Time-series data

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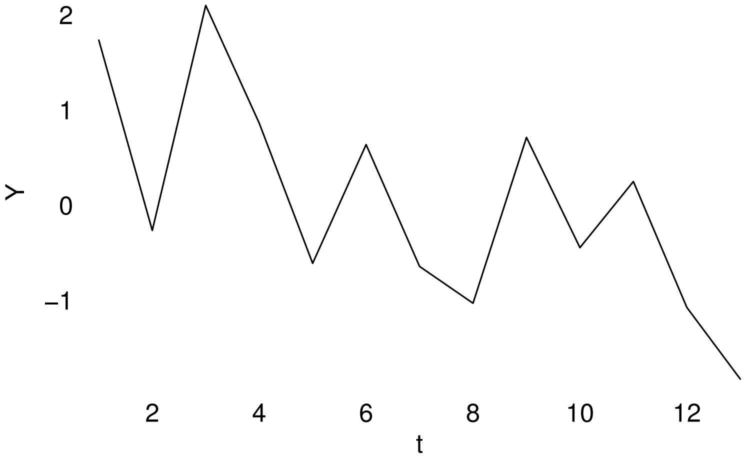
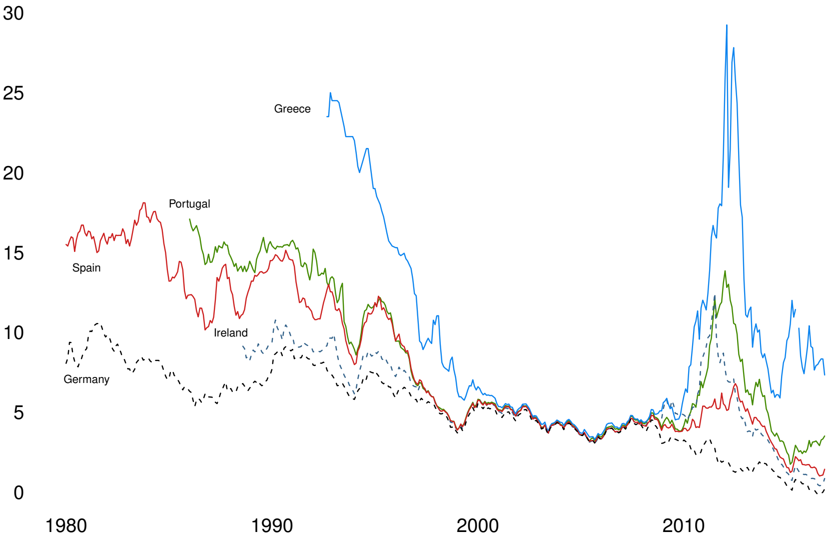
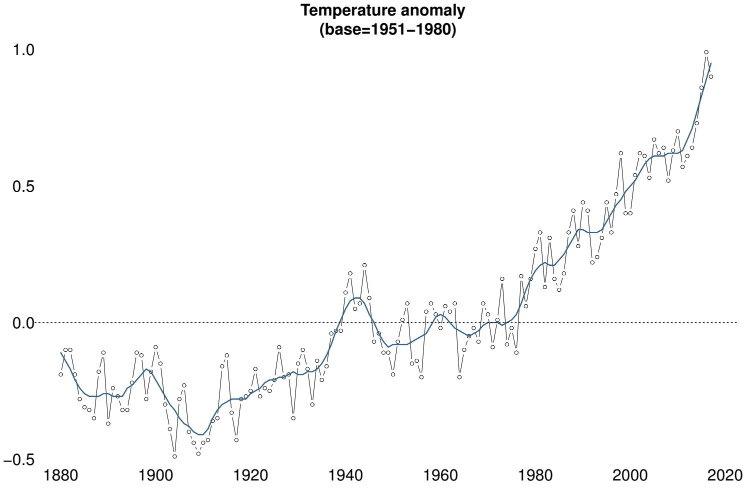
Empirical macroeconomics

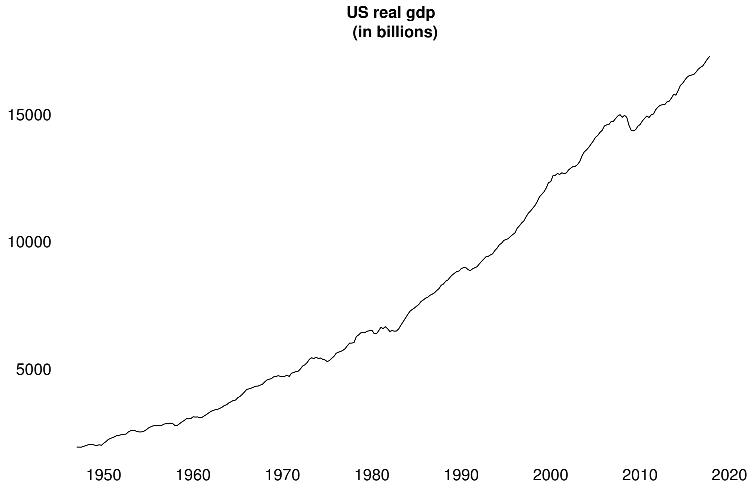
1. Data description: Describe and summarize macroeconomic data
2. Forecasting: Make macroeconomic forecasts
3. Structural inference: Quantify what we know and don't know about the true structure of the macro economy
4. Policy analysis: Advise on macroeconomic policy


2
1
0
−1
Y
2
4
6
8
10
12
t

Bond yields (percentage)



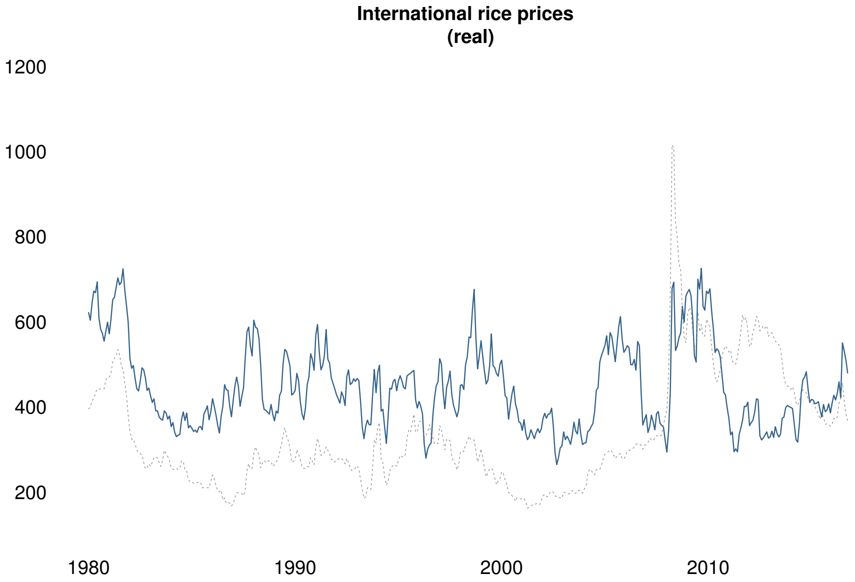

Temperature anomaly
(base=1951−1980)
1.0
0.5
0.0
−0.5
1880
1900
1920
1940
1960
1980
2000
2020

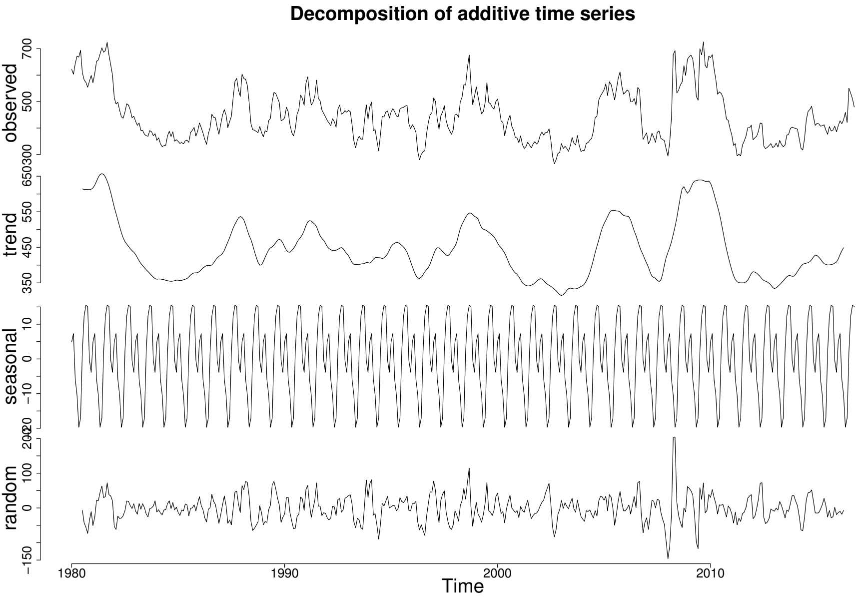

US real gdp
(in billions)
15000
10000
5000
1950
1960
1970
1980
1990
2000
2010
2020

In general, time-series y_t for $t = 1, 2, ..., T$ consists of

1. Trend component τ_t
2. Cyclical component c_t
3. Error component ϵ_t

$$y_t = \tau_t + c_t + \epsilon_t \tag{1}$$


International rice prices
(real)
1200
1000
800
600
400
200
1980
1990
2000
2010



Decomposition of additive time series
observed
300
500
700
trend
350
450
550
650
seasonal
−20
−10
0
10
random
−150
0
100
200
1980
1990
2000
2010
Time

Concerning macroeconomic data what you generally do is

1. Subtract non-stationary long-run trend from data
2. Analyse stationary cyclical component that remains

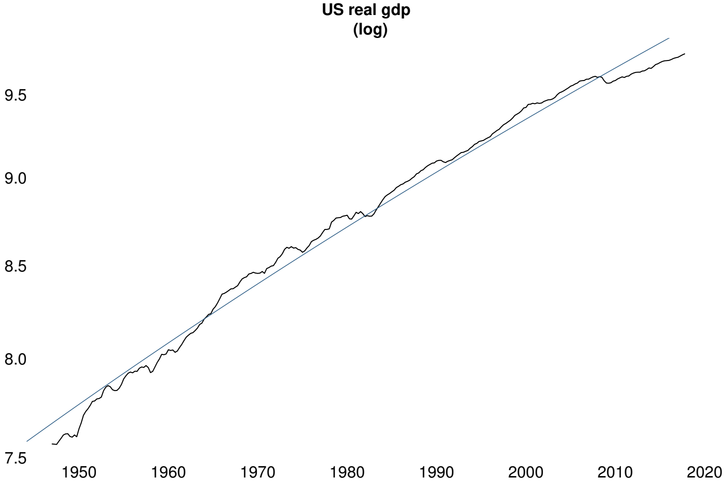
The short-term fluctuations comprising #2 is the business cycle

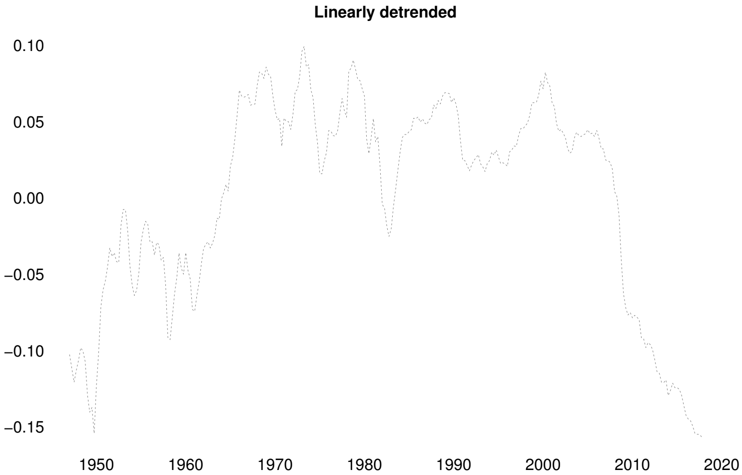
One of the simplest ways of detrending time-series data is using a log-linear model

$$log\ Y_t = y_t = \alpha + gt + \epsilon_t \tag{2}$$

$\alpha + gt$ is the trend component

ϵ is the stationary cyclical component (with zero mean)


US real gdp
(log)
9.5
9.0
8.5
8.0
7.5
1950
1960
1970
1980
1990
2000
2010
2020


Linearly detrended
0.10
0.05
0.00
–0.05
–0.10
–0.15
1950
1960
1970
1980
1990
2000
2010
2020

Taking first-differences of log-transformed data will give something equivalent to the growth rate Δy_t which comprises of

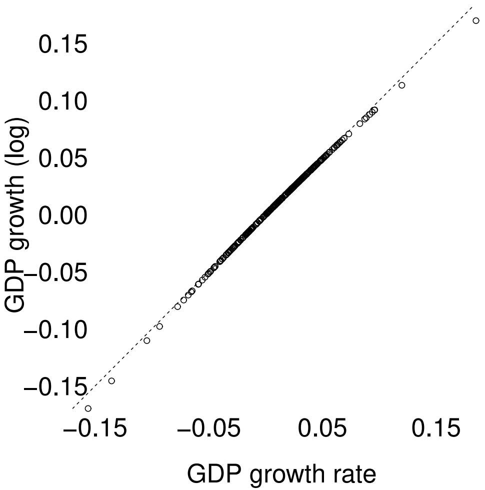
1. Constant trend growth g
2. Change in cyclical component $\Delta \epsilon_t$

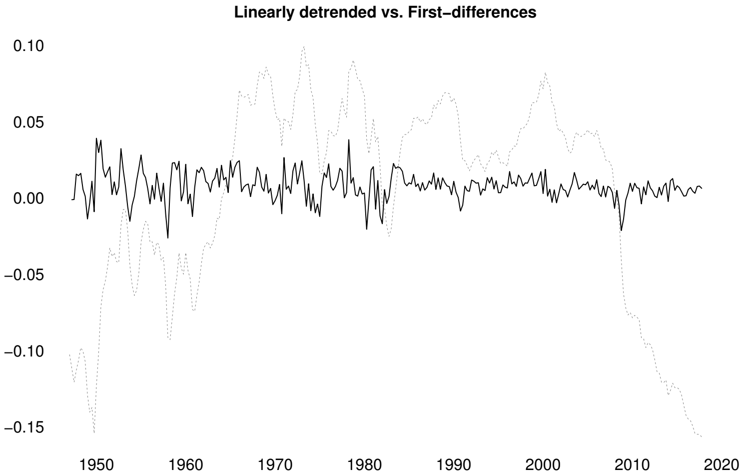
Let y_t be our time-series, can write growth as

$$\Delta y_t = \frac{y_t - y_{t-1}}{y_{t-1}} \approx log\ y_t - log\ y_{t-1} \tag{3}$$

$$\begin{aligned} \frac{y_t - y_{t-1}}{y_{t-1}} &= \frac{y_t}{y_{t-1}} - \frac{y_{t-1}}{y_{t-1}} \\ &= \frac{y_t}{y_{t-1}} - 1 \\ &\approx log\left(\frac{y_t}{y_{t-1}}\right) \\ &= log\ y_t - log\ y_{t-1} \end{aligned} \tag{4}$$

NB- If y is close to 1, this means that *log* y will be close to $y - 1$, and $\frac{y_t}{y_{t-1}}$ is likely close to 0


0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
GDP growth (log)
−0.15
−0.05
0.05
0.15
GDP growth rate


Linearly detrended vs. First−differences
0.10
0.05
0.00
−0.05
−0.10
−0.15
1950
1960
1970
1980
1990
2000
2010
2020

Although straightforward, there is a caveat using a straight line to detrend the data. Suppose that the correct model is

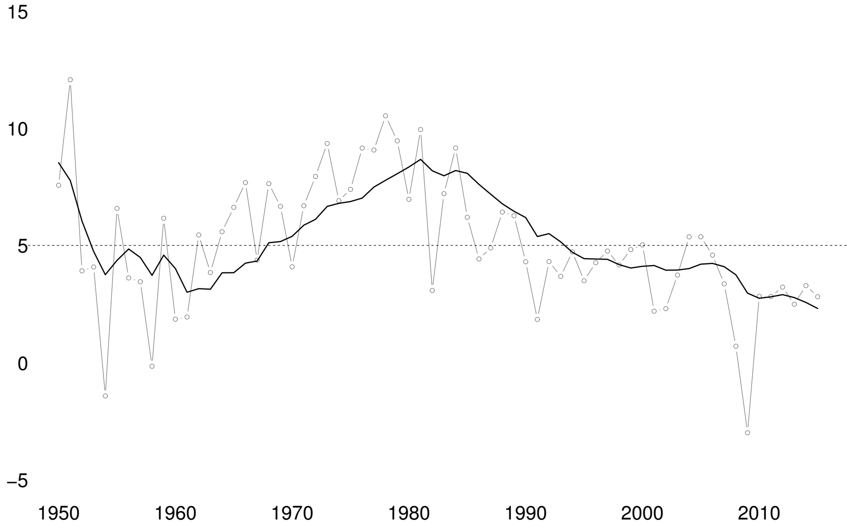
$$y_t = g + y_{t-1} + \epsilon_t \tag{5}$$

Growth has a constant component g and random component ϵ_t:

two important implications

1. Cycles are the accumulation of all random shocks over time that affected Δy_t
2. Expected growth will be g; irrespective of what happened in the past

NB-Another issue is medium-run changes in g


15
10
5
0
–5
1950
1960
1970
1980
1990
2000
2010

Since Δy_t is stationary, taking first-differences will remove non-stationary stochastic trend component (y_{t-1}) in the data

$$\Delta y_t = y_t - y_{t-1} = g + \epsilon_t \tag{6}$$

Therefore, fitting a log-linear line to the data, there might appear to be a mean-reverting cyclical component, which is not there.

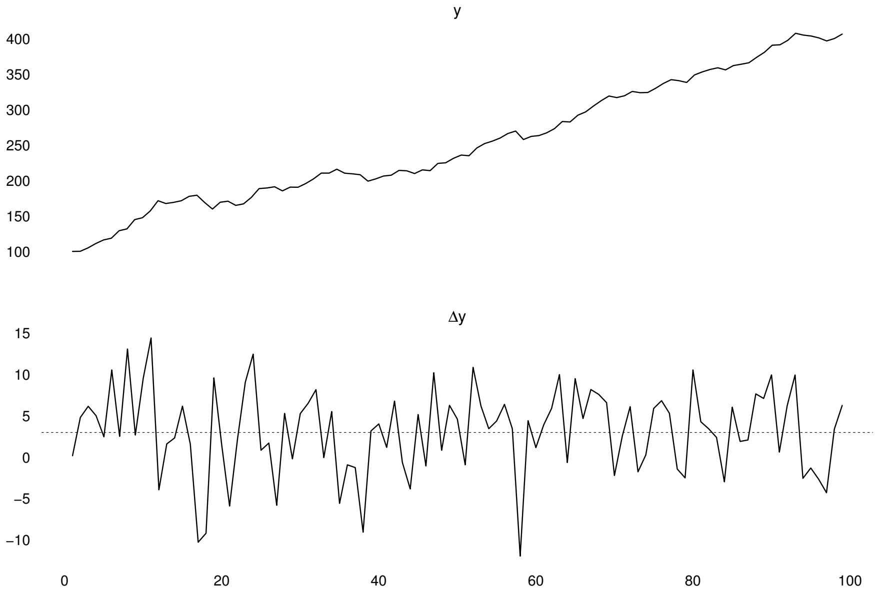
An example: data is randomly generate for following model

$$y_t = g + y_{t-1} + \epsilon_t \tag{7}$$

g=3; starting value for y is 100

$\epsilon \sim N(0, 5)$
Take first-differences

$$\Delta y = y_t - y_{t-1} \tag{8}$$


y
400
350
300
250
200
150
100
Δy
15
10
5
0
−5
−10
0
20
40
60
80
100

Incorrect usage of time-series dynamics can lead to errors in identification. An interesting example comes from a paper on the link between precipitation, economic growth, and conflict by Miguel et al. (2004) who use the following IV-2SLS model

$$\Delta Y_{it} = a_{1i} + b_1 X'_{it} + c_{1,0}\Delta R_{it} + c_{1,1}\Delta R_{it-1} + d_{1i} y_t + \epsilon_{1it} \quad (9)$$

$$C_{it} = \alpha_{2i} + \beta_2 X'_{it} + \gamma_{2,0}\Delta \hat{Y}_{it} + \gamma_{2,1}\Delta \hat{Y}_{it-1} + \delta_{2,1} + \epsilon_{2it} \quad (10)$$

Ciccone (2011) argues the following; consider the following model to link conflict and precipitation

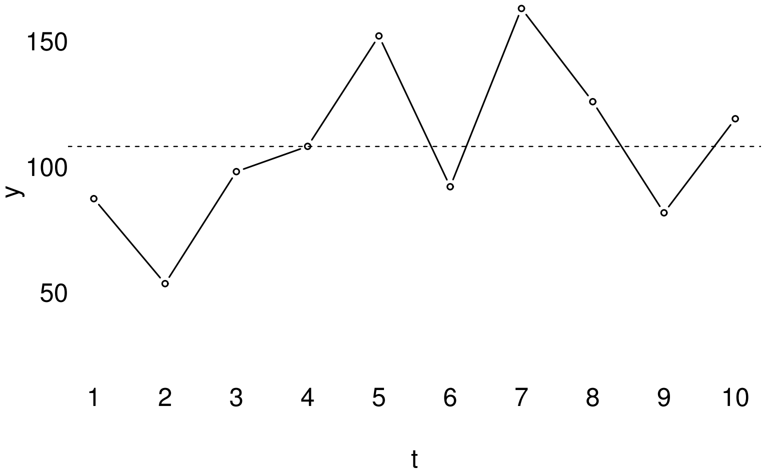
$$P(conflict_t) = a\Delta R_t + b\Delta R_{t-1} \tag{11}$$

In levels this becomes

$$P(conflict_t) = \alpha_0 \log\ R_t + \alpha_1 log\ R_{t-1} + \alpha_2 log\ R_{t-2} \tag{12}$$

With $\Delta R_t = log\ R_t - log\ R_{t-1}$, parameters in (11) become mixture of those in (12)

$$a = \frac{2\alpha_0 - (\alpha_1 + \alpha_2)}{3}; b = \frac{(\alpha_0 + \alpha_1) - 2\alpha_2}{3} \tag{13}$$


150
100
50
y
1
2
3
4
5
6
7
8
9
10
t

A commonly use method to detrend data is the Hodrick-Prescott (HP) filter; taking y_t the filter minimises

$$\sum_{t=1}^{N}[(y_t - y_t^*)^2 + \lambda(\Delta y_t^* - \Delta y_{t-1}^*)] \tag{14}$$

Here y^* is the time-varying trend; λ is a penalty parameter

- For quarterly data $\lambda = 1,600$

The HP filter does two things:

1. Minimise the sum of squared deviations between output and its trend

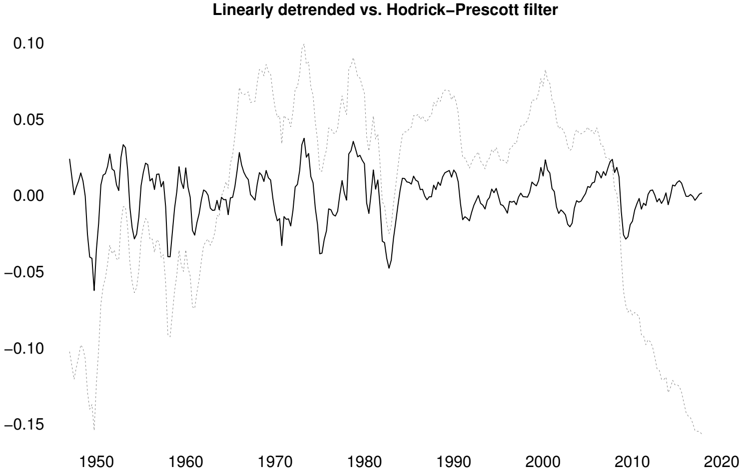
$$(Y_t - Y_t^*)^2 \tag{15}$$

2. Minimising the change in the trend growth rate

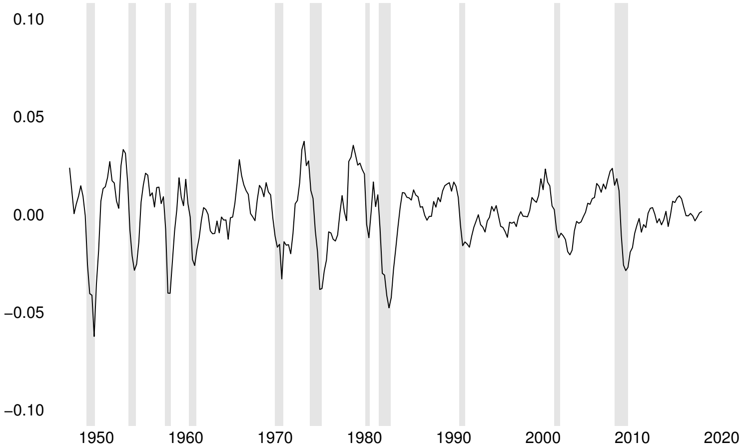
$$\lambda(\Delta Y_t^* - \Delta Y_{t-1}^*) \tag{16}$$

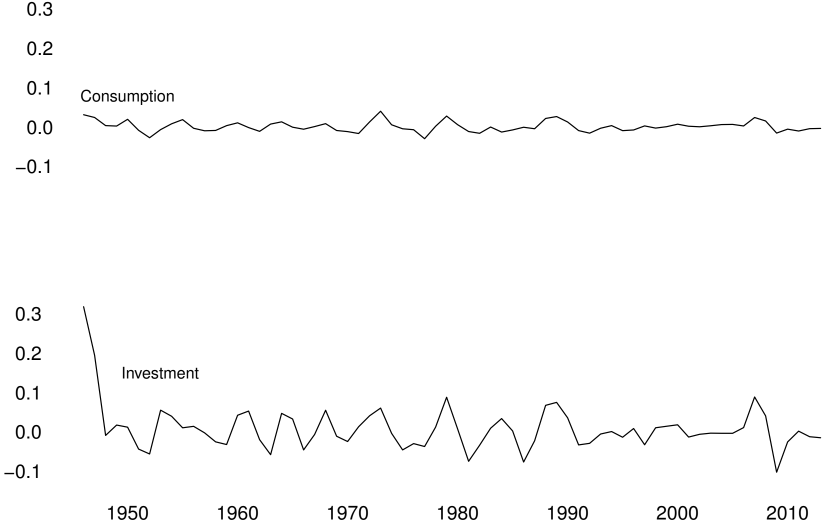
Important is that the HP-filter let's the growth rate very over time.

- Larger λ means smoother trend


Linearly detrended vs. Hodrick−Prescott filter
0.10
0.05
0.00
−0.05
−0.10
−0.15
1950
1960
1970
1980
1990
2000
2010
2020

HP filter and recessions

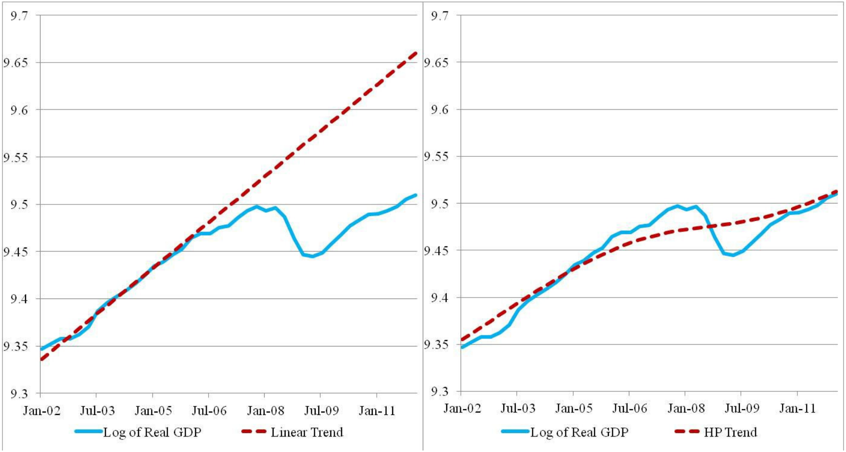




0.3
0.2
0.1
0.0
−0.1
Consumption
Investment
1950
1960
1970
1980
1990
2000
2010

Can use the HP-filter to look at cycles in different components of GDP (this case UK): shows that consumption tends to be more stable

1. The consumption of specifically non-durable goods such as food is stable over time
2. People tend to spend a constant amount of money regardless of temporary shocks (permanent income hypothesis)



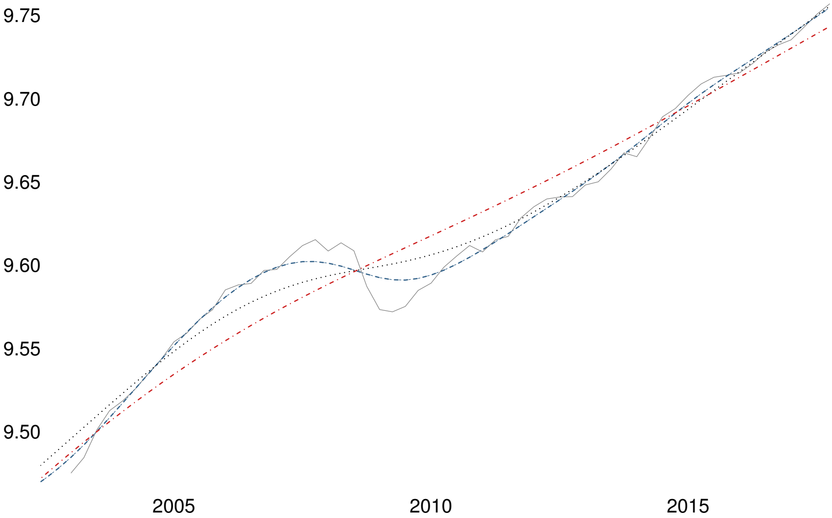
Source: James Bullard

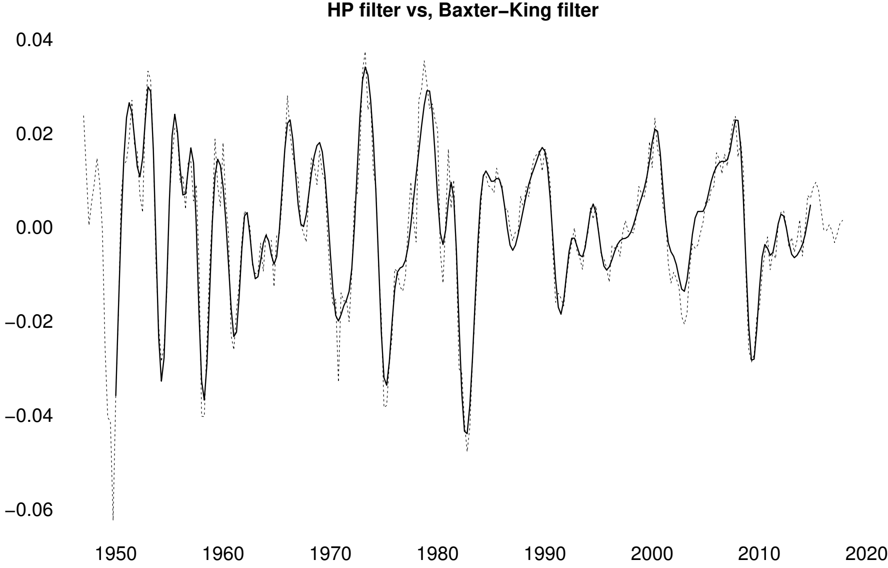
Statistical technique is only useful if assumptions reflect economic reality. But HP-filter assumes that deviations from trend are short term and correct themselves quickly

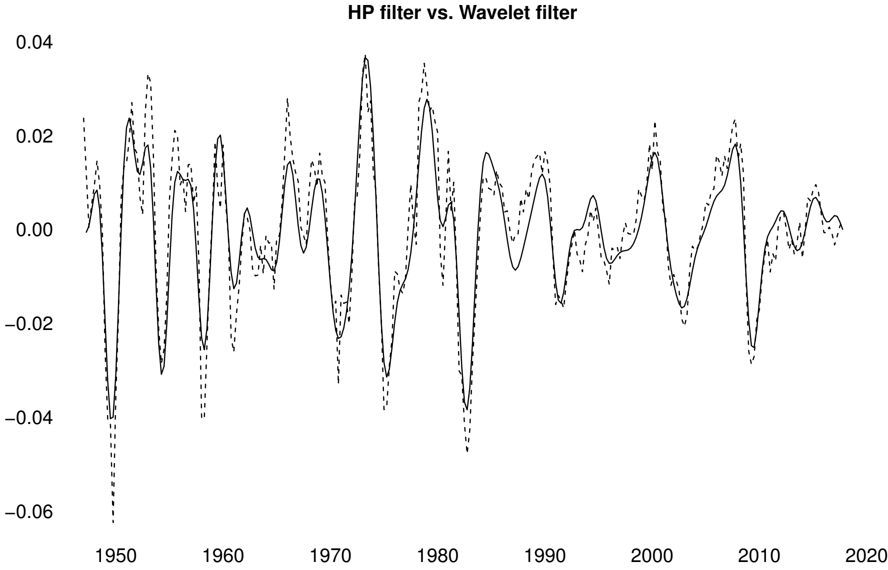
- Prolonged periods below potential GDP not possible

There is an endpoint problem in the filter

- Smoothed series close to observed data at beginning and end or period


9.75
9.70
9.65
9.60
9.55
9.50
2005
2010
2015


HP filter vs, Baxter−King filter
0.04
0.02
0.00
−0.02
−0.04
−0.06
1950
1960
1970
1980
1990
2000
2010
2020


HP filter vs. Wavelet filter
0.04
0.02
0.00
−0.02
−0.04
−0.06
1950
1960
1970
1980
1990
2000
2010
2020

AR(1) model: Cyclical components in time-series data can be autocorrelated and exhibit random-looking fluctuations. Simple way to capture these dynamics is through an Autoregression (AR) model, e.g. AR(1)

$$y_t = \rho y_{t-1} + \epsilon_t \tag{17}$$

ρ is propagation mechanism

- Determines at which speed a shock in ϵ fade away

Impulse Response Function: Time path of y after a shock in ϵ:
t follows

$$\epsilon_t + 1, \epsilon_{t+1}, \epsilon_{t+2}, \ldots \tag{18}$$

Instead of

$$\epsilon_t, \epsilon_{t+1}, \epsilon_{t+2}, \ldots \tag{19}$$

This means there is an incremental effect in all future periods of a unit shock today.

Imagine AR(1) series starting at 0 with shock $\epsilon_t = 1$ followed by 0 shocks for $t+1$. We get that for period t

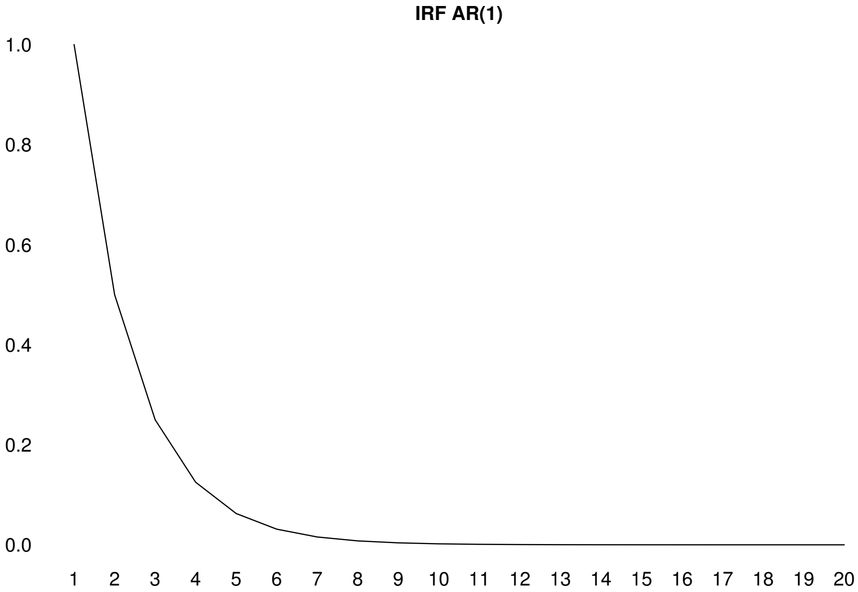
$$y_t = 1 \tag{20}$$

For period $t+1$

$$y_{t+1} = \rho \tag{21}$$

For period $t+n$

$$y_{t+n} = \rho^n \tag{22}$$


IRF AR(1)
1.0
0.8
0.6
0.4
0.2
0.0
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

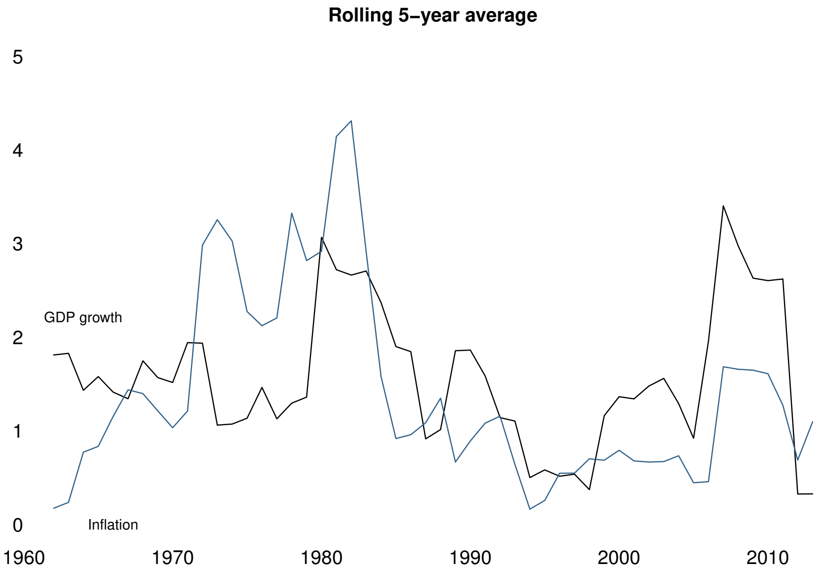
Volatility:

The long run variance of y_t is the same as the long-run variance of y_{t-1} and $Var(\epsilon_t) = \sigma_\epsilon^2$. Variance of y_t is

$$\begin{aligned}\sigma_y^2 &= \rho^2\sigma_y^2 + \sigma_\epsilon^2 \\ &= \frac{\sigma_\epsilon^2}{1-\rho^2}\end{aligned} \tag{23}$$

Variance determined by

1. Size of shock in ϵ
2. Strength of propagation mechanism ρ


Rolling 5−year average
5
4
3
2
1
0
GDP growth
Inflation
1960
1970
1980
1990
2000
2010

the Great Moderation: Since mid-1980s output and inflation have become less volatile, which could be due to

1. Smaller shocks (lower values for ϵ_t)
    - Less random policy shocks
    - Smaller shocks from goods and/or financial markets
    - Smaller supply shocks

2. Weaker propagation mechanisms (lower values for ρ)
    - Policy became more stabilizing
    - More stable economy
    - Stabilisation of economy due to financial modernisation

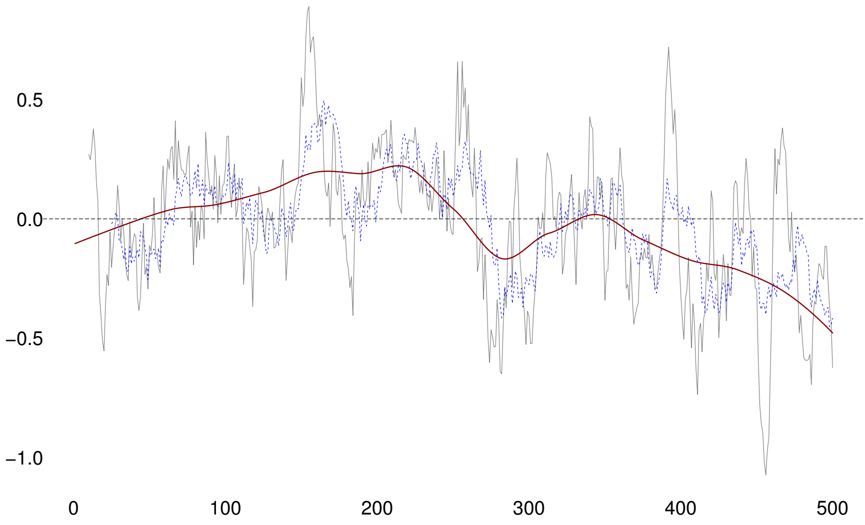
Slutsky effect: when data are uncorrelated smoothing can introduce appearance of irregular oscillations (Kelly & O'Grada, 2014). There are two different definitions, the formal one

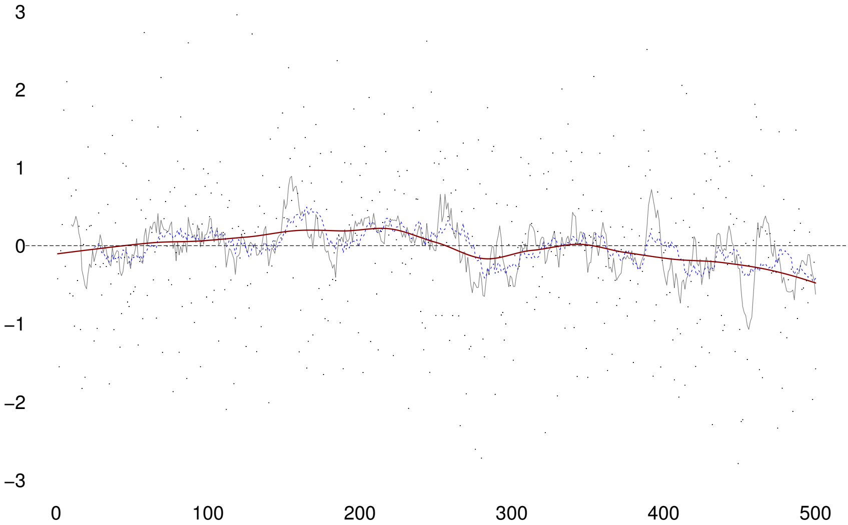
$$f(\omega) = \frac{1}{m^2} \frac{1 - cos\ m\omega}{1 - cos\ \omega} \tag{24}$$

This is the transfer function for a moving average of m periods.

And there is the colloquial one

> *Applying a moving average to a white noise series will generate the appearance of irregular oscillations, as the filter is distorted by runs of high or low observations.*


0.5
0.0
−0.5
−1.0
0
100
200
300
400
500


3
2
1
0
–1
–2
–3
0
100
200
300
400
500

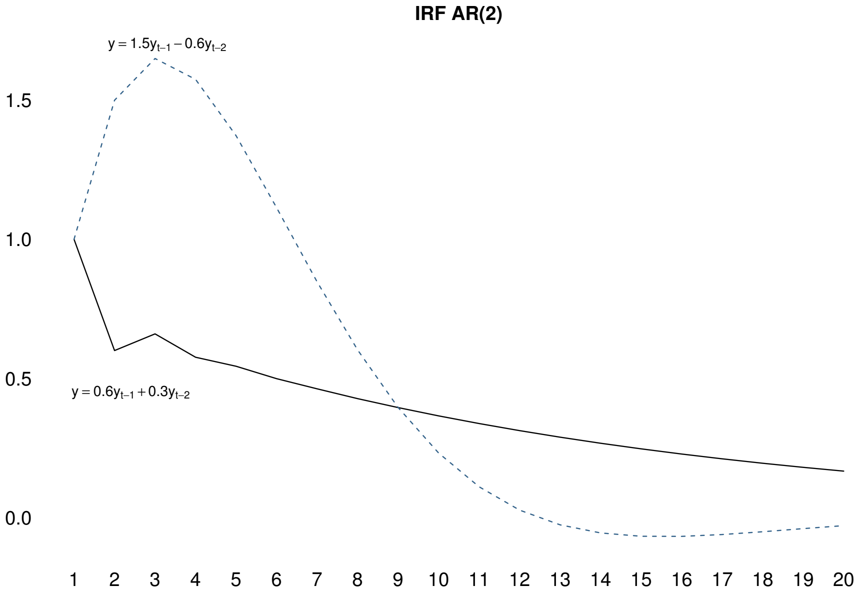
AR(p) model

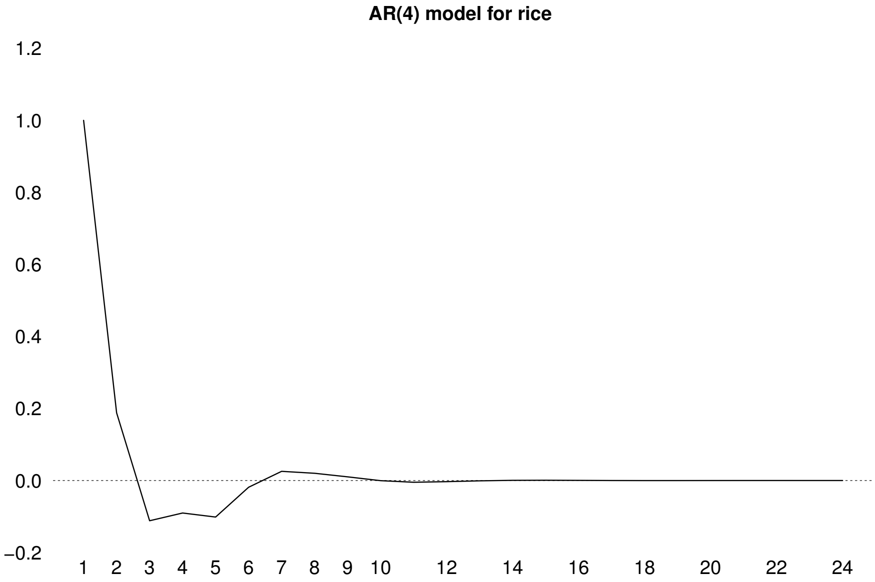
Often macroeconomic dynamics extend beyond AR(1) process; consider AR(2) model

$$y_t = \alpha + \rho_1 y_{t-1} + \rho_2 y_{t-2} + \epsilon_t \tag{25}$$

IRF can take on various forms based on ρ_1, ρ_2 values

- Dynamic properties of model depend on number of lags p included in model


IRF AR(2)
$y = 1.5y_{t-1} - 0.6y_{t-2}$
$y = 0.6y_{t-1} + 0.3y_{t-2}$
1.5
1.0
0.5
0.0
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20


AR(4) model for rice
1.2
1.0
0.8
0.6
0.4
0.2
0.0
–0.2
1
2
3
4
5
6
7
8
9
10
12
14
16
18
20
22
24

Lag operator: For more complex models we can use the lag operator L for model notation.

- The operator moves the time-series back in time

$$y_{t-1} = Ly_t \tag{26}$$

Similarly

$$L^2 y_t = y_{t-2} \tag{27}$$

Can use the lag operator in model specification when the model includes a number of lags. Can write model

$$y_t = a_1 y_{t-1} + a_2 y_{t-2} + \epsilon_t \tag{28}$$

as

$$y_t = A(L) y_t + \epsilon_t \tag{29}$$

$$A(L) = a_1 L + a_2 L^2 \tag{30}$$

Alternatively you can write it as

$$B(L) y_t = \epsilon_t \tag{31}$$

$$B(L) = 1 - a_1 L + a_2 L^2 \tag{32}$$

Vector Autoregression

AR models are useful in understanding the dynamics of individual variables

- But they ignore the relationships between variables

VAR models dynamics between n different variables; allowing each variable to depend on lagged values of all variables

- Can examine IRF of n variables to n shocks

Simplest VAR model has two variables and one lag

$$\begin{aligned} y_{1t} &= a_{11}y_{1,t-1} + a_{12}y_{2,t-1} + \epsilon_{1t} \\ y_{2t} &= a_{21}y_{1,t-1} + a_{22}y_{2,t-1} + \epsilon_{2t} \end{aligned} \tag{33}$$

Shocks:

1. Policy changes: those not captured by the systematic component of the VAR equation
2. Changes in preferences: work vs. leisure or spending vs. saving
3. Technology shocks: random changes in the productivity of firms
4. Shocks to various frictions: changes in the efficiency with which markets (labour, goods, financial) work

Time-series perspective is central to economic fluctuations in macroeconomics

- Cycles are determined by various random shocks, propagated throughout the economy over time

VARs are commonly used for modeling macroeconomic dynamics and the effect of shocks

- Can help explain *how* things work, but not why things work the way they do

Dynamics Stochastic General Equilibrium model build upon VAR framework, but dynamics are derived from economic theory

- In this framework agents are i) rational and ii) optimising