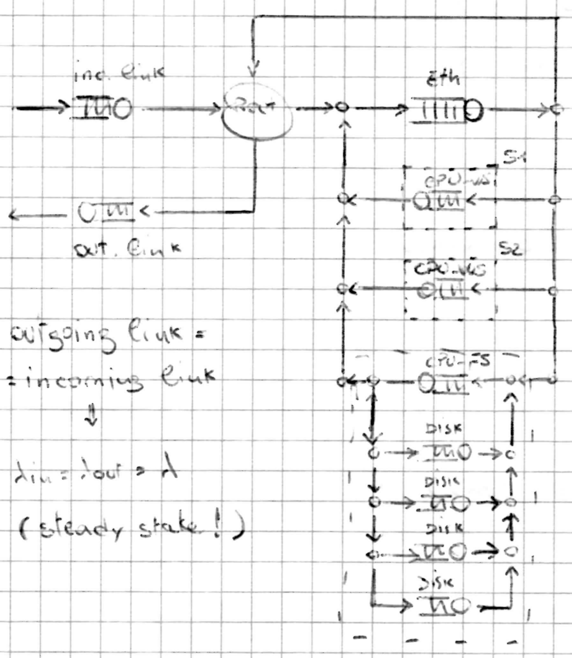
Exam 06.06.2012

Exercise 3



outgoing link = incoming link

⇓

$\lambda_{in} = \lambda_{out} = \lambda$

(steady state!)

$\lambda = 10$ (req/sec);

$n = 2$ (server);

$B = 1$ (GB/sec);

$L_{rout} \approx \phi$ (router latency);

$F = 10$ (Kbyte); $\{10 \cdot 1024$ (byte)$\}$

$R = 300$ (byte); (request)

$P_{hit} = 80\% = 0{,}8$ | $P_{miss} = 20\% = 0{,}2$

$S_{CPU-WS} = 10$ (ms) | $S_{CPU-WS} = 20$ (ms)

$S_{CPU-FS} = 10$ (ms)

$S_{DISK} = 20$ (ms)

MTTF = 10'000 (hours);

MTTR = 10 (hours);

$\}$ ∀ component

- Residence time:

in general:

$D_i = S_i \cdot V_i$ service demand ∀ i = component (CPU-WS, CPU-FS, DISK)

In this case, ∃ the $P_{hit/miss}$ and the load balancer. So:

$$D_i = \frac{(P_{hit} \cdot D_{i,hit}) + (P_{miss} \cdot D_{i,miss})}{n° \text{ of } i \text{ components}}$$

Hence:

$$D_{CPU-WS} = \frac{(0{,}8 \cdot 0{,}01) + (0{,}2 \cdot 0{,}02)}{2} = 0{,}006 \text{ (sec)} = 6 \text{ (ms)}$$

$$D_{CPU-FS} = \frac{\cancel{(0{,}8 \cdot \phi)} + (0{,}2 \cdot 0{,}01)}{1} = 0{,}002 \text{ (sec)} = 2 \text{ (ms)}$$

$$D_{DISK} = \frac{\cancel{(0{,}8 \cdot \phi)} + (0{,}2 \cdot 0{,}02)}{4} = 0{,}001 \text{ (sec)} = 1 \text{ (ms)}$$

- Max arrival rate:

Which is the maximum rate of the incoming requests that saturate the system?

$$\lambda_s = \frac{1}{\max\{D_i\}} = \frac{1}{D_{CPU-WS}} = \frac{1}{0{,}006} = 166{,}67 \left(\frac{req}{sec}\right)$$ (beyond this value, the system saturates)

$\lambda_s > \lambda \Rightarrow$ I consider λ for my next calculations!

$\boxed{X_0 = \lambda}$ $\{$ if $\lambda_s < \lambda$ then $X_0 = \lambda_s$ naturally $\}$

↑ for the steady state condition

Utilization factor: $U_i = X_0 \cdot D_i$

$U_{CPU-WS} = X_0 \cdot D_{CPU-WS} = 10 \cdot 0{,}006 = 0{,}06 = 60$ (ms); 60 %

$U_{CPU-FS} = X_0 \cdot D_{CPU-FS} = 10 \cdot 0{,}002 = 0{,}02 = 20$ (ms); 20 %

$U_{DISK} = X_0 \cdot D_{DISK} = 10 \cdot 0{,}001 = 0{,}01 = 10$ (ms); 10 %

Residence time: $R_i = \frac{D_i}{1 - U_i}$

$R_{CPU-WS} = \frac{D_{CPU-WS}}{1 - U_{CPU-WS}} = \frac{0{,}006}{1-0{,}06} = 0{,}00638$ (sec) $= 6{,}38$ (ms)

$R_{CPU-FS} = \frac{D_{CPU-FS}}{1 - U_{CPU-FS}} = \frac{0{,}002}{1-0{,}02} = 0{,}00204$ (sec) $= 2{,}04$ (ms)

$R_{DISK} = \frac{D_{DISK}}{1 - U_{DISK}} = \frac{0{,}001}{1-0{,}01} = 0{,}00101$ (sec) $= 1{,}01$ (ms)

- NETWORK TIME Hypothesis = The TCP layer doesn't know the local MTU (MTU_{ETH})

REQUEST

$\# segment = \left\lceil \frac{R}{MTU_{TCP}} \right\rceil = \left\lceil \frac{300}{65'515} \right\rceil = 1$;

$\# datagram = \left\lceil \frac{R + (\# segment \cdot TCP_{ovhd})}{MSS} \right\rceil = \left\lceil \frac{300 + (1 \cdot 20)}{1480} \right\rceil = 1$ $\left\{ MSS = MTU_{ETH} - IP_{ovhd} = 1500 - 20 = 1480 \right\}$

frame = # datagram (because the payload of the frame is equal at the MSS!)

$Overhead_{req} = (\# segment \cdot TCP_{ovhd}) + [\# datagram (IP_{ovhd} + Eth_{ovhd})] = (1 \cdot 20) + [1 \cdot (20+18)] = 58$ byte

$S_{req} = \frac{R + Overhead}{B} = \frac{358}{1 \cdot 10^9} = 0{,}358 \cdot 10^{-6}$ (sec) $= 0{,}000358$ (ms)

RESPONSE

$\# segment = \left\lceil \frac{F}{MTU_{TCP}} \right\rceil = \left\lceil \frac{10 \cdot 1024}{65'515} \right\rceil = 1$,

$\# datagram = \left\lceil \frac{F + \# segment (TCP_{ovhd})}{MSS} \right\rceil = \left\lceil \frac{(10 \cdot 1024) + (1 \cdot 20)}{1480} \right\rceil = 7$;

frame = # datagram divided as:

ETH	IP	TCP	
18	20	20	1460

ETH	TCP	
18	20	1480

× 5

ETH	TCP	
18	20	1380

$Overhead_{resp} = (\# segment \cdot TCP_{ovhd}) + \# datagram (IP_{ovh} + Eth_{ovhd})$
$= (1 \cdot 20) + 7 (20 + 18) = 286$ byte

$S_{resp} = \frac{F + Overhead_{resp}}{B} = \frac{10'526 \text{ (byte)}}{1 \cdot 10^9 \text{ (byte/sec)}} = 10{,}526 \cdot 10^{-6}$ (sec)
$= 0{,}010526$ (ms)

$\{(1 \cdot 1460) + 5(1480) = 8'860 \Rightarrow 10'240 - 8'860 = 1380\}$

~~20/06~~ 06 2012 (2)

HANDSHAKING

$\# segment = 6 \ (\text{is fixed}) = \# datagram \neq \# frame \quad \left\{ \begin{array}{|c|c|c|} ETH & IP & TCP \\ 18 & 20 & 20 \end{array} \times 6 \right\}$

$Overhead_{HS} = \# frame \, (TCP_{ovhd} + IP_{ovhd} + Eth_{ovhd}) = 6 \cdot 58 \, (byte) = 348 \text{ byte}$

$S_{hs} = \frac{overhead}{B} = \frac{348}{1 \cdot 10^9} = 0{,}348 \cdot 10^{-6} \, (sec) = 0{,}000348 \, (ms)$

$D_{eth} = (S_{req} + S_{resp} + S_{hs}) \cdot \overset{V_{eth}}{1} = 11{,}232 \cdot 10^{-6} \, (sec) \quad P_{hit} \left[(S_{req} + S_{resp} + S_{hs}) \cdot 1 \right] +$

$10 \quad 0{,}000013 \qquad + P_{miss} \left[(S_{req} + S_{resp} + S_{hs}) \cdot 2 \right] = 0{,}013 \, (ms)$

$U_{eth} = X_0 \cdot D_{eth} = \cancel{11{,}232 \cdot 10^{-6}} \, (req_i) \rightarrow = 0{,}00013$

in caso di miss passo 2 volte sulla ethernet!

$0{,}013$

$R'_{eth} = \frac{D_{eth}}{1 - U_{eth}} = \frac{\cancel{11{,}232 \cdot 10^{-6}}}{0{,}999} \simeq \cancel{0{,}0000130 \cdot 10^{-6}} \quad 0{,}013 \, (ms)$

1) Average ~~response~~ service time

$R_{tot} = R_{CPU-WS} + R_{CPU-FS} + R_{Disk} + R_{eth}$

2) The bottleneck is given by the CPU-WS (it has the maximum ~~response~~ residence time)

3) If the LAN/router/CPU-FS fails, the system doesn't work anymore...

$A_i = \frac{MTTF}{MTTF + MTTR} = \frac{10'000}{10'010} = 0{,}999 \quad \forall i = \text{CPU-FS, eth, router, disk, CPU-WS}$

- 2 WS & 4 disks : $(A_{CPU-WS})^2 \cdot (A_{Disk})^4 \cdot A_{CPU-FS} \cdot A_{rout} \cdot A_{eth}$
- 2 WS & 3 disks : " $\cdot \binom{4}{1} \left[(A_{Disk})^3 (1 - A_{Disk}) \right] \cdot$ " $\cdot$ " $\cdot$ "
- 2 WS & 2 disk : " $\cdot \binom{4}{2} \left[(A_{Disk})^2 (1 - A_{Disk})^2 \right] \cdot$ " $\cdot$ " $\cdot$ "
- 2 WS & 1 disk : " $\cdot \binom{4}{3} \left[A_{Disk} (1 - A_{Disk})^3 \right] \cdot$ " $\cdot$ " $\cdot$ "
- 1 WS & 4 disks : $\binom{2}{1} \left[A_{CPU_{WS}} (1 - A_{CPU_{WS}}) \right] \cdot (A_{Disk})^4 \cdot$ " $\cdot$ " $\cdot$ "
- 1 WS & 3 disks : $\binom{2}{1} \left[A_{CPU_{WS}} (1 - A_{CPU_{WS}}) \right] \cdot \binom{4}{1} \left[(A_{Disk})^3 (1 - A_{Disk}) \right] \cdot$ " $\cdot$ " $\cdot$ "

⋮

For every cases, I have to calculate the new residence time for the component that has suffered the fault

- 2 ws & 4 disks: Already discussed

- 2 ws & 3 disks: $D_{DISK_{2,3}} = \frac{\phi + (0{,}2 \cdot 0{,}02)}{3} = 0{,}0013$ (sec);

$U_{DISK_3} = X_0 \cdot D_{isk_3} = 0{,}013$; $\Rightarrow R'_{DISK_3} = \frac{D_{DISK_3}}{1 - U_{DISK_3}}$

$R_{2,3} = R'_{CPU-WS} + R'_{DISK_3} + R'_{eth} + R'_{CPU-FS} + R$

- 2 ws & 2 disks: $D_{isk_2} = \frac{0{,}2 \cdot 0{,}02}{2} = 0{,}002$

⋮

- 2 ws & 1 disk: $D_{DISK_1} = 0{,}2 \cdot 0{,}02 = 0{,}004$

⋮

- 1 ws & 4 disk: $D_{CPU-WS_1} = \frac{(0{,}8 \cdot 0{,}01) + (0{,}2 \cdot 0{,}02)}{1} = 0{,}012$ (sec)

$U_{CPU-WS_1} = X_0 \cdot D_{CPU-WS_1} = 0{,}12$; $\Rightarrow R'_{CPU-WS_1} = \frac{D_{CPU-WS_1}}{1 - U_{CPU-WS_1}}$

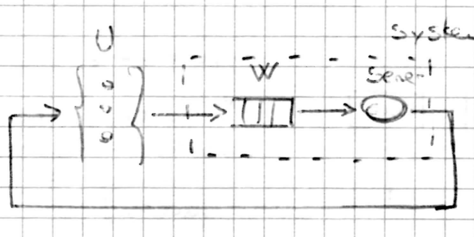
$R_{1,4} = R'_{CPU-WS_1} + R'_{DISK} + R'_{eth} + R'_{CPU-FS}$

⋮

Finally, I have to calculate the average response time based on the probability of stay in all of these cases:

$$\bar{R} = \frac{\sum_{i=1}^{2} \sum_{j=1}^{4} P_{ij} (R_{ij})}{1 - P_{fault}} = \frac{(P_{2,4} \cdot R_{2,4}) + (P_{2,3} \cdot R_{2,3}) + (P_{2,2} \cdot R_{2,2}) + \ldots}{}$$

Exercise 2



$U = 8$;

$W = 5$;

$Z = 30$ (sec);

$S = 20$ (sec) $\rightarrow \mu = \frac{1}{20} = 0{,}05$ $\left(\frac{req}{sec}\right)$

$n = 1$ (server)

STD

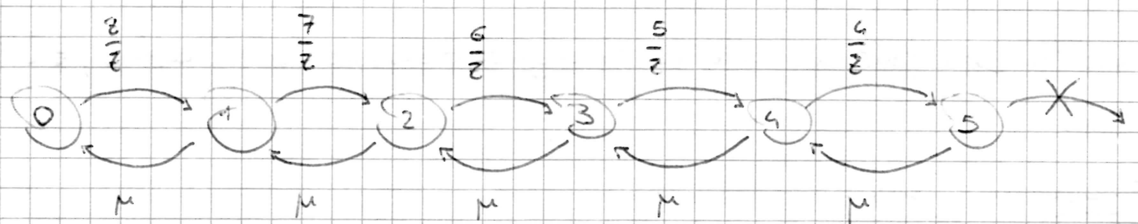
If k are the request subjected to the server ⇒ k users are into the system ⇒ U−k users are in the think state ⇒ the rate at which each of them generate a new request is:

$$\lambda_k = \frac{U-k}{Z}$$

The service rate depends on the n° of server ~~that~~ on which the system is composed as given ~~that~~ by:

$$\mu_k = \begin{cases} k\cdot\mu & 1 \le k \le n \\ n\cdot\mu & n < k \le W \end{cases} \longrightarrow \mu_k = \mu \quad \forall\, k = 0, \dots, W;$$

STD (M/M/1/5/8)



Flow in = Flow out

$$\begin{cases} P_0 \cdot \frac{8}{Z} = P_1 \cdot \mu \\ P_1 \cdot \frac{7}{Z} = P_2 \cdot \mu \\ P_2 \cdot \frac{6}{Z} = P_3 \cdot \mu \\ P_3 \cdot \frac{5}{Z} = P_4 \cdot \mu \\ P_4 \cdot \frac{4}{Z} = P_5 \cdot \mu \\ \sum_{i=0}^{5} P_i = 1 \end{cases}$$

$P_1 = P_0\left(\frac{8}{Z}\cdot\frac{1}{\mu}\right) = P_0\,(0{,}27\cdot 20) = P_0\,(5{,}4)$

$P_2 = P_1\left(\frac{7}{Z}\cdot\frac{1}{\mu}\right) = P_1\,(4{,}6) = P_0\,(5{,}4\cdot 4{,}6) = P_0\,(24{,}84)$

$P_3 = \dots$

$P_0 + P_1 + P_2 + P_3 + P_4 + P_5 = 1 \Rightarrow$

$P_4 = \dots$

$P_5 = \dots \Rightarrow P_0\left[1 + 5{,}4 + 24{,}84 + \dots\right] = 1 \Rightarrow P_0 = \frac{1}{(1 + 5{,}4 + 24{,}84)}$

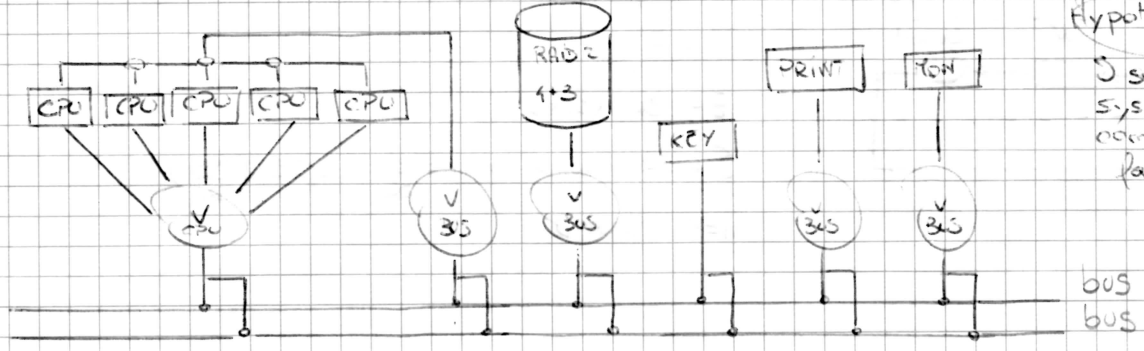
$$\bar{X} = \sum_{i=1}^{W} P_i \cdot \mu = (P_1 + P_2 + P_3 + P_4 + P_5)\,\mu$$

$$N = \sum_{i=0}^{W} P_i \cdot i = P_1 + 2P_2 + 3P_3 + 4P_4 + 5P_5$$

$R = \frac{N}{\bar{X}}$ (From Little's Law)

Exercise 1

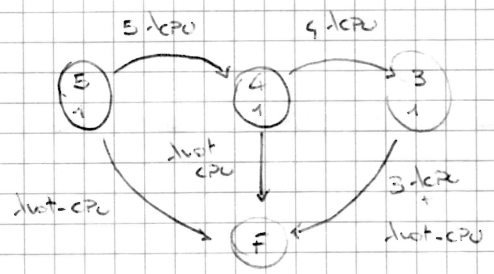
$\log_2 4 = 3 \Rightarrow 4+3$!



Hypothesis:
I switch off the system when a component is faulty!

RELIABILITY

- CPU + VOTER CPU

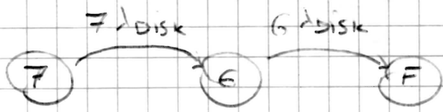


$$\begin{cases} P'_{5,1}(t) = -P_{5,1}(t)\,(5\lambda_{cpu} + \lambda_{vot\text{-}cpu}) \\ P'_{4,1}(t) = -P_{4,1}(t)\,(4\lambda_{cpu} + \lambda_{vot\text{-}cpu}) + P_{5,1}(t)\,(5\lambda_{cpu}) \\ P'_{3,1}(t) = -P_{3,1}(t)\,(3\lambda_{cpu} + \lambda_{vot\text{-}cpu}) + P_{4,1}(t)\,(4\lambda_{cpu}) \\ P'_F(t) = +[P_{5,1}(t) + P_{4,1}(t) + P_{3,1}(t)(3\lambda_{cpu})]\,\lambda_{vot\text{-}cpu} \\ P_{5,1}(t) + P_{4,1}(t) + P_{3,1}(t) + P_F(t) = 1 \\ P_{5,1}(0) = 1 \end{cases}$$

$$R_{CPU}(t) = 1 - P_F(t)$$

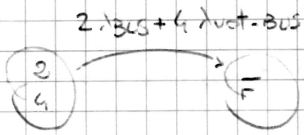
- RAID

Since I have the same fault rate either for the original disks and for the copies, I consider them together!



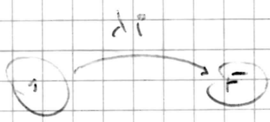
$$R_{RAID}(t) = 1 - P_F(t)$$

- BUS + VOTER BUS



$$R_{BUS}(t) = 1 - P_F(t)$$

- MONITOR / KEYBOAR / PRINTER

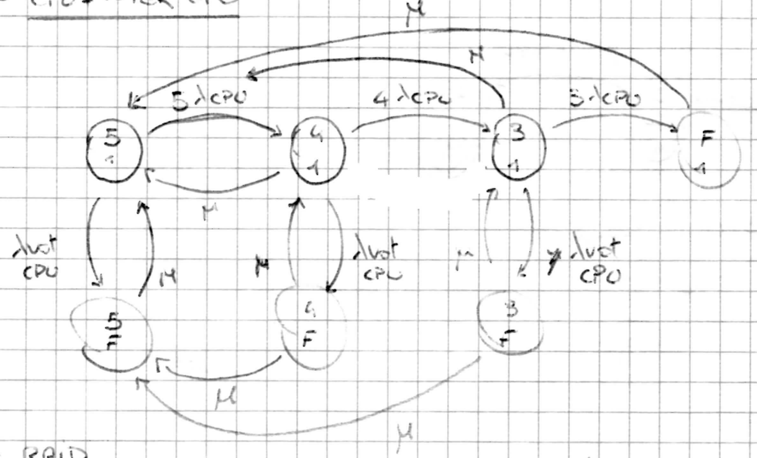


$$R_i(t) = 1 - P_F(t)$$

$$R_{tot}(t) = R_{CPU}(t) \cdot R_{RAID}(t) \cdot R_{BUS}(t) \cdot R_{Pri}(t) \cdot R_{Key}(t) \cdot R_{Mon}(t)$$

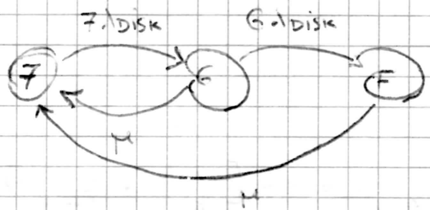
AVAILABILITY

- CPU + VOTER CPU



$$A_{CPU}(t) = 1 - \left[P_{5,F}(t) + P_{4,F}(t) + P_{3,F}(t) + P_{F,1}(t) \right]$$

- RAID



$$P'_7(t) = 0 = -P_7(t)(7\lambda_{DISK}) + \mu\left(P_6(t) + P_F(t)\right)$$

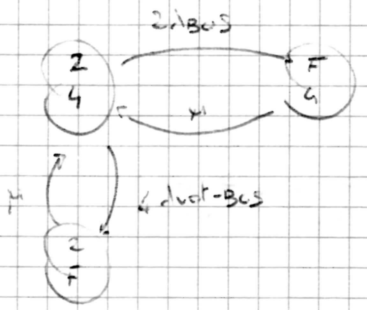
$$P'_6(t) = 0 = +P_7(t)(7\lambda_{DISK}) - P_6(t)(6\lambda_{DISK} + \mu)$$

$$P'_F(t) = 0 = P_6(t)(6\lambda_{DISK}) - P_F(t)\,\mu$$

$$P_7(t) + P_6(t) + P_F(t) = 1; \quad P_7(0) = 1;$$

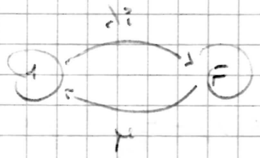
$$A_{RAID}(t) = 1 - P_F(t)$$

- BUS + VOTER BUS



$$A_{BUS}(t) = 1 - \left[P_{F,4}(t) + P_{2,F}(t) \right]$$

- MONITOR/KEYBOARD/PRINTER

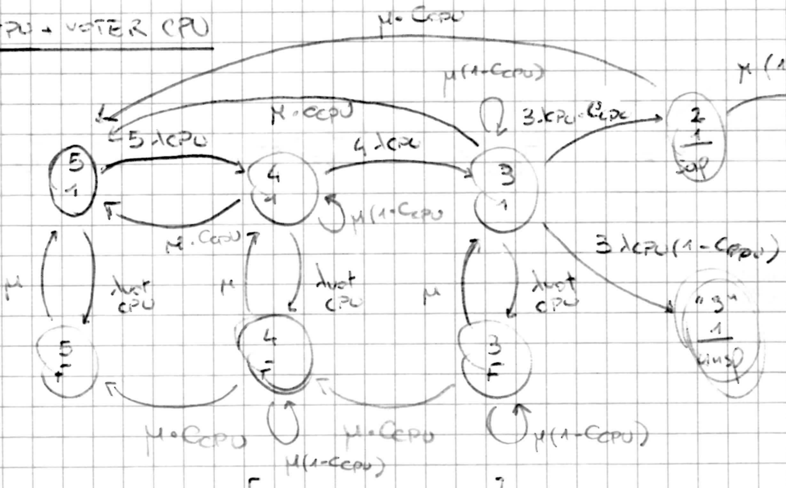


$$A_i(t) = 1 - P_F(t)$$

$$A_{tot}(t) = A_{CPU}(t) \cdot A_{RAID}(t) \cdot A_{BUS}(t) \cdot A_{Pri}(t) \cdot A_{Mon}(t) \cdot A_{Key}(t)$$

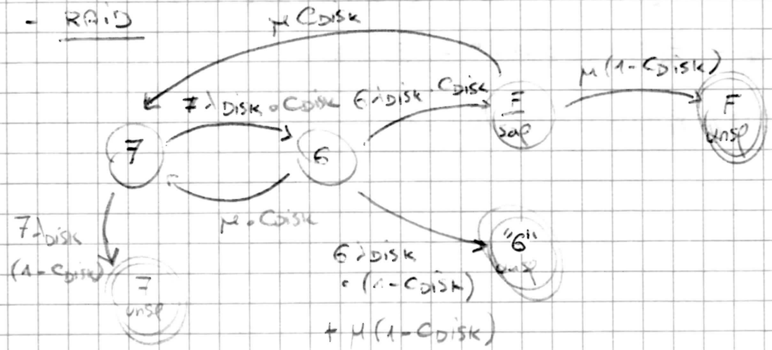
SAFETY

- CPU + VOTER CPU



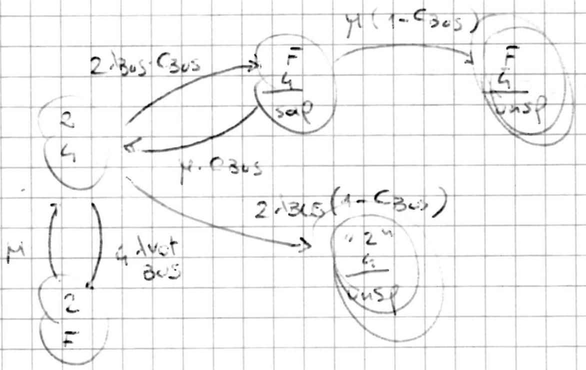
$$S_{CPU}(t) = 1 - \left[P_{"3",1}(t) + P_{\substack{2,1\\unsp}}(t) \right]$$

- RAID



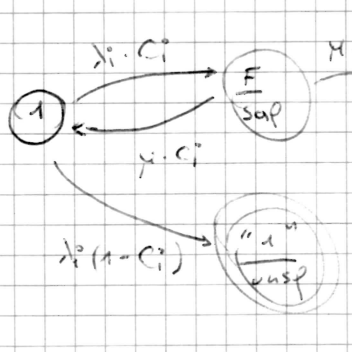
$$S_{RAID}(t) = 1 - \left[P_{"6"}(t) + P_{F,unsp}(t) \right]$$

- BUS + VOTER BUS



$$S_{BUS}(t) = 1 - \left[P_{"2",4}(t) + P_{F,4,unsp}(t) \right]$$

- MONITOR/PRINTER/KEYBOARD



$$\begin{cases} P_1'(t) = -P_1(t)\left(\lambda_i C_i + \lambda_i(1-C_i)\right) + P_{F,sp}(t)(\mu C_i) \\ P_{F,sp}'(t) = -P_{F,sp}(t)\left[\mu\left[C_i + (1-C_i)\right]\right] + P_1(t)\lambda_i C_i \\ P_{"1"}'(t) = +P_1(t)\lambda_i(1-C_i) \\ P_{F,unsp}'(t) = +P_{F,sp}(t)\left[\mu(1-C_i)\right] \\ P_1(t) + P_{F,sp}(t) + P_{F,usp}(t) + P_{"1"}(t) = 1; \quad P_1(0) = 1 \end{cases}$$

$$S_i(t) = 1 - \left[P_{"1"}(t) + P_{F,unsp}(t) \right]$$

$$S_{tot}(t) = S_{CPU}(t) \cdot S_{RAID}(t) \cdot S_{BUS}(t) \cdot S_{PRI}(t) \cdot S_{MON}(t) \cdot S_{KEY}(t).$$