formal methods

EXERCISE 1:

$q_1(x) :- r(x,x), b(x,y), b(y,x)$

$q_2(x) :- r(x,y), b(y,z), b(z,x)$

check whether q_1 is contained in q_2, explaining the technique used and show the homomorphism between the databases.

Process is described in slide 32.

1) Freeze variables, the free ones. x is the free variable here.
   When we want to do containment, both queries should have the same free variables.

   Freeze → fresh constant. So, $q_1(x) \subseteq q_2(x)$ becomes $q_1(a) \subseteq q_2(a)$ and:

   $q_1(a) :- r(a,a), b(a,y), b(y,a)$

   $q_2(a) :- r(a,y), b(y,z), b(z,a)$

2) Compute canonical interpretation for $q_1(a)$.

$$\left.\begin{array}{l} \Delta^I = \{a, y\} \\ r^I = \{(a,a)\} \\ b^I = \{(a,y), (y,a)\} \end{array}\right\} I_{q_1(a)}$$

: remember that a and y are strings, 'a' and "y"

Two tables:

r	1	2
✓	a	a

b	1	2
✓	a	y
✓	y	a

3) Evaluate $I_{q_1(a)} \models q_2(a)$.

   We follow these steps:

   a) Cross assignment of existential variables.
   b) check all atoms of CQ

$a(y) = a|y$
$a(z) = a|y$

There's only one entry in r so $r(a,y)$ must be assigned to that entry and we guess:

$a(y) = a$

$b(y,z)$ becomes $b(a,z)$ and there's only one element in the second table with b as 1st value, and we guess:

$a(z) = y$

Now, is $b(z,a)$ satisfied? It becomes $b(y,a)$ and infact there's a matching entry in the second table, so we conclude that it is a satisfied assignment.

We can follow a second method:

a) compute I_q
b) guess homo $h: I_q \to I$
c) check h is homo

$$I_{q2}(a): \begin{cases} \Delta^{I_2} = \{a,y,z\} \\ r^{I_2} = \{(a,y)\} \\ b^{I_2} = \{(y,z),(z,a)\} \end{cases}$$

r	1	2
/	a	y

b	1	2
/	y	z
/	z	a

$h(a) = a$
$h(y) = x(y) = a$
$h(z) = x(z) = h$

(... x goes to x ... we keep the relationship $q_1(x) \subseteq q_2(x)$)
→ we used the CM theorem

But we can also guess.

$h(y) =$ we look at r^{I_2}, we have (a,y). There's only one tuple in r^{I_1} that matches, it says assign to 'y', 'a'. So, $h(y) = a$.

$h(z) =$ Take (y,z), and consider the tuples in b^{I_1}. We remember that we assigned 'a' to 'y', therefore we're looking for (a, something) in b^{I_1}. We find (a,y), so $h(z) = y$.

Can we find (z,a) in b^{I_1}? Yes! It's (y,a). So it's an homomorphism.