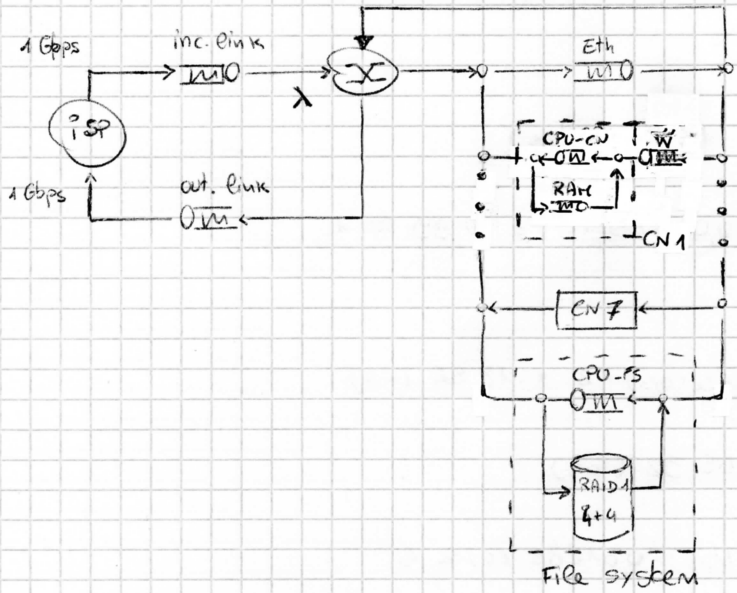
Exam 11.06.2014.



DATA:

$B_{eth} = 1$ Gbps $= B_{line}$;

$L = 200$ (μsec/pkt)

$\lambda = 100$ (req/sec)

$R = 400$ (byte)

$F = 80$ (Kbyte)

$= 80 \cdot 1024 = 81'920$ (byte)

$W = 4$ (users)

$P_{hit} = 30\%$ | $P_{miss} = 70\%$

$D^{hit}_{CPU-CN} = 10$ (ms) | $D^{miss}_{CPU-CN} = 30$ (ms)

$D_{CPU-FS} = 5$ (ms)

$S_{DISK} = 10$ (ms · 10 Kbyte)

1) Average response time in case of faulty-free.

First, I calculate the service demand according to the $P_{hit/miss}$:

$$D_{CPU-CN} = \frac{P^{hit}[D^{hit}_{CPU-CN} \cdot V^{hit}_{CPU-CN}] + P^{miss}[D^{miss}_{CPU-CN} \cdot V^{miss}_{CPU-CN}]}{7} =$$

because I have to store the file into the RAM, so I pass twice in CPU-CN!

$$= \frac{0{,}3[10\,(ms) \cdot 1] + 0{,}7[30\,(ms) \cdot 2]}{7} = \frac{3 + 42}{7}\,(ms) = 6{,}43\,(ms)$$

1 time for seeing that there isn't the file
2 time for storing the file in RAM

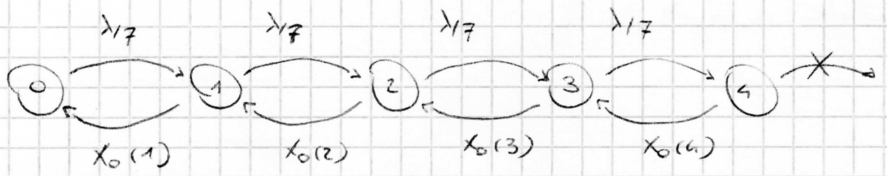
$$D_{CPU-FS} = \emptyset + P^{miss}[D_{CPU-FS} \cdot V_{CPU-FS}] = 0{,}7[5\,(ms) \cdot 1] = 3{,}5\,(ms)$$

because $F = 80$ Kbyte $\Rightarrow 10$ (ms · 10 Kbyte) · 8 = 80 (ms · 80 K byte)

$$D_{DISK} = \frac{\emptyset + P^{miss}[D_{DISK} \cdot V_{DISK}]}{8} = \frac{0{,}7[10\,(ms) \cdot 8]}{8} = 7\,(ms)$$

Now, according to this value, I apply the MVA at the single computing node seen that it has a finite queue for the users. It's a closed network!

STD:



Each state represent the n° of users in the CPU-CN's queue.

The arrival rate is indipendent of this number and, according to the load balancer, is divided for 7.

The throughput instead depends on the states, and for calculating it, I have to use the MVA.

$\boxed{n=1}$

$R'_{CPU-CN}(1) = D_{CPU-CN} = 6{,}43\ (ms)$

$X_0(1) = 1 / R'_{CPU-CN}(1) = 1 / 6{,}43 \cdot 10^{-3}\ (s) = 155{,}52\ (tps)$

$n_{CPU-CN}(1) = X_0(1) \cdot R'_{CPU-CN}(1) = 155{,}52\ (tps) \cdot 0{,}00643\ (s) = 0{,}99 \approx 1$

$\boxed{n=2}$

$R'_{CPU-CN}(2) = D_{CPU-CN}[1 + n_{CPU-CN}(1)] = 6{,}43\ (ms) \cdot 2 = 12{,}86\ (ms)$

$X_0(2) = 2 / R'_{CPU-CN}(2) = 2 / 12{,}86 \cdot 10^{-3}\ (s) = 155{,}52\ (tps)$

$n_{CPU-CN}(2) = X_0(2) \cdot R'_{CPU-CN}(2) = 155{,}52\ (tps) \cdot 0{,}01286\ (s) \approx 2$

$\boxed{n=3}$

$R'_{CPU-CN}(3) = D_{CPU-CN}[1 + n_{CPU-CN}(2)] = 6{,}43\ (ms) \cdot 3 = 19{,}29\ (ms)$

$X_0(3) = 3 / R'_{CPU-CN}(3) = 3 / 19{,}29 \cdot 10^{-3} = 155{,}52\ (tps)$

$n_{CPU-CN}(3) = X_0(3) \cdot R'_{CPU-CN}(3) = 155{,}52 \cdot 0{,}01929 \approx 3$

$\boxed{n=4}$

$R'_{CPU-CN}(4) = D_{CPU-CN}[1 + n_{CPU-CN}(3)] = 6{,}43 \cdot 4 = 25{,}72$

$X_0(4) = 4 / R'_{CPU-CN}(4) = 4 / 25{,}72 \cdot 10^{-3} = 155{,}52\ (tps)$

Now, I apply the flow-in = flow-out principle:

$$\begin{cases} p_0 \cdot \frac{\lambda}{7} = p_1 \cdot X_0(1) \\ p_1 \cdot \frac{\lambda}{7} = p_2 \cdot X_0(2) \\ p_2 \cdot \frac{\lambda}{7} = p_3 \cdot X_0(3) \\ p_3 \cdot \frac{\lambda}{7} = p_4 \cdot X_0(4) \\ \sum_{i=0}^{4} p_i = 1 \end{cases}$$

$\rightsquigarrow$ calculate p_0, p_1, p_2, p_3, p_4

Finally, I calculate for the single computing node:

$X(0) = \sum_{i=1}^{4} p_i \cdot X_0(i)$; (average throughput)

$N(0) = \sum_{i=0}^{4} i \cdot p_i$; (average n° of users)

$R'(0) = \frac{N(0)}{X(0)}$; (average residence time) $\rightarrow R'_{CPU-CN}$

$\hookrightarrow$ fault-free

The others components are open network, so for them I apply the open network formulas.
But first, I've to calculate the maximum arrival rate:

$$X_0 = \frac{1}{\max D_i} = \frac{1}{D_{disk}} = \frac{1}{0{,}007\ (s)} = 142{,}86\ \left(\frac{req}{sec}\right)$$

Beyond this value, the system saturates.
For the steady state condition, $X_0 \overset{!}{=} \lambda = 100\ (req/sec)$
So:

$$U_{CPU-FS} = X_0 \cdot D_{CPU-FS} = 100\left(\frac{req}{sec}\right) \cdot 0{,}0035\ (s) = 0{,}35\ (35\%)$$

$$U_{disk} = X_0 \cdot D_{disk} = 100 \cdot 0{,}007 = 0{,}7\ (7\%)$$

$$R'_{CPU-FS} = \frac{D_{CPU-FS}}{1 - U_{CPU-FS}} = \frac{3{,}5\ (ms)}{0{,}65} = 1{,}54\ (ms)$$

$$R'_{disk} = \frac{D_{disk}}{1 - U_{disk}} = \frac{7\ (ms)}{0{,}3} = 23{,}33\ (ms)$$

About the networks, we have: {Hypothesis: The TCP layer doesn't know the Eth MTU!}

$\boxed{\text{REQUEST}}$

$$\#segment = \left\lceil \frac{R}{MTU_{TCP}} \right\rceil = \left\lceil \frac{400}{65'515} \right\rceil = 1$$

$$\#datagram = \left\lceil \frac{R + \#Seg\,(TCP_{ovhd})}{MSS} \right\rceil = \left\lceil \frac{400 + 1(20)}{1480} \right\rceil = 1 \quad \{MSS = MTU_{ETH} - IP_{ovhd} = 1500 - 20\}$$

$$\#frame = \#datagram$$

$$Overhead = \#segment\,(TCP_{ovhd}) + \#datagram\,(IP_{ovhd} + Eth_{ovhd}) = (1 \cdot 20) + 1\,(20 + 18) = 58\ (byte)$$

$$S^{req}_{eth} = \frac{(R + Overhead)\,8}{B_{eth}} = \frac{(458)\,8\ (bit)}{1 \cdot 10^9\ \left(\frac{bit}{sec}\right)} = 3'664 \cdot 10^{-9}\ (sec) = 0{,}0037\ (ms)$$

$$= S^{req}_{line}$$

$$S^{req}_{rout} = \#datagram \cdot L = 1\ (pkt) \cdot 200\ \left(\frac{\mu sec}{pkt}\right) = 0{,}2\ (ms)$$

RESPONSE

$$\#segment = \left\lceil \frac{F}{MTU_{TCP}} \right\rceil = \left\lceil \frac{81'920}{65'515} \right\rceil = 2;$$

$$\#datagram = \#frame = \left\lceil \frac{F + \#segment\,(TCP_{ovhd})}{MSS} \right\rceil = \left\lceil \frac{81'920 + (2 \cdot 20)}{1480} \right\rceil = 56;$$

$$Overhead = \#segment\,(TCP_{ovhd}) + \#datagram\,(IP_{ovhd} + Eth_{ovhd}) =$$

$$= 2(20) + 56(20+18) = 2168 \text{ (byte)}$$

$$S_{line}^{resp} = \frac{(F + Overhead)\,8}{B_{line}} = S_{eth}^{resp} = \frac{(81'920 + 2168)\,8}{1 \cdot 10^9} = 0{,}67 \text{ (ms)}$$

$$S_{rout}^{resp} = \#datagram \cdot L = 11{,}2 \text{ (ms)}$$

HANDSHAKING

$$\#segment = \#datagram = \#frame = 6$$

$$Overhead = \#frame\,(TCP_{ovhd} + IP_{ovhd} + Eth_{ovhd}) = 6(20+20+18) = 348 \text{ (byte)}$$

$$S_{line}^{hs} = S_{eth}^{hs} = \frac{(\phi + Overhead)\,8}{1 \cdot 10^9} = 0{,}0028 \text{ (ms)}$$

$$S_{rout}^{hs} = 6 \cdot 200 \text{ (}\mu s/pkt\text{)} = 1{,}2 \text{ (ms)}$$

Then:

$$D_{line} = (S_{line}^{req} + S_{line}^{resp} + S_{line}^{hs}) \cdot V_{line} = (0{,}0037 + 0{,}67 + 0{,}0028) \cdot 1 = 0{,}6765 \text{ (ms)}$$

$$D_{eth} = P_{hit}\left[(S_{eth}^{hs} + S_{eth}^{req} + S_{eth}^{resp}) \cdot V_{eth}^{hit}\right] + P_{miss}\left[(S_{eth}^{hs} + S_{eth}^{req} + S_{eth}^{resp}) \cdot V_{eth}^{miss}\right] =$$

because I have to request the piece at the FS

$$= 0{,}3\left[(0{,}6765) \cdot 1\right] + 0{,}7\left[(0{,}6765) \cdot 2\right] = 0{,}20295 + 0{,}9471 = 1{,}15005 \text{ (ms)}$$

$$D_{rout} = (S_{rout}^{hs} + S_{rout}^{req} + S_{rout}^{resp}) \cdot 1 = 12{,}6 \text{ (ms)}$$

$$U_{line} = X_0 \cdot D_{line} = 100\left(\frac{req}{sec}\right) \cdot 0{,}0006765 \text{ (sec)} = 0{,}06765 \approx 0{,}68$$

$$U_{eth} = X_0 \cdot D_{eth} = 100 \cdot 0{,}00115005 = 0{,}115 \approx 0{,}12$$

$$U_{rout} = X_0 \cdot D_{rout} = 100 \cdot 0{,}0126 = 1{,}26 \;?$$

$$R'_{line} = \frac{D_{line}}{1 - U_{line}} = \ldots$$

$$R'_{eth} = \frac{D_{eth}}{1 - U_{eth}} = \ldots$$

$$R'_{rout} = \frac{D_{rout}}{1 - U_{rout}} = \ldots$$

So, the average response time is given by:

$$R = R'_{CPU-CN} + R'_{CPU-FS} + R'_{DISK} + R'_{file} + R'_{rout} + R'_{eth}$$

2) Availability:

I know that: $A_i = \dfrac{MTTF_i}{MTTF_i + MTTR_i}$ for i = CPU-CN, CPU-FS, eth, DISK, router

So, for every possible cases, I have:

$\{7, 4+4\} = (A_{CPU-CN})^7 \cdot (A_{DISK})^8 \cdot A_{CPU-FS} \cdot A_{eth} \cdot A_{rout} \Rightarrow$ fault-free

$\{7, 3+4\} = \quad '' \quad \cdot \{\binom{4}{1}[(A_{DISK})^3(1-A_{DISK})]\}(A_{DISK})^4 \cdot '' \cdot '' \cdot ''$

$\{7, 3+3\} = \quad '' \quad \cdot \{ \quad '' \quad \}\{\binom{4}{1}[A_{DISK}^3(1-A_{DISK})]\} \cdot '' \cdot '' \cdot ''$

$\vdots$

$\{7,$

$\{7, 2+2+2+2\} = (A_{CPU-CN})^7 \cdot (A_{DISK})^2 \cdot A_{CPU-FS} \cdot A_{eth} \cdot A_{rout} \Rightarrow$ fault-free case

$\{7, 1+2+2+2\} = (A_{CPU-CN})^7 \cdot \{2[A_{DISK}(1-A_{DISK})] \cdot (A_{DISK})^6\} \cdot '' \cdot '' \cdot ''$

$\vdots$

$\{7, 1+1+2+2\} = \quad '' \quad \cdot \{4[A_{DISK}(1-A_{DISK})]^2(A_{DISK})^4\} \cdot '' \cdot '' \cdot ''$

$\vdots$

$\{7, 1+1+1+1\} = \quad '' \quad \cdot \{8[A_{DISK}(1-A_{DISK})]^4\} \cdot '' \cdot '' \cdot ''$

$\{6, 2+2+2+2\} = \{\binom{7}{1}[(A_{CPU-CN})^6(1-A_{CPU-CN})] \cdot '' \cdot '' \cdot '' \cdot ''$

$\{5, 2+2+2+2\} = \binom{7}{2}[(A_{CPU-CN})^5(1-A_{CPU-CN})^2] \cdot '' \cdot '' \cdot '' \cdot ''$

$\vdots$

$\{1, 2+2+2+2\} = \binom{7}{6}[A_{CPU-CN}(1-A_{CPU-CN})^6] \cdot '' \cdot '' \cdot '' \cdot ''$

$\vdots$

$\forall$ combination

3) Average response time in presence of faults and reparations

- $\{7, 1+2+2+2\} = \{7, 2+1+2+2\} = \{7, 2+2+1+2\} = \{7, 2+2+2+1\}$:

  $D_{DISK} = \dfrac{P_{miss}(D_{DISK} \cdot V_{DISK})}{7} \Rightarrow$ new $U_{DISK} \Rightarrow$ new $R'_{DISK} \Rightarrow$ new R (replacing at the old D_{DISK} the new one).

- $\{7, 1+1+2+2\} = \{7, 1+2+1+2\} = \{7, 1+2+2+1\} = \{7, 2+1+2+1\} = \{7, 2+1+1+2\}$

  $= \{7, 2+2+1+1\}$:

  $D_{DISK} = \dfrac{P_{miss}(D_{DISK} \cdot V_{DISK})}{6} \Rightarrow$ ecc...

  ⋮

- $\{7, 1+1+1+1\}$: $D_{DISK} = \dfrac{P_{miss}(D_{DISK} \cdot V_{DISK})}{4} \Rightarrow$ ecc...

- $\{6, 2+2+2+2\}$: $D_{CPU-CN} = \dfrac{P_{hit}[D_{CPU-CN} \cdot V_{CPU-CN}] + P_{miss}[D_{CPU-CN} \cdot V_{CPU-CN}]}{6} \Rightarrow$ ecc...

  ⋮

- $\{1, 2+2+2+2\}$: $D_{CPU-CN} = \dfrac{\ldots}{1} \Rightarrow$ ecc...

- $\{6, 1+2+2+2\} = \{6, 2+1+2+1\} = \{6, 2+2+1+2\} = \{6, 2+2+2+1\}$:

  $D_{DISK} = \dfrac{P_{miss}(D_{DISK} \cdot V_{DISK})}{7} \Rightarrow$ new $U_{DISK} \Rightarrow$ new R'_{DISK}

  $D_{CPU} = \dfrac{P_{hit}(\ldots) + P_{miss}(\ldots)}{6} \Rightarrow$ new $U_{CPU-CN} \Rightarrow$ new R'_{CPU-CN}

  $\Rightarrow$ new R

  ⋮

$\forall$ combination

The total average response time is given by:

$$\bar{R} = \frac{P\{7, 2+2+2+2\}R + P\{7, 1+2+2+2\}R + \ldots + P\{6, 2+2+2+2\}R + \ldots + P\{1, 1+1+1+1\}R}{1 - P_{fault}}$$

probability to stay in the case ...