Comp Arch HW 3

10/3/17

1. $91_{10} + C6_{16}$

Convert 91_{10} to binary

$$91 - 2^6 = 91 - 64 = 27$$
$$27 - 2^4 = 27 - 16 = 11$$
$$11 - 2^3 = 11 - 8 = 3$$
$$3 - 2^1 = 11 - 2 = 1$$
$$1 - 2^0 = 1 - 1 = 0$$

1
0
1
1
0
1
1

$91_{10} = 1011011_2$

Convert $C6_{16}$ to binary

C_{16}	6_{16}
↓	↓
12_{10}	6_{10}
↓	↓
$2^3_{10} + 2^2_{10}$	$2^2 + 2^1$
↓	↓
1100	0110

Since $2^4_{10} = 16_{10}$ we can examine each hex bit individually and convert them into sets of four binary bits.

$C6_{16} = 11000110_2$

$$\begin{array}{r} 1011011_2 \\ 11000110_2 \\ \hline 100100001_2 \end{array}$$

↓ convert to decimal

$$2^8_{10} + 2^5_{10} + 2^1_{10} = 256_{10} + 32_{10} + 1_{10} = \boxed{289_{10}}$$

2. $11_8 - 11_{10}$

Convert 11_8 to binary

1_8	1_8
001_2	001_2

Since $8 = 2^3$, we can convert octal to binary by converting each individual bit into three binary bits.

$11_8 = 1001_2$

Convert 11_{10} to binary

$11_{10} - 2_{10}^{3} = 3_{10}$ 1

3_{10} is less than 2^2 0

$3_{10} - 2_{10}^{1} = 1_{10}$ 1

$1_{10} - 2_{10}^{0} = 0_{10}$ 1

$11_{10} = 1011_2$

^ Because we want to Subtract a positive number, we need to turn it into a negative using Two's complement with 5 bits

$01011_2 \to 1_{10}$

↓convert to negative by flipping bits and adding one

10101_2

$-11_{10} = 10101_2$

$$\begin{array}{r} 01001_2 \\ +\,10101_2 \\ \hline 11110_2 \end{array}$$

This is -2_{10} in Two's Complement, becau it is the value away from all 1 bits, which is always -1.

So $11_8 - 11_{10} = \boxed{-2_{10}}$

3. $12.3125_{10} + 0110_{I2Q2}$

Start by converting both to I5Q4

$12.3125_{10} \to I5Q4$

	0
$12.3125 - 2^3 = 4.3125$	1
$4.3125_{10} - 2^2 = 0.3125$	1
0.3125_{10} is less than 2^1 and 2^0	0 0
0.3125_{10} is less than 2^{-1}	0
$0.3125_{10} - 2^{-2} = .0625$	1
0.0625 is less than 2^{-3}	0
$0.0625 - 2^{-4} = 0$	1

011000101

$0110_{I \& Q2}$ $\xrightarrow{\text{converted}}$ 000011000_{I5Q4}

$$
\begin{array}{r}
011000101 \\
+\ 000011000 \\
\hline
011011101
\end{array}
$$

Convert 011011101_{I5Q4} to base 10

$(2^3+2^2+2^0+2^{-1}+2^{-2}+2^{-4})_{10}$

$=(8+4+1+0.5+0.25+0.0625)_{10}$

$\boxed{=13.8125_{10}}$

5 $9_{10} \cdot 3_{10}$

Convert both to binary

$9_{10} = 2^3+2^0$ } easy to figure out in head

$9_{10} = 1001_2$

$3_{10} = 2^1+2^0$

$3_{10} = 11_2$

Convert both to I5Q0

$=9_{10} = 01001_{I5Q0}$

$=3_{10} = 00011_{I5Q0}$

Multiply

$$
\begin{array}{r}
01001 \\
*\ 00011 \\
\hline
01001 \\
01001 \\
\hline
000011011_{I9Q0}
\end{array}
$$

} After these two, all other rows will be only zeros

Convert back to decimal

$011011_{I9Q0} = 2^0_{10}+2^1_{10}+2^3_{10}+2^4_{10}$

$\boxed{=27_{10}}$

6. $(-5)_{10} \cdot (-6)_{16}$

Convert both to I4Q0

- Invert, add 1 to find negative

$6_{16} = 6_{10} = 0110_{I4}$

$\bar{6}_{10} = 1001_{I4}$

$\bar{6}_{10} + 1 = -6_{10} = 1010_{I4}$

$5_{10} = 0101_{I4}$

$\bar{5}_{10} = 1010_{I4}$

$\bar{5}_{10} + 1 = 1011_{I4}$

$-6_{16} = 1010_{I4}$

$-5_{10} = 1011_{I4}$

Multiply with sign extending

```
        11111010
        11111011
        --------
        11111010
       11111010
      00000000
     11111010
    11111010
   11111010
  11111010
11111010
----------------
        00011110 I8
```

cutoff here

Convert back to decimal

$00011110_{I8Q0} = 2^1_{10} + 2^2_{10} + 2^3_{10} + 2^4_{10}$

$= (2 + 4 + 8 + 16)_{10}$

$\boxed{= 30_{10}}$

7. $9.5_{10} \cdot 2.625_{10}$

Convert to I_5Q_3

$9.5_{10} - 2^3_{10} = 1.5_{10}$

$1.5_{10} - 2^0_{10} = 0.5_{10}$

$0.5_{10} - 2^{-1}_{10} = 0$

0	1	0	0	1	1	0	0
sign	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}

$9.5_{10} = 01001100_{I5Q3}$

$2.625_{10} - 2^1_{10} = 0.625_{10}$

$0.625_{10} - 2^{-1} = 0.125_{10}$

$0.125_{10} - 2^{-3} = 0_{10}$

0	0	0	1	0	1	0	1
sign	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}

$2.625_{10} = 00010101_{I5Q3}$

~~because there~~ Multiply

```
        01001100 I5Q3
        00010101 I5Q3
        --------
        01001100
      100000000
      01001100
     00000000
    01001100
   00000000
  00000000
 00000000
----------------
0000110000111100 I4Q6
```

convert to decimal

$= 2^4_{10} + 2^3_{10} + 2^{-1}_{10} + 2^{-2}_{10} + 2^{-3}_{10} + 2^{-4}_{10}$

$= 16 + 8 + 0.5 + 0.25 + 0.125 + 0.625 = \boxed{24.9375_{10}}$

Optional

1. Represent -5.6875_{10} in single-precision floating point

- sign bit is 1
- $5_{10} = 101_2$
- $0.6875 = 2^{-1} + 2^{-3} + 2^{-4}$

so

$5.6875_{10} = 101.1011_2$

since radix point is here, exponent value must be 2_{10}

$2_{10} = exp - 127_{10}$

$exp = 129_{10}$

$= 2^7 + 2^0$

$= 10000001_2$

significand will be 011011 trailing 0's

$\underbrace{1}_{s}\ \underbrace{10000001}_{\text{exponent}}\ \underbrace{01101100000000000000000}_{\text{significand}}$