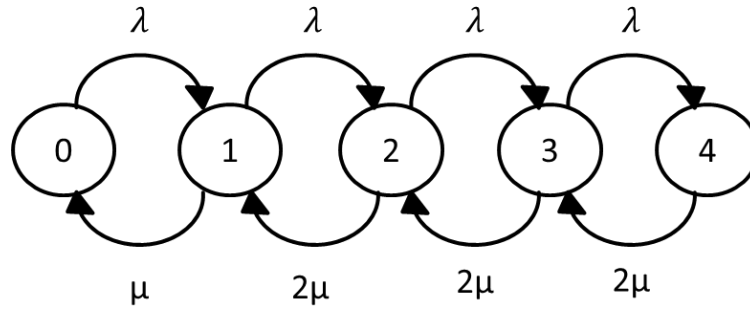


Obtain the markovian process of a queue M/M/2/4 (2 servers, 4 users in the system). Then calculate the probability that a user request is rejected, the throughput and the utilization factor, assuming that the arrival rate is $\lambda = 0.5$ req/sec and the service rate is $\mu = 0.66$ req/sec.



Flow-in = Flow-out :

$$\left\{ \begin{array}{l} p_0 \lambda = p_1 \cdot \mu \\ p_1 \lambda = p_2 \cdot 2\mu \\ p_2 \lambda = p_3 \cdot 2\mu \\ p_3 \lambda = p_4 \cdot 2\mu \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} p_1 = p_0 \left(\frac{\lambda}{\mu} \right) \\ p_2 = p_0 \left(\frac{\lambda}{\mu} \right)^2 \cdot \frac{1}{2} \\ p_3 = p_0 \left(\frac{\lambda}{\mu} \right)^3 \cdot \frac{1}{2} \cdot \frac{1}{2} \\ p_4 = p_0 \left(\frac{\lambda}{\mu} \right)^4 \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \end{array} \right.$$

$$\left\{ \begin{array}{l} p_1 = p_0 \left(\frac{0.5}{0.66} \right) \\ p_2 = p_0 \left(\frac{0.5}{0.66} \right)^2 \cdot \frac{1}{2} \\ p_3 = p_0 \left(\frac{0.5}{0.66} \right)^3 \cdot \frac{1}{4} \\ p_4 = p_0 \left(\frac{0.5}{0.66} \right)^4 \cdot \frac{1}{8} \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} p_1 = p_0 \cdot 0.757 \\ p_2 = p_0 \cdot 0.2869 \\ p_3 = p_0 \cdot 0.108 \\ p_4 = p_0 \cdot 0.0410 \end{array} \right.$$

Using the constraint $\sum_{k=0}^4 p_k = 1$, we can solve the above system of equations.

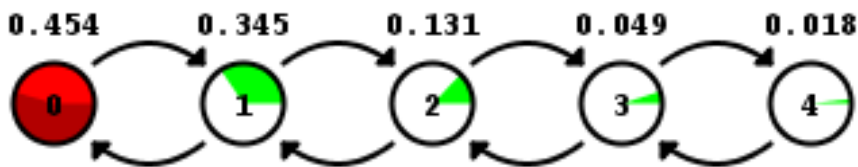
$$p_0 = 0.454$$

$$p_1 = p_0 \cdot 0.7575 = 0.344$$

$$p_2 = p_0 \cdot 0.2869 = 0.131$$

$$p_3 = p_0 \cdot 0.108 = 0.049$$

$$p_4 = p_0 \cdot 0.0410 = 0.018 \quad \leftarrow \text{Probability that a user request is rejected}$$



$$X = p_1 \cdot \mu + \sum_{i=2}^4 p_i \cdot 2\mu = 0,23 + 0,172 + 0,064 + 0,023 = 0,49 \text{ req/sec}$$