Programma Online Markets

- **Game theory** basics
    - Dominant strategy and dominated strategies
    - Nash Equilibrium
    - Randomized (mixed) strategies
    - Mixed Nash Equilibrium (**Always exists**)
    - Pareto Optimality
    - Social Welfare
    - **Price of stability**: best equilibrium over optimal state
    - **Price of anarchy**: worst equilibrium over optimal state

- **Congestion games:** resources with costs that depend on the number of players using them. Strategies are subsets of resources, the cost of a player is the total latency he experiences from the resources he uses.

- **Potential functions:** for every pair of states that differ on the strategy of a single player, the difference in the value of the potential and the difference of the cost of this player have the same sign.
    - If a game admits a potential function, it has a pure Nash equilibrium.
    - Rosenthal's function is a potential function for all congestion games.

- Social cost of a state: the total cost of the players
- **PoS bounds** for potential games: find a relation between the potential function and the social cost, and use it to bound the PoS

$$\lambda \cdot \mathrm{SC}(\boldsymbol{s}) \leq \Phi(\boldsymbol{s}) \leq \mu \cdot \mathrm{SC}(\boldsymbol{s})$$

$$\mathrm{SC}(\boldsymbol{s}) \leq \frac{1}{\lambda} \cdot \Phi(\boldsymbol{s}) \leq \frac{1}{\lambda} \cdot \Phi(\boldsymbol{s}_{OPT}) \leq \frac{\mu}{\lambda} \cdot \mathrm{SC}(\boldsymbol{s}_{OPT}) \Rightarrow \text{PoS} \leq \frac{\mu}{\lambda}$$

    - PoS of linear congestion games: at most 2

- **PoA bounds**: compare the equilibrium state with the deviating strategies to get PoA bounds.
    - PoA of linear congestion games: tight bound of 5/2

- **Auctions**: allocation rule + payment rule
    - An allocation rule is **implementable** is there exists a payment rule, so that together they define a truthful auction
    - An allocation rule is **monotone**, if larger bids give more stuff
- **Single-item auctions**: first-price is not truthful, second-price is truthful and maximizes the social welfare (sells to the bidder with the highest value)
- **Sponsored search auctions**: generalized second-price auction is not truthful
- **Myerson's Lemma**:

(a) An allocation rule $\boldsymbol{x}$ is implementable if and only if it is monotone
(b) For every allocation rule $\boldsymbol{x}$, there exists a unique payment rule $\boldsymbol{p}$ such that $(\boldsymbol{x}, \boldsymbol{p})$ is a truthful auction

    - Using Myerson's Lemma we can design a truthful sponsored search auction

- **Voting**: a way to make decisions
- **Social choice functions**: take as input the preferences of the agents, output a single winning alternative
- **Social welfare functions**: output a ranking over all alternatives
- **Unanimity**: If all agents have exactly the same preferences over the alternatives, then the output should be what everyone wants
- **Independence of Irrelevant Alternatives** (IIA): the relative order of two alternatives does not depend on other alternatives
- Positional scoring rules
    - Plurality rule
    - Veto rule
    - Borda rule
    - Copeland
    - Ranked pairs

    - Dictatorship (the only one that can satisfy unanimity and independence of irrelevant alternatives (for at least 3 alternatives))
    - Only dictatorship cannot be manipulated by the agents (for at least 3 alternatives)

- Agents have implicit values for the alternatives. These values induce the preference rankings
    - Many different valuation profiles can induce the same ordinal profile

- **Distortion**: worst case ratio over all valuation profiles between the social welfare of the optimal outcome over the social welfare of the outcome chosen by the voting rule
- **Deterministic rules**: distortion is Ω m^2
- **Randomized rules**: distortion is between Ω m and O m log* m

- Improving distortion knowing ordinal profile and some info about cardinal profile: use Binary Search
- **Fair Division of Indivisible Items**
    - Envy-Freeness
    - Proportionality
    - Envy-Freeness up to one item (**EF1**) **Allocation always exists**
    - Envy-Freeness up to any item (**EFX**)
        - EFX allocations and 2 agents: Always exist
        - EFX allocations with n agents and identical values: Always exist
    - Almost Approximate Proportional Fairness (**APS**)
        - APS allocations and 2 agents: Always exist
        - APS allocations and n>2 agents: No guarantee of existence
        - Round Robin
        - Envy Cycle Elimination
    - Pairwise APS Fairness (PAPS)
- Kidney Exchange
    - Top Trading Cycle Algorithm
    - Stable Matching algorithm