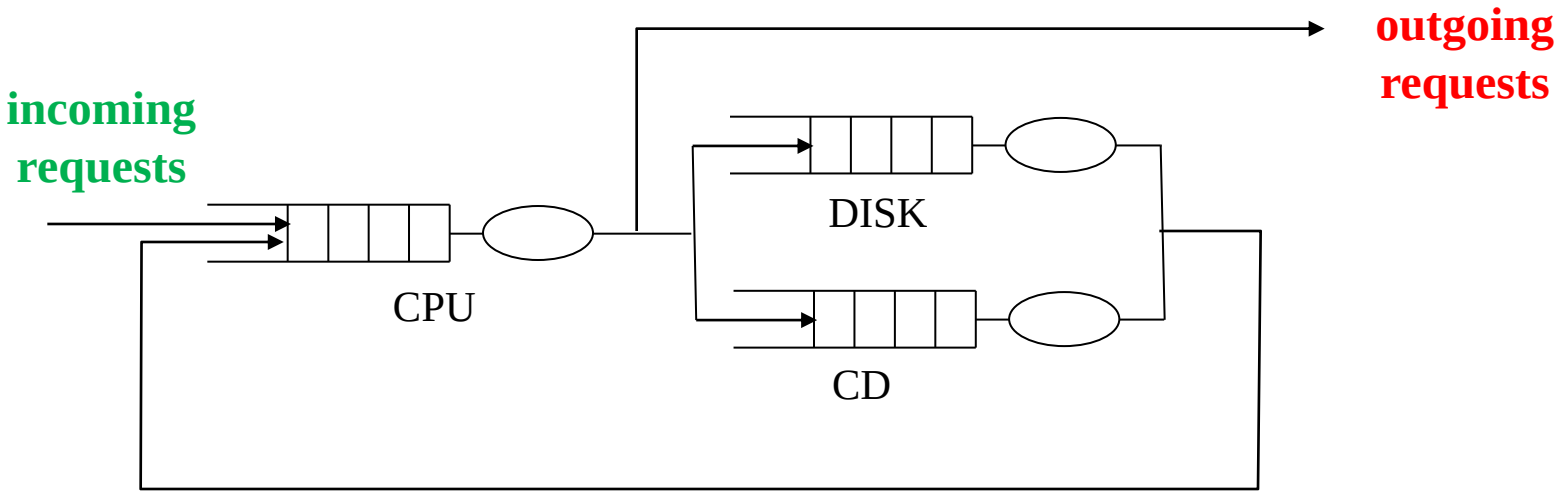
Multiple class queueing networks
Mean Value Analysis

- Open queueing networks

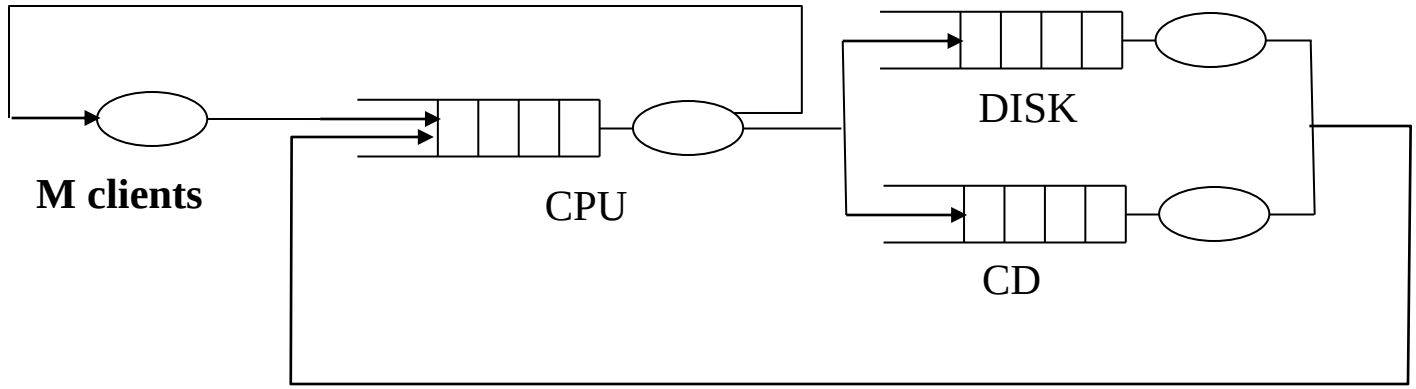
- Closed queueing networks

Open queueing network



Closed queueing network

(finite number of users)



Incoming request class

Different kind of requests should be in a system (queueing network) that need different services by the servers, i.e:

- a database server is subject to two type of transactions:
  - simple query (that needs only read activities on the disks)
  - updating transactions (that needs read and write activities on the disks)
- a web server is subject to two type of requests:
  - Read of a little file
  - Uploading of a big file

Definitions

K: number of queues

i: queue identification

r: class identification (from 1 to R)

λ_r**:** arrival rate for class r request

$\boldsymbol{\lambda = (\lambda_1, \lambda_2, ..., \lambda_R)}$

$V_{i,r}$: average number of visits a class **r** request makes to server ***i*** from its generation to its completion (request goes out from the system if open network)

Definitions

$\mathbf{S_{i,r}}$: average class **r** request service time at the server ***i***

$\mathbf{W_{i,r}}$: average class **r** request waiting time in the queue *i*

$\mathbf{R_{i,r}}$: average class **r** request response time in the queue *i*

$$\mathbf{R_{i,r} = S_{i,r} + W_{i,r}}$$

Definitions

$\mathbf{R'_{i,r}}$: average class **r** request residence time in the queue ***i*** from its creation to its service completion time (request goes out from the system in case of open network)

$$\mathbf{R'_{i,r} = V_{i,r} R_{i,r}}$$

$\mathbf{D_{i,r}}$: request class **r** service demand to a server in a queue *i* from its creation to its service completion time (request goes out from the system in case of open network)

$$\mathbf{D_{i,r} = V_{i,r} S_{i,r}}$$

Formulas for multiple class open QNs

Input parameters

$D_{i,r}$, λ_r

Equations

- $U_{i,r}(\lambda) = \lambda_r V_{i,r} S_{i,r} = \lambda_r D_{i,r}$
- $U_i(\lambda) = \Sigma^{R}_{r=1} U_{i,r}(\lambda)$ **total utilization factor**
- $R'_{i,r}(\lambda) = D_{i,r}$ **delaying resource**

  $R'_{i,r}(\lambda) = D_{i,r} / (1 - U_i(\lambda))$ **queuing resource**

Formulas for multiple class open QNs

- $R_{0,r}(\lambda) = \Sigma^{K}_{i=1} R'_{i,r}(\lambda)$

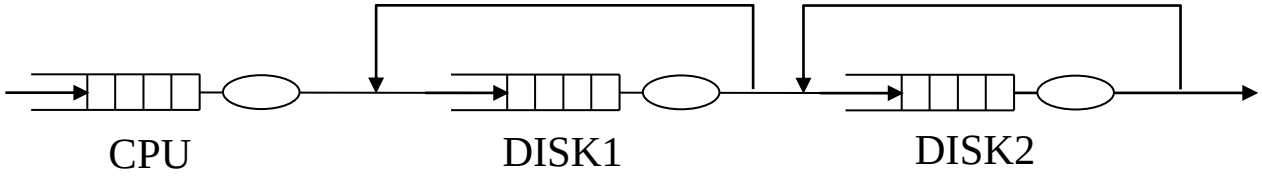
- $n_{i,r}(\lambda) = U_{i,r}(\lambda) / (1- U_i(\lambda))$

- $n_i(\lambda) = \Sigma^{R}_{r=1} n_{i,r}(\lambda)$

NOTE: total utilization in the denominator

DB Server

(example 9.5)



Class 1 trx: query

λ_1 = 5 requests per second (tps)

D_{CPU} = 0,1 sec — Service demand at CPU
D_{DISK1} = 0.08 — Service demand at disk 1
D_{DISK2} = 0.07 — Service demand at disk 2

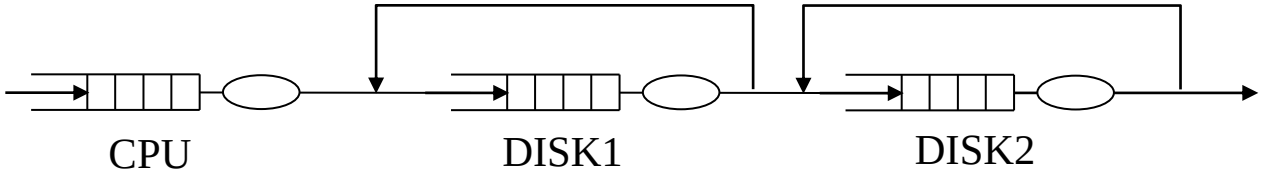
Class 1 trx: updating trx

λ_1 = 2 requests per second (tps)

D_{CPU} = 0,15 sec — Service demand at CPU
D_{DISK1} = 0.20 — Service demand at disk 1
D_{DISK2} = 0.10 — Service demand at disk 2

DB Server

(example)

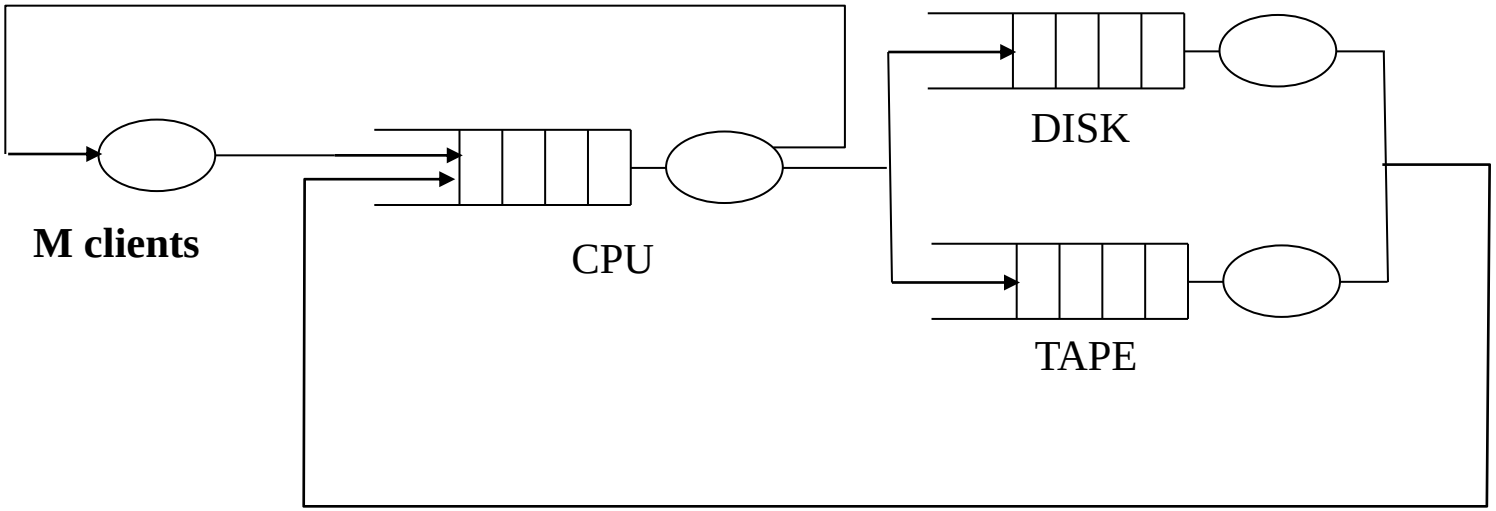


Service demand x	**Query**	**Updates**
• **CPU**	0,1	0,15
• **DISK1**	0,08	0,20
• **DISK2**	0,07	0,10

Utilizations (%)		
CPU	50	30
Disk1	40	40
Disk 2	35	20
Residence times (sec)		
CPU	0,50	0.75
Disk1	0,40	1,00
Disk 2	0,016	0,22
Response times (sec)	1,06	1,97

Multiclass closed queue networks

(finite number of users)



Notations

N_r : fixed number of requests in the system for each class (r)

$\boldsymbol{N}$: $(N_1, N_2, \ldots, N_R)$

$\boldsymbol{1}_r$: vector where all components are zero except for the r-th component, which is equal to 1

Formulas

-> **Residence Time Equation for class r**

$$R'_{i,r}(\boldsymbol{N}) = D_{i,r}[1 + n_i(\boldsymbol{N} - \boldsymbol{1_r})]$$

-> **Throughput equation for class r**

$$X_{0,r} = N_r \,/\, \Sigma^{K}_{r=1} R'_{i,r}(\boldsymbol{N})$$

-> **Queue lenght equation for class r**

$$n_{i,r}(N) = X_{0,r}(N)\, R'_{i,r}$$

-> **Queue equation**

$$n_i(\boldsymbol{N}) = \Sigma^{R}_{r=1} n_{i,r}(\boldsymbol{N})$$

Example with 2 classes

Residence Time Equation for class r

$$R'_{i,r}(\boldsymbol{N}) = D_{i,r}[1 + n_i(\boldsymbol{N} - \mathbf{1}_{\boldsymbol{r}})]$$

for example, to evaluate the formulas, when the state is N=(3,4), i.e. 3 customers of class 1 and 4 customers of class 2, we need to know:

- the average number of users in queue i when there are 2 customers of class 1 and 4 customer of class 2
- the average number of users in queue i when there are 3 customers of class 1 and 4 customer of class 3

$$R'_{i,1}(\boldsymbol{3,4}) = D_{i,r}[1 + n_i(\boldsymbol{2,4})]$$

$$R'_{i,2}(\boldsymbol{3,4}) = D_{i,r}[1 + n_i(\boldsymbol{3,3})]$$

A markov process with 2 classes and 2 users x class