#### Query Answering with Incomplete Information: Conjunctive Queries over Naive Tables

#### Formal Methods

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### Incomplete information and query answering

- Incomplete information in data: missing / unknown / partially specified data
- Query answering
  - Over usual databases (complete information): QA by evaluation (or "model checking")

$$D \models Q$$

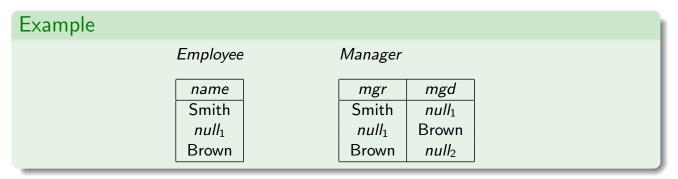
i.e., D is seen as an interpretation (for simplicity we assume the query to be boolean, no free variables)

Over incomplete databases (incomplete information): QA by logical implication (or "entailment")

$$\forall \mathcal{I}.\mathcal{I} \models D \text{ implies } \mathcal{I} \models Q$$

### Incomplete databases

A common form of incomplete databases are the so-called "naive tables", which include values and "labelled nulls" (standing for unknown values) [IL84].



- Const: we have infinite constants, corresponding to domain objects as usual;
- Nulls: we have a countably infinite set of nulls, corresponding to variables ranging over Cons;
- Tables are incomplete, i.e., more tuples may belong to them, corresponding to the so called "open-world-assumption" or OWA. (For example null<sub>2</sub> belongs to Employee though not reported in the table.)

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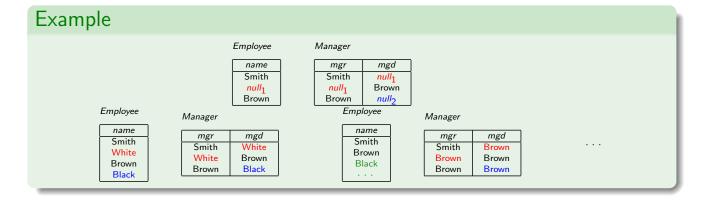
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## Incomplete databases: semantics

Semantics of incomplete databases:

- A valuation function for nulls is a assignment function  $\sigma : Nulls \to Const$  (essentially nulls are considered as individual variables in logic).
- We denote by  $\mathcal{I}, \sigma \models D$  the fact that for every tuple  $(t_1, \ldots, t_n) \in P$  for each table P we have  $\mathcal{I}, \sigma \models P(t_1, \ldots, t_n)$ .
- We define in logic the set of databases completing D as

 $Models(D) = \{\mathcal{I} | \text{ there exists a } \sigma \text{ such that } \mathcal{I}, \sigma \models D\}$ 



#### Certain answers to a query

An incomplete database acts like a logical theory: it selects models.

#### Query answering in complete databases

The answer to a query  $q(\vec{x})$  over a complete database D, denoted  $q^D$ , is the set of tuples  $\vec{c}$  of constants of *Const* such that the  $\vec{c} \in q^D$  is to true in D.

#### Query answering in incomplete databases

The certain answer to a query  $q(\vec{x})$  over an incomplete database D, denoted cert(q, D), is the set of tuples  $\vec{c}$  of constants of Const such that  $\vec{c} \in q^{\mathcal{I}}$ , for every model  $\mathcal{I}$  of D.

#### Note:

- It q is boolean, and D is incomplete: we write  $D \models q$  iff q evaluates to true in every model  $\mathcal{I}$  of D, (otherwise we write  $D \not\models q$ .
- We use the same notation as for query answering based on evaluation: the difference is in the incompleteness of the database.

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#### Query languages for incomplete databases

Which query language to use?

- Full SQL (or equivalently, first-order logic)
  - ▶ NO: in the presence of incomplete information, query answering becomes undecidable (FOL validity). (Notice this holds already for an empty incomplete database!)
- Conjunctive queries (or better union of conjunctive queries)
  - Conjunctive queries are well behaved wrt containment. Can they be used for query answering in presence of incomplete information. YES! See what follows.

## Conjunctive queries and incomplete databases

A conjunctive query (CQ) is a first-order query of the form

$$q(\vec{x}) \leftarrow \exists \vec{y}. R_1(\vec{x}, \vec{y}) \land \cdots \land R_k(\vec{x}, \vec{y})$$

where each  $R_i(\vec{x}, \vec{y})$  is an atom using (some of) the free variables  $\vec{x}$ , the existentially quantified variables  $\vec{y}$ , and possibly constants.

We will also use the simpler Datalog notation:

$$q(\vec{x}) \leftarrow R_1(\vec{x}, \vec{y}), \dots, R_k(\vec{x}, \vec{y})$$

#### Note:

- CQs contain no disjunction, no negation, no universal quantification.
- Correspond to SQL/relational algebra select-project-join (SPJ) queries the most frequently asked queries.
- A Boolean CQ is a CQ without free variables  $\Rightarrow q() \leftarrow \exists \vec{y}. R_1(\vec{y}) \land \cdots \land R_k(\vec{y}).$

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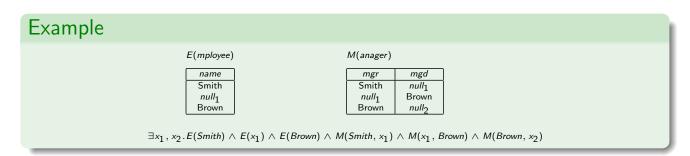
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## Conjunctive queries and incomplete databases

Containment of conjunctive queries  $q_1 \subseteq q_2$  is decidable: and LOGSPACE in  $q_1$  and NP-complete in  $q_2$  [CM77].

Given an incomplete database D as above we can construct in linear time a (boolean) conjunctive query  $q_D$  that fully captures it.

- For each tuple in a table of D becomes an atom in the conjunctive query  $q_D$ .
- For each labelled nulls occurring in D becomes an existentially quantified variable in q<sub>D</sub>.



# Conjunctive queries and incomplete databases

# Theorem ([IL84])

Let D be a database with incomplete information as above (naive tables),  $q_D$  the corresponding conjunctive query constructed as above, and q a boolean (union) of conjunctive query. Then:

$$D \models q \text{ iff } q_D \subseteq q$$

Proof.

- ① Observe that the models of D by construction coincide with that of the formula  $q_D$ : that is  $\forall \mathcal{I}.\mathcal{I} \models D$  iff  $\mathcal{I} \models q_D$ .
- ② Moreover,  $q_D \subseteq q$  in the case of boolean queries stands for  $\forall \mathcal{I}.\mathcal{I} \models q_D$  implies  $\mathcal{I} \models q$ , or simply  $q_D \models q$ .
- **3** Hence, by (1)  $D \models q$  iff  $q_D \models q$ .

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# Conjunctive queries and incomplete databases

Using Chandra & Merlin Theorem [CM77], we get:

### Theorem ([IL84])

Let D be a database with incomplete information as above (naive tables),  $q_D$  the corresponding conjunctive query constructed as above,  $\mathcal{I}_{q_D}$  its canonical database, and q a boolean (union) of conjunctive query. Then:

$$D \models q \; \textit{iff} \; \mathcal{I}_{q_D} \models q$$

Note:  $\mathcal{I}_{q_D}$  is exactly D with nulls interpreted as additional constants!

Compute certain answers of non boolean CQs over incomplete databases

Given a non boolean (U)CQ q and an incomplete database D:

- Evaluate q over D as it was a complete database
- ② filter out all answers where null appears (certain answers are constituted by tuples of constants in Const)

## Conjunctive queries and incomplete databases

As a consequence of the above theorem we have:

Computing certain answers for (union) of conjunctive queries over databases with incomplete information (naive tables) is:

- LOGSPACE in data complexity
- NP-complete in query complexity and combined complexity

Note1: Exactly as for the case of complete information!

Note2: Use of CQs is crucial, since for full FOL we get undecidability!

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# Examples of CQs over an incomplete database

### Example

E(mployee)

M(anager)

name
Smith
null<sub>1</sub>
Brown

 $\begin{array}{c|c} \textit{mgr} & \textit{mgd} \\ \hline \textit{Smith} & \textit{null}_1 \\ \textit{null}_1 & \textit{Brown} \\ \hline \textit{Brown} & \textit{null}_2 \\ \end{array}$ 

- Queries:  $q_1(x,y) \leftarrow M(x,y)$  $q_2(x) \leftarrow \exists y. M(x,y)$ 
  - $q_3(x) \leftarrow \exists y_1, y_2, y_3. M(x, y_1) \land M(y_1, y_2) \land M(y_2, y_3)$  $q_4(x, y_3) \leftarrow \exists y_1, y_2. M(x, y_1) \land M(y_1, y_2) \land M(y_2, y_3)$
- Answers:  $q_1$ :  $\{\}$   $q_2$ :  $\{\$ Smith, Brown  $\}$   $q_3$ :  $\{\$ Smith  $\}$ 
  - $q_4$ :  $\{ \}$

#### Conclusion

#### Incomplete information

Several other forms of incomplete information have been studied in the literature of Databases and especially in the literature of Knowledge Representation and Reasoning in Artificial Intelligence.

#### These include:

- Knowledge Bases
- Ontologies, Description Logics, Semantic Technologies
- Reasoning about Actions (incomplete information also on the dynamics)
- ...

#### Note

Only in very few cases dealing with incomplete information can be done through query evaluation techniques.

If interested, take the course on Knowledge Representation and Semantic Technologies.

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### References

[CM77] A. K. Chandra and P. M. Merlin. Optimal implementation of conjunctive queries in relational data bases. In Proc. of the 9th ACM Symp. on Theory of Computing (STOC'77), pages 77-90, 1977.

T. Imielinski and W. J. Lipski. [IL84] Incomplete information in relational databases. J. of the ACM, 31(4):761–791, 1984.