisation

Use lower-case letters to define log-deviations of variables from steady state values

$$x_t = \log X_t - \log X^* \quad (39)$$

Key is that every variable can be written as

$$X_t = X^* \frac{X_t}{X^*} = X^* e^{x_t} \quad (40)$$

Big trick: first-order Taylor approximation for e^{x_t} given by

$$e^{x_t} \approx 1 + x_t \quad (41)$$

Can write variable as

$$X_t \approx X^* (1 + x_t) \quad (42)$$

Second trick is that you set

$$x_t y_t = 0 \quad (43)$$

For variables multiplying each other

$$X_t Y_t \approx X^* Y^* (1 + x_t)(1 + y_t) \quad (44)$$

$$\approx X^* Y^* (1 + x_t + y_t) \quad (45)$$

Multiplying small deviations from steady-state will produce term close to zero anyway.

Note

We have assumed that technology, the source of all long-run growth, is given by

$$a_t = \rho a_{t-1} + \epsilon_t \quad (46)$$

Meaning there is no trend growth in this economy

- ▶ Entails all steady-state variables are constants

Example: Income

$$Y_t = C_t + I_t \quad (47)$$

Rewrite

$$Y^* e^{y_t} = C^* e^{c_t} + I^* e^{i_t} \quad (48)$$

Use first-order approximation

$$Y^*(1 + y_t) = C^*(1 + c_t) + I^*(1 + i_t) \quad (49)$$

Steady-state terms must obey identities

$$Y^* = C^* + I^* \quad (50)$$

Canceling terms on both sides

$$Y^* y_t = C^* c_t + I^* i_t \quad (51)$$

Which we can write

$$y_t = \frac{C^*}{Y^*} c_t + \frac{I^*}{Y^*} i_t \quad (52)$$

Example: Production function

$$Y_t = A_t K_{t-1}^{\alpha} N_t^{1-\alpha} \quad (53)$$

Re-write in terms of steady-state and log deviations

$$Y^* e^{y_t} = (A^* e^{a_t})(K^*)^{\alpha} e^{\alpha k_{t-1}} (N^*)^{1-\alpha} e^{(1-\alpha)n_t} \quad (54)$$

Steady-state must obey identities

$$Y^* = A^* (K^*)^{\alpha} (N^*)^{1-\alpha} \quad (55)$$

Canceling terms we get

$$e^{y_t} = e^{a_t} e^{\alpha k_{t-1}} e^{(1-\alpha)n_t} \quad (56)$$

Use first-order Taylor approximation

$$(1 + y_t) = (1 + a_t)(1 + \alpha k_{t-1})(1 + (1 - \alpha)n_t) \quad (57)$$

Ignore cross-products of log-deviations: simplifies to

$$y_t = a_t + \alpha k_{t-1} + (1 - \alpha)n_t \quad (58)$$

Log-linearised system

$$y_t = \frac{C^*}{Y^*} c_t + \frac{I^*}{Y^*} i_t$$

$$y_t = a_t + \alpha k_{t-1} + (1 - \alpha) n_t$$

$$k_t = \frac{I^*}{K^*} i_t + (1 - \delta) k_{t-1}$$

$$n_t = y_t - \eta c_t$$

$$c_t = \mathbb{E}_t c_{t+1} - \frac{1}{\eta} \mathbb{E}_t r_{t+1}$$

$$r_t = \left(\frac{\alpha}{R^*} \frac{Y^*}{K^*} \right) (y_t - k_{t-1})$$

$$a_t = \rho a_{t-1} + \epsilon_t$$

Calculating steady-state

Three steady-state variables that need to be calculated; involves terms

$$\frac{C^*}{Y^*}, \frac{I^*}{Y^*}, \frac{I^*}{K^*}, \frac{\alpha}{R^*} \frac{Y^*}{K^*} \quad (59)$$

1. Take original non-linearised RBC system
2. Figure out what it looks like along zero growth path

Recall

$$y_t = y_{t+1} = y^* \quad (60)$$

Therefore

$$\frac{y_t}{y_{t+1}} = 1 \quad (61)$$

Return on capital

Linked to consumption behaviour via Keynes-Ramsey condition

$$C_t^{-\eta} = \beta \mathbb{E}_t(C_{t+1}^{-\eta} R_{t+1}) \quad (62)$$

$$1 = \beta \mathbb{E}_t \left(\left(\frac{C_t}{C_{t+1}} \right)^\eta R_{t+1} \right) \quad (63)$$

No trend growth in technology: constant steady-state values, no uncertainty

$$C_t^* = C_{t+1}^* = C^* \quad (64)$$

Ergo

$$R^* = \beta^{-1} \quad (65)$$

In a no-growth economy, the rate of return on capital is determined by the rate of time preference.

$$R_t = \alpha \frac{Y_t}{K_{t-1}} + 1 - \delta \quad (66)$$

In steady-state becomes

$$R^* = \beta^{-1} = \alpha \frac{Y^*}{K^*} + 1 - \delta \quad (67)$$

Re-arranging

$$\frac{Y^*}{K^*} = \frac{\beta^{-1} + \delta - 1}{\alpha} \quad (68)$$

Telling us that

$$\frac{\alpha}{R^*} \frac{Y^*}{K^*} = \alpha \beta \left(\frac{\beta^{-1} + \delta - 1}{\alpha} \right) \quad (69)$$

$$= 1 - \beta(1 - \delta) \quad (70)$$

Only have to find **investment-capital** and **investment-output** ratio: Can use identity

$$K_t = I_t + (1 - \delta)K_{t-1} \quad (71)$$

In steady-state we have $K_t^* = K_{t-1}^* = K^*$ so we get

$$K^* = I^* + (1 - \delta)K^* \quad (72)$$

$$K^* = I^* + K^* - \delta K^*$$

$$I^* = \delta K^*$$

$$\frac{I^*}{K^*} = \delta$$

We have $\frac{Y^*}{K^*}$ and $\frac{I^*}{K^*}$, combining these

$$\begin{aligned}\frac{I^*}{Y^*} &= \frac{\frac{I^*}{K^*}}{\frac{Y^*}{K^*}} = \frac{\delta}{\frac{\beta^{-1} + \delta - 1}{\alpha}} \\ &= \frac{\alpha\delta}{\beta^{-1} + \delta - 1}\end{aligned}\tag{73}$$

Follows that **consumption-output** ratio is given by

$$\frac{C^*}{Y^*} = 1 - \frac{\alpha\delta}{\beta^{-1} + \delta - 1}\tag{74}$$

Final system

$$y_t = \left(1 - \frac{\alpha\delta}{\beta^{-1} + \delta - 1}\right) c_t + \left(\frac{\alpha\delta}{\beta^{-1} + \delta - 1}\right) i_t \quad (75)$$

$$y_t = a_t + \alpha k_{t-1} + (1 - \alpha)n_t \quad (76)$$

$$k_t = \delta i_t + (1 - \delta)k_{t-1} \quad (77)$$

$$n_t = y_t - \eta c_t \quad (78)$$

$$c_t = E_t c_{t+1} - \frac{1}{\eta} E_t r_{t+1} \quad (79)$$

$$r_t = (1 - \beta(1 - \delta))(y_t - k_{t-1}) \quad (80)$$

$$a_t = \rho a_{t-1} + \epsilon_t \quad (81)$$

Simulation

1. Make assumption about underlying parameter values

$$\alpha, \beta, \delta, \eta, \rho$$

2. Use Binder-Pesaran algorithm to get reduced-form solution
3. Simulate model

Can check model parameterisation and simulate IRFs

$$\alpha = \frac{1}{3} \quad (82)$$

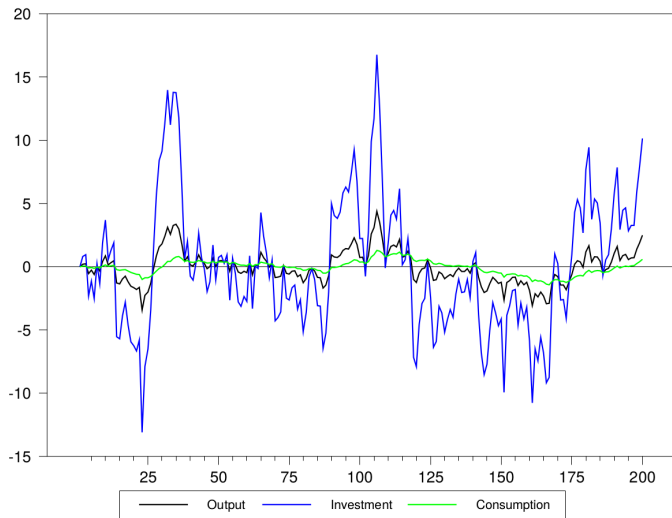
$$\beta = 0.99 \quad (83)$$

$$\delta = 0.015 \quad (84)$$

$$\rho = 0.95 \quad (85)$$

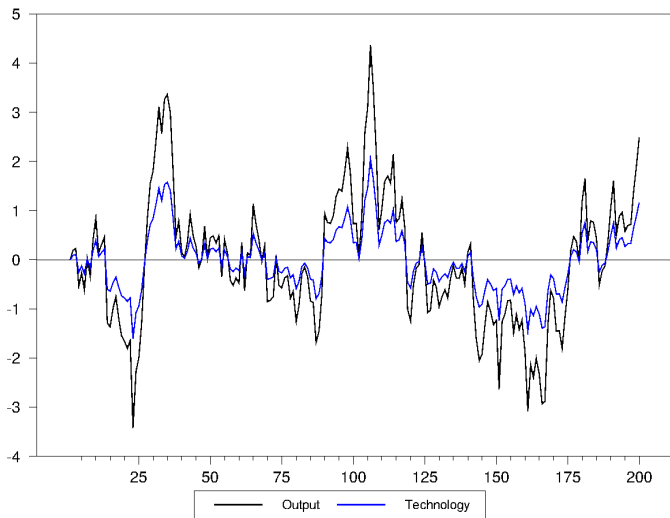
$$\eta = 1 \quad (86)$$

Simulate quarterly data for 200 periods



RBC's main feature: being able to generate business cycles

1. Model roughly matches observed fluctuations in output
2. Model reflects fact that investment cycles are more volatile than consumption

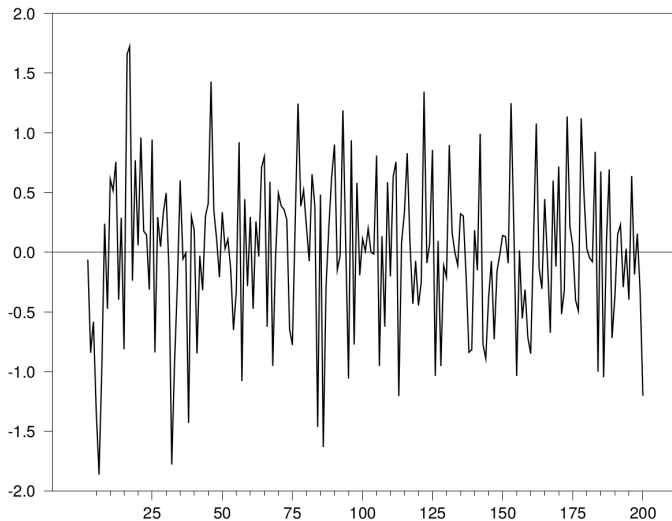


Technology shocks relate to business cycles as increases would lead to extra output through

1. Capital accumulation
2. Inducing people to work more

RBC model contains important propagation mechanisms turning technology shocks into business cycles

- ▶ In world with identical technology level, RBC model would still generate business cycles
- ▶ Mechanisms quite weak; output fluctuations follow technology fluctuations quite closely

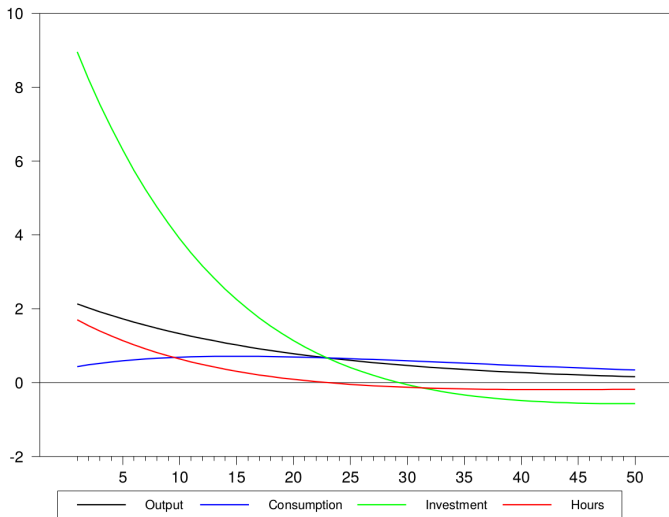


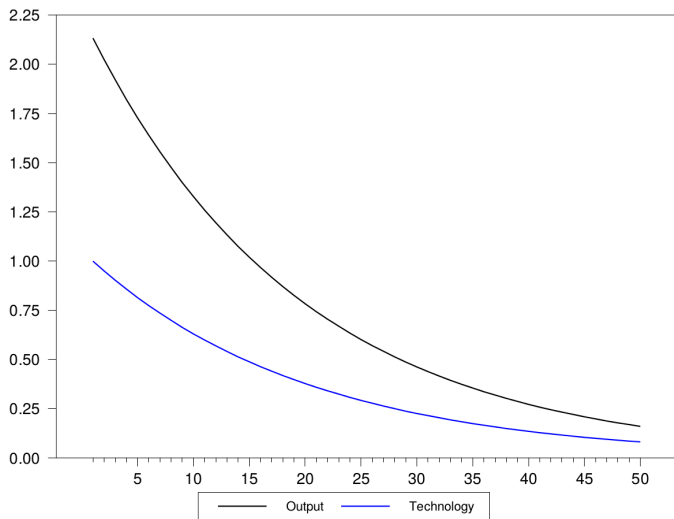
Cogley & Nason (1995) note that RBC models do not match positively autocorrelated growth

- ▶ Autocorrelation coefficient is 0.34 in real life

Authors argue that IRFs need to be hump-shaped

- ▶ Growth rate increase followed by another growth rate increase
- ▶ Response to technology shocks do not emulate this
- ▶ No other source of shocks in model





Extending RBC approach

Model fails to explain labour market response to technology shocks

- ▶ Hours worked tends to decline following positive technology shock

Fixing deficiencies

1. Strengthen propagation mechanism
 - ▶ Adjusting market clearing approach
 - ▶ e.g. lags in investment projects, habit persistence consumer utility
2. More systematic departure from RBC
 - ▶ Adding sticky prices/wages