

# A Game of Tiers: Exploring the Formal Properties of TSL Languages

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Computational Phonology Workshop Dec 12, 2016

### The Talk in a Nutshell

### Subregular Hypothesis for Phonology

- Phonotactic patterns can be described by classes in the subregular hierarchy;
- Pin-pointing the right class will be useful for typology, learnability, cognitive predictions ...

### Current Hypothesis

- ► Tier-based Strictly Local seems to be the right fit;
- ▶ But ... several problematic patterns have been reported!

#### Idea

We can explore (minimal) extensions to TSI

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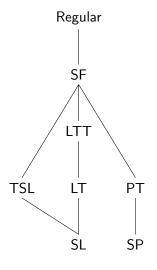
- Tier-based Strictly Local seems to be the right fit;
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We can explore (minimal) extensions to TSL

## Outline

- 1 Preliminaries
- 2 TSL Limits
- 3 Extending the TSL Class
- 4 Conclusions



## Subregular Hypothesis

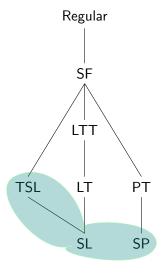
- Phonology is subregular;
- Local phonotactic dependencies are Strictly Local.

#### SL: Example

Word-final devoicing:  $*[+voice] \times$ 

$$\times$$
 rad  $\times$ 

× rat ×



## Subregular Hypothesis

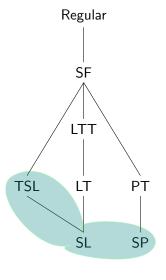
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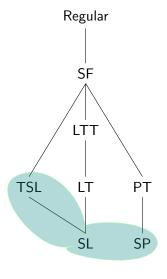
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### SL: Example

Word-final devoicing:  $*[+voice] \ltimes$ 

 $\times$  rad  $\times$ 

w rat w



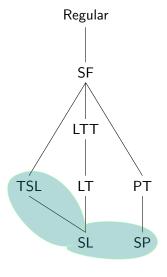
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### SL: Example

Word-final devoicing:  $*[+voice] \times$ 

$$ok \times r \text{ a t } |$$

## Long Distance Dependencies as Tier-based Strictly Local

- ► Problem: Unbounded processes cannot be captured by Strictly Local Grammars
- Solution: Select a subset of segments and enforce constraints only over it.

### Tier-based Strictly Local (TSL) Grammars

- ▶ a projection function  $E: \Sigma \to T$  with  $T \subseteq \Sigma$   $\Rightarrow$  projection on a tier is determined just by the "shape" of the segment (no structural or "proximity" information);
- strictly local constraints over T;

### Grammar

$$T = \{s, z, g, f\} S = \{*gs, *gg, *gf, *f\} \}$$

\* a: e r s e

 $^{ok}$  аа: е г  $\int$  е

### Grammar

$$T = \{s, z, g, f\} S = \{*gs, *sg, *sf, *fs\}$$

3

- T: sibilant harmony
- \*a: erse

### Grammar

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#### Grammar

$$\mathsf{T} = \{\mathsf{s},\,\mathsf{z},\,\mathsf{g},\,\mathsf{f}\}\;\mathsf{S} = \{\mathsf{*gs},\,\mathsf{*sg},\,\mathsf{*sf},\,\mathsf{*fs}\;\}$$

```
T: sibilant harmony
 a: e r s e
```

T: sibilant harmony a: e r∫e

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а a: е

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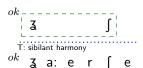
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## Sibilant Harmony in IMDLAWN TASHLHIYT (McMullin2016)

```
1) Underlying causative prefix /s(:)-/
    Base
           Causative
                     "be evacuated"
   uga sː-uga
a.
   as:twa s-as:twa "settle, be levelled"
b.
```

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2) Sibilant harmony
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   fia[r [- fia[r
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3) Sibilant voicing harmony blocked
    Base Causative
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```
a. ukz s:-ukz "recognize"
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b. quasi J- quasi "be dislocated, broken"

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3) Sibilant voicing harmony blocked

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Base
        Causative
   ukz sı-ukz "recognize"
a.
   q:u3:i [- qu3:i "be dislocated, broken"
b.
```

Can we write a TSL grammar to capture this pattern?

### Generalization (1/2)

Sibilants must agree in anteriority and voicing.

#### Grammar

$$T = \{ \mathbf{z}, \mathbf{s}, \mathbf{z}, \mathbf{f} \}$$

$$S = \{ *\mathsf{sz}, *\mathsf{sz}, *\mathsf{sf}, *\mathsf{zs}, *\mathsf{fs}, *\mathsf{zs}, *\mathsf{zf}, *\mathsf{zz}, *\mathsf{fz}, *\mathsf{fz}, *\mathsf{zf}, *\mathsf{zf}, *\mathsf{zz} \}$$

$$^{ok}$$
 a m: a d a w  $\mid$ 

### Generalization (1/2)

Sibilants must agree in anteriority and voicing.

#### Grammar

Z

.....

\*z m: a d a w l

 $^{ok}$   $\mathtt{a}$   $\mathtt{m}$ :  $\mathtt{a}$   $\mathtt{d}$   $\mathtt{a}$   $\mathtt{w}$   $\mathtt{I}$ 

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#### Grammar

 $^{ok}$  র m: র d a w |

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#### Grammar

z 3 \* z m: 3 d a w l

 $^{ok}$   $\mathtt{z}$   $\mathsf{m}$ :  $\mathtt{z}$   $\mathsf{d}$   $\mathsf{a}$   $\mathsf{w}$   $\mathsf{l}$ 

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Sibilants must agree in anteriority and voicing.

#### Grammar

```
z 3
.....*
* z m: 3 d a w |
```

 $^{ok}$   $\mathtt{a}$   $\mathsf{m}$ :  $\mathtt{a}$   $\mathsf{d}$   $\mathsf{a}$   $\mathsf{w}$   $\mathsf{l}$ 

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z 3 \* z m: 3 d a w I

 $^{ok}$   $\mathtt{am}$ :  $\mathtt{adaw}$   $\mathtt{l}$ 

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z 3 \* z m: 3 d a w |

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z a T \* z m: a d a w |

<sup>ok</sup> аm:аdaw∣

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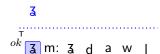
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### Generalization (1/2)

Sibilants must agree in anteriority and voicing.

### Generalization (2/2)

Voiceless obstruents block agreement in voicing.

$$\begin{split} T &= \{ \text{ $\it{g}$, s, z,f, q} \} \\ S &= \{ \text{ *sg, *sz, *sf, *gs, *fs, *zs, *zf, *zg, *fz, *fg, * gf, *gz } \} \end{split}$$

$$^{ok}$$
  $\int$  q u  $\mathbf{g}$ :  $\mathbf{i}$ 

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```
∫ q
;;; * s q u д; ; * s q u д;
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```
∫ q
;;
; ok ∫ q u z; i * s q u z;
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```
\int q \overline{3}:
\stackrel{\text{T}}{ok} \int q u \overline{3}: i * s q u \overline{3}:
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$$\begin{bmatrix} ok & \begin{bmatrix} & & & & & \\ & & & & & \end{bmatrix} & \mathbf{Z} \end{bmatrix}$$

\* S Q U 3:

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### Generalization (2/2)

Voiceless obstruents block agreement in voicing.

$$T = \{ \text{ $\it{g}$, $\it{s}$, $\it{z}$,$\it{f}$, $\it{q}$} \\ S = \{ \text{ *sg, *sg, *sf, *gs, *sf, *zs, *zf, *zg, *fz, *fg, * gf, *gz, *} \}$$

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#### Grammar

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No TSL grammar can block voicing and enforce anteriority!

## Sibilant Harmony in SAMALA (McMullin2016)

1) Unbounded sibilant harmony

```
a. /k-su-ʃojin/ kʃuʃojin "I darken it"
```

b. /k-su-k'ili-mekeken-ʃ/ kʃuk'ilimekeketʃ "I straighten up"

```
2) /s/\rightarrow [\int] when preceding (adjacent) [t, n, l]
```

```
a. /s-lok'in/ flok'in he cuts it
```

3) Long-distance agreement overrides local disagreement

```
a. /s-iʃt-tiʃti-jep-us/ sististijepus "they show him"
```

b. /s-net-us/ snetus "he does it to hi

Can we write a TSL grammar to capture this pattern?

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```
a. /s-lok'in/ flok'in he cuts it
```

- b. /s-tepu?/ "he gambles"
- Long-distance agreement overrides local disagreement

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a. /s-iʃt-tiʃti-jep-us/ sististijepus "they show him"
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## Sibilant Harmony in SAMALA (McMullin2016)

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```
a. /k-su-ʃojin/ kʃuʃojin "I darken it"
```

b. /k-su-k'ili-mekeken-∫/ k∫uk'ilimekeket∫ "I straighten up"

2)  $/s/\rightarrow$  [ʃ] when preceding (adjacent) [t, n, l]

```
a. \sqrt{s-lok'in} flok'in he cuts it
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3) Long-distance agreement overrides local disagreement

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a. /s-i[t-ti[ti-jep-us/ sististijepus "they show him"
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Can we write a TSL grammar to capture this pattern?

# Sibilant Harmony in SAMALA (1/2)

### Generalization

- Anticipatory Sibilant harmony
- ► Local restriction agains [\*sn, \*st, \*sl]

$$T = \{s, f, n, t, l\} S = \{*sf, *fs, *sn, *st, *sl\}$$

$$^{ok}$$
 k  $\int$  u  $\int$  o  $j$  in  $^{ok}$   $\int$   $\int$  O  $k$ ' in

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$$\overset{\mathsf{o}}{\mathsf{k}}$$
 [ u [ o j i n  $\overset{\mathsf{o}k}{\mathsf{k}}$  [ | o k' i n

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$$\begin{pmatrix} ok_{1} - - \frac{ok_{1}}{1} - - \frac{ok_{1}}{1} - \frac{ok_{1}}{1} \\ \frac{1}{1} - \frac{1}{1} - \frac{ok_{1}}{1} - \frac{ok_{1}}{1} \\ \frac{1}{1} - \frac{ok_{1}}{1} - \frac{ok_{1}}{1} - \frac{ok_{1}}{1} \\ \frac{1}{1} - \frac{ok_{1}}{1} - \frac{ok_{1}}{1} - \frac{ok_{1}}{1} - \frac{ok_{1}}{1} \\ \frac{1}{1} - \frac{ok_{1}}{1} - \frac{ok_{1}}{1} - \frac{ok_{1}}{1} - \frac{ok_{1}}{1} \\ \frac{1}{1} - \frac{ok_{1}}{1} - \frac{ok$$

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$$\stackrel{ok}{\stackrel{\vdash}{\bigcup}} \stackrel{\vdash}{\bigcup} \stackrel{\vdash}{\bigcup} \qquad \mathsf{n}$$

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- anticipatory sibilant harmony
- palatalization to avoid local restriction
- sibilant harmony overides palatalization

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$$^{ok}$$
 s n e t u s

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$$\overset{\mathsf{T}}{\overset{\mathsf{T}}{ok}}$$
  $\overset{\mathsf{T}}{\mathsf{s}}$   $\mathsf{n}$   $\mathsf{e}$   $\mathsf{t}$   $\mathsf{u}$   $\mathsf{s}$ 

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$$^{\mathsf{T}}$$
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s n .... 
$$\overset{\mathsf{T}}{ok} \mathsf{s} \mathsf{n} \; \mathbf{e} \; \mathsf{t} \; \mathsf{u} \; \mathsf{s}$$

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$$\overset{\mathsf{r}}{\overset{\mathsf{r}}{o^k}}$$
 s n e t u s

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```
* s n t s

T

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```

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```
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```

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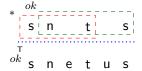
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### Grammar

$$T = \{s, f, n, t, l\} S = \{*sf, *sf, *sn, *st, *sl \}$$



No TSL grammar can capture this patter...

## Interim Summary

### TSL is a good fit for long distance dependencies:

- projection of a subset of segments on a tier T;
- strictly local constraints enforced on T.

### Not every attested pattern can be described this way:

- overlapping constraints cannot work on the same tier.
- not enough information is used when projecting elements.

## Generalizing the TSL class

### TSL languages are characterized by:

- ▶ a 1-local projection function that projects one tier T;
- strictly k-local constraints applied on T.

#### Idea

### What if ..

- we could apply different projection functions to project multiple tiers?
- the locality of the projection function was higher than 1?

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## Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

$$^{ok}$$
  $\int$  q u  $\mathbf{g}$ :  $\mathbf{i}$ 

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$$ok \begin{bmatrix} - & - & & \\ & & & & \\ & & & \end{bmatrix}$$
  $\mathbf{q} \begin{bmatrix} & & & \\ & & & \end{bmatrix}$   $\mathbf{g} \begin{bmatrix} & & & \\ & & & \\ & & & \end{bmatrix}$ 

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$$\begin{array}{c} ok \\ ok \\ - & - & - & - \\ & & &$$

#### Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

$$T_1 = \{ \mathbf{z}, \; \mathbf{s}, \; \mathbf{z}, \mathbf{f}, \; \mathbf{q} \} \; S_1 = \{ \mathbf{*sz}, \; \mathbf{*sz}, \; \mathbf{*zs}, \; \mathbf{*zs}, \; \mathbf{*fz}, \; \mathbf{*fz}, \; \mathbf{*zf} \}$$

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Unbounded agreement in anteriority:

S

T<sub>1</sub>: sibilant voicing

\*
S Q U 3: j

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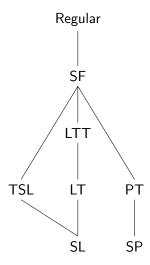
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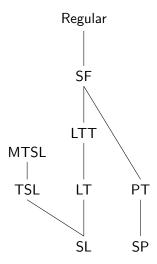
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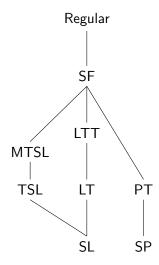
### MTSL: Relations to other Classes



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# Generalizing the TSL class (Reprise)

#### TSL languages are characterized by:

- ▶ a 1-local projection function that projects one tier T;
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#### Idea

#### What if ...

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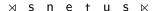
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s n e t u s

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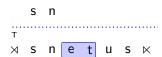
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```
s n
.....
⊤
⋊ s n e t u s ⋉
```

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```
s n
....
⊤
⋈ s n e t u s ⋉
```

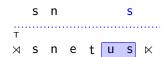
- anticipatory sibilant harmony
- palatalization to avoid local restrictions
- sibilant harmony overrides palatalization



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```
s n
...
x s n e t u s ×
```

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```
* S n e t II S K
```

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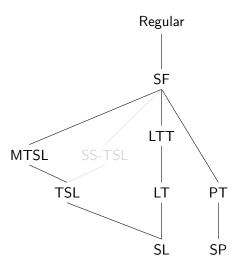
## SAMALA Sibilant Harmony (Revisited)

- anticipatory sibilant harmony
- palatalization to avoid local restrictions
- sibilant harmony overrides palatalization

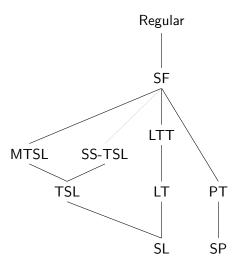
#### Grammar

$$T = \{s, f \land \{n, t, l\} \triangleright s\} \ S = \{*sf, *sf, *sn(\neg s), *st(\neg s), *sl(\neg s)\}$$

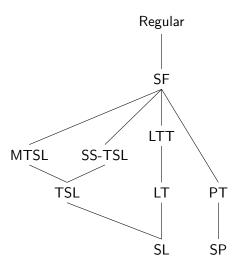
## SS -TSL: Relations to other Classes

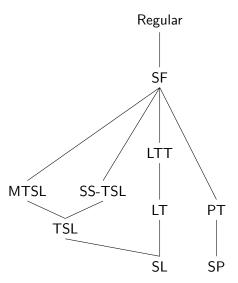


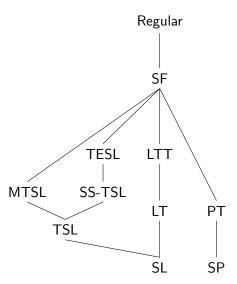
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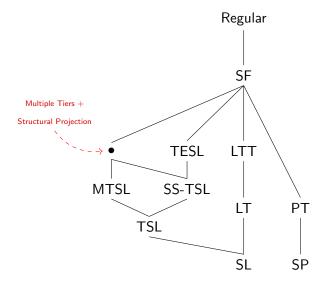


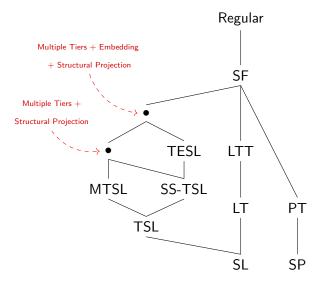
## SS -TSL: Relations to other Classes











#### Conclusions and Future Work

## **Tracing Back our Steps**

- subregular hypothesis as a strong computational theory of language complexity. Phonology is SL + SP + TSL
- but there are patterns that are unaccounted for!

#### In this Talk

- ► TSL is not the right fit, but it seems to be close!
- minor changes lead to interesting new classes: MTSL, SS-TSL

#### Future Work

- ▶ further study of the TSL neighborhood
- ightharpoonup learnability ightarrow learning algorithms, AGL experiments ...

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#### Selected References

- **Chandlee, Jane**. 2014. *Strictly Local Phonological Processes*. Ph.D. thesis, University of Delaware.
- Planta Jeffrey, Chetan Rawal, and Herbert G. Tanner. 2011. Tier-based strictly local constraints in phonology. In Proceedings of ACL 49th, 58–64.
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- McMullin, Kevin. 2016. Tier-based locality in long-distance phonotactics: learnability and typology. Doctoral dissertation, University of British Columbia.

**Appendix** 

$$T = \{ s, f, tf^h \}, S = \{ * sf, *fs, *stf^h, *tf^h \}$$

$$\int a p \mid t \int^h o \mid u \int w a \int$$

$$T = \{ s, f, tf^h \}, S = \{ * sf, *fs, *stf^h, *tf^h \}$$

```
f_1: anticipatory harmony f_2 a f_3 f_4 f_5 f_6 f_7 f_8 f_8
```

$$T = \{ s, f, tf^h \}, S = \{ sf, sf, stf^h, tf^h \}$$

```
\int_{1: \text{ anticipatory harmony}} \mathsf{T}_1: \mathsf{anticipatory harmony} \int \mathsf{a} \mathsf{P} \; \mathsf{i} \; \mathsf{t} \mathsf{j}^h \; \mathsf{o} \; \mathsf{l} \; \mathsf{u} \; \mathsf{j} \; \mathsf{w} \; \mathsf{a} \; \mathsf{j}
```

$$T = \{ s, f, tf^h \}, S = \{ sf, sf, stf^h, tf^h \}$$

```
egin{array}{lll} f T_1:& {
m anticipatory\ harmony} \ & f a & f P & {
m it} f f^h & {
m old} & {
m ull} & {
m was} & {
m for} \ & {
m old} & {
m vall} & {
m old} & {
m vall} \end{array}
```

$$T = \{ s, f, tf^h \}, S = \{ sf, sf, stf^h, tf^h \}$$

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```
\int t \int^h t_1: \text{ anticipatory harmony}
\int a p i t \int^h o | u \int w a \int
```

$$T = \{ s, f, tf^h \}, S = \{ sf, sf, stf^h, tf^h \}$$

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```
\int \qquad \mathsf{t} \int^h \mathsf{t}_{1: \text{ anticipatory harmony}} \\ \int \mathsf{a} \; \mathsf{p} \; \mathsf{i} \; \mathsf{t} \int^h \mathsf{o} \; \mathsf{I} \; \mathsf{u} \; \int \mathsf{w} \; \mathsf{a} \; \int
```

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```
\int \qquad \qquad t \int^h \qquad \qquad \int \int a \ p \ i \ t \int^h o \ | \ u \ \int w \ a \ \int
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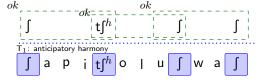
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#### Anticipatory Harmony in SAMALA

$$T = \{ s, f, tf^h \}, S = \{ * sf, *fs, *stf^h, *tf^h \}$$

$$\mathsf{T} = \{\sigma \colon \sigma \in \{\mathsf{s}, \, \mathsf{f}, \, \mathsf{t} \mathsf{f}^h\} \, \, \land (\rtimes \sigma \vee \sigma \ltimes) \} \, \, \mathsf{S} = \{\mbox{$^*$sf, $^*$stf}^h, \, \mbox{$^*$tf}^h \, \, \mathsf{s} \}$$

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$$\rtimes \int a P i t \int^h o | u s w a \int \ltimes$$

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# Closure Properties of Subregular Classes

	SL	TSL	MTSL	SS-TSL	SF	Reg
$\cup$	×	×	×	×	$\checkmark$	$\checkmark$
$\cap$	$\checkmark$	×	$\checkmark$	×	$\checkmark$	$\checkmark$
Relabeling	$\times$	$\times$	×	×	$\times$	$\checkmark$
Complement	×	$\times$	×	×	$\checkmark$	$\checkmark$