Computer Architecture Homework 3

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1. $91_{10} + C6_{16}$

$$
\begin{aligned}
91_{10} + C6_{16} &= (64_{10} + 16_{10} + 8_{10} + 2_{10} + 1_{10}) + (C_{16} \times 2^4 + 6_{16}) \\
&= \quad 001011011_2 \ (U9) \\
&+ \ \underline{011000110_2} \ (U9) \\
&= \quad 100100001_2 \ (U9) \\
&= \quad 256_{10} + 32_{10} + 1_{10} \\
&= \quad \boxed{289_{10}}
\end{aligned}
$$

2. $11_8 - 11_{10}$

$$
\begin{aligned}
11_8 - 11_{10} &= (1 \times 2^3 + 1) - (2^3 + 2^1 + 1) \\
&= \quad 01001_2 - 01011_2 \ (I5) \\
&= \quad 01001_2 \ (I5) \\
&+ \ \underline{10101_2} \ (I5) \\
&= \quad 11110_2 \ (I5) \\
&= \quad \boxed{-2_{10}}
\end{aligned}
$$

3. $12.3125_{10} + 0110_{I2Q2}$

$$
\begin{aligned}
12.3125_{10} + 0110_{I2Q2} &= (8_{10} + 4_{10} + 0.25_{10} + 0.0625_{10}) + 01.10_2 \ (I2Q2) \\
&= \quad 01100.0101_2 \ (I5Q4) \\
&+ \ \underline{00001.1000_2} \ (I5Q4) \\
&= \quad 01101.1101_2 \ (I5Q4) \\
&= \quad 8_{10} + 4_{10} + 1_{10} + 0.5_{10} + 0.25_{10} + 0.0625_{10} \\
&= \quad \boxed{13.8125_{10}}
\end{aligned}
$$

4. $5.75_{10} - 7.125_{10}$

$$
\begin{aligned}
5.75_{10} - 7.125_{10} &= (4_{10} + 1_{10} + 0.5_{10} + 0.25_{10}) - (7_{10} + 0.125_{10}) \\
&= 101.11(I3Q2) - 0111.001(I4Q3) \\
&= 101.11(I3Q2) - 1000.111(I4Q3) \\
&= 0101.110 \ (I4Q3) \\
& + \underline{1000.111} \ (I4Q3) \\
&= 1110.101_2 \ (I4Q3) \\
&= -2_{10} + 0.5_{10} + 0.125_{10} \\
&= \boxed{-1.375_{10}}
\end{aligned}
$$

5. $9_{10} \cdot 3_{10}$

$$
\begin{aligned}
9_{10} \cdot 3_{10} &= (8_{10} + 1_{10}) \cdot (2_{10} + 1_{10}) \\
&= 1001_2 \ (U4) \\
& \times \underline{0011_2} \ (U4) \\
&= 01001_2 \ (U5) \\
& + \underline{10010_2} \\
&= 11011_2 \ (U5) \\
&= \boxed{27_{10}}
\end{aligned}
$$

6. $(-5)_{10} \cdot (-6)_{16}$

$$
\begin{aligned}
(-5)_{10} \cdot (-6)_{16} &= ((-0101)_2 \cdot (-0110)_2) \ (I4) \\
&= 1011_2 \times 1010_2 \ (I4) \\
&= 11111011_2 \ (I8) \\
& \times \underline{11111010_2} \ (I8) \\
& 00000000 \\
& 11110110 \\
& 00000000 \\
& 11011000 \\
& 10110000 \\
& 01100000 \\
& 11000000 \\
& + \underline{10000000} \\
&= 00011110_2 \ (I8) \\
&= \boxed{30_{10}}
\end{aligned}
$$

7. $9.5_{10} \cdot 2.625_{10}$

$$
\begin{aligned}
9.5_{10} \cdot 2.625_{10} &= (8_{10} + 1_{10} + 0.5_{10}) \cdot (2_{10} + 0.5_{10} + 0.125_{10}) \\
&= 1001.1_2 \ (U4Q1) \times 10.101_2 \ (U2Q3) \\
&= 001001.1000_2 \ (U6Q4) \\
& \times \ \underline{000010.1010_2} \ (U6Q4) \\
& 000001.0011 \\
& 000100.1100 \\
& + \ \underline{010011.0000} \\
&= 011000.1111_2 \ (U6Q4) \\
&= \boxed{24.9375_{10}}
\end{aligned}
$$

8. $(-1.25)_{10} \cdot 3.5_{10}$

$$
\begin{aligned}
(-1.25)_{10} \cdot 3.5_{10} &= (-1_{10} - 0.25_{10}) \cdot (2_{10} + 1_{10} + 0.5_{10}) \\
&= (-1.01_2) \cdot 11.1_2 \\
&= 10.11_2 \ (I2Q2) \times 011.1 \ (I3Q1) \\
&= 11110.110_2 \ (I5Q3) \\
& \times \ \underline{00011.100_2} \ (I5Q3) \\
& 11111.011 \\
& 11110.110 \\
& + \ \underline{11101.100} \\
&= 11011.101_2 \ (I5Q3) \\
&= \boxed{-4.375_{10}}
\end{aligned}
$$

Optional Problems

1. -5.6875_{10}

$$
\begin{aligned}
-5.6875 &= (-1) \times 2^2 \times 1.421875_{10} \\
&= (-1) \times 2^2 \times \left(1 + \frac{27_{10}}{64_{10}}\right) \\
&= (-1) \times 2^2 \times \left(1 + \frac{16_{10}}{64_{10}} + \frac{8}{64_{10}} + \frac{2}{64_{10}} + \frac{1}{64_{10}}\right) \\
&= (-1) \times 2^2 \times (1 + 2^{-2} + 2^{-3} + 2^{-5} + 2^{-6})
\end{aligned}
$$

3

Sign bit: b1
Exponent bits: $(127 + 2)_{10} = 129_{10} = $ b10000001
Fraction bits: b01101100000000000000000
Binary Expression: b11000000101101100000000000000000
Hexadecimal Expression: $\boxed{0xc0b60000}$

2. 0x44fc2000
   0x44fc2000 = b01000100111111000010000000000000
   Sign bit: b0
   Exponent bits: b10001001 $= 128_{10} + 8 + 1 = 137_{10}$
   Fraction bits: b11111000010000000000000

   Exponent $= 137_{10} - 127_{10} = 10_{10}$
   Fraction $= 1 + 2^{-1} + 2^{-2} + 2^{-3} + 2^{-4} + 2^{-5} + 2^{-10}$

   Decimal Expression:

   $$
   \begin{aligned}
   & 2^{10} \times (1 + 2^{-1} + 2^{-2} + 2^{-3} + 2^{-4} + 2^{-5} + 2^{-10}) \\
   &= 2^{10} + 2^9 + 2^8 + 2^7 + 2^6 + 2^5 + 2^0 \\
   &= 1024_{10} + 512_{10} + 256_{10} + 128_{10} + 64_{10} + 32_{10} + 1 \\
   &= \boxed{2017.0_{10}}
   \end{aligned}
   $$