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# Variable-lag Granger Causality for Time Series Analysis

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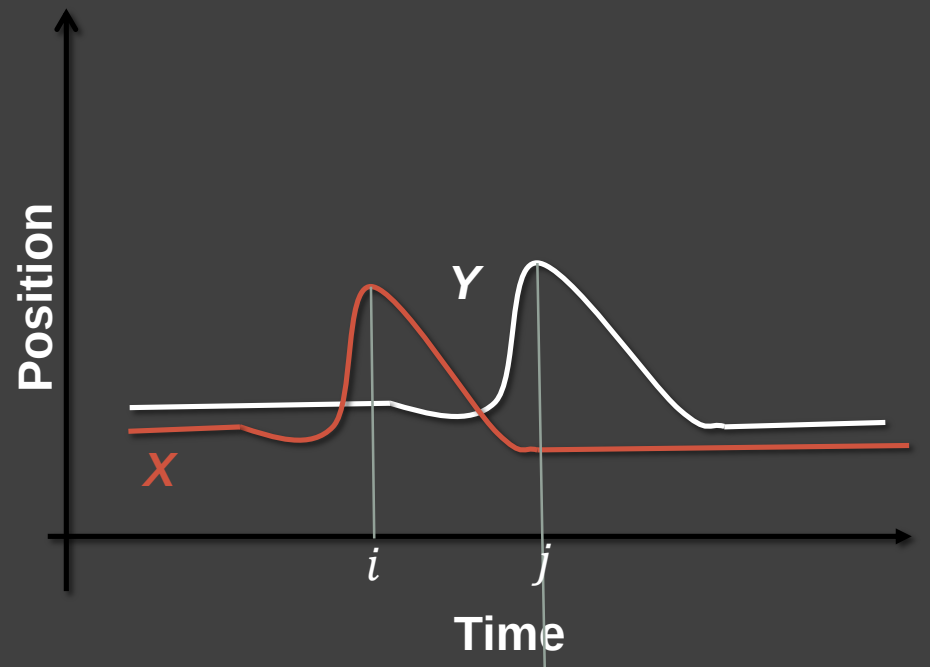
# Get to know Granger Causality

Invented by C. W. J. Granger in 1969

Definition: Given time series $X$ and $Y$

"X Granger-causes Y if past information on X predicts the behavior of Y better than using Y 's past information alones"



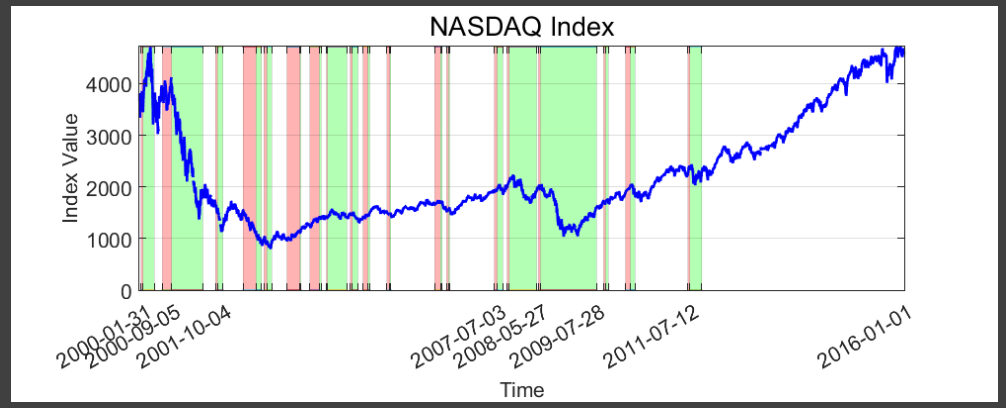

$Y$ follows $X$ with a time delay $\Delta t$

$X$ Granger-causes $Y$

# Applications of Granger causality

**Finding key factors that initiate collective patterns**

- Group Decision Making Process
  - Human behaviors: e.g. time series of actions in social network
  - Animal behaviors: e.g. time series of positions from GPS collars
- Collective events
  - Stock market coordination event: time series of closing prices



# Fixed vs. Arbitrary lag follower

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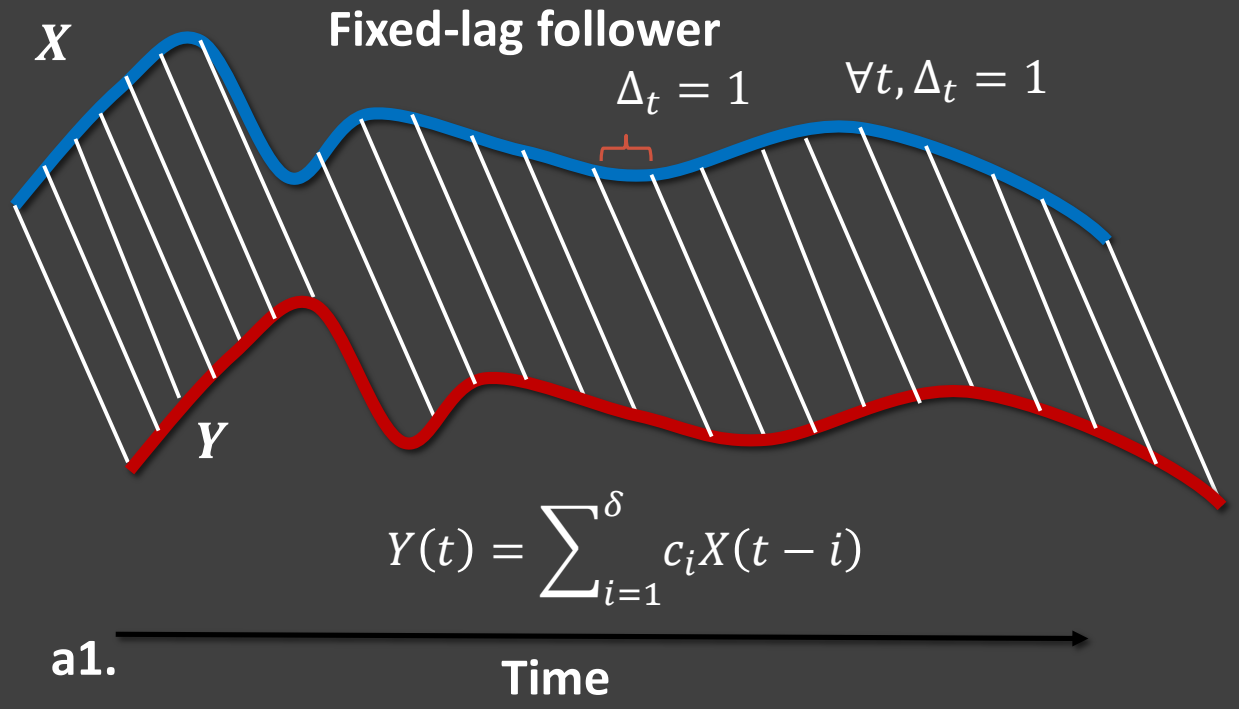
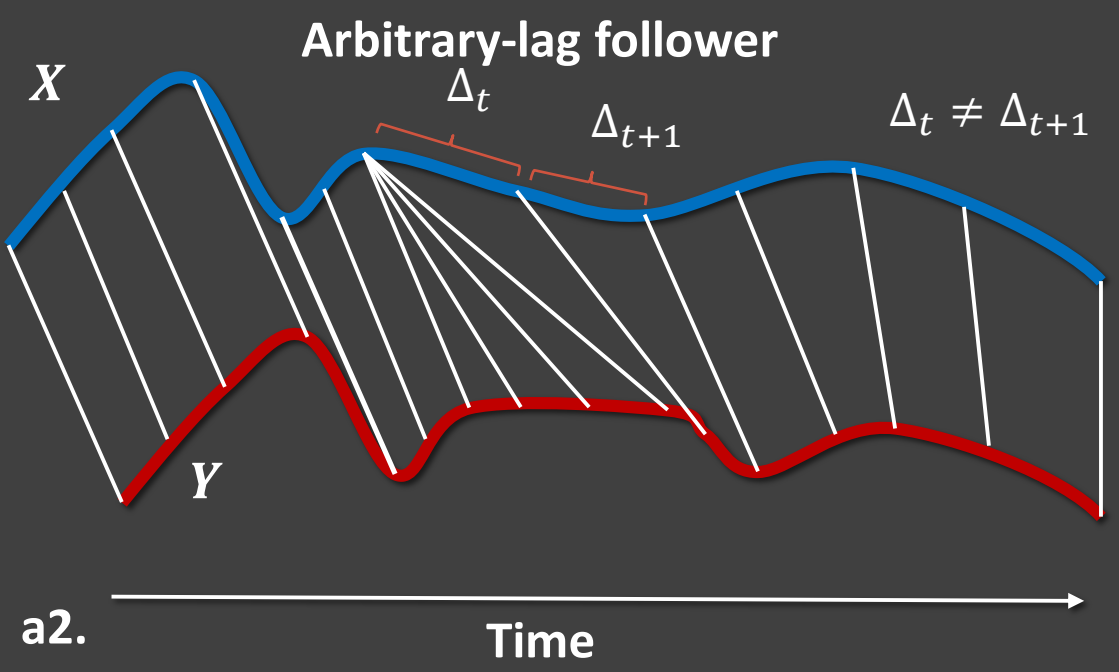


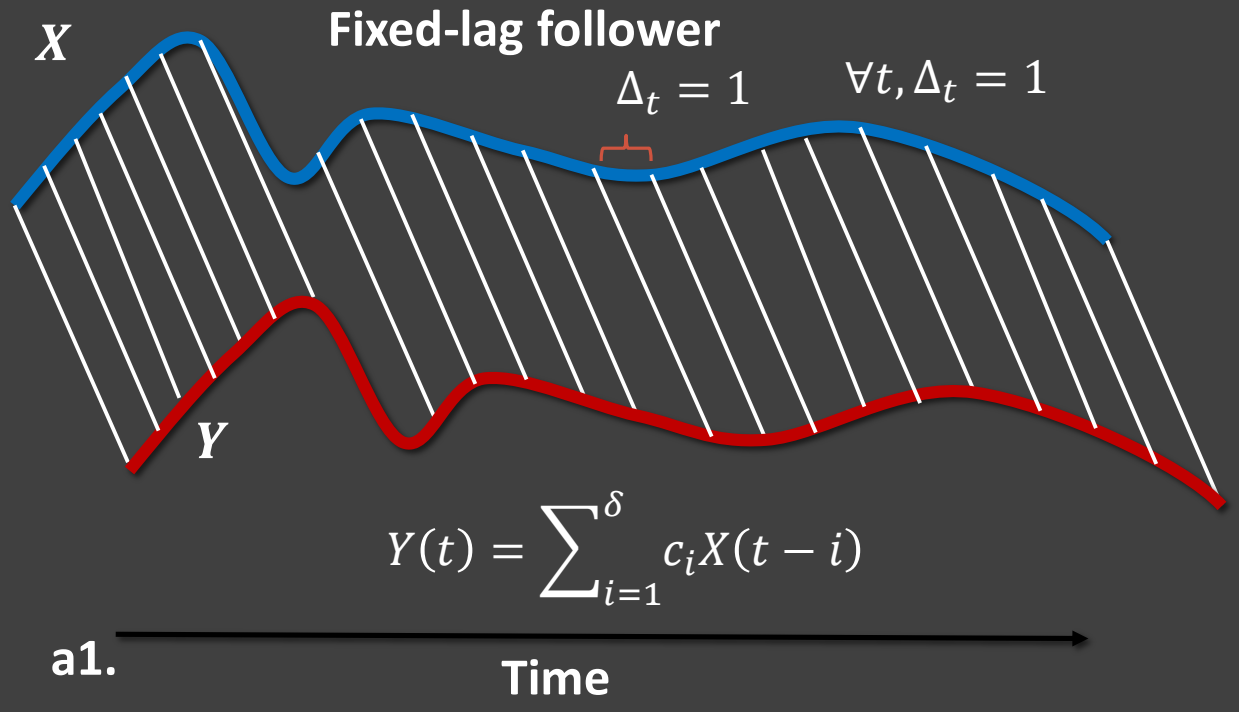



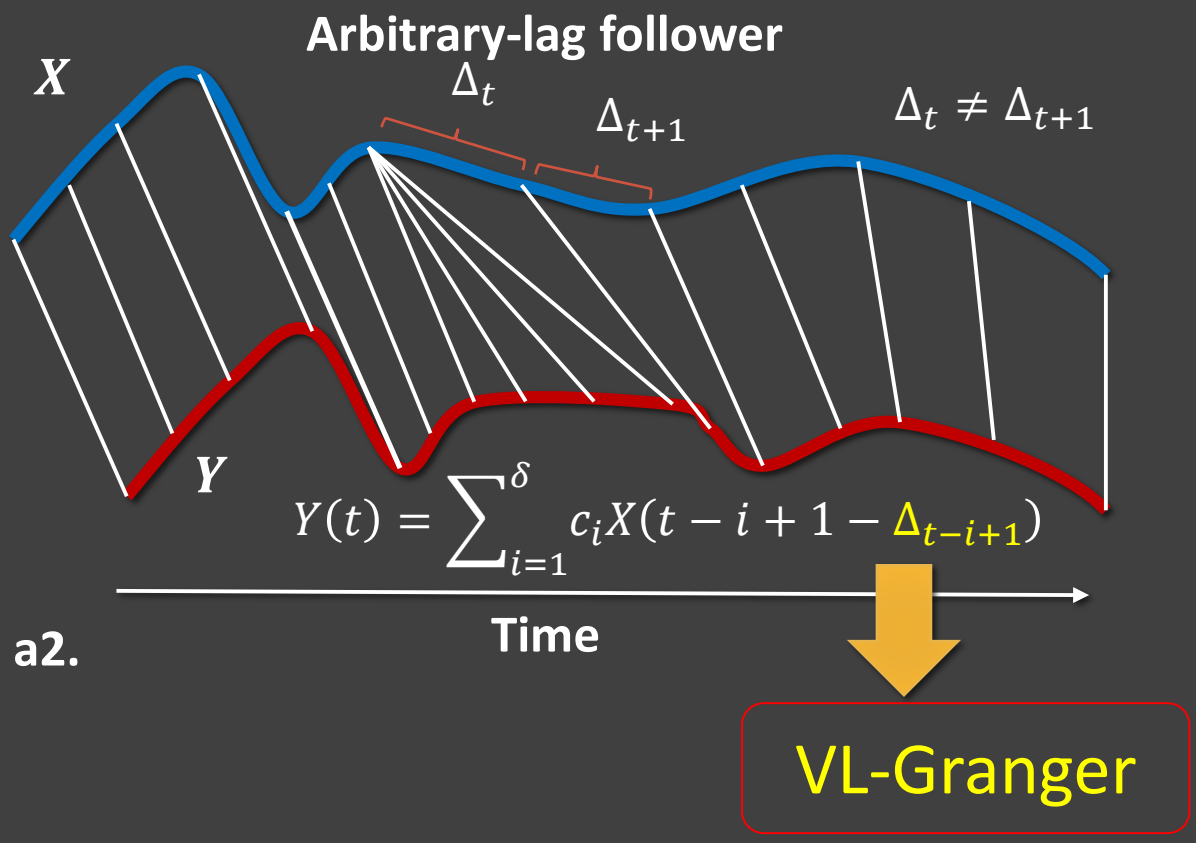

Granger: Lockstep marching

Reality: Walking and running

# Method

# Fixed vs. Arbitrary lag follower



# Traditional vs. VL- Granger

Algorithm: Traditional Granger causality

For all $t$ do:

1. Regret $Y(t)$ on $Y(t-1), \dots, Y(t-\delta_{max})$ to get errors of prediction (sum of residuals) for null model $H_0$
2. Regret $Y(t)$ on $Y(t-1), \dots, Y(t-\delta_{max})$ and X$(t-1), \dots, X(t-\delta_{max})$ to get errors of prediction for alternative model $H_1$
3. Compare errors of $H_0$ and $H_1$:
   - If $H_1$ reduces errors compared to $H_0$ significantly (e.g. by F-test), then $X$ Granger-causes $Y$
   - Else $X$ doesn't Granger-cause $Y$

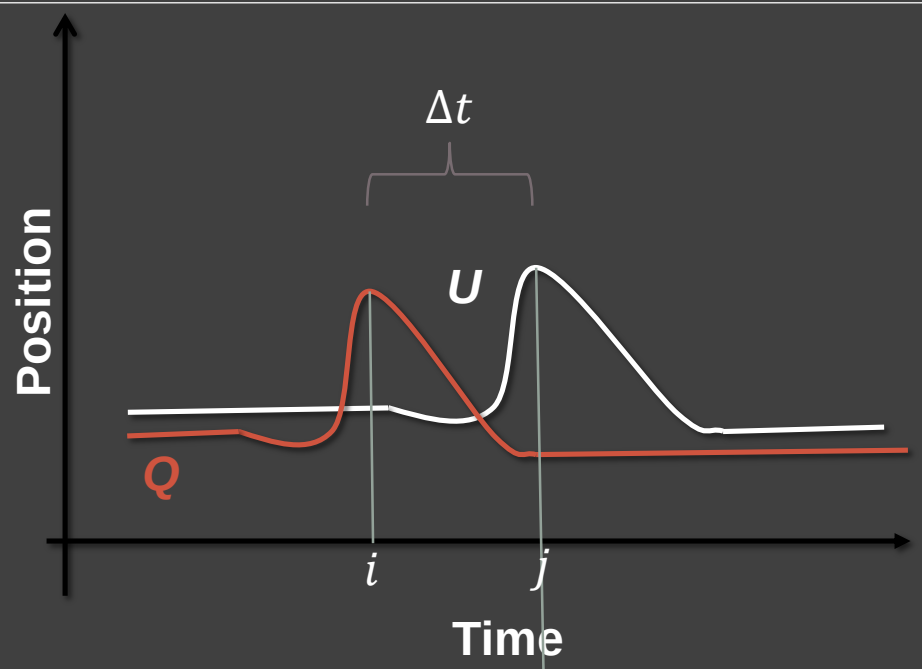
# Traditional vs. VL- Granger

Algorithm: VL-Granger causality

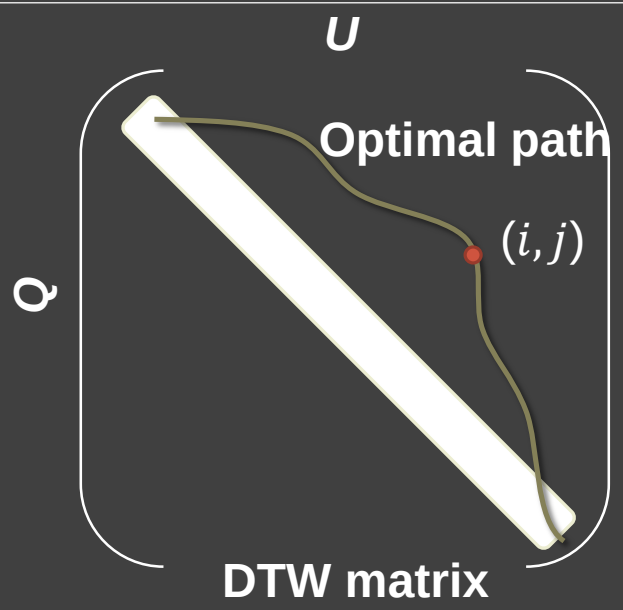
For all $t$ do:

1. Regret $Y(t)$ on $Y(t-1), \dots, Y(t-\delta_{max})$ to get errors of prediction (sum of residuals) for null model $H_0$
2. Regret $Y(t)$ on $Y(t-1), \dots, Y(t-\delta_{max})$ and $X(t-1), \dots, X(t-\delta_{max})$ to get errors of prediction for alternative model $H_1$
3. Regret $Y(t)$ on $Y(t-1), \dots, Y(t-\delta_{max})$ and $X(t-\Delta_1), \dots, X(t-\Delta_{\delta_{max}})$ to get errors of prediction for alternative model $H_2$ s.t. $\Delta_i$ is inferred by Dynamic Time Warping (DTW)
4. Compare errors of $H_0$, $H_1$ and $H_2$:
   - If $H_2$ reduces errors compared to $H_0$ and $H_1$ significantly (e.g. by F-test), then $X$ VL-Granger-causes $Y$
   - Else if $H_1$ reduces errors from $H_0$ significantly, then $X$ Fixed-Lag-Granger-causes $Y$
   - Else $X$ doesn't Granger-cause $Y$

# Dynamic Time Warping




$U$ follows $Q$ with a time delay $\Delta t$




$$j - i = \Delta t$$

# Results - five case studies

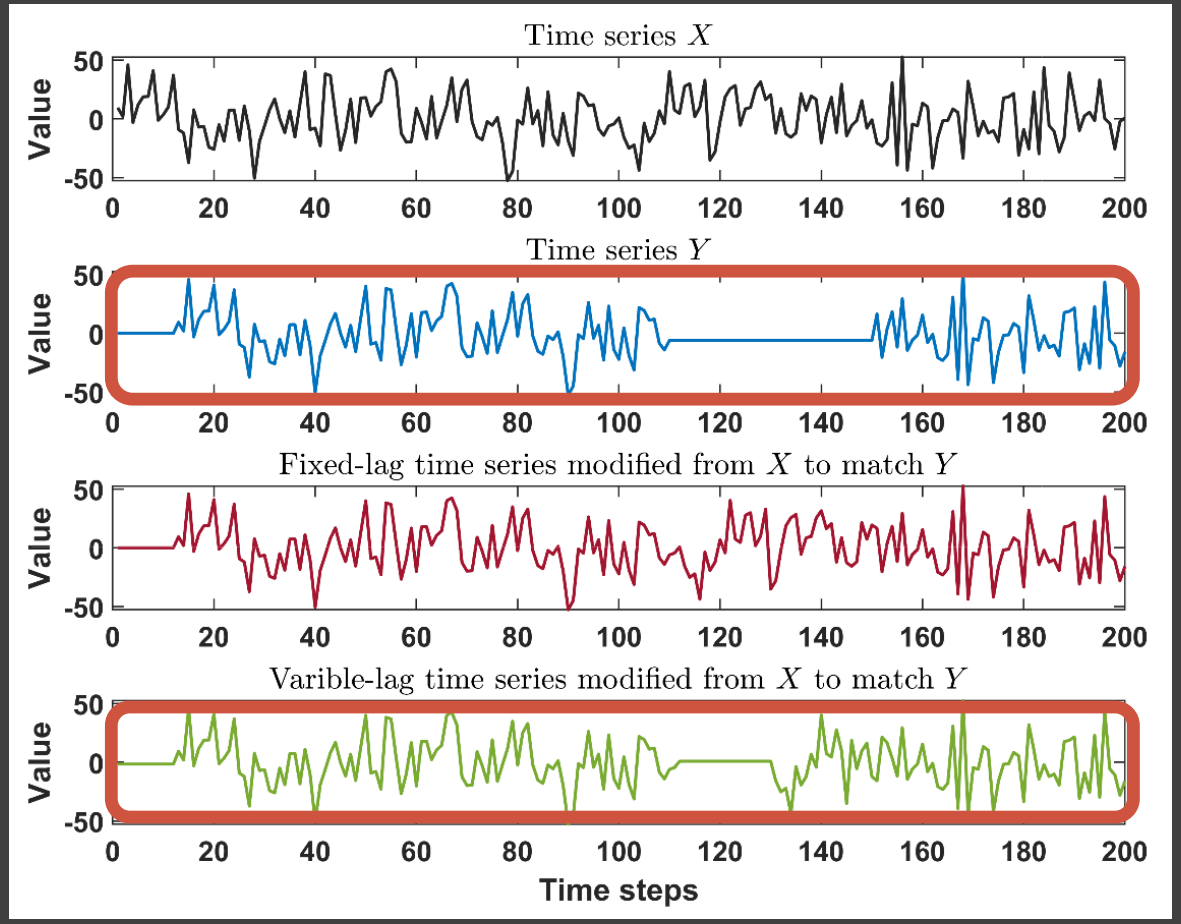
# Case study: Variable-lag follower.

Initiator

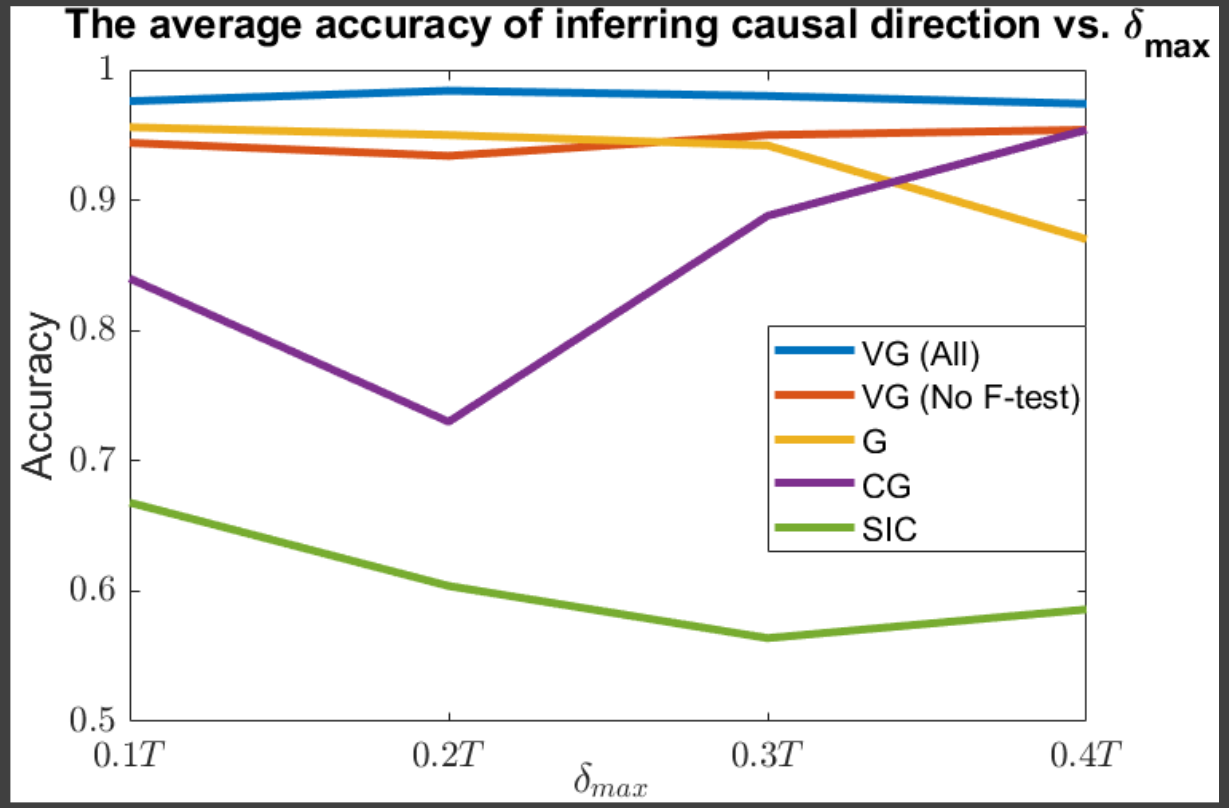
Follower

Fixed-Inferred-Follower

VL-Inferred-Follower



# Case study: Simulation & Sensitivity



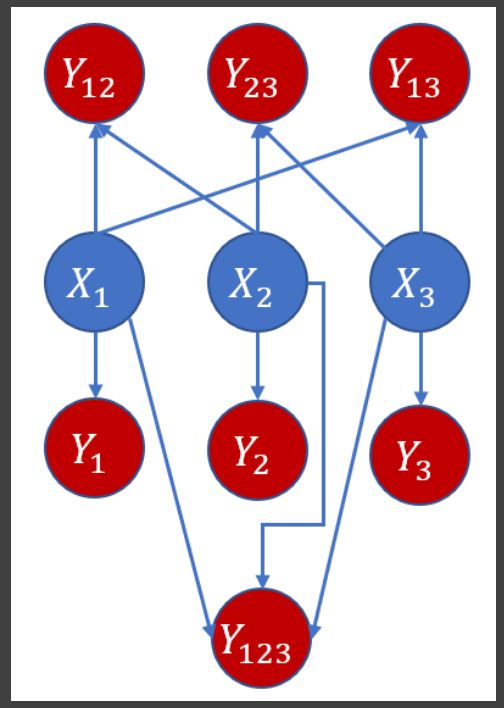

Methods

Having causation?

| | VG All | VG No F-test | G | CG | SIC |
|---|---|---|---|---|---|
| $\mathcal{N}$:$X \prec Y$ | 1.00 | 1.00 | 1.00 | 0.79 | 0.70 |
| $\mathcal{N}$:$X \not\prec Y$ | 1.00 | 1.00 | 0.92 | 0.90 | 0.51 |
| A.:$X \prec Y$ | 0.97 | 0.97 | 1.00 | 0.82 | 0.65 |
| A.:$X \not\prec Y$ | 0.95 | 0.79 | 0.78 | 0.85 | 0.49 |
| $\mathcal{N}$:$X$, A.:$Y$ | 0.99 | 0.99 | 0.95 | 0.91 | 0.68 |
| Group $\mathcal{N}$ | 1.00 | 1.00 | 1.00 | 0.12 | 0.53 |
| Group A. | 1.00 | 1.00 | 1.00 | 0.30 | 0.50 |

**F1 scores Table**

# Case study: Inferring causal graph

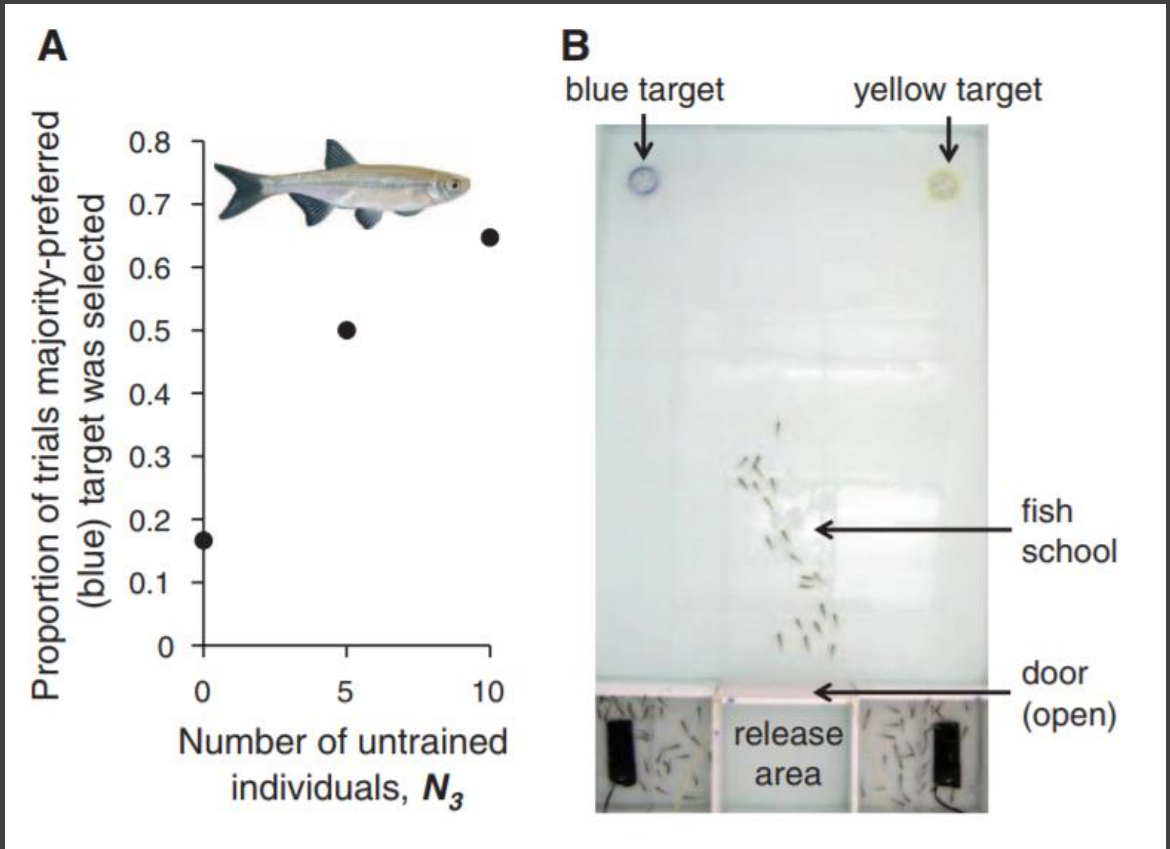



***Causal Graph***

**$X_i$ function generator**

| | $\mathcal{N}$ | | | ARMA | | |
|---|---|---|---|---|---|---|
| | Prec. | Rec. | F1 | Prec. | Rec. | F1 |
| VG (All) | 0.98 | 0.90 | 0.93 | 0.66 | 0.95 | 0.77 |
| VG (No F-test) | 0.96 | 0.90 | 0.93 | 0.57 | 0.95 | 0.71 |
| G | 0.91 | 0.97 | 0.93 | 0.38 | 0.71 | 0.49 |
| CG | 0.55 | 0.68 | 0.59 | 0.42 | 0.64 | 0.47 |
| SIC | 0.15 | 0.55 | 0.23 | 0.14 | 0.53 | 0.23 |

# Case study: School of fish movement coordination– 24 traces



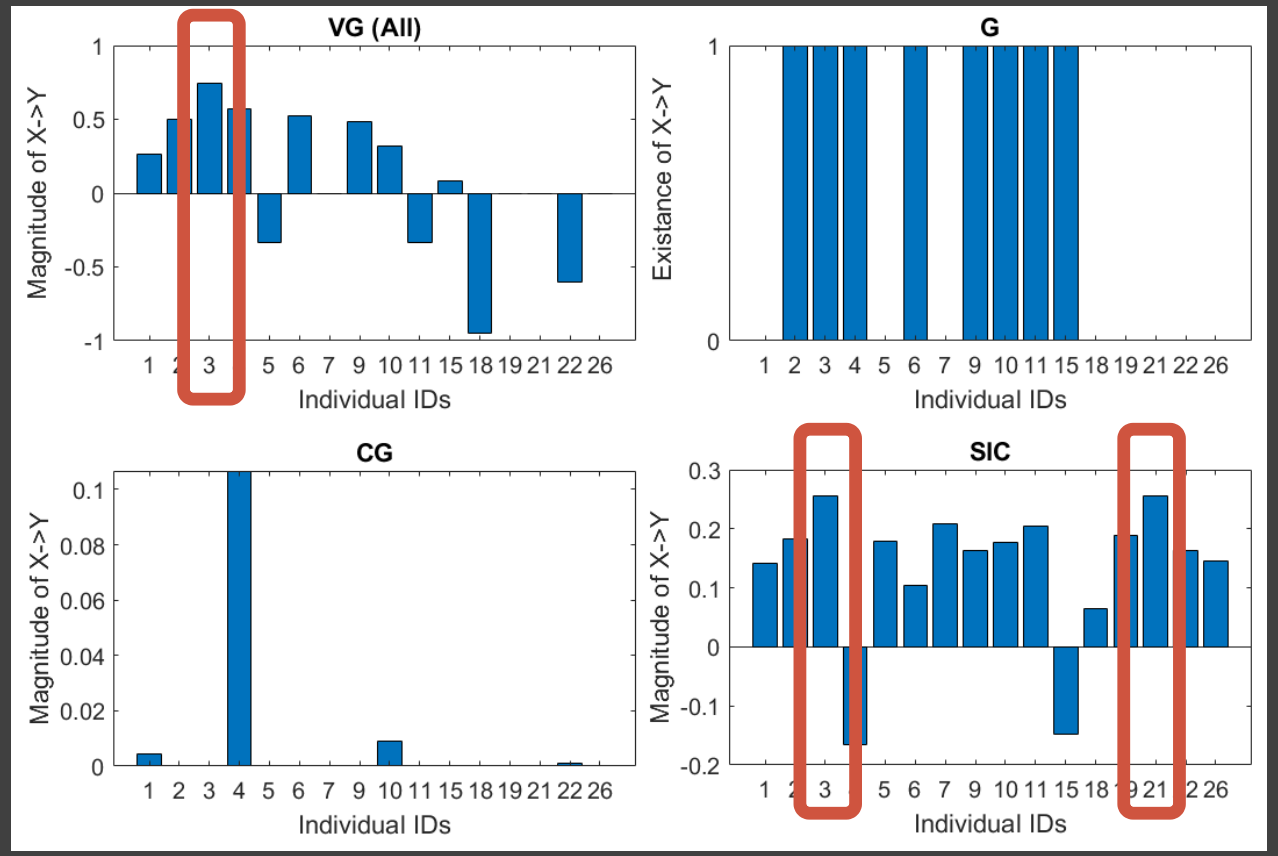

Methods

| $\delta_{max}$ | VG All | VG No F-test | G | CG | SIC |
|---|---|---|---|---|---|
| $0.1T$ | 14 | 19 | 2 | 17 | 19 |
| $0.2T$ | 18 | 19 | 2 | 11 | 21 |
| $0.3T$ | 16 | 18 | 5 | 11 | 20 |
| $0.4T$ | 18 | 18 | 5 | 11 | 19 |

Couzin, Iain D., Christos C. Ioannou, Güven Demirel, Thilo Gross, Colin J. Torney, Andrew Hartnett, Larissa Conradt, Simon A. Levin, and Naomi E. Leonard. "Uninformed individuals promote democratic consensus in animal groups." *science* 334, no. 6062 (2011): 1578-1580.

# Case study: 16 Baboons movement coordination

Ground truth: ID3 is the initiator




A. Strandburg-Peshkin, D. R. Farine, I. D. Couzin, and M. C. Crofoot, “Shared decision-making drives collective movement in wild baboons,” Science , vol. 348, no. 6241, pp. 1358–1361, 2015.

# Summary

We formalized Variable-Lag Granger Causality

- Relaxed fixed-lag assumption of Granger causality

Propose VL-Granger Causality Framework in Time Series

1. Identifying VL-Granger time series of pair-wise time series
2. Identifying VL-Granger time series of group time series

Our results show that,

- Simulation: VL-Granger framework performs better than baseline approaches in various settings
- School of Fish: VL-Granger framework performs better than the traditional approach.
- Baboons: VL-Granger framework can infer the true initiator of coordination movement while several methods fail.

# Q&A

# Caution! When Granger causality cannot find the true causes

1. When all time series we consider share the same cause that we can't observe
   - Issue: Suppose the political situation causes all stocks to fall down, then we can't infer causality from the stock closing price alone.
   - Sol: We should apply Granger causality in various political situations and other potential factors.
2. When all time series are periodic and have no causal relation, Granger causality just picks the earliest one that has a strong trend
   - Issue: Suppose the profit gains of companies depend heavily on seasons and no causality, then Granger causality just picks the first company that initiates the trend w.r.t. the season.
   - Sol: We should apply Granger causality in various intervals to find the consistency among different intervals.