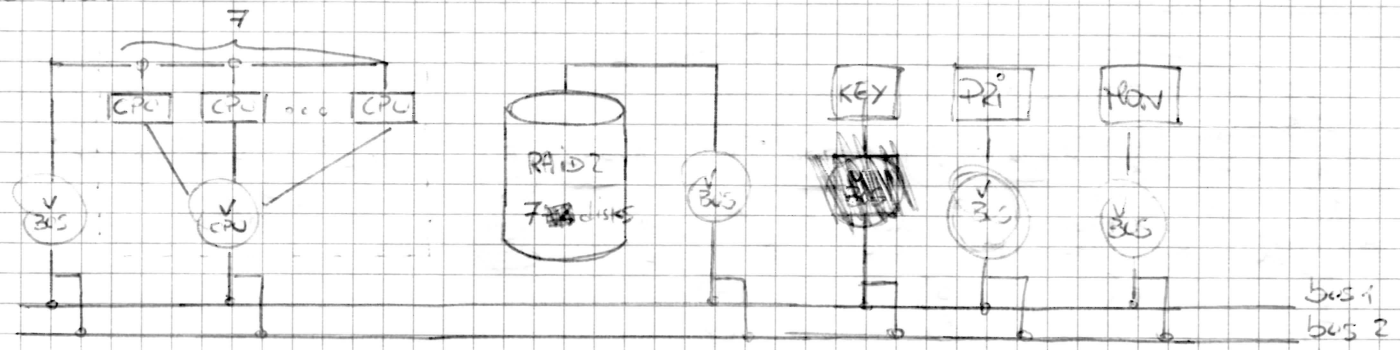
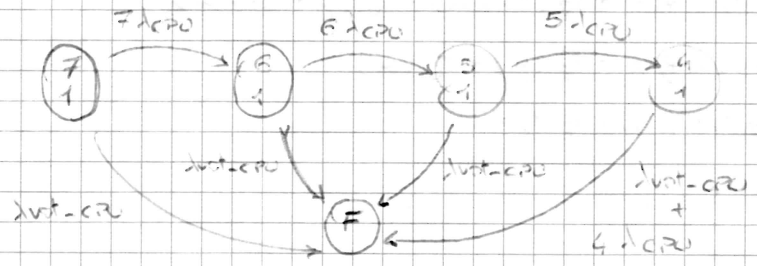
Exam 20.02.2013

Exercise 1



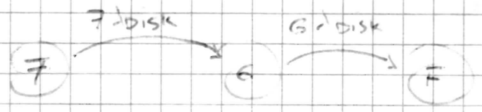
RELIABILITY

- CPU + VOTER CPU



$R_{CPU}(t) = 1 - P_F(t)$

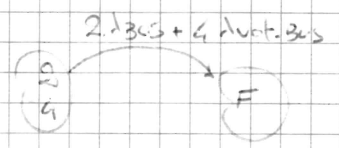
- RAID



$R_{RAID}(t) = 1 - P_F(t)$

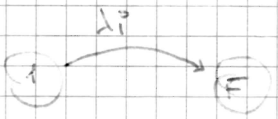
$$\begin{cases} P_7'(t) = -P_7(t)(7\lambda_{DISK}) \\ P_6'(t) = +P_7(t)(7\lambda_{DISK}) - P_6(t)(6\lambda_{DISK}) \\ P_F'(t) = +P_6(t)(6\lambda_{DISK}) \\ P_F(t) + P_6(t) + P_7(t) = 1 \\ P_7(0) = 1 \end{cases}$$

- BUS + VOTER BUS



$R_{BUS}(t) = 1 - P_F(t)$

- PRINTER / MONITOR / Keyboard

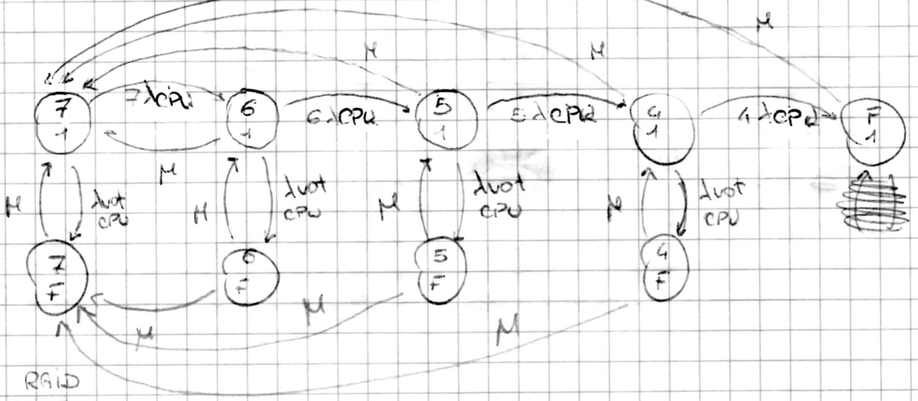


$R_i(t) = 1 - P_F(t)$ i = pri, mon, key

$$R(t) = R_{CPU}(t) \cdot R_{RAID}(t) \cdot R_{BUS}(t) \cdot R_{pri}(t) \cdot R_{mon}(t) \cdot R_{key}(t)$$

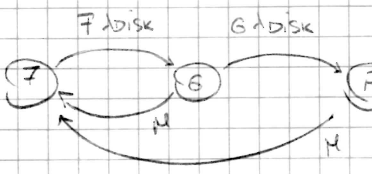
AVAILABILITY

- CPU + VOTER CPU



$$A_{CPU}(t) = 1 - \left[P_{7,F}(t) + P_{6,F}(t) + P_{5,F}(t) + P_{4,F}(t) + P_{F,1}(t) \right]$$

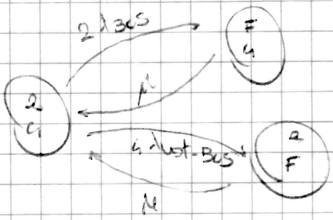
- RAID



$$\begin{cases} P_7'(t) = 0 = -P_7(7\lambda_{DISK}) + \mu(P_6 + P_F) \\ P_6'(t) = 0 = +P_7(7\lambda_{DISK}) - P_6(\mu + 6\lambda_{DISK}) \\ P_F'(t) = 0 = -P_F(\mu) + P_6(\mu) \\ P_7(t) + P_6(t) + P_F(t) = 1 \\ P_7(0) = 1; \end{cases}$$

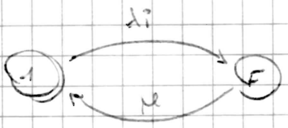
$$A_{RAID}(t) = 1 - P_F(t)$$

- BUS + VOTER BUS



$$A_{BUS}(t) = 1 - \left[P_{F,1}(t) + P_{2,F}(t) \right]$$

- PRINTER / MONITOR / KEYBOARD

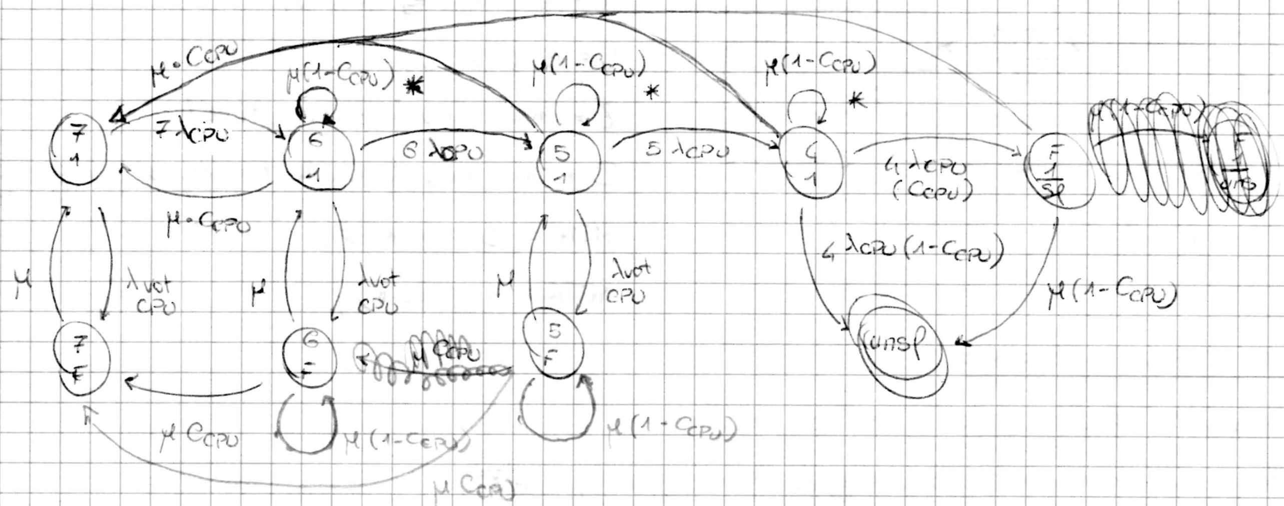


$$A_i(t) = 1 - P_F(t) \qquad i = pri / mon / key$$

$$A(t) = A_{CPU}(t) \cdot A_{RAID}(t) \cdot A_{BUS}(t) \cdot A_{PRI}(t) \cdot A_{mon}(t) \cdot A_{key}(t).$$

SAFETY ← Non era da fare ...

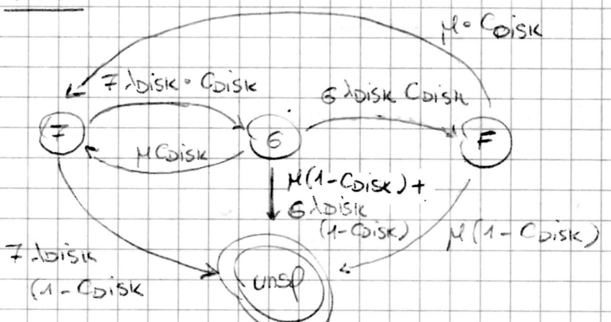
- CPU + VOTER CPU



* Se 3 repair bad, 3 remain in the same state because 3 have the majority of CPU and the voter is active. So this event is captures and so 3 remain in a safe state!

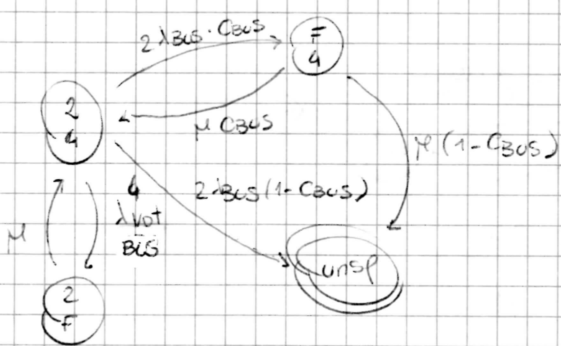
$S_{CPU}(t) = 1 - P_{unsf}(t)$.

- RAID



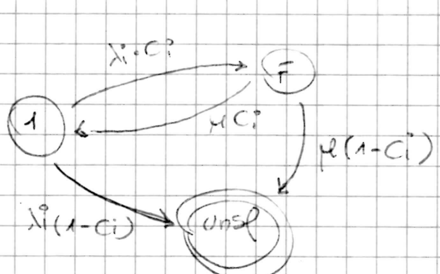
$S_{RAID}(t) = 1 - P_{unsf}(t)$

- BUS + VOTER BUS



$S_{BUS}(t) = 1 - P_{unsf}(t)$

- PRINTER / KEYBOARD / MONITOR

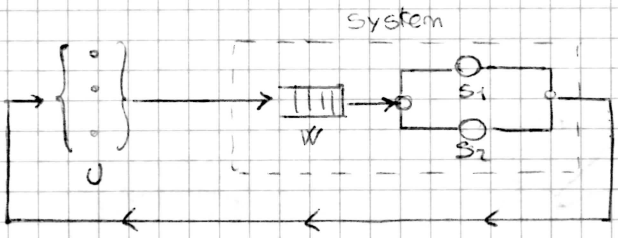


$$\begin{cases} P_1'(t) = -P_1(t)\left[\lambda_i C_i + \lambda_i(1-C_i)\right] + P_F(t)(\mu C_i) \\ P_F'(t) = -P_F(t)\left[\mu\left(C_i + (1-C_i)\right)\right] + P_1(t)(\lambda_i C_i) \\ P_{unsf}'(t) = +P_1(t)\left[\lambda_i(1-C_i)\right] + P_F(t)\left[\mu(1-C_i)\right] \\ P_1(t) + P_F(t) + P_{unsf}(t) = 1 \\ P_1(0) = 1 \end{cases}$$

$S_i(t) = 1 - P_{unsf}(t)$.

$$S(t) = S_{CPU}(t) \cdot S_{RAID}(t) \cdot S_{BUS}(t) \cdot S_{PRI}(t) \cdot S_{MON}(t) \cdot S_{KEY}(t)$$

Exercise 2



$\overline{R} = ?$ $\quad \overline{X} = ?$

$n = 2$ (server)

$W = 4$ (users queue);

$U = 6$ (n° users);

$Z = 100$ (sec) {thinking time};

$S = 1$ (sec) {service time}; $\Rightarrow \mu = \frac{1}{S}$ $\left(\frac{req}{sec}\right)$

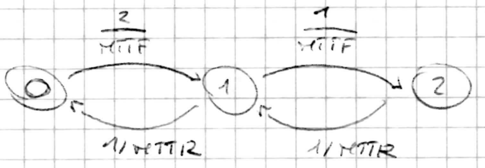
MTTF = 3 (weeks);

MTTR = 1 (hours);

- NUMBER OF FAULTY SERVERS

If I have "k" faulty server, "n-k" are active.

The rate on which the active server fault is given by: $\frac{n-k}{MTTF}$;



$$\begin{cases} p_0 \cdot \frac{2}{MTTF} = p_1 \cdot \frac{1}{MTTR} \\ p_1 \cdot \frac{1}{MTTF} = p_2 \cdot \frac{1}{MTTR} \\ p_0 + p_1 + p_2 = 1 \end{cases}$$

Calculate of p_0, p_1, p_2:

$$p_1 = p_0 \cdot \frac{2}{MTTF} \cdot MTTR = p_0 \frac{MTTR}{MTTF} \cdot 2 = p_0 \cdot \frac{1}{504} \frac{(hours)}{(hours)} 2 = p_0 (0,004)$$

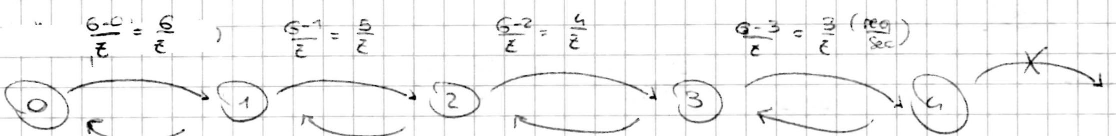
$$p_2 = p_1 \cdot \frac{MTTR}{MTTF} = p_0 (0,004) \frac{1}{504} = p_0 (0,004)(0,002) = p_0 (0,000008)$$

$$p_0 + p_1 + p_2 = 1 \Rightarrow p_0 [1 + 0,004 + 0,000008] = 1 \Rightarrow p_0 = \frac{1}{1,004008} = 0,1 ;$$

So: $p_1 = 0,1 \cdot 0,004 = 0,004$

$p_2 = 0,1 \cdot 0,000008 = 0,0000008$

- SYSTEM SCENARIO WITH "K" FAULTY SERVER



$\boxed{k=0}$	μ	2μ	2μ	2μ
$\boxed{k=1}$	μ	μ	μ	μ

For $k=0$:

$$\begin{cases} q_0^0 \frac{6}{Z} = q_1^0 \cdot \mu \\ q_1^0 \frac{5}{Z} = q_2^0 \cdot 2\mu \\ q_2^0 \frac{4}{Z} = q_3^0 \cdot 2\mu \\ q_3^0 \frac{3}{Z} = q_4^0 \cdot 2\mu \\ \sum_{i=0}^{W} q_i^0 = 1 \end{cases}$$

For $k=1$:

$$\begin{cases} q_0^1 \frac{6}{Z} = q_1^1 \cdot \mu \\ q_1^1 \frac{5}{Z} = q_2^1 \cdot \mu \\ q_2^1 \frac{4}{Z} = q_3^1 \cdot \mu \\ q_3^1 \frac{3}{Z} = q_4^1 \cdot \mu \\ \sum_{i=0}^{W} q_i^1 = 1 \end{cases}$$

- The throughput of a single server, in case of "k" faulty servers, is given by:

$$X(k) = \sum_{i=1}^{W} q_i^k \cdot X_k(i) \quad \text{where } i = n° \text{ requests into the } \sout{\text{system}} \text{ server} ; \quad \text{for } k = 0, 1;$$

So:

$$\begin{cases} X(0) = \sum_{i=1}^{4} q_i^0 \cdot X_0(i) = q_1^0 \mu + q_2^0 2\mu + q_3^0 2\mu + q_4^0 2\mu \\ X(1) = \sum_{i=1}^{4} q_i^1 \cdot X_1(i) = q_1^1 \mu + q_2^1 \mu + q_3^1 \mu + q_4^1 \mu \end{cases}$$

The average throughput of the whole system is given by:

$$\bar{X} = \sum_{i=0}^{n-1} p_i [X(i)] \quad \text{where } i = n° \text{ faulty servers}$$

So:

$$\bar{X} = p_0[X(0)] + p_1[X(1)]$$

- The n° of users of a single server, in case of "k" faulty servers, is given by:

$$N(k) = \sum_{i=0}^{W} q_i^k \cdot i \quad \text{where } i = n° \text{ of requests into the server} ; \quad \text{for } k = 0, 1;$$

So:

$$\begin{cases} N(0) = \sum_{i=0}^{4} q_i^0 \cdot i = \phi + q_1^0 + 2q_2^0 + 3q_3^0 + 4q_4^0 \\ N(1) = \sum_{i=0}^{4} q_i^1 \cdot i = \phi + q_1^1 + 2q_2^1 + 3q_3^1 + 4q_4^1 \end{cases}$$

The average n° of users in the whole system, is given by:

$$\bar{N} = \sum_{i=0}^{n-1} p_i \cdot N(i) \quad \text{where } i = n° \text{ of faulty server}$$

So:

$$\bar{N} = p_0 \cdot N(0) + p_1 \cdot N(1)$$

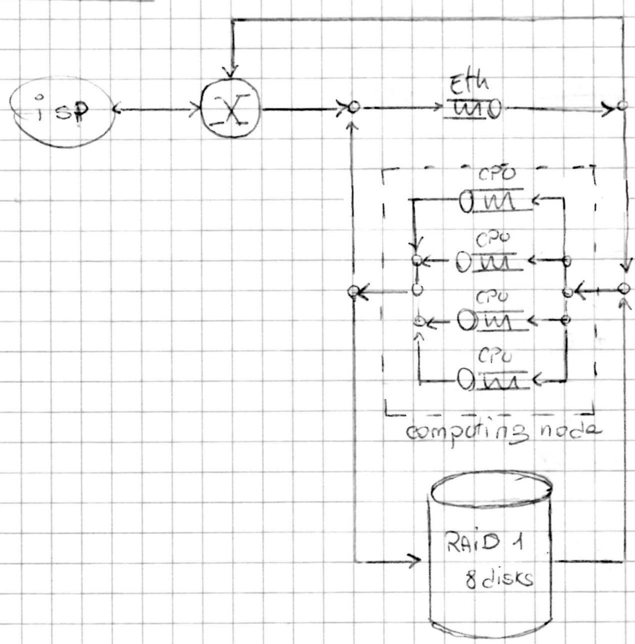
- The response time of a single server, in case of "k" faulty servers, is given by:

$$R(k) = \frac{N(k)}{X(k)} \quad \text{for } k = 0, 1;$$

The average response time of the whole system, is given by:

$$\bar{R} = \frac{\sum_{i=0}^{n-1} p_i R(i)}{1 - P_{fault}} = \frac{p_0 R(0) + p_1 R(1)}{1 - p_2}$$

Exercise 3



$\lambda = 50$ (req/sec);

$B_{eth} = 100$ (Mbit/sec);

$L = 100$ (μsec/pkt);

$B_{isp} = 1$ (Gbit/sec);

$R = 400$ (byte);

$F = 100'000$ (byte);

$S_{CPU} = 20$ (msec)

$S_{DISK} = 10$ (msec) $\forall$ 10'000 (byte)

{ $\Rightarrow$ for having 100'000, $V_{DISK} = 10$ }

$$D_{CPU} = \frac{S_{CPU} \cdot V_{CPU}}{4} = \frac{20\,(msec) \cdot 1}{4} = 5\,(msec);$$

$$D_{RAID} = \frac{S_{DISK} \cdot V_{DISK}}{8} = \frac{10\,(msec) \cdot 10}{8} = 12{,}5\,(msec);$$

$$X_0 = \frac{1}{\max D_i} = \frac{1}{D_{RAID}} = \frac{1}{0{,}0125\,(sec)} = 80\,(req/sec) \rightarrow \text{max arrival rate}$$

For the steady-state condition, $X_0 = \lambda = 50$ (req/sec)

$$U_{CPU} = X_0 \cdot D_{CPU} = 50\left(\frac{req}{\cancel{sec}}\right) \cdot 0{,}005\,(\cancel{sec}) = 0{,}25$$

$$U_{RAID} = X_0 \cdot D_{RAID} = 50 \cdot 0{,}0125 = 0{,}625$$

$$R'_{CPU} = \frac{D_{CPU}}{1 - U_{CPU}} = \frac{5\,(msec)}{0{,}75} = 6{,}67\,(ms);$$

$$R'_{RAID} = \frac{D_{RAID}}{1 - U_{RAID}} = \frac{12{,}5}{0{,}375} = 33{,}30\,(ms);$$

About the network:

$\boxed{\text{REQUEST}}$

$$\#segment = \left\lceil \frac{R}{MTU_{TCP}} \right\rceil = \left\lceil \frac{400}{65'515} \right\rceil = 1;$$

$$\#datagram = \left\lceil \frac{R + \#segment(TCP_{ovhd})}{MSS} \right\rceil = \left\lceil \frac{400 + (1 \cdot 20)}{1480} \right\rceil = 1 = \#frame$$

$$Overhead = \#segment(TCP_{ovhd}) + \#datagram(IP_{ovhd} + Eth_{ovhd}) = (1 \cdot 20) + 1(20 + 18) = 58$$

$$S_{isp}^{req} = \frac{(R+Overhead)\,8}{B_{isp}} = \frac{3'664\ (\cancel{bit})}{1\cdot 10^{9}\ (\frac{\cancel{bit}}{sec})} = 3'664\cdot 10^{-9}\ (s)$$

$$= 0{,}003664 \approx 0{,}0037;$$

$$S_{eth}^{req} = \frac{(R+Overhead)\,8}{B_{eth}} = \frac{3'664}{100\cdot 10^{6}} = 36{,}64\cdot 10^{-6}\ (s) \approx 0{,}037;$$

$$S_{rout}^{req} = \#datagram \cdot L = 1\ (\cancel{pkt}) \cdot 100\ (\mu sec/\cancel{pkt}) = 0{,}1\ (ms);$$

$\boxed{\text{REPLY}}$

$$\#segment = \left\lceil \frac{F}{MTU_{TCP}} \right\rceil = \left\lceil \frac{100'000}{65'515} \right\rceil = 2;$$

$$\#datagram = \#frame = \left\lceil \frac{F + \#segment\,(TCP_{ovhd})}{MSS} \right\rceil = \left\lceil \frac{100'000 + (2\cdot 20)}{1480} \right\rceil = 68;$$

$$Overhead = \#segment\,(TCP_{ovhd}) + \#datagram\,(IP_{ovhd} + Eth_{ovhd}) =$$

$$= (2\cdot 20) + 68\,(20+18) = 2'624\ (byte);$$

$$S_{isp}^{rep} = \frac{(F+Overhead)\,8}{B_{isp}} = \frac{820'992}{1\cdot 10^{9}}\ (s) \approx 0{,}82\ (ms)$$

$$S_{eth}^{rep} = \frac{(F+Overhead)\,8}{B_{eth}} = \frac{820'992}{100\cdot 10^{6}}\ (s) \approx 8{,}201\ (ms);$$

$$S_{rout}^{rep} = \#datagram \cdot L = 68 \cdot 100 = 6'800\ (\mu s) = 6{,}8\ (ms);$$

$\boxed{\text{HANDSHAKING}}$

$$\#segment = \#datagram = \#frame = 6$$

$$Overhead = \#frame\,(TCP_{ovhd} + IP_{ovhd} + Eth_{ovhd}) = 6\,(20+20+18) = 348\ (byte).$$

$$S_{isp}^{hs} = \frac{(\phi + Overhead)\,8}{B_{isp}} = \frac{2'784}{1\cdot 10^{9}}\ (s) \approx 0{,}0028\ (ms);$$

$$S_{eth}^{hs} = \frac{(\phi + Overhead)\,8}{B_{eth}} = \frac{2'784}{100\cdot 10^{6}}\ (s) = 0{,}028\ (ms);$$

$$S_{rout}^{hs} = \#datagram \cdot L = 6\cdot 100 = 600\ (\mu sec) = 0{,}6\ (ms);$$

So:

$$D_{isp} = (S_{isp}^{hs} + S_{isp}^{req} + S_{isp}^{rep}) \cdot 1 \approx 0{,}83 \text{ (ms)};$$

$$D_{eth} = (S_{eth}^{hs} + S_{eth}^{req} + S_{eth}^{rep}) \cdot 1 \approx 8{,}28 \text{ (ms)};$$

$$D_{rout} = (S_{rout}^{hs} + S_{rout}^{req} + S_{rout}^{rep}) \cdot 1 \approx 7{,}5 \text{ (ms)};$$

$$U_{isp} = X_0 \cdot D_{isp} = 50 \left(\frac{req}{sec}\right) \cdot 0{,}00083 \text{ (s)} = 0{,}0415 \approx 0{,}042$$

$$U_{eth} = X_0 \cdot D_{eth} = 50 \cdot 0{,}00828 = 0{,}414 \approx 0{,}41$$

$$U_{rout} = X_0 \cdot D_{rout} = 50 \cdot 0{,}0075 = 0{,}375 \approx 0{,}38$$

$$R'_{isp} = \frac{D_{isp}}{1 - U_{isp}} = \frac{0{,}83 \text{ (ms)}}{0{,}958} \approx 0{,}87 \text{ (ms)};$$

$$R'_{eth} = \frac{D_{eth}}{1 - U_{eth}} = \frac{8{,}28 \text{ (ms)}}{0{,}59} \approx 14{,}03 \text{ (ms)};$$

$$R'_{rout} = \frac{D_{rout}}{1 - U_{rout}} = \frac{7{,}5 \text{ (ms)}}{0{,}62} \approx 12{,}10 \text{ (ms)};$$

The average response time is given by:

$$\bar{R} = R'_{CPU} + R'_{RAID} + R'_{isp} + R'_{rout} + R'_{eth} = 66{,}87 \text{ (ms)};$$

The bottleneck of the system is the maximum residence time and then: R'_{RAID}