Smets-Wouters model

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Smets & Wouters (2007) "Shocks and Frictions in US Business Cycles : A Bayesian DSGE Approach"

> *Using a Bayesian likelihood approach, we estimate a dynamic stochastic general equilibrium model for the US economy using seven macroeconomic time series. The model incorporates many types of real and nominal frictions and seven types of structural shocks. We show that this model is able to compete with Bayesian Vector Autoregression models in out-of-sample prediction. We investigate the relative empirical importance of the various frictions. Finally, using the estimated model, we address a number of key issues in business cycle analysis: What are the sources of business cycle fluctuations? Can the model explain the cross correlation between output and inflation? What are the effects of productivity on hours worked? What are the sources of the Great Moderation?*

Main features

- Sticky nominal price and wage settings
- Habit formation in consumption and investment adjustment costs
- Variable capital utilization and fixed production costs

Dynamics in model driven by seven orthogonal structural shocks

- TFP shock
- Two shocks affecting intertemporal margin
  - risk premium, investment-specific technology shock
- Two shocks affecting intratemporal margin
  - wage and price mark-up shocks
- Two policy shocks
  - exogenous spending and monetary policy shocks

- Data: US 1966:1-2004:4
- Bayesian estimation methodology
    1. Maximise log-posterior; estimate mode posterior distribution
    2. Evaluate marginal likelihood; use Metropolis-Hastings algorithm to get posterior

Note use of deterministic growth rate, driven by labour-augmenting technological process

- Means that data does not have to be detrended

Demand side

$$y_t = c_y c_t + i_y i_t + z_y z_t + \epsilon_t^g \tag{1}$$

y_t is GDP
c_t is consumption
i_t is investment
ϵ_t^g is exogenous spending

z_t is included in the resource constraint because of the assumption that there are costs associated with having high rates of capital utilisation.

Variables with subscript y are steady-state shares

$$c_y = 1 - g_y - i_y \tag{2}$$

$$i_y = (\gamma - 1 + \delta) k_y \tag{3}$$

$$z_y = R_*^k k_y \tag{4}$$

Exogenous spending is assumed to develop over time according to

$$\epsilon_t^g = \rho\epsilon_{t-1}^g + \eta_t^g + \rho_{ga}\eta_t^a \tag{5}$$

Exogenous spending is assumed to have two components

1. Government spending
2. Element related to productivity

Consumption

$$c_t = c_1 c_{t-1} + (1 - c_1) E_t c_{t+1} + c_2 (l_t - E_t l_{t+1}) - c_3 (r_t - E_t \pi_{t+1} + \epsilon_t^b) \tag{6}$$

c_1, c_2, c_3 are constant parameters (functions of deeper structural parameters)

r_t is the interest rate on a one-period safe bond (quarterly)

ϵ_t^b is a risk premium shock determining the willingness of a household to hold the one-period bond

- Preference shock that influence short-term consumption-saving decisions

$$\epsilon_t^b = \rho_b \epsilon_{t-1} + \eta_t^b \tag{7}$$

Two other important things concerning consumption equation

- Backward looking consumption term represent habit forming
- Equation allows for substitution of consumption with labour input

Investment

$$i_t = i_t i_{t-1} + (1 - i_1) E_t i_{t+1} + i_2 q_t + \epsilon_t^i \tag{8}$$

Main driver of investment: q_t

$$q_t = q_1 E_t q_{t+1} + (1 - q_1) r_{t+1}^k - (r_t - E_t \pi_{t+1} + \epsilon_t^b) \tag{9}$$

Supply side

$$y_t = \phi_p(\alpha k_t^s + (1-\alpha) l_t + \epsilon_t^a) \tag{10}$$

y_t is GDP
k_t^s is capital in use: determined by lagged level of capital and a capacity utilisation variable

$$k_t^s = k_{t-1} + z_t \tag{11}$$

l_t is labour input
ϵ_t^a is total factor productivity

z_t linked to marginal productivity of capital; function of capital to labour ratio and the real wage

$$z_t = z_1 r_t^k; z_1 = (1-\psi)/\psi \tag{12}$$

$$r_t^k = -(k_t - l_t) + w_t \tag{13}$$

Total factor productivity evolves over time according to

$$\epsilon_t^a = \rho \epsilon_{t-1}^a + \eta_t^a \tag{14}$$

Prices

$$\mu_t^p = mpl_t - w_t = \alpha(k_t - l_t) + \epsilon_t^a - w_t \tag{15}$$

Equation accounts for

1. Diminishing marginal productivity of capital
2. Productivity shocks effect on costs
3. Real wage

Inflation

$$\pi_t = \pi_1 \pi_{t-1} + \pi_2 E_t \pi_{t+1} - \pi_3 \mu_t^p + \epsilon_t^p \qquad (16)$$

- Adjusted to account for lagged inflation
- Most firms will index their prices based on past inflation levels and can only set an optimal price occasionally

ϵ_t^p is a price mark-up disturbance

$$\epsilon_t^p = \rho^p \epsilon_{t-1}^p + \eta_t^p - \mu_p \eta_{t-1}^p \qquad (17)$$

Shock affects both current and lagged inflation in order to get a temporary price level shock.

Wages

$$\mu_t^w = w_t - mrs_t \tag{18}$$

$$= w_t - \left(\sigma_l l_t - \frac{1}{1-\lambda/\gamma}(c_t - \lambda/c_{t-1})\right) \tag{19}$$

μ_t^w is the wage mark-up

- Gap between real wage and marginal rate of substitution between working and consuming
- Sort of sticky: wages adjust gradually to equate the marginal costs and benefits of working

$$
\begin{aligned}
w_t = {} & w_1 w_{t-1} \\
& + (1 - w_1) E_t (w_{t+1} + \pi_{t+1}) \\
& - w_2 \pi_t + w_3 \pi_{t-1} - w_4 \mu_t^w + \epsilon_t^w
\end{aligned}
\tag{20}
$$

$$
\epsilon_t^w = \rho^w \epsilon_{t-1}^w + \eta_t^w - \mu_w \eta_{t-1}^w \tag{21}
$$

Monetary policy

$$r_t = \rho r_{t-1} + (1-\rho)(r_\pi \pi_t + r_y(y_t - y_t^p)) + \\ r_{\Delta y}[(y_t - y_t^p) - (y_{t-1} - y_{t-1}^p)] + \epsilon_t^r \tag{22}$$

$$\epsilon_t^r = \rho^r \epsilon_{t-1}^r + \eta_t^r \tag{23}$$

Central bank sets short-term interest rate according to

1. Last period's interest rate
2. Gradual adjustment towards target interest rate
    - Depends on inflation and output gap
3. Output gap growth rate

Potential output defined as the level of output that would prevail if prices and wages were fully flexible

- Model effectively needs to be expanded to add shadow flexible-price economy

Fourteen endogenous variables

$$y_t, c_t, i, q_t, k_t^s, k_t, z_t, r_t^k, \mu_t^p, \pi_t, u_t^w, l_t, r_t$$

Seven exogenous disturbances

$$\epsilon_t^a, \epsilon_t^i, \epsilon_t^b, \epsilon_t^g, \epsilon_t^p, \epsilon_t^w, \epsilon_t^r$$

VAR system

$$Y_t = \begin{pmatrix} dlGDP_t \\ dlCONS_t \\ dlINV_t \\ dlWAG_t \\ lHOURS_t \\ dlP_t \\ FEDFUNDS_t \end{pmatrix} = \begin{pmatrix} \overline{\gamma} \\ \overline{\gamma} \\ \overline{\gamma} \\ \overline{\gamma} \\ \bar{l} \\ \overline{\pi} \\ \bar{r} \end{pmatrix} + \begin{pmatrix} y_t - y_{t-1} \\ c_t - c_{t-1} \\ i_t - i_{t-1} \\ w_t - w_{t-1} \\ l_t \\ \pi_t \\ r_t \end{pmatrix} \tag{24}$$

l and dl stand for 100 times log and log-difference.

Additional features compared to RBC or NK model:

- Adjustment costs for investment
- Capacity utilisation cost
- Habit persistence
- Price indexation
- Wage indexation
- All kinds of autocorrelated shock terms

Fixes included to overcome shortcomings previous model: slow things down

- Give random shocks longer lasting effects
- Make development of variables more sluggish

Velocity major shortcoming of RBC: wage/price indexation addresses NK shortcoming

- Failed to deal with inflation persistence

Adjustment are largely ad hoc: no clear theoretical grounding.

Priors

Stochastic processes are harmonised as much as possible

- Rather loose priors are used
- Various distributions: Gamma, Gaussian, Beta

Five parameters are fixed

$$\gamma = 0.025 \tag{25}$$
$$g_y = 18\% \tag{26}$$
$$\lambda_w = 1.5 \tag{27}$$
$$\epsilon_p = \epsilon_w = 10 \tag{28}$$

Table 1A—Prior and Posterior Distribution of Structural Parameters

	Prior distribution			Posterior distribution			
	Distr.	Mean	St. Dev.	Mode	Mean	5 percent	95 percent
φ	Normal	4.00	1.50	5.48	5.74	3.97	7.42
σ_c	Normal	1.50	0.37	1.39	1.38	1.16	1.59
h	Beta	0.70	0.10	0.71	0.71	0.64	0.78
ξ_w	Beta	0.50	0.10	0.73	0.70	0.60	0.81
σ_l	Normal	2.00	0.75	1.92	1.83	0.91	2.78
ξ_p	Beta	0.50	0.10	0.65	0.66	0.56	0.74
ι_w	Beta	0.50	0.15	0.59	0.58	0.38	0.78
ι_p	Beta	0.50	0.15	0.22	0.24	0.10	0.38
ψ	Beta	0.50	0.15	0.54	0.54	0.36	0.72
Φ	Normal	1.25	0.12	1.61	1.60	1.48	1.73
r_π	Normal	1.50	0.25	2.03	2.04	1.74	2.33
ρ	Beta	0.75	0.10	0.81	0.81	0.77	0.85
r_y	Normal	0.12	0.05	0.08	0.08	0.05	0.12
$r_{\Delta y}$	Normal	0.12	0.05	0.22	0.22	0.18	0.27
$\bar{\pi}$	Gamma	0.62	0.10	0.81	0.78	0.61	0.96
$100(\beta^{-1} - 1)$	Gamma	0.25	0.10	0.16	0.16	0.07	0.26
$\bar{l}$	Normal	0.00	2.00	−0.1	0.53	−1.3	2.32
$\bar{\gamma}$	Normal	0.40	0.10	0.43	0.43	0.40	0.45
α	Normal	0.30	0.05	0.19	0.19	0.16	0.21

Note: The posterior distribution is obtained using the Metropolis-Hastings algorithm.

Table 1B—Prior and Posterior Distribution of Shock Processes

	Prior distribution			Posterior distribution			
	Distr.	Mean	St. Dev.	Mode	Mean	95 percent	5 percent
σ_a	Invgamma	0.10	2.00	0.45	0.45	0.41	0.50
σ_b	Invgamma	0.10	2.00	0.24	0.23	0.19	0.27
σ_g	Invgamma	0.10	2.00	0.52	0.53	0.48	0.58
σ_I	Invgamma	0.10	2.00	0.45	0.45	0.37	0.53
σ_r	Invgamma	0.10	2.00	0.24	0.24	0.22	0.27
σ_p	Invgamma	0.10	2.00	0.14	0.14	0.11	0.16
σ_w	Invgamma	0.10	2.00	0.24	0.24	0.20	0.28
ρ_a	Beta	0.50	0.20	0.95	0.95	0.94	0.97
ρ_b	Beta	0.50	0.20	0.18	0.22	0.07	0.36
ρ_g	Beta	0.50	0.20	0.97	0.97	0.96	0.99
ρ_I	Beta	0.50	0.20	0.71	0.71	0.61	0.80
ρ_r	Beta	0.50	0.20	0.12	0.15	0.04	0.24
ρ_p	Beta	0.50	0.20	0.90	0.89	0.80	0.96
ρ_w	Beta	0.50	0.20	0.97	0.96	0.94	0.99
μ_p	Beta	0.50	0.20	0.74	0.69	0.54	0.85
μ_w	Beta	0.50	0.20	0.88	0.84	0.75	0.93
ρ_{ga}	Beta	0.50	0.20	0.52	0.52	0.37	0.66

Note: The posterior distribution is obtained using the Metropolis-Hastings algorithm.

Out-of-sample predictions

1. Estimate model, using data 1966:1-1989:4
2. Forecast data series for 1990:1-2004:4

VAR models were re-estimated each quarter, DSGE model each year.

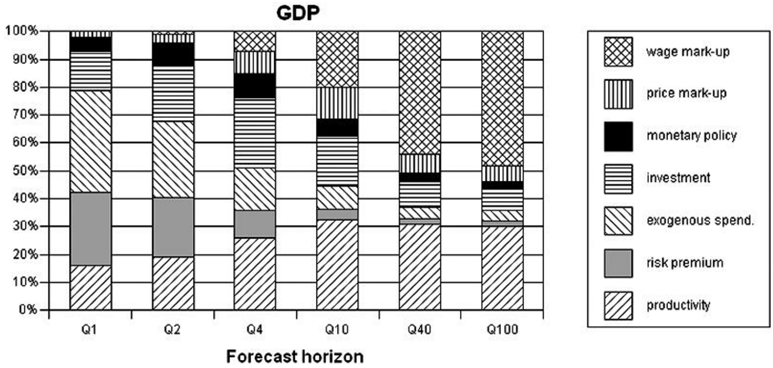
Table 3—Out-of-Sample Prediction Performance

	GDP	dP	Fedfunds	Hours	Wage	CONS	INV	Overall
VAR(1)	*RMSE-statistic for different forecast horizons*							
1q	0.60	0.25	0.10	0.46	0.64	0.60	1.62	−12.87
2q	0.94	0.27	0.18	0.78	1.02	0.95	2.96	−8.19
4q	1.64	0.34	0.36	1.45	1.67	1.54	5.67	−3.25
8q	2.40	0.53	0.64	2.13	2.88	2.27	8.91	1.47
12q	2.78	0.63	0.79	2.41	4.09	2.74	10.97	2.36
BVAR(4)	*Percentage gains (+) or losses (−) relative to VAR(1) model*							
1q	2.05	14.14	−1.37	−3.43	2.69	12.12	2.54	3.25
2q	−2.12	15.15	−16.38	−7.32	−0.29	10.07	2.42	0.17
4q	−7.21	31.42	−12.61	−8.58	−3.82	1.42	0.43	0.51
8q	−15.82	33.36	−13.26	−13.94	−8.98	−8.19	−11.58	−4.10
12q	−15.55	37.59	−13.56	−4.66	−15.87	−3.10	−23.49	−9.84
DSG	*Percentage gains (+) or losses (−) relative to VAR(1) model*							
1q	5.68	2.05	−8.24	0.68	5.99	20.16	9.22	3.06
2q	14.93	10.62	−17.22	10.34	6.20	25.85	16.79	2.82
4q	20.17	46.21	1.59	19.52	9.21	26.18	21.42	6.82
8q	22.55	68.15	28.33	22.34	15.72	21.82	25.95	11.50
12q	32.17	74.15	40.32	27.05	21.88	23.28	41.61	13.51

Notes: All models are estimated starting in 1966:1. The forecast period is 1990:1–2004:4. VAR(1) and BVAR(4) models are reestimated each quarter, the DSGE model each year. The overall measure of forecast performance is the log determinant of the uncentered forecast error covariance matrix. Gains and losses in the overall measure are expressed as the difference in the overall measure divided by the number of variables and by two to convert the variance to standard errors (times 100).

Model is used to answer four questions

1. What are the sources of business cycle fluctuations?
2. Can the model explain the cross correlation between output and inflation?
3. What are the effects of productivity on hours worked?
4. What are the sources of the Great Moderation?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Forecast horizon
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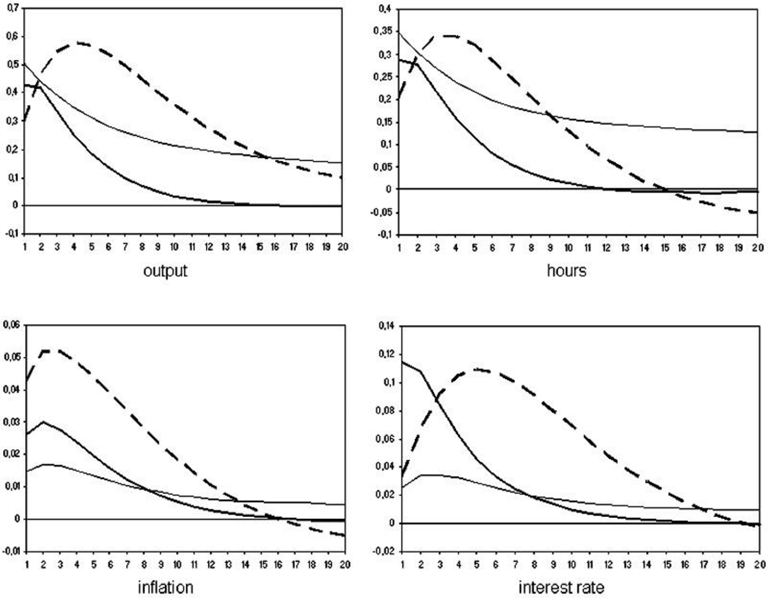
risk premium
productivity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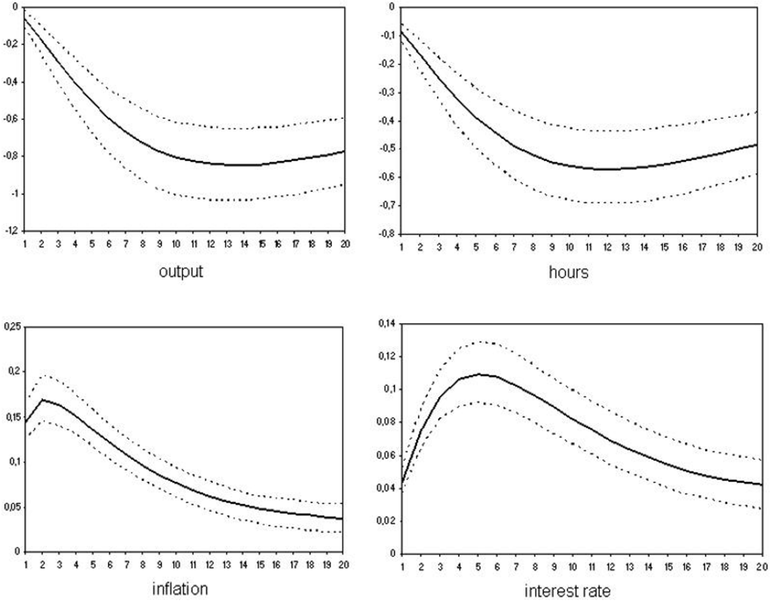
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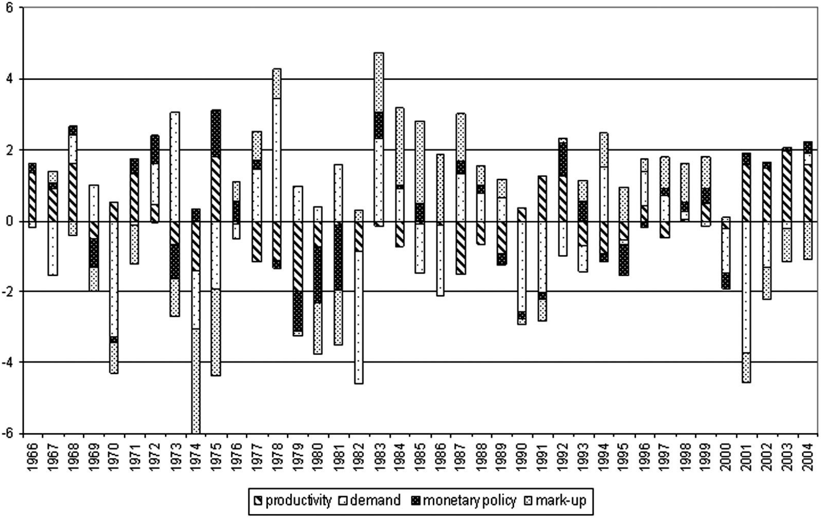
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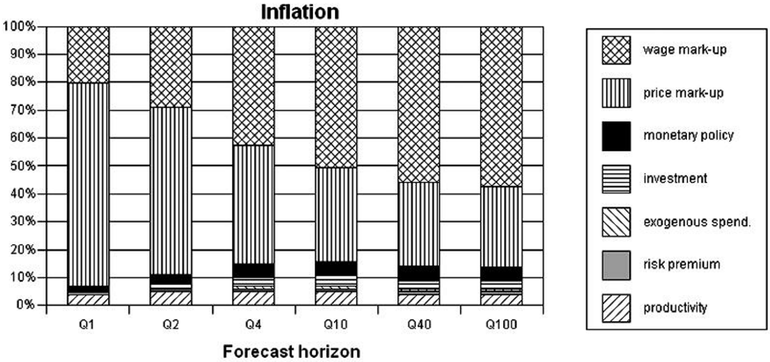
Inflation
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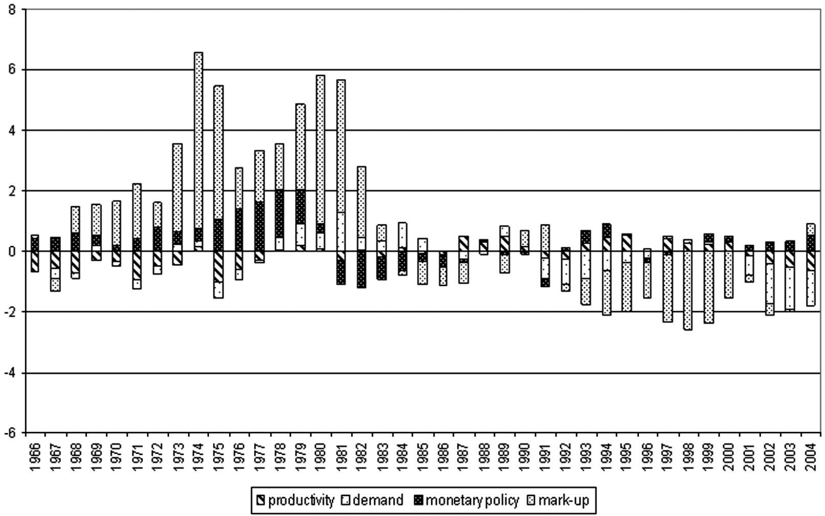
Forecast horizon
wage mark-up
price mark-up
monetary policy
investment
exogenous spend.
risk premium
productivity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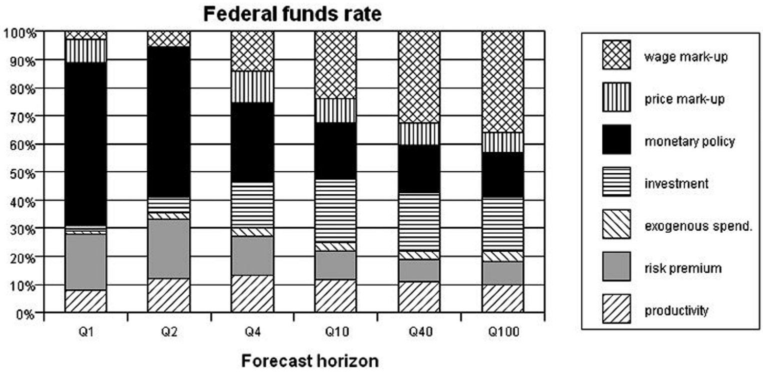
productivity
demand
monetary policy
mark-up

Two reasons for limited effect of demand/productivity shocks on inflation

1. Estimated slope NKPC is small
2. Fed responds aggressively to emerging output gaps and impact on inflation (according to estimation)


Federal funds rate
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Forecast horizon
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monetary policy
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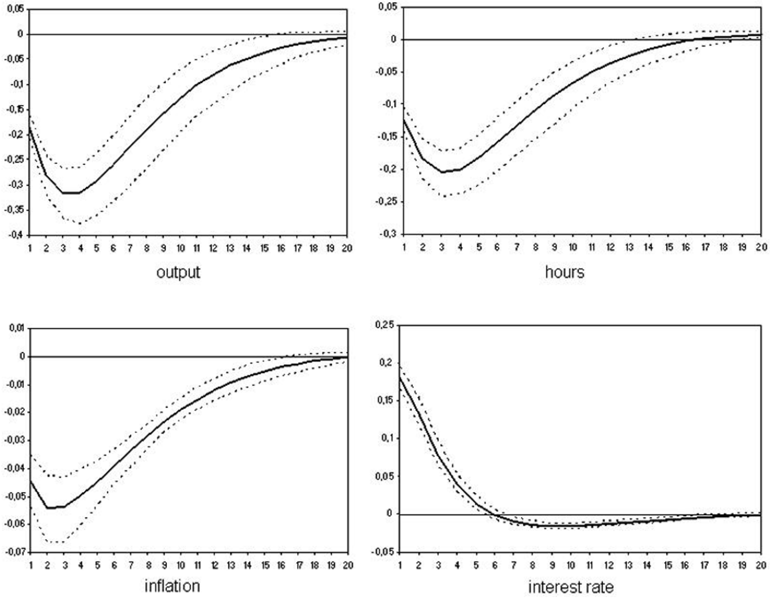
exogenous spend.
risk premium
productivity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hours
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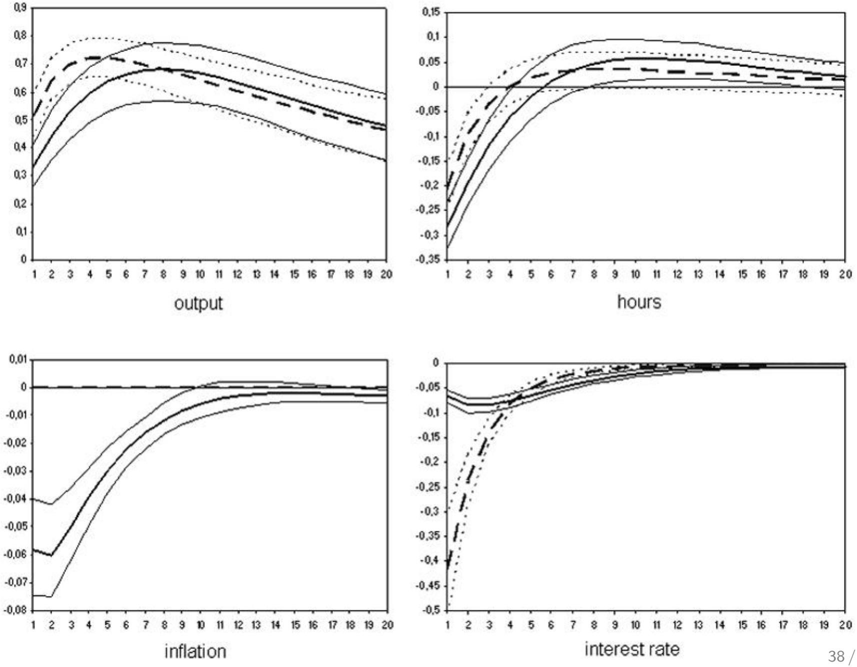
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Test stability of results estimate model for two subsamples

1. Great Inflation 1966:2-1979:2
2. Great Moderation 1984:1-2004:4

Table 5—Subsample Estimates

	Structural parameters					Shock processes			
	1966:1–1979:2		1984:1–2004:4			1966:1–1979:2		1984:1–2004:4	
	Mode	SD	Mode	SD		Mode	SD	Mode	SD
φ	3.61	1.03	6.23	1.12	σ_a	0.58	0.05	0.35	0.02
σ_c	1.39	0.22	1.47	0.13	σ_b	0.22	0.04	0.18	0.02
h	0.63	0.07	0.68	0.04	σ_g	0.54	0.05	0.41	0.03
ξ_w	0.65	0.07	0.74	0.13	σ_I	0.52	0.09	0.39	0.05
σ_l	1.52	0.65	2.30	0.67	σ_r	0.20	0.02	0.12	0.01
ξ_p	0.55	0.08	0.73	0.04	σ_p	0.22	0.03	0.11	0.01
ι_w	0.58	0.13	0.46	0.16	σ_w	0.20	0.02	0.21	0.03
ι_p	0.45	0.18	0.21	0.09	ρ_a	0.97	0.01	0.94	0.02
ψ	0.34	0.13	0.69	0.11	ρ_b	0.39	0.17	0.14	0.08
Φ	1.43	0.09	1.54	0.09	ρ_g	0.91	0.03	0.96	0.01
r_π	1.65	0.19	1.77	0.29	ρ_I	0.60	0.10	0.64	0.07
ρ	0.81	0.03	0.84	0.02	ρ_r	0.22	0.10	0.29	0.10
r_y	0.17	0.03	0.08	0.05	ρ_p	0.51	0.24	0.74	0.13
$r_{\Delta y}$	0.20	0.03	0.16	0.02	ρ_w	0.96	0.02	0.82	0.15
$\bar{\pi}$	0.72	0.11	0.67	0.10	μ_p	0.46	0.20	0.59	0.18
$\beta^{-1} - 1$	0.14	0.06	0.12	0.05	μ_w	0.84	0.07	0.62	0.17
$\bar{l}$	0.03	0.62	−0.55	1.21	ρ_{ga}	0.58	0.11	0.39	0.11
$\bar{\gamma}$	0.33	0.04	0.44	0.02					
α	0.19	0.02	0.21	0.02					

Note: SD stands for standard deviation.

TABLE 6—ACTUAL, MODEL-BASED, AND COUNTERFACTUAL STANDARD DEVIATIONS OF GDP GROWTH AND INFLATION

	1966:1–2004:4		1966:1–1979:2		1984:1–2004:4		Counterfactual 1984:1–2004:4		
	Actual	Model	Actual	Model	Actual	Model	Shocks	Policy	Structure
Growth	0.86	0.94	1.01	1.13	0.59	0.73	1.21	0.70	0.75
Inflation	0.62	0.57	0.55	0.81	0.25	0.34	1.30	0.39	0.32

Notes: "Actual" refers to the data-based standard deviations over the indicated sample; "model" refers to the standard deviations generated by the DSGE model estimated over the indicated sample. The counterfactual standard deviations for the period 1984:1–2004:4 refer to the standard deviations that would have occurred in this period if the shock processes ("shocks"), the monetary policy rule ("policy"), or the structural parameters ("structure") would have been the same as the ones estimated in the 1966:1–1979:2 sample.