## **Arm Constraints**

Kevin Harrington, Ryan Benasutti May 2019

#### 1 Arm Constraints

#### 1.1 Features

The feature matrix of a motor is given by

$$F_{m} = \begin{bmatrix} \tau_{stall}^{(0)} & \tau_{stall}^{(1)} & \tau_{stall}^{(2)} \\ \omega_{free}^{(0)} & \omega_{free}^{(1)} & \omega_{free}^{(2)} \\ P^{(0)} & P^{(1)} & P^{(2)} \\ M^{(0)} & M^{(1)} & M^{(2)} \end{bmatrix}$$

$$(1)$$

where  $au_{stall}^{(i)}$  is the stall torque in Newton-meters for motor i,  $\omega_{free}^{(i)}$  is the free speed in radians per second for motor i,  $P^{(i)}$  is the price of motor i, and  $M^{(i)}$  is the mass in kilograms of motor i.

#### 1.2 Required Tip Torque

F is the applied force on the tip (given).  $R_i$  is the length of link i.

$$\tau_{stall} \ge \begin{bmatrix} F \cdot (R_1 + R_2) \\ (F \cdot (R_1 + R_2) + (M_1 \cdot G \cdot R_1)) \\ F \cdot R_2 \end{bmatrix}$$
 (2)

### 1.3 Required Tip Velocity

V is the tip velocity (given).

$$\boldsymbol{\omega} = \begin{bmatrix} \frac{V_y}{R_1 + R_2} \\ \frac{V_z}{R_1 + R_2} \\ \frac{V_z}{R_2} \end{bmatrix} \tag{3}$$

The torque-speed curve of a motor is given by:

$$t(\omega_i) = \tau_{stall} - \frac{\omega_i \cdot \tau_{stall}}{\omega_{free}} \tag{4}$$

We can then describe the minimum required torque  $( au_r, ext{ given})$  at the applied speed  $\omega$ .

$$t(\boldsymbol{\omega}) \ge \boldsymbol{\tau_r} \tag{5}$$

# 1.4 Maximum Price

 $P_r$  is the maximum allowable price (given).

$$P_r \ge \sum_{i=0}^{2} P_i \tag{6}$$