# **Arm Constraints**

Kevin Harrington, Ryan Benasutti June 20, 2019

# 1 Arm Constraints

#### 1.1 Features

The feature matrix of a motor module is given by

$$F_{m} = \begin{bmatrix} \tau_{stall}^{(1)} & \tau_{stall}^{(2)} & \tau_{stall}^{(3)} \\ \omega_{free}^{(1)} & \omega_{free}^{(2)} & \omega_{free}^{(3)} \\ P^{(1)} & P^{(2)} & P^{(3)} \\ M^{(1)} & M^{(2)} & M^{(3)} \end{bmatrix}$$

$$(1)$$

where  $au_{stall}^{(i)}$  is the stall torque in Newton-meters for motor i,  $\omega_{free}^{(i)}$  is the free speed in radians per second for motor i,  $P^{(i)}$  is the price of motor i, and  $M^{(i)}$  is the mass in kilograms of motor i.

## 1.2 Required Tip Force and Velocity

V is the tip velocity (given as requiredTipVelocityMeterPerSec). F is the tip force (given as requiredTipForceNewtons).  $R_j$  is the r parameter of link j (the dh-A parameter). The arm is mounted  $90 \deg$  off vertical.

The torque-speed curve of a motor is given by:

$$\omega_i(\tau; \tau_{stall}^{(i)}, \omega_{free}^{(i)}) = \frac{(\tau_{stall}^{(i)} - \tau)\omega_{free}^{(i)}}{\tau_{stall}^{(i)}}$$
(2)

$$\tau_1 \ge F(R_1 + R_2 + R_3) + G(M_2R_1 + M_3(R_1 + R_2)) \tag{3}$$

$$\tau_2 \ge F(R_2 + R_3) + M_3 G R_2 \tag{4}$$

$$\tau_3 \ge FR_3 \tag{5}$$

$$\omega_1(\tau_1) \ge \frac{V}{R_1 + R_2 + R_3}$$
(6)

$$\omega_2(\tau_2) + \omega_3(\tau_3) \ge \frac{V}{R_2 + R_3} \tag{7}$$

$$\omega_3(\tau_3) \ge \frac{V}{R_3} \tag{8}$$

### 1.3 Optimization Goal

We want to optimize for price (lowest price).