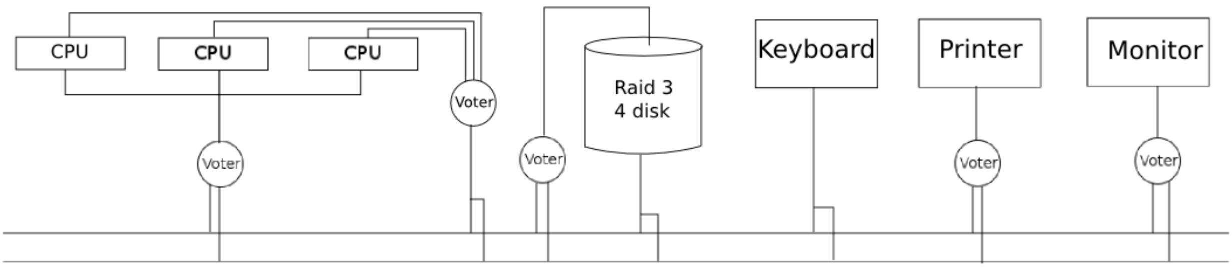
Exercise n.1

Evaluate the reliability, the steady state availability and the safety of a system composed of three CPU (working in parallel and whose output is given by a voter), one RAID 3 system with four disks, two bus system (working in parallel and whose outputs are given by voters), one keyboard, one printer and one monitor, with the hypothesis that the faults happen according to an exponential distribution with rate equal to □CPU, □DISK□BUS□KEY□MON,□PRI, □VOTER-CPU, □VOTER-BUS, with covering factors equal to C_{CPU}, C_{DISK}, C_{BUS}, C_{KEY}, C_{MON}, C_{PRI} (the covering factor for the voters is equal to one) and with the same repair rate (□ for all kind of components. A single "repair technician for component type" is available and the repair rate is independently by the number of fault occurrences of the same type.

<u>N.B. Advice:</u> evaluate each global index analyzing by single sub system.

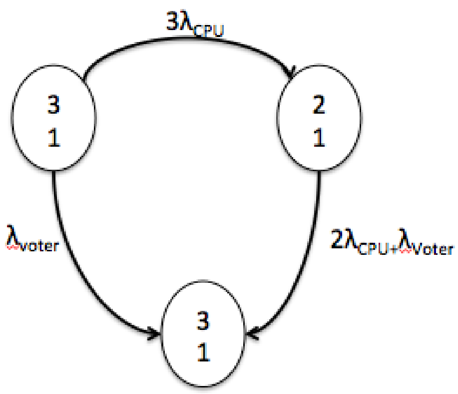
Solution



Hypothesis: the subsystem is turned off when a subsystem goes into fault.

Reliability

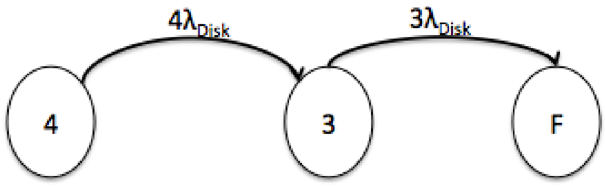
Sub System CPU



$$
\begin{cases}
P'_{3,1}(t) = -(3\lambda_{cpu} + \lambda_{Voter-CPU}) P_{3,1}(t) \\
P'_{2,1}(t) = -(2\lambda_{cpu} + \lambda_{Voter-CPU}) P_{2,1}(t) + 3\lambda_{CPU} P_{3,1}(t) \\
P'_{F}(t) = \lambda_{Voter-CPU} P_{3,1}(t) + (2\lambda_{cpu} + \lambda_{Voter-CPU}) P_{2,1}(t) \\
P_{3,1}(t) + P_{2,1}(t) + P_F(t) = 1 \\
P_{3,1}(0) = 1
\end{cases}
$$

$$R_{SubCpu}(t) = 1 - P_F(t)$$

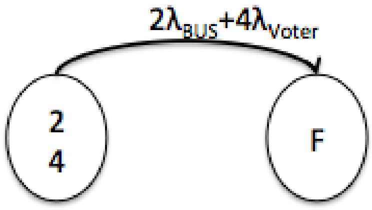
Sub System Raid3



$$\begin{cases} P_4'(t) = -4\lambda_{Disk} P_4(t) \\ P_3'(t) = -3\lambda_{Disk} P_3(t) + 4\lambda_{Disk} P_4(t) \\ P_F'(t) = 3\lambda_{Disk} P_3(t) \\ P_4(t) + P_3(t) + P_F(t) = 1 \\ P_4(0) = 1 \end{cases}$$

$$R_{SubRaid3}(t) = 1 - P_F'(t)$$

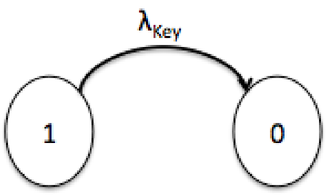
Sub System BUS



$$\begin{cases} P_{2,4}'(t) = -(2\lambda_{BUS} + 4\lambda_{Voter-BUS}) P_{2,4}(t) \\ P_F'(t) = (2\lambda_{BUS} + 4\lambda_{Voter-BUS}) P_{2,4}(t) \\ P_{2,4}(t) + P_F(t) = 1 \\ P_{2,4}(0) = 1 \end{cases}$$

$$R_{SubBUS}(t) = 1 - P_F(t)$$

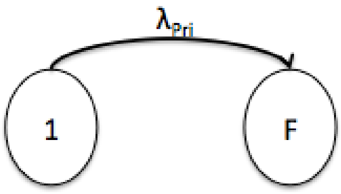
Sub System Keyboard



$$\begin{cases} P_1'(t) = -\lambda_{Key} P_1(t) \\ P_F'(t) = \lambda_{Key} P_1(t) \\ P_1(t) + P_F(t) = 1 \\ P_1(0) = 1 \end{cases}$$

$$R_{SubKey}(t) = 1 - P_F(t)$$

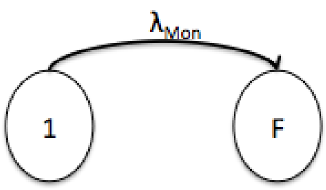
Sub System Printer



$$\begin{cases} P_1'(t) = -\lambda_{Pri} P_1(t) \\ P_F'(t) = \lambda_{Pri} P_1(t) \\ P_1(t) + P_F(t) = 1 \\ P_1(0) = 1 \end{cases}$$

$$R_{SubPri}(t) = 1 - P_F(t)$$

Sub System Monitor



$$\begin{cases} P_1'(t) = -\lambda_{Mon} P_1(t) \\ P_F'(t) = \lambda_{Mon} P_1(t) \\ P_1(t) + P_F(t) = 1 \\ P_1(0) = 1 \end{cases}$$

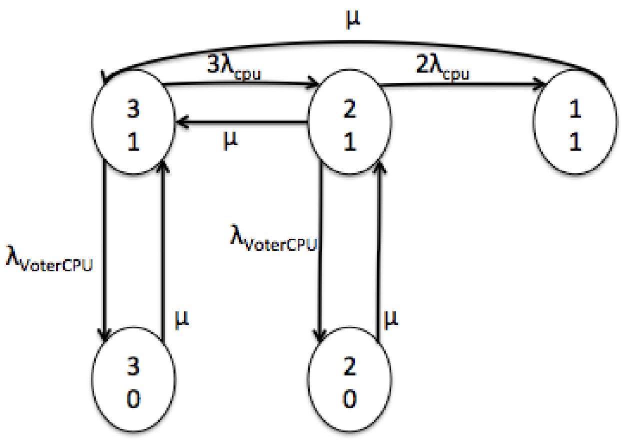
$$R_{SubMon}(t) = 1 - P_F(t)$$

System

$$R_{Tot}(t) = R_{SubCpu}(t) R_{SubRaid3}(t) R_{SubBUS}(t) R_{SubKey}(t) R_{SubPri}(t) R_{SubMon}(t)$$

Avability

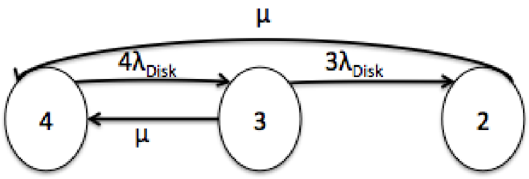
Sub System CPU



$$\begin{cases} P_{3,1}'(t) = -(3\lambda_{CPU} + \lambda_{VoterCPU}) P_{3,1}(t) + \mu(P_{2,1}(t) + P_{1,1}(t) + P_{3,0}(t)) \\ P_{3,0}'(t) = -\mu P_{3,0}(t) + \lambda_{VoterCPU} P_{3,1}(t) \\ P_{2,1}'(t) = -(2\lambda_{CPU} + \lambda_{VoterCPU} + \mu) P_{2,1}(t) + 3\lambda_{CPU} P_{3,1}(t) + \mu P_{2,0}(t) \\ P_{2,0}'(t) = -\mu P_{2,0}(t) + \lambda_{VoterCPU} P_{2,1}(t) \\ P_{1,1}'(t) = -\mu P_{1,1}(t) + 2\lambda_{CPU} P_{2,1}(t) \\ P_{3,1}(t) + P_{3,0}(t) + P_{2,1}(t) + P_{2,0}(t) + P_{1,1}(t) = 1 \\ P_{3,1}(0) = 1 \end{cases}$$

$$A_{SubCPU}(t) = 1 - (P_{3,0}(t) + P_{2,0}(t) + P_{1,1}(t))$$

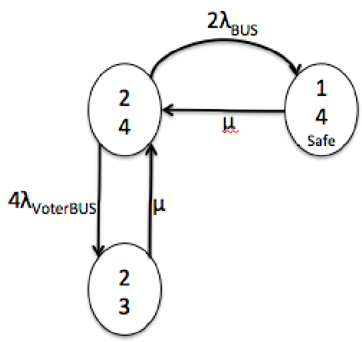
Sub System Raid3



$$
\begin{cases}
P_4'(t) = -4\lambda_{Disk} P_4(t) + \mu\left(P_3(t) + P_2(t)\right) \\
P_3'(t) = -\left(3\lambda_{Disk} + \mu\right) P_3(t) + 4\lambda_{Disk} P_4(t) \\
P_2'(t) = -\mu P_2(t) + 3\lambda_{Disk} P_3(t) \\
P_4(t) + P_3(t) + P_2(t) = 1 \\
P_4(0) = 1
\end{cases}
$$

$$A_{SubRaid3}(t) = 1 - P_2(t)$$

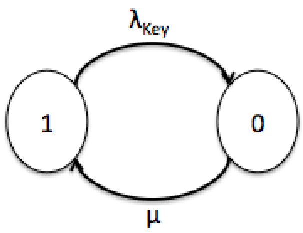
Sub System BUS



$$
\begin{cases}
P_{2,4}'(t) = -\left(2\lambda_{Bus} + 4\lambda_{VoterBus}\right) P_{2,4}(t) + \mu\left(P_{1,4}(t) + P_{2,3}(t)\right) \\
P_{2,3}'(t) = -\mu P_{2,3}(t) + 4\lambda_{VoterBus} P_{2,4}(t) \\
P_{1,4}'(t) = -\mu P_{1,4}(t) + 2\lambda_{Bus} P_{2,4}(t) \\
P_{2,4}(t) + P_{2,3}(t) + P_{1,4}(t) = 1 \\
P_{2,4}(0) = 1
\end{cases}
$$

$$A_{SubBus}(t) = 1 - \left(P_{2,4}(t) + P_{1,4}(t)\right)$$

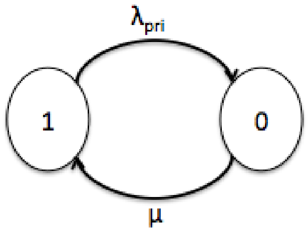
Sub System Keyboard



$$
\begin{cases}
P_1'(t) = -\lambda_{Key} P_1(t) + \mu P_0(t) \\
P_0'(t) = -\mu P_0(t) + \lambda_{Key} P_1(t) \\
P_1(t) + P_0(t) = 1 \\
P_1(0) = 1
\end{cases}
$$

$$A_{SubKey}(t) = 1 - P_0(t)$$

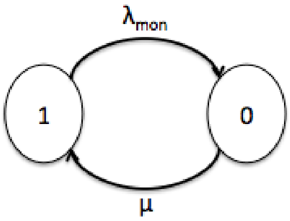
Sub System Printer



$$
\begin{cases}
P_1'(t) = -\lambda_{Pri} P_1(t) + \mu P_0(t) \\
P_0'(t) = -\mu P_0(t) + \lambda_{Pri} P_1(t) \\
P_1(t) + P_0(t) = 1 \\
P_1(0) = 1
\end{cases}
$$

$$A_{SubPri}(t) = 1 - P_0(t)$$

Sub System Monitor



$$\begin{cases} P_1^{'}(t) = -\lambda_{Mon} P_1(t) + \mu P_0(t) \\ P_0^{'}(t) = -\mu P_0(t) + \lambda_{Mon} P_1(t) \\ P_1(t) + P_2(t) = 1 \\ P_1(0) = 1 \end{cases}$$

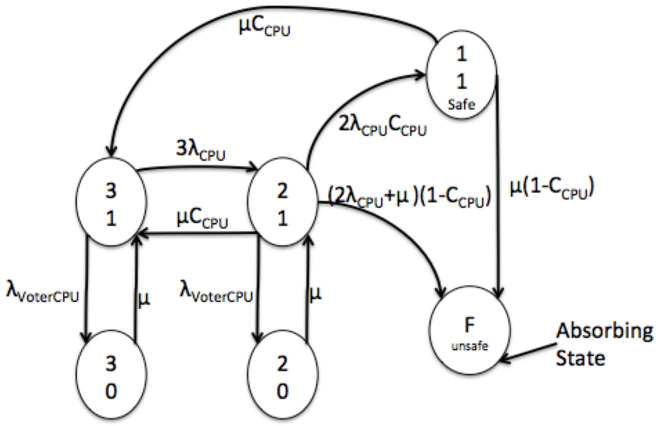
$$A_{¿Mon}(t) = 1 - P_0(t)$$

System

$$A_{Tot}(t) = A_{SubCpu}(t)\, A_{SubRaid3}(t)\, A_{SubBUS}(t)\, A_{SubKey}(t)\, A_{SubPri}(t)\, A_{SubMon}(t)$$

Safety

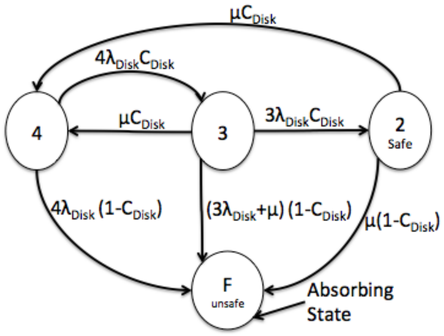
Sub System CPU



$$\begin{cases} P'_{3,1}(t) = -(3\lambda_{CPU} + \lambda_{VoterCPU}) P_{3,1}(t) + \mu C_{CPU} (P_{2,1}(t) + P_{1,1}(t)) + \mu P_{3,0}(t) \\ P'_{3,0}(t) = -\mu P_{3,0}(t) + \lambda_{VoterCPU} P_{3,1}(t) \\ P'_{2,1}(t) = -(\mu + \lambda_{VoterCPU} + 2\lambda_{CPU} C_{CPU} + 2\lambda_{CPU}(1 - C_{CPU})) P_{2,1}(t) + 3\lambda_{CPU} P_{3,1}(t) + \mu P_{2,0}(t) \\ P'_{2,0}(t) = -\mu P_{2,0}(t) + \lambda_{VoterCPU} P_{2,1}(t) \\ P'_{1,1}(t) = -\mu P_{1,1}(t) + 2\lambda_{CPU} C_{CPU} P_{2,1}(t) \\ P'_{F}(t) = (2\lambda_{CPU} + \mu)(1 - C_{CPU}) P_{2,1}(t) + \mu (1 - C_{CPU}) P_{1,1} \\ P_{3,1}(t) + P_{3,0}(t) + P_{2,1}(t) + P_{2,0}(t) + P_{1,1}(t) + P_{F}(t) = 1 \\ P_{3,1}(0) = 1 \end{cases}$$

$$S_{SubCPU}(t) = 1 - P_F(t)$$

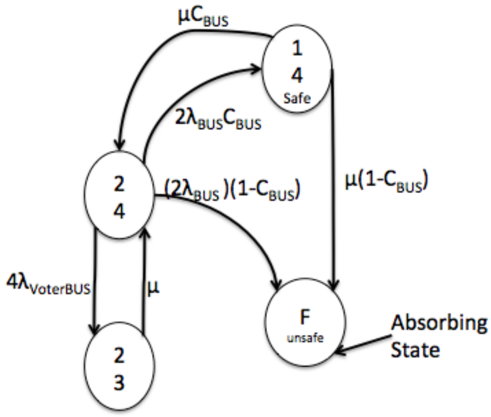
Sub System Raid3



$$\begin{cases} P_4^{'}(t)=-4\lambda_{Disk}P_4(t)+\mu C_{Disk}(P_3(t)+P_2(t)) \\ P_3^{'}(t)=-(\mu+3\lambda_{Disk}C_{Disk}+3\lambda_{Disk}(1-C_{Disk}))P_3(t)+4\lambda_{Disk}C_{Disk}P_4(t) \\ P_2^{'}(t)=-\mu P_2(t)+3\lambda_{Disk}C_{Disk}P_3(t) \\ P_F^{'}(t)=(3\lambda_{Disk}+\mu)(1-C_{Disk})P_3(t)+4\lambda_{Disk}C_{Disk}P_4(t)+\mu(1-C_{Disk})P_2(t) \\ P_4(t)+P_3(t)+P_2(t)+P_F(t)=1 \\ P_4(0)=1 \end{cases}$$

$$S_{SubRaid3}(t)=1-P_F(t)$$

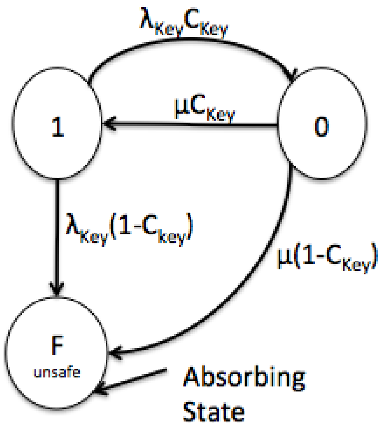
Sub System BUS



$$\begin{cases} P_{2,4}^{'}(t)=-(4\lambda_{VoterBus}+2\lambda_{Bus}C_{Bus}+2\lambda_{Bus}(1-C_{Bus}))P_{2,4}(t)+\mu P_{2,3}(t)+\mu C_{Bus}P_{1,4}(t) \\ P_{2,3}^{'}(t)=-\mu P_{2,3}(t)+4\lambda_{VoterBus}P_{2,4}(t) \\ P_{1,4}^{'}(t)=-\mu P_{1,4}(t)+2\lambda_{Bus}C_{Bus}P_{2,4}(t) \\ P_{\mathrm{F}}^{'}(t)=2\lambda_{Bus}(1-C_{Bus})P_{2,4}(t)+\mu(1-C_{Bus})P_{1,4}(t) \\ P_{2,4}(t)+P_{2,3}(t)+P_{1,4}(t)+P_{\mathrm{F}}(t)=1 \\ P_{2,4}(0)=1 \end{cases}$$

$$S_{SubBUS}(t)=1-P_{\mathrm{F}}(t)$$

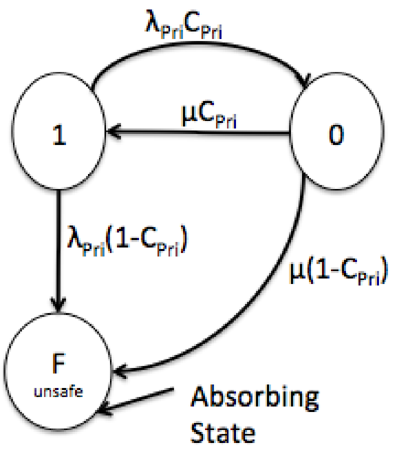
Sub System Keyboard



$$\begin{cases} P_1'(t) = -\left(\lambda_{Key} C_{Key} + \lambda_{Key}\left(1 - C_{Key}\right)\right) P_1(t) + \mu C_{Key} P_0(t) \\ P_0'(t) = -\mu P_0(t) + \lambda_{Key} C_{Key} P_1(t) \\ P_F'(t) = \lambda_{Key}\left(1 - C_{Key}\right) P_1(t) + \mu\left(1 - C_{Key}\right) P_0(t) \\ P_1(t) + P_0(t) + P_1(t) = 1 \\ P_1(0) = 1 \end{cases}$$

$$S_{SubKey}(t) = 1 - P_F(t)$$

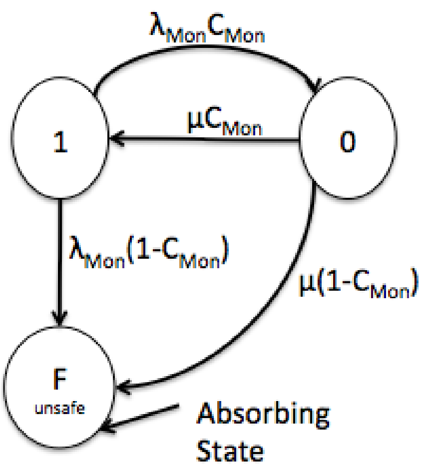
Sub System Printer



$$\begin{cases} P_1'(t) = -\left(\lambda_{Pri} C_{Pri} + \lambda_{Pri}(1 - C_{Pri})\right) P_1(t) + \mu C_{Pri} P_0(t) \\ P_0'(t) = -\mu P_0(t) + \lambda_{Pri} C_{Pri} P_1(t) \\ P_F'(t) = \lambda_{Pri}(1 - C_{Pri}) P_1(t) + \mu (1 - C_{Pri}) P_0(t) \\ P_1(t) + P_0(t) + P_F(t) = 1 \\ P_1(0) = 1 \end{cases}$$

$$S_{SubPri}(t) = 1 - P_F(t)$$

Sub System Monitor



$$\begin{cases} P_1'(t) = -\left(\lambda_{Mon} C_{Mon} + \lambda_{Mon}(1 - C_{Mon})\right) P_1(t) + \mu C_{Mon} P_0(t) \\ P_0'(t) = -\mu P_0(t) + \lambda_{Mon} C_{Mon} P_1(t) \\ P_F'(t) = \lambda_{Mon}(1 - C_{Mon}) P_1(t) + \mu (1 - C_{Mon}) P_0(t) \\ P_1(t) + P_0(t) + P_F(t) = 1 \\ P_1(0) = 1 \end{cases}$$

$$S_{SubMon}(t) = 1 - P_F(t)$$

$$S_{Tot}(t)=S_{SubCPU}(t)\,S_{SubRaid\,3}(t)\,S_{SubBUS}(t)\,S_{SubKey}(t)\,S_{SubPri}(t)\,S_{SubMon}(t)$$

Exercise n.2

Evaluate the service time to transmit a TCP segment (10.000 bytes) over an Ethernet LAN given that: 18 byte (overhead) are for the frame Ethernet header, 1.500 bytes (max data area) and 20 Mbyte/sec (bandwidth).
<u>N.B Advice:</u> draw the frames showing the number of bytes for each field

Solution

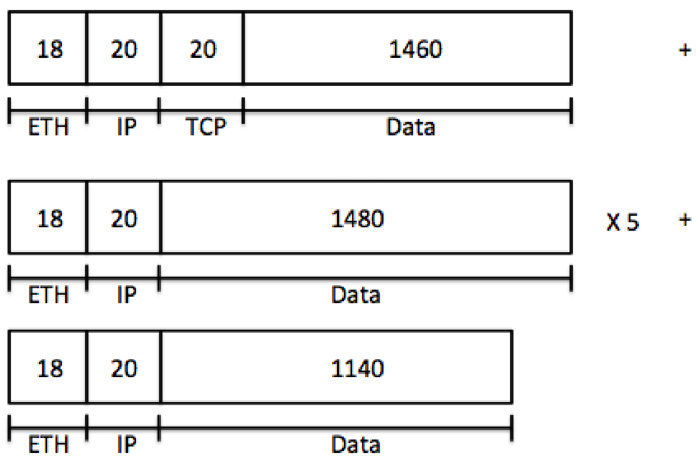
TCP segment has size 65535 and we need only one segment for transmit this segment. This segment is encapsulate in ip datagram, this have 20B for the header and 1480 for the data.
It possible sees than we have more that one datagram. We can calculate this with the following formula:

$$N_{Datagram}=\lceil \frac{¿\,Document+N_{segment}(TCP_{OH})}{min\,MTU-I\,P_{OH}}\rceil$$

In this exercise:

$$N_{Datagram}=\lceil \frac{10000+20}{1500-20}\rceil=\lceil \frac{10020}{1480}\rceil=\lceil 6{,}77\rceil=7$$

All datagrams are encapsulated in Ethernet packet; the size is 1518B, 1500 for data and 18 for



header. It possible sees that one datagram is encapsulated in one packet, so we have 7 packets for transmit the TCP segment.
In the following figure we have a global view of how the segment is fragmented.
Total Overhead:

$$OH=N_{segment}\,TCP_{OVH}+N_{datagram}\,IP_{OVH}+N_{packet}\,ETH_{OVH}$$

In this exercise:

$$OH=1*20+7*20+7*18=20+140+126=286$$

Now it is possible evaluate the Service time:

$$S=\frac{¿\,Document+OH}{Bandwith}$$

In this exercise:

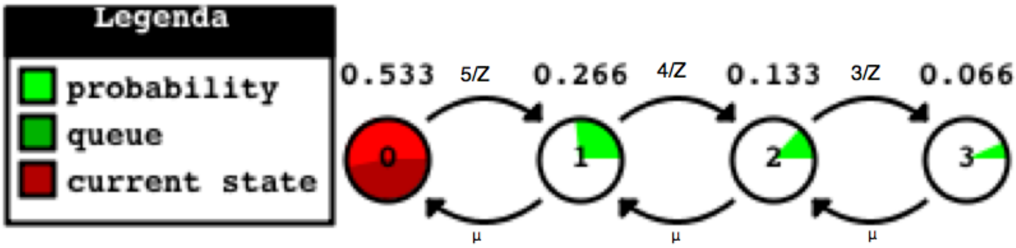
$$S=\frac{10000+286}{10^{6}*20}=514{,}3*10^{-6}\,s=0{,}5\,ms$$

Exercise n.3

Evaluate the average response time and throughput for a system composed of a single server with a finite queue (at most 3 users in the system), with a finite number of users (number of users equal to 5), given that the think time for each user is 100 sec and the service time is 10 sec.

Solution

- M=5 user
- Max number of users in the system =3
- μ=1/10 req/s
- Z=100 s



Throughput:

$$X=\sum_{k=1}^{3}\mu P_k=\frac{0{,}266}{10}+\frac{0{,}133}{10}+\frac{0{,}066}{10}=\frac{0{,}665}{10}=0{,}0665\,req/s$$

Mean Number of users:

$$\acute{N}=\sum_{k=1}^{3}K\,P_k=(1*0{,}266)+(2*0{,}133)+(3*0{,}066)=0{,}266+0{,}266+0{,}198=0{,}730$$

Average response time:

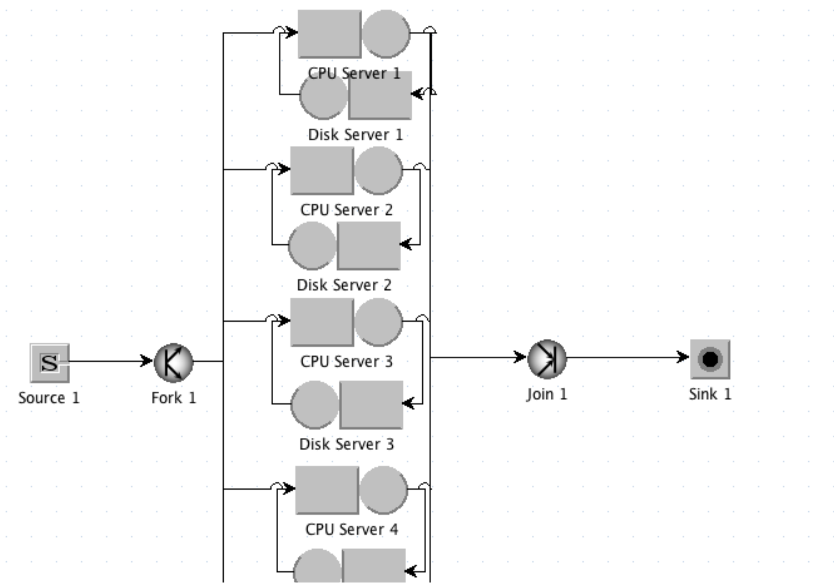
$$R=\frac{\acute{N}}{X}=\frac{0{,}730}{0{,}0665}=10{,}9774\,s$$

Exercise n.4

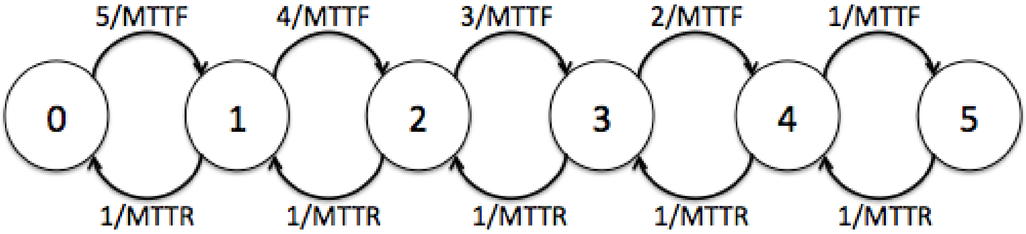
A Web site receives 25 requests per second. These requests are served by a cluster of 5 identical servers. A workload balancer divides in equal parts the load among the servers. Every request needs 20 ms of CPU and 10 I/O requests to a disk, the time for each I/O is 5 ms. Every server can manage at most 3 users at the same time and has a MTTF equal to 1.000 hours and a MTTR equal to 10 hours.
Calculate the average service time, the average throughput and the percentage of requests refused.
N.B Advice: A time shown the methodology you have to evaluate at least the average service time the average throughput and the percentage of requests refused when all the servers are fault-free.

Solution

- λ=25 req/s
- W=3
- D_{cpu}= 10ms
- D_{Disk} =10*5ms=50ms
- MTTF=1000 hours
- MTTR=10 hours



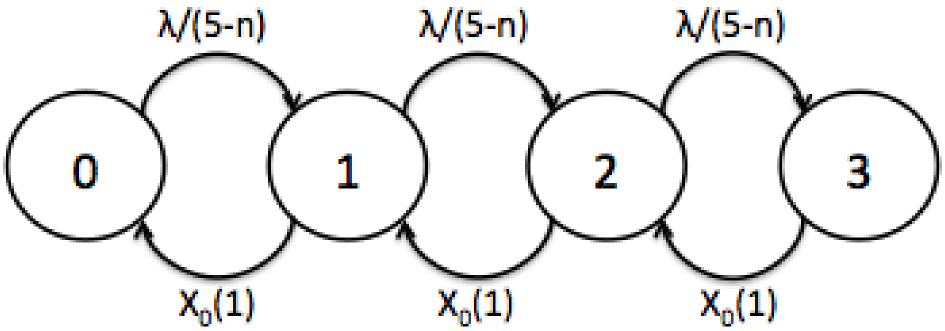
Number of Faulty Server



Flow-in=Flow-out

$$\begin{cases} P_0\frac{5}{1000}=P_1\frac{1}{10} \\ P_1\frac{4}{1000}=P_2\frac{1}{10} \\ P_2\frac{3}{1000}=P_3\frac{1}{10} \\ P_3\frac{2}{1000}=P_4\frac{1}{10} \\ P_4\frac{1}{1000}=P_5\frac{1}{10} \\ P_0+P_1+P_2+P_3+P_4+P_5=1 \end{cases} = \begin{cases} P_0=0{,}9505150352 \\ P_1=0{,}0475257517 \\ P_2=0{,}0019010300 \\ P_3=0{,}0000570309 \\ P_4=0{,}0000011406 \\ P_5=0{,}0000000114 \end{cases}$$

Single Server scenario with n faulty server



Use Mean Value Analysis for Single class:
there are three equations:

- Residence Time equation:
  $$R_i'=D_i[1+n_i(n-1)]$$
- Throughput equation:
  $$X_0=n/\sum_{i=1}^{K}R_i'(n)$$
- Queue lenght equation:
  $$n_i(n)=X_0(n)R_i'(n)$$

$$\left|\begin{array}{c} R'_{CPU}(1)=10\,ms \\ R'_{Disk}(1)=50\,ms \\ R_0(1)=60\,ms \\ X_0(1)=1/60\,ms=16{,}7\,tx/s \\ n_{CPU}(1)=0{,}167 \\ n_{Disk}(1)=0{,}835 \end{array}\right|\begin{array}{c} R'_{CPU}(2)=11{,}67\,ms \\ R'_{Disk}(2)=91{,}75\,ms \\ R_0(2)=103{,}42\,ms \\ X_0(2)=2/103{,}42\,ms=19{,}3\,tx/s \\ n_{CPU}(2)=0{,}225 \\ n_{Disk}(2)=1{,}77 \end{array}$$

$$\left| \begin{array}{l} R'_{CPU}(3)=12{,}25\,ms \\ R'_{Disk}(3)=138{,}5\,ms \\ R_0(3)=150{,}75\,ms \\ X_0(3)=3/150{,}75\,ms=19{,}9\,tx/s \end{array} \right.$$

Flow in= Flow out

$$\begin{cases} q_0^n \dfrac{\lambda}{5-n} = q_1^n X_0(1) \\ q_1^n \dfrac{\lambda}{5-n} = q_2^n X_0(2) \\ q_2^n \dfrac{\lambda}{5-n} = q_3^n X_0(3) \\ q_0^n + q_1^n + q_2^n + q_3^n = 1 \end{cases}$$

$$X^n = \sum_{j=1}^{3} q_j^n X_0(j)$$

$$N^n = \sum_{j=1}^{3} j\, q_j^n$$

$$R^n = \frac{N^n}{X^n}$$

For n=0 we have:

$$\begin{cases} q_0^0 \dfrac{\lambda}{5} = q_1^0 X_0(1) \\ q_1^0 \dfrac{\lambda}{5} = q_2^0 X_0(2) \\ q_2^0 \dfrac{\lambda}{5} = q_3^0 X_0(3) \\ q_0^0 + q_1^0 + q_2^0 + q_3^0 = 1 \end{cases} \quad \begin{cases} q_0^0 \dfrac{25}{5} = q_1^0\, 16{,}7 \\ q_1^0 \dfrac{25}{5} = q_2^0\, 19{,}3 \\ q_2^0 \dfrac{25}{5} = q_3^0\, 19{,}9 \\ q_0^0 + q_1^0 + q_2^0 + q_3^0 = 1 \end{cases} \quad \begin{cases} q_0^0 = 0{,}71 \\ q_1^0 = 0{,}21 \\ q_2^0 = 0{,}05 \\ q_3^0 = 0{,}03 \end{cases}$$

$$X^0 = 0{,}21 * 16{,}7 + 0{,}05 * 19{,}3 + 0{,}03 * 19{,}9 = ¿$$
$$¿\,3{,}507 + 0{,}965 + 0{,}597 = 5{,}069$$
$$N^0 = 1 * 0{,}21 + 2 * 0{,}05 + 3 * 0{,}03 = ¿$$
$$¿\,0{,}21 + 0{,}1 + 0{,}09 = 0{,}4$$
$$R^0 = \frac{0{,}4}{5{,}069} = 0{,}078$$

The trhoughput with the faulty server is:

$$X = P_0 X^0 + P_1 X^1 + P_2 X^2 + P_3 X^3 + P_4 X^4$$

The service time:

$$R = \frac{1}{1-P_5}\left[P_0 R^0 + P_1 R^1 + P_2 R^2 + P_3 R^3 + P_4 R^4\right]$$

Fraction of Rejected Request:

$$\frac{25-X}{25}$$